#### Administrivia

Part I Meetings this week
10 teams have NOT signed up...

Your app needs to read, write, and edit data
Using open data great to populate your DB
Users should be able to add new data

# L4 Relational Algebra

Eugene Wu Fall 2018

### Supplemental Materials

#### Helpful References

https://en.wikipedia.org/wiki/Relational\_algebra

#### Overview

Last time, learned about pre-relational models an informal introduction to relational model an introduction to the SQL query language.

Learn about formal relational query languages
Relational Algebra (algebra: perform operations)
Relational Calculus (logic: are statements true?)

Keys to understanding SQL and query processing

#### Who Cares?

Clean query semantics & rich program analysis

Helps/enables optimization

Opens up rich set of topics

Materialized views

Data lineage/provenance

Query by example

Distributed query execution

• • •

# What's a Query Language?

Allows manipulation and retrieval of data from a database.

Traditionally: QL != programming language

Doesn't need to be turing complete

Not designed for computation

Supports easy, efficient access to large databases

#### Recent Years

Scaling to large datasets is a reality

Query languages are a powerful way to

think about data algorithms scale

think about asynchronous/parallel programming

## What's a Query Language?

Re-use permitted when acknowledging the original © Daniel Abadi, Shivnath Babu, Fatma Ozcan, and Ippokratis Pandis (2015)

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#### MapReduce is **not** the answer

- MapReduce is a powerful primitive to do many kinds of parallel data processing
- BUT
  - Little control of data flow
  - Fault tolerance guarantees not always necessary
  - Simplicity leads to inefficiencies
  - Does not interface with existing analysis software
  - Industry has existing training in SQL



#### SQL interface for Hadoop critical for mass adoption

SQL-on-Hadoop Tutorial

9002015

Tutorial at VLDB 2015 Abadi et al. http://www.slideshare.net/abadid/sqlonhadoop-tutorial

# Formal Relational Query Languages

Formal basis for real languages e.g., SQL

Relational Algebra

Operational, used to represent execution plans

Relational Calculus

Logical, describes what data users want (not operational, fully declarative)

#### **Prelims**

Query is a function over relation instances

$$Q(R_1,...,R_n) = R_{result}$$

Schemas of input and output relations are *fixed* and well defined by the query Q.

Positional vs Named field notation

Position easier for formal defs

one-indexed (not 0-indexed!!!)

Named more readable

Both used in SQL

#### **Prelims**

Relation (for this lecture)
Instance is a set of tuples
Schema defines field names and types (domains)
Students(sid int, name text, major text, gpa int)

How are relations different than generic sets ( $\mathbb{R}$ )? Can assume item structure due to schema Some algebra operations (x) need to be modified Will use this later

### Relational Algebra Overview

Core 5 operations

PROJECT  $(\pi)$ 

SELECT  $(\sigma)$ 

UNION (U)

SET DIFFERENCE (-)

CROSSPRODUCT (x)

Additional operations

RENAME (p)

INTERSECT (∩)

JOIN (⋈)

DIVIDE (/)

## Instances Used Today: Library

Students, Reservations

RI

sid	rid	day
I	101	10/10
2	102	11/11

Use positional or named field notation

SI

sid	name	gpa	age
I	eugene	4	20
2	barak	3	21
3	trump	2	88

Fields in query results are inherited from input relations (unless specified)

**S2** 

sid	name	gpa	age
4	aziz	3.2	21
2	barak	3	21
3	trump	2	88
5	rusty	3.5	21

### Project

$$\pi_{\langle attr1,...\rangle}(A) = R_{result}$$

Pick out desired attributes (subset of columns)

Schema is subset of input schema in the projection list

 $\pi_{\langle a,b,c\rangle}(A)$  has output schema (a,b,c) w/ types carried over

## Project

**S2** 

sid	name	gpa	age
4	aziz	3.2	21
2	barak	3	21
3	trump	2	88
5	rusty	3.5	21

$$\pi_{\text{name,age}}(S2) =$$

name	age
aziz	21
barak	21
trump	88
rusty	21

### Project

**S2** 

sid	name	gpa	age
4	aziz	3.2	21
2	barak	3	21
3	trump	2	88
5	rusty	3.5	21

$$\pi_{age}(S2) = \frac{21}{88}$$

Where did all the rows go? Real systems typically don't remove duplicates. Why?

