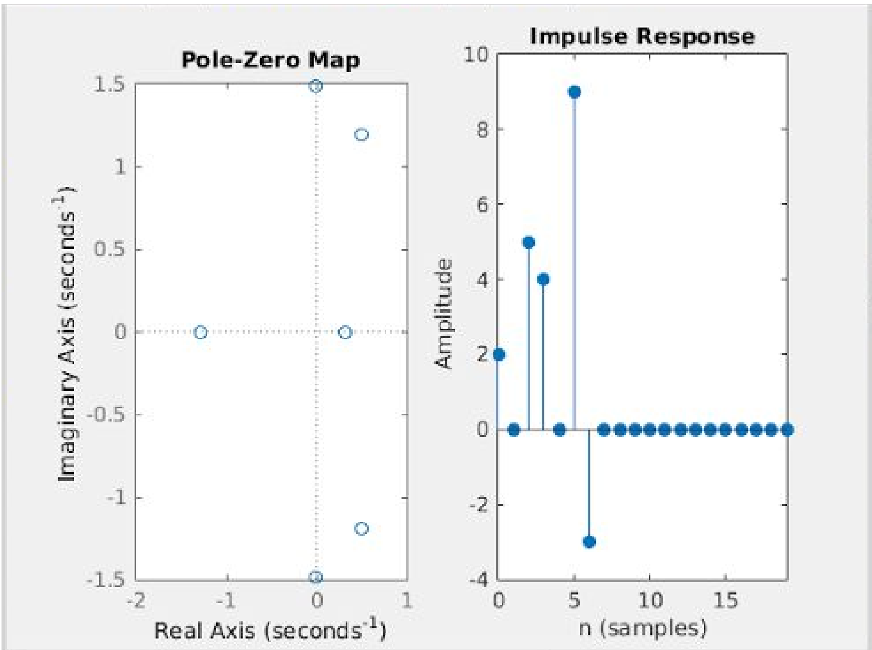
1. **Z-Transform**

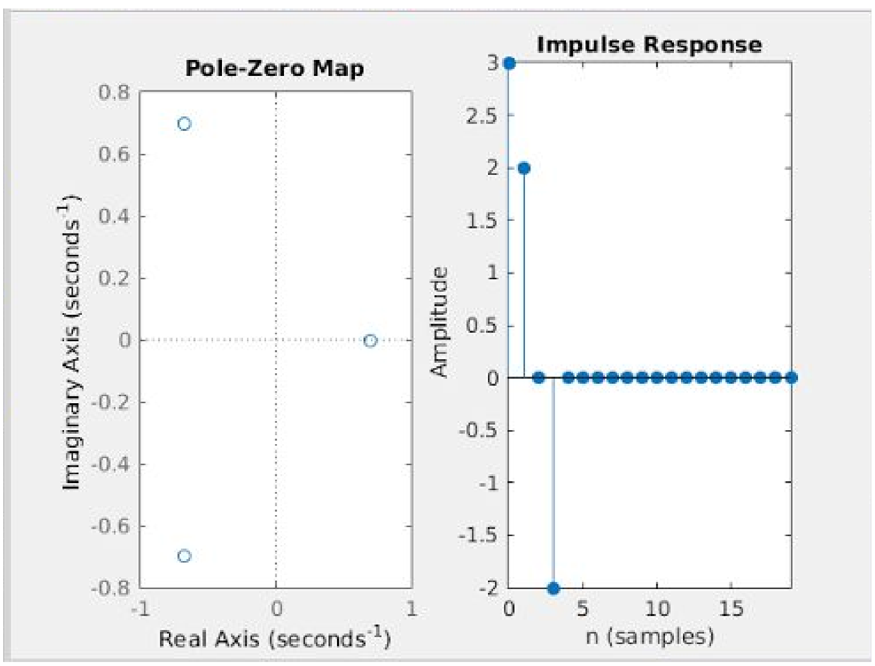
a). Plot poles, zeros, and impulse response of length 20

H1:



