Course Name: Algorithm Design and Analysis

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CoPA Questions

For problems 1-3, you should solve them using **CoPA**
and document the **entire** process (questions and answers) in a .md file.

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1. Question1

   The Nine Links is a traditional intellectual game originating from China, featuring nine circular rings threaded onto a "sword," which are interlinked with one another. In the traditional Nine Links, the kth ring ($k \geq 2$) can either be placed on the sword (denoted as 1) or removed from it (denoted as 0), and this can happen only if the $k-1$th ring is on the sword and all previous rings are off. Notably, the first ring can move freely.

   In this problem, we will discuss a more general case, although this simple Nine Links may not necessarily be feasible in a physical sense.

   A simple Nine Links can be viewed as two binary strings: a rule string s and a state string t, with the condition $|s| = |t| - 1$. Here, $t_i = 1$ indicates that the ith ring is placed on the sword, while $t_i = 0$ indicates that it is removed. The string s remains constant throughout the game, while t changes one position at a time (either from 0 to 1 or from 1 to 0). The simple Nine Links is removed when all t_i are 0, and it is placed on when all t_i are 1.

   In the simple Nine Links, t_i can change only if $t_{1\sim i-1}$ is a **suffix** of s. It can be observed that the traditional Nine Links is a special case where s is 00...01.

   Given a string s, the task is to determine the minimum number of steps required to transition from the removed state to the placed state. The answer should be taken modulo $10^9 + 7$.

   Input:
   1. An integer n, representing the length of s. **Note that this is not the number of rings.**
   2. A binary string s.

   Output:
   One line containing an integer that represents the value of the answer modulo $10^9 + 7$.

2. Question2

   Given a 01 sequence $\{a_n\}$ of length n, you need to find the minimum number of operations required to transform the sequence into a "good sequence".

   A "good sequence" is defined as follows:
   1. There exist k intervals $\{(l_k, r_k)\}$, where within each interval all values are 1, i.e.,

   $$\forall i \in [1, n], a_i = 1 \text{ if and only if } \exists j \in [1, k], i \in [l_j, r_j]$$

   2. The sequence is considered "good" if and only if the lengths of these intervals are strictly increasing from left to right, meaning:

   $$r_i - l_i < r_{i+1} - l_{i+1}, \forall i \in [1, k-1]$$

   You are allowed to perform the following operation: select two different positions i and j ($i \neq j$), and swap the values a_i and a_j.

Your task is to find the minimum number of operations needed to transform the sequence $\{a_n\}$ into a "good sequence."

Input:
A single line contains a 01 sequence $\{a_n\}$ of length n ($n \leq 800$).

Output:
The minimum number of swaps required to transform the sequence into a "good sequence".

3. Question3

You are given an array $a_1, a_2, ..., a_n$ of positive integers. You can color some elements of the array red, but there cannot be two adjacent red elements.

Your score is the maximum value of a red element, plus the minimum value of a red element, plus the number of red elements. Find the maximum score you can get.

Input:
Each test contains multiple test cases. The first line contains the number of test cases $t(1 \leq t \leq 10^4)$. The description of the test cases follows.
For each test case:
1. The first line contains a single integer $n(1 \leq n \leq 2 \times 10^5)$ – the length of the array.
2. The second line contains n integers $a_1, a_2, ..., a_n(1 \leq a_i \leq 10^9)$ – the given array.

It is guaranteed that the sum of n over all test cases does not exceed 2×10^5.

Output:
For each test case, output a single integer: the maximum possible score you can get after coloring some elements red according to the statement.