#### Select

$$\sigma_{}(A) = R_{result}$$

Select subset of rows that satisfy condition p
Won't have duplicates in result. Why?
Result schema same as input

#### Select

SI sid gpa name age 4 20 eugene 2 barak 3 21 3 88 2 trump

$$\sigma_{\text{age}<30}$$
 (S1) =

sid	name	gpa	age
I	eugene	4	20
2	barak	3	21

$$\pi_{\text{name}}(\sigma_{\text{age} < 30} \text{ (S1)}) = \begin{bmatrix}
\text{name} \\
\text{eugene}
\end{bmatrix}$$

$$A + B = B + A$$

$$A * B = B * A$$

$$A + (B * C) = (B * C) + A$$

$$A + (B + C) = (A + B) + C$$
  
 $A + (B * C) = (A + B) * C$ 

$$A + B = B + A$$

$$A * B = B * A$$

$$A + (B * C) = (B * C) + A$$

$$A + (B + C) = (A + B) + C$$
  
 $A + (B * C) = (A + B) * C$ 

$$\pi_{age}(\sigma_{age < 30} (SI))$$

	sid	name	gpa	age	I,
	I	eugene	4	20	
$\sigma_{age < 30}$	2	barak	3	21	
	3	trump	2	88	

sid	name	gpa	age
I	eugene	4	20
2	barak	3	21

$$\pi_{age}(\sigma_{age < 30} (SI))$$

	sid	name	gpa	age	
$\pi_{age}$	I	eugene	4	20	
480	2	barak	3	21	

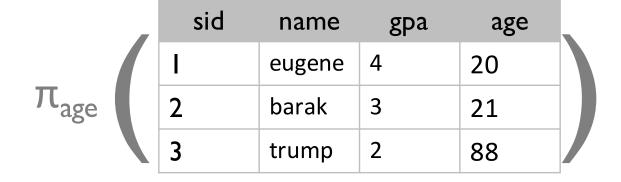
age
20
21

$$\sigma_{age < 30}(\pi_{age}(SI))$$

		sid	name	gpa	age	
		1	eugene	4	20	
$\pi_{age}$		2	barak	3	21	
	1	3	trump	2	88	

age	
20	
21	
88	

$$\sigma_{\text{age} < 30}(\pi_{\text{age}}(SI))$$



Does Project and Select commute?

$$\pi_{\text{age}}(\sigma_{\text{age} < 30} (SI)) = \sigma_{\text{age} < 30}(\pi_{\text{age}}(SI))$$

What about

$$\pi_{\text{name}}(\sigma_{\text{age}<30} (SI))$$
?

Does Project and Select commute?

$$\pi_{\text{age}}(\sigma_{\text{age} < 30} (SI)) = \sigma_{\text{age} < 30}(\pi_{\text{age}}(SI))$$

What about

$$\pi_{\text{name}}(\sigma_{\text{age} < 30} \text{ (SI)}) \mathrel{!=} \sigma_{\text{age} < 30}(\pi_{\text{name}}(\text{SI}))$$

Does Project and Select commute?

$$\pi_{\text{age}}(\sigma_{\text{age} < 30} (SI)) = \sigma_{\text{age} < 30}(\pi_{\text{age}}(SI))$$

What about

$$\pi_{\text{name}}(\sigma_{\text{age} < 30} (SI)) := \sigma_{\text{age} < 30}(\pi_{\text{name, age}}(SI))$$

Does Project and Select commute?

$$\pi_{\text{age}}(\sigma_{\text{age} < 30} (SI)) = \sigma_{\text{age} < 30}(\pi_{\text{age}}(SI))$$

What about

$$\pi_{\text{name}}(\sigma_{\text{age} < 30} (SI)) = \pi_{\text{name}}(\sigma_{\text{age} < 30}(\pi_{\text{name, age}}(SI)))$$

OK!

#### Union, Set-Difference

A op 
$$B = R_{result}$$

A, B must be union-compatible

Same number of fields

Field i in each schema have same type

Result Schema borrowed from first arg (A)

A(big int, poppa int) U B(thug int, life int) = ?