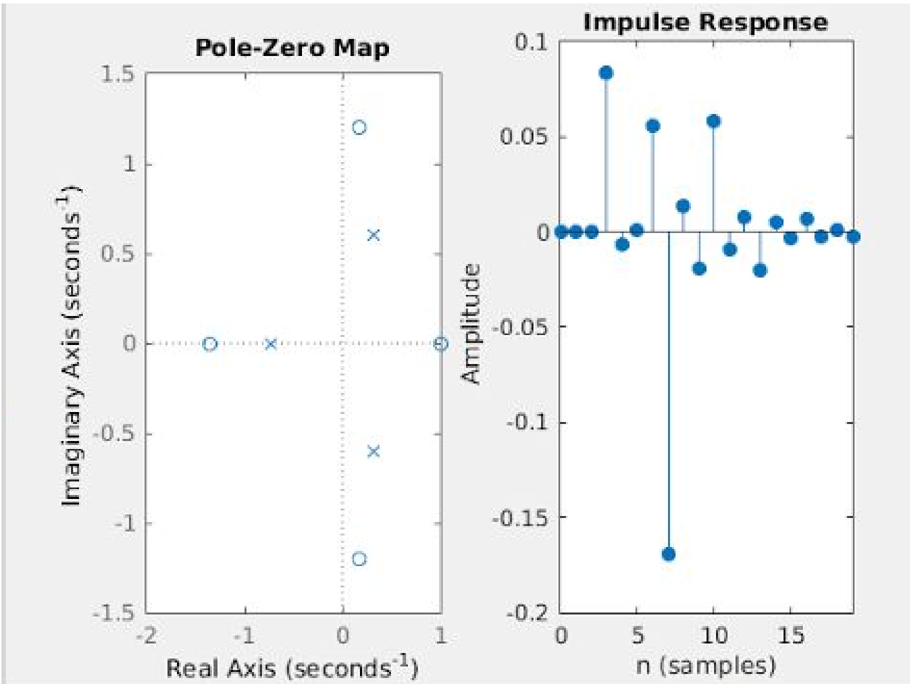
The system is stable since no poles are outside the unit circle.

H2:



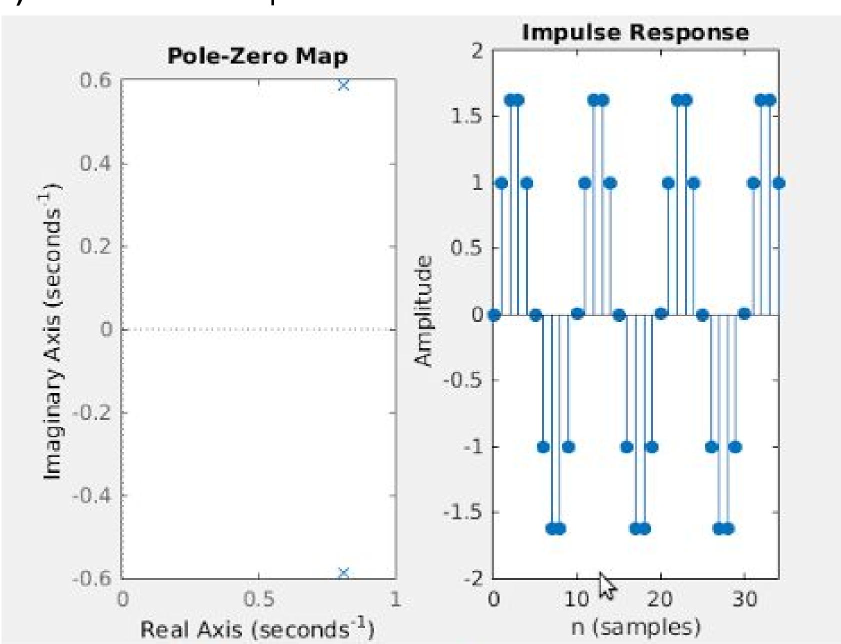
The system is stable since no poles are outside the unit circle.