#### Union, Set-Difference

A op 
$$B = R_{result}$$

A, B must be union-compatible

Same number of fields

Field i in each schema have same type

Result Schema borrowed from first arg (A)

A(big int, poppa int) U B(thug int, life int) = R<sub>result</sub>(big int, poppa int)

#### Union, Intersect, Set-Difference

SI

sid	name	gpa	age
I	eugene	4	20
2	barak	3	21
3	trump	2	88

**S2** 

sid	name	gpa	age
4	aziz	3.2	21
2	barak	3	21
3	trump	2	88
5	rusty	3.5	21

SIUS2 =

sid	name	gpa	age	
1	eugene	4	20	
4	aziz	3.2	21	
5	rusty	3.5	21	
3	trump	2	88	
2	barak	3	21	

#### Union, Intersect, Set-Difference

SI

sid	name gpa		age
I	eugene	4	20
2	barak	3	21
3	trump	2	88

**S2** 

sid	name	gpa	age
4	aziz	3.2	21
2	barak	3	21
3	trump	2	88
5	rusty	3.5	21

$$SI-S2 =$$

sid	name	gpa	age
1	eugene	4	20

#### Note on Set Difference & Performance

Notice that most operators are monotonic increasing size of inputs  $\rightarrow$  outputs grow if  $A \supseteq B \rightarrow Q(A,T) \supseteq Q(B,T)$  can compute incrementally

Set Difference is not monotonic

if 
$$A \supseteq B \rightarrow T-A \subseteq T-B$$
  
e.g.,  $5 > I \rightarrow 9-5 < 9-I$ 

Set difference is blocking:

For T – S, must wait for all S tuples before any results

#### Cross-Product

$$A(a_1,...,a_n) \times B(a_{n+1},...,a_m) = R_{result}(a_1,...,a_m)$$

Each row of A paired with each row of B

Result schema concats A and B's fields, inherit if possible

Conflict: students and reservations have sid field

Different than mathematical "X" by flattening results:

math A x B = { (a, b) | a 
$$\in$$
 A ^ b  $\in$  B }  
e.g., {1, 2} X {3, 4} = { (1, 3), (1, 4), (2, 3), (2, 4) }  
what is {1, 2} x {3, 4} x {5, 6}?

#### Cross-Product

SI

sid	name gpa		age
1	eugene	4	20
2	barak	3	21
3	trump	2	88

RI

sid	rid	day
	101	10/10
2	102	11/11

SIxR1 =

(sid)	name	gpa	age	(sid)	rid	day
I	eugene	4	20	I	101	10/10
2	barak	3	21	1	101	10/10
3	trump	2	88	1	101	10/10
I	eugene	4	20	2	102	11/11
2	barak	3	21	2	102	11/11
3	trump	2	88	2	102	11/11

#### Rename

Explicitly defines/changes field names of schema

$$p(C(1 \rightarrow sid1, 5 \rightarrow sid2), S1 \times R1)$$

	Sidi	name	gpa	age	SidZ	rid	day
<b>C</b> =	I	eugene	4	20		101	10/10
	2	barak	3	21		101	10/10
	3	trump	2	88	I	101	10/10
	I	eugene	4	20	2	102	11/11
	2	barak	3	21	2	102	11/11
	3	trump	2	88	2	102	11/11

Project 
$$\pi($$
  $) =$   $Select$   $\sigma($   $) =$   $Cross product$   $X =$   $=$   $Union$   $U =$   $Union$   $Union$   $U =$   $Union$   $Union$   $U =$   $Union$   $Union$ 

### Compound/Convenience Operators

INTERSECT (∩)

JOIN (⋈)

DIVIDE (/)

$$A \cap B = R_{result}$$

A, B must be union-compatible

SI

sid	name	gpa	age
I	eugene	4	20
2	barak	3	21
3	trump	2	88

**S2** 

sid	name	gpa	age
4	aziz	3.2	21
2	barak	3	21
3	trump	2	88
5	rusty	3.5	21

SI∩S2 =

sid	name	gpa	age
2	barak	3	21
3	trump	2	88

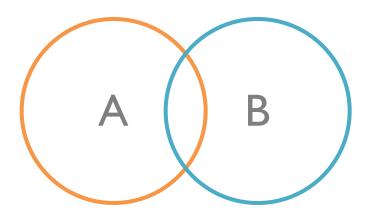
$$A \cap B = R_{result}$$

A, B must be union-compatible

Can we express using core operators?

$$A \cap B = ?$$

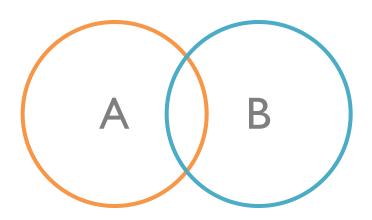
$$A \cap B = R_{result}$$



Can we express using core operators?

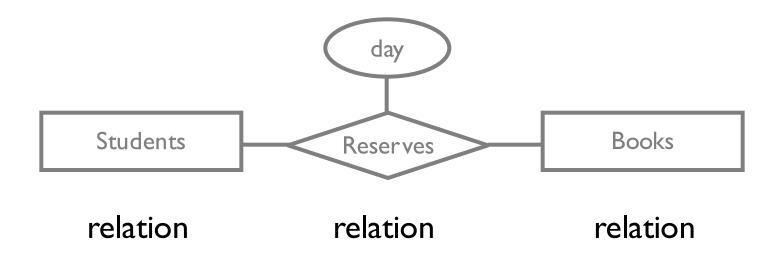
 $A \cap B = A - ?$  (think venn diagram)

$$A \cap B = R_{result}$$



Can we express using core operators?

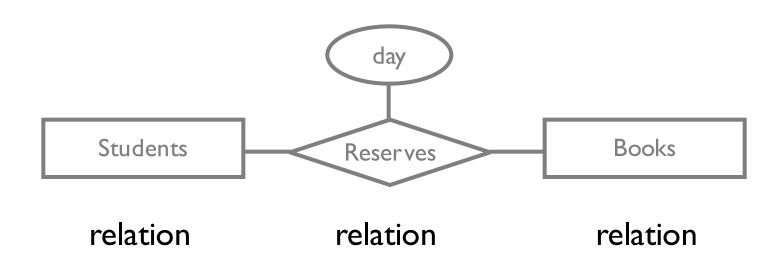
$$A \cap B = A - (A - B)$$



What if you want to query across all three tables? e.g., all names of students that reserved "The Purple Crayon"

#### Need to combine these tables

Cross product? But that ignores foreign key references



SI

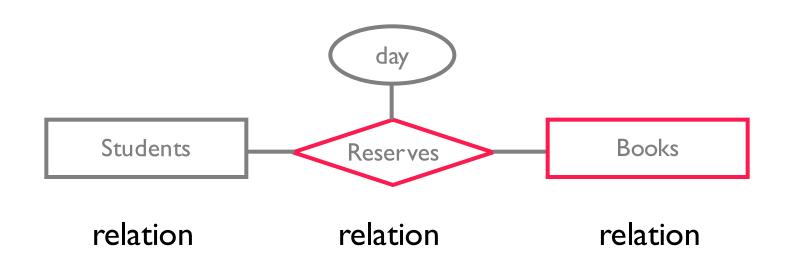
sid	name	gpa	age
I	eugene	4	20
2	barak	3	21
3	trump	2	88

RI

sid	rid	day
I	101	10/10
2	102	11/11

BI

rid	name
101	The Purple Crayon
102	1984



SI

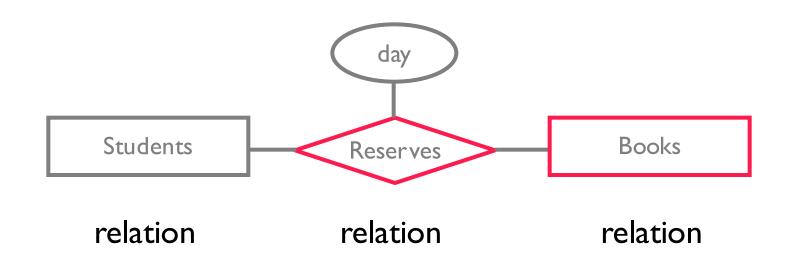
sid	name	gpa	age
I	eugene	4	20
2	barak	3	21
3	trump	2	88