H3:



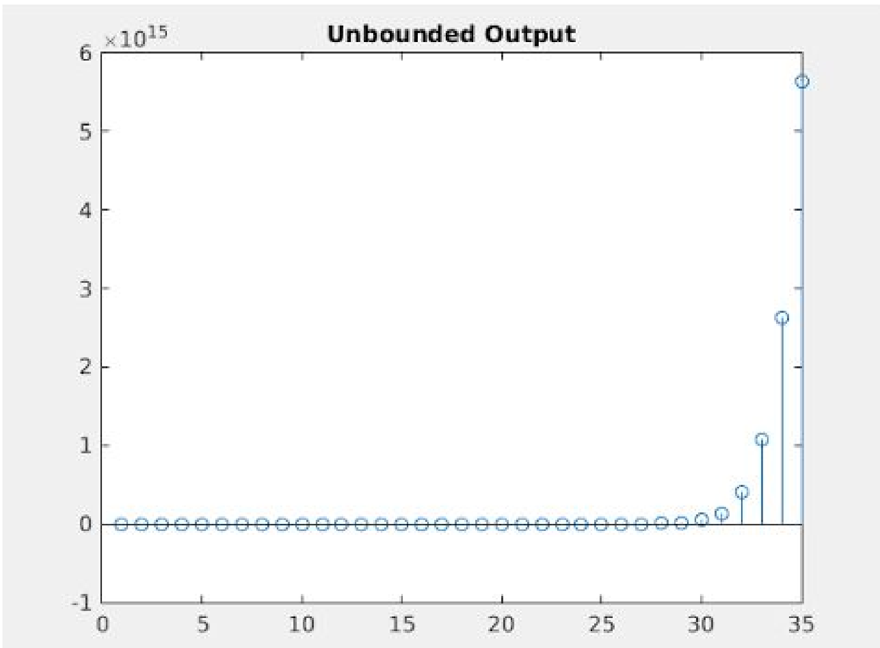
The system is stable since no poles are outside the unit circle.
We can determine if a system is stable or not by looking at the impulse response of the system. If the impulse response is absolutely summable, the system is stable. This is because it implies that any given bounded input will result a bounded output.

b). Plot and answer questions

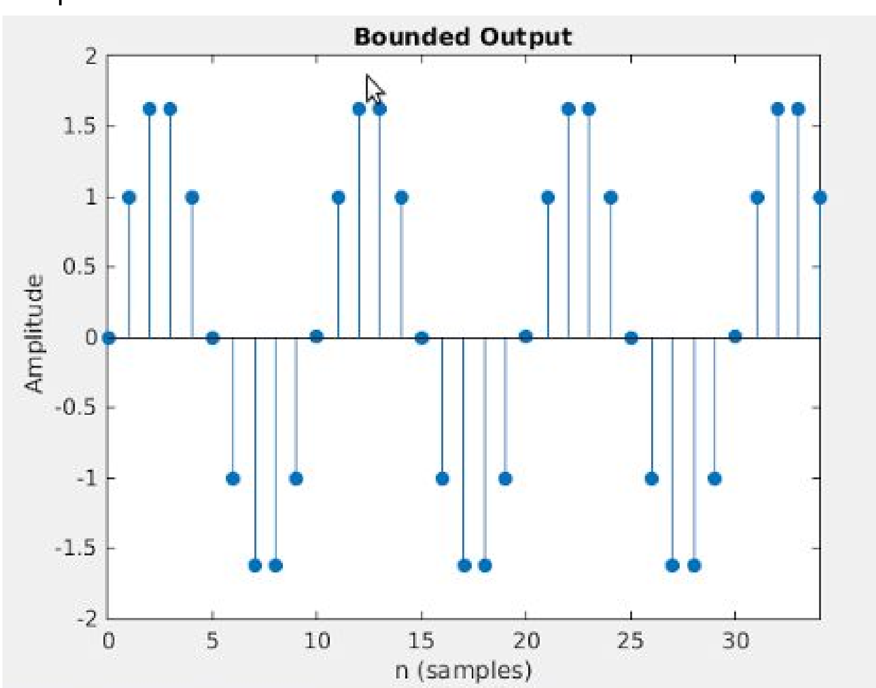


The system is unstable since not all poles are within the unit circle, and the impulse response is not absolutely summable.