RI

sid	rid	day
	101	10/10
2	102	11/11

BI

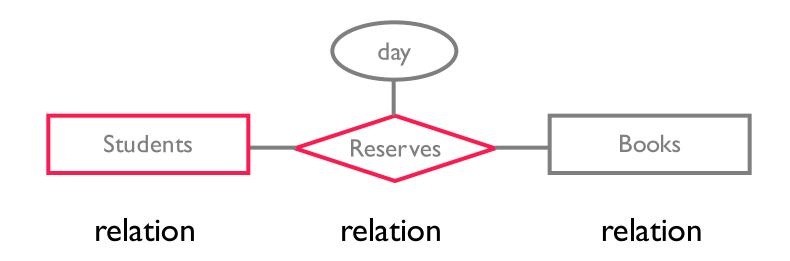
rid	name
101	The Purple Crayon
102	1984



SI RBI

sid	name	gpa	age
	eugene	4	20
2	barak	3	21
3	trump	2	88

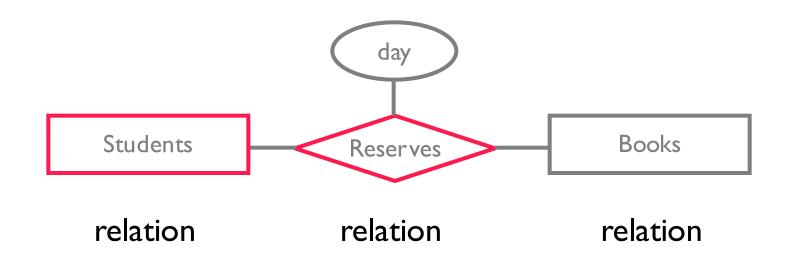
sid	(rid)	day	(rid)	name
I	101	10/10	101	The Purple Crayon
2	102	11/11	102	1984



SI RBI

sid	name	gpa	age
I	eugene	4	20
2	barak	3	21
3	trump	2	88

sid	(rid)	day	(rid)	name
1	101	10/10	101	The Purple Crayon
2	102	11/11	102	1984



#### SRBI

(si	id)	(name)	gpa	age	(sid)	(rid)	day	(rid)	(name)
1		eugene	4	20	ı	101	10/10	101	The Purple Crayon
2		barak	3	21	2	102	11/11	102	1984

## theta $(\theta)$ Join

$$A \bowtie_{c} B = \sigma_{c}(A \times B)$$

Most general form

Result schema same as cross product

Often far more efficient to compute than cross product

Commutative

$$(A\bowtie_c B)\bowtie_c C = A\bowtie_c (B\bowtie_c C)$$

## theta $(\theta)$ Join

SI

sid	name	gpa	age
1	eugene	4	20
2	barak	3	21
3	trump	2	88

RI

sid	rid	day
	101	10/10
2	102	11/11

$$SI \bowtie_{SI.sid} \leq RI.sid RI =$$

(sid)	name	gpa	age	(sid)	rid	day
I	eugene	4	20	1	101	10/10
I	eugene	4	20	2	102	11/11
2	barak	3	21	2	102	11/11

### Equi-Join

$$A\bowtie_{attr} B = \pi_{all\ attrs\ except\ B.attr}(A\bowtie_{A.attr\ =\ B.attr} B)$$

Special case where the condition is attribute equality Result schema only keeps *one copy* of equality fields Natural Join (AMB):

Equijoin on all shared fields (fields w/ same name)

## Equi-Join

SI

sid	name	gpa	age
I	eugene	4	20
2	barak	3	21
3	trump	2	88

RI

sid	rid	day
I	101	10/10
2	102	11/11



sid	name	gpa	age	rid	day
	eugene	4	20	101	10/10
2	barak	3	21	102	11/11

### Division

Let us have relations A(x, y), B(y)

$$A/B = \{ <_{x}> \mid \forall y \in B <_{x,y}> \in A \}$$

Find all students that have reserved all books

A/B = all x (students) s.t. for every y (reservation),  $\langle x,y \rangle \in A$ 

Good to ponder, not supported in most systems (why?)

Generalization

y can be a list of fields in B

x U y is fields in A

Α

sid	rid
I	I
ı	2
I	3
1	4
2	I
2	2
3	2
4	2
4	4

RI

	rid	
2		

R2

	rid
2	
4	

R3

	rid	
2		
4		

A/RI

A/R2

A

sid	rid
I	
I	2
	3
I	4
2	I
2	2
3	2
4	4
4	4

RI

rid 2

sid

2

3

4

A/RI

R2

rid 2 4 R3

rid I

2

4

A/R2

Α

sid	rid
I	I
	2
	3
I	4
2	I
2	2
3	2
4	2
4	4

RI

	rid	
2		

sid

2	
3	
4	

A/RI

R2

	rid	
2		
4		

R3

	rid
I	
2	
4	

sid

4	

A/R2

sid	rid
	1
	2
I	3
I	4
2	I
2	2
3	2
4	2
4	4

RI

	rid	
2		

sid

2	
3	
4	

A/RI

R2

	rid
2	
4	



R3

	rid	
1		
2		

sid

4		
4		

A/R2

sid

### Is A/B a Fundamental Operation?

No. Shorthand like Joins

joins so common, it's natively supported

Hint: Find all xs not 'disqualified' by some y in B. x value is disqualified if

- I. by attaching y value from B (e.g., create  $\langle x, y \rangle$ )
- 2. we obtain an  $\langle x,y \rangle$  that is not in A.

Α

sid	rid
I	I
ı	2
I	3
1	4
2	I
2	2
3	2
4	2
4	4

В

	rid	
2		
4		

Disqualified = A/B =

A

sid	rid
I	I
I	2
I	3
I	4
2	I
2	2
3	2
4	2
4	4

В

	rid
2	
4	

$$\pi_{sid}(A) \times B$$

sid	rid
1	2
I	4
2	2
2	4
3	2
3	4
4	2
4	4

Disqualified = 
$$(\pi_{sid}(A) \times B)$$
  
A/B =

A

sid	rid
I	I
I	2
	3
I	4
2	I
2	2
3	2
4	2
4	4

B

	rid
2	
4	

$$\pi_{sid}(A) \times B$$

sid	rid
1	2
I	4
2	2
2	4
3	2
3	4
4	2
4	4

$$(\pi_{sid}(A) \times B) - A$$

sid	rid
2	4
3	4

Disqualified = 
$$((\pi_{sid}(A) \times B) - A)$$
  
A/B =

A

sid	rid
I	I
I	2
I	3
1	4
2	I
2	2
3	2
4	2
4	4

B

	rid
2	
4	

$$\pi_{sid}(A) \times B$$

sid	rid
1	2
I	4
2	2
2	4
3	2
3	4
4	2
4	4

$$(\pi_{sid}(A) \times B) - A$$

sid	rid
2	4
3	4

A/B

Disqualified = 
$$\pi_{sid}((\pi_{sid}(A) \times B) - A)$$
  
A/B =  $\pi_{sid}(A)$  - Disqualified

$$\pi_{\text{name}}(\sigma_{\text{rid}=2} (R1) \bowtie SI)$$

# Equivalent Queries

p(tmp1, 
$$\sigma_{rid=2}$$
 (R1))  
p(tmp2, tmp1  $\bowtie$  SI)  
 $\pi_{name}$ (tmp2)

$$\pi_{\text{name}}(\sigma_{\text{rid}=2}(\text{R1} \bowtie \text{SI}))$$

Book(rid, type) Reserve(sid, rid) Student(sid)

Need to join DB books with reserve and students  $\sigma_{type='db'}$  (Book)

Book(rid, type) Reserve(sid, rid) Student(sid)

Need to join DB books with reserve and students

 $\sigma_{type='db'}$  (Book)  $\bowtie$  Reserve

Book(rid, type) Reserve(sid, rid) Student(sid)

Need to join DB books with reserve and students

 $\sigma_{\text{type='db'}}$  (Book)  $\bowtie$  Reserve  $\bowtie$  Student

Book(rid, type) Reserve(sid, rid) Student(sid)

Need to join DB books with reserve and students

 $\pi_{\text{name}}(\sigma_{\text{type='db'}} \text{ (Book)} \bowtie \text{Reserve} \bowtie \text{Student})$ 

Book(rid, type) Reserve(sid, rid) Student(sid)

### Need to join DB books with reserve and students

 $\pi_{name}(\sigma_{type='db'})$  (Book)  $\bowtie$  Reserve  $\bowtie$  Student)

### More efficient query

 $\pi_{name}(\pi_{sid}((\pi_{rid} \sigma_{type='db'} (Book)) \bowtie Reserve) \bowtie Student)$ 

Book(rid, type) Reserve(sid, rid) Student(sid)

### Need to join DB books with reserve and students

 $\pi_{\text{name}}(\sigma_{\text{type='db'}} \text{ (Book)} \bowtie \text{Reserve} \bowtie \text{Student})$ 

### More efficient query

 $\pi_{\text{name}}(\pi_{\text{sid}}((\pi_{\text{rid}} \sigma_{\text{type='db'}} (\text{Book}))) \bowtie \text{Reserve}) \bowtie \text{Student})$ 

Book(rid, type) Reserve(sid, rid) Student(sid)

### Need to join DB books with reserve and students

 $\pi_{name}(\sigma_{type='db'})$  (Book)  $\bowtie$  Reserve  $\bowtie$  Student)

### More efficient query

 $\pi_{\text{name}}(\pi_{\text{sid}}((\pi_{\text{rid}} \sigma_{\text{type='db'}} (\text{Book}))) \bowtie \text{Reserve}) \bowtie \text{Student})$ 

Book(rid, type) Reserve(sid, rid) Student(sid)

### Need to join DB books with reserve and students

 $\pi_{name}(\sigma_{type='db'})$  (Book)  $\bowtie$  Reserve  $\bowtie$  Student)

### More efficient query

 $\pi_{\text{name}}(\pi_{\text{sid}}((\pi_{\text{rid}} \sigma_{\text{type='db'}} (\text{Book})) \bowtie \text{Reserve}) \bowtie \text{Student})$ 

#### Students that reserved DB or HCl book

- Find all DB or HCI books
- 2. Find students that reserved one of those books

p(tmp, 
$$(\sigma_{type='DB' \ v \ type='HCI'}(Book))$$
  
 $\pi_{name}(tmp \bowtie Reserve \bowtie Student)$ 

#### **Alternatives**

define tmp using UNION (how?) what if we replaced v with ^ in the query?

#### Students that reserved a DB and HCl book

Does previous approach work?

p(tmp, (
$$\sigma_{type='DB' \land type='HCI'}$$
 (Book))  
 $\pi_{name}$ (tmp  $\bowtie$  Reserve  $\bowtie$  Student)



#### Students that reserved a DB and HCI book

#### Does previous approach work?

- Find students that reserved DB books
- 2. Find students that reversed HCl books
- 3. Intersection

```
p(tmpDB, \pi_{sid}(\sigma_{type='DB'} Book) \bowtie Reserve)
p(tmpHCl, \pi_{sid}(\sigma_{type='HCl'} Book) \bowtie Reserve)
\pi_{name}((tmpDB\cap tmpHCl) \bowtie Student)
```

### Students that reserved all books

Use division

Be careful with schemas of inputs to / !

p(tmp, (
$$\pi_{sid,rid}$$
 Reserves) / ( $\pi_{rid}$  Books))  
 $\pi_{name}$ (tmp  $\bowtie$  Student)

What if want students that reserved all horror books?

```
p(tmp, (\pi_{sid,rid} Reserves) / (\pi_{rid}(\sigma_{type='horror'}, Book)))
```

### Let's step back

Relational algebra is expressiveness benchmark

A language equal in expressiveness as relational algebra is relationally complete

But has limitations

nulls

aggregation

recursion

duplicates

### Equi-Joins are a way of life

Matching of two sets based on shared attributes

Yelp: Join between your location and restaurants

Market: Join between consumers and suppliers

High five: Join between two hands on time and space

Comm.: Join between minds on ideas/concepts

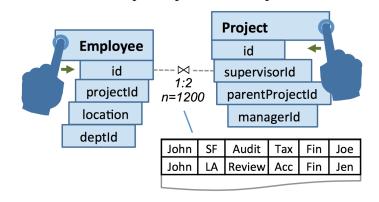


### What can we do with RA?

### Query by example

Here's my data and examples of the result, generate the query for me

### Novel relationally complete interfaces



GestureDB. Nandi et al.

### Summary

Relational Algebra (RA) operators

Operators are closed inputs & outputs are relations

Multiple Relational Algebra queries can be equivalent

It is operational

Same semantics but different performance

Forms basis for optimizations

### **Next Time**

Relational Calculus

SQL