Unbounded output: in order to have unbounded output, X(z) need to have single poles on U.C. that matches the poles in H(z). Therefore, x[n] can be exp(e^(j8*pi/10))u[n].
The output plot is:



If the input signal is the unit impulse function δ[n] (which is bounded), the output signal is unbounded. Z transform of δ[n] is 1. Therefore, the output signal has its origin poles.
The plot is:



2. Filter Function in Matlab

a). Determine X(z)

$$x[n] = \left[\left(\tfrac{1}{2}\right)^n + \left(-\tfrac{1}{3}\right)^n\right] u[n]$$

$$X(z) = \frac{1}{1-\frac{1}{2}z^{-1}} + \frac{1}{1+\frac{1}{3}z^{-1}}$$

$$= \frac{z}{(z-\frac{1}{2})} + \frac{z}{(z+\frac{1}{3})}$$

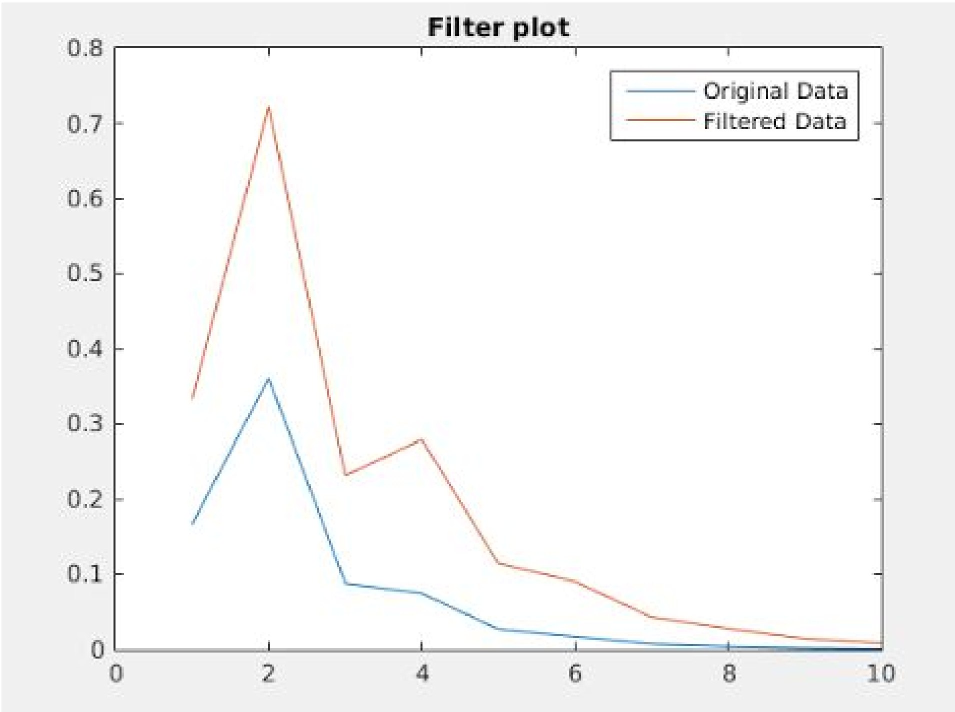
$$= \frac{2z}{2z-1} + \frac{3z}{3z+1}$$

$$= \frac{2z(3z+1) + 3z(2z-1)}{(2z-1)(3z+1)}$$

$$= \frac{6z^2+2z+6z^2-3z}{6z^2-z-1} = \frac{12z^2-z}{6z^2-z-1}$$

$$= \frac{12-z^{-1}}{6-z^{-1}-z^{-2}}$$

b). Verify X(z)



The filtered data is in the shape of the original data. Therefore, the analytical result is correct.

3. LSI System Response

a). Find the impulse response h[n] for the LSI system

$$x[n] = u[n]$$

$$X(z) = \frac{1}{1-z^{-1}}, \quad |z| > 1$$

$$y[n] = 2\left(\tfrac{1}{3}\right)^n u[n]$$

$$Y(z) = 2\,\frac{1}{1-\frac{1}{3}z^{-1}}, \quad |z| > \left|\tfrac{1}{3}\right|$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{2}{1-\frac{1}{3}z^{-1}}\cdot(1-z^{-1}) = \frac{2(1-z^{-1})}{1-\frac{1}{3}z^{-1}}$$

$$= \frac{2z-2}{z-\frac{1}{3}} = \frac{6z-6}{3z-1}$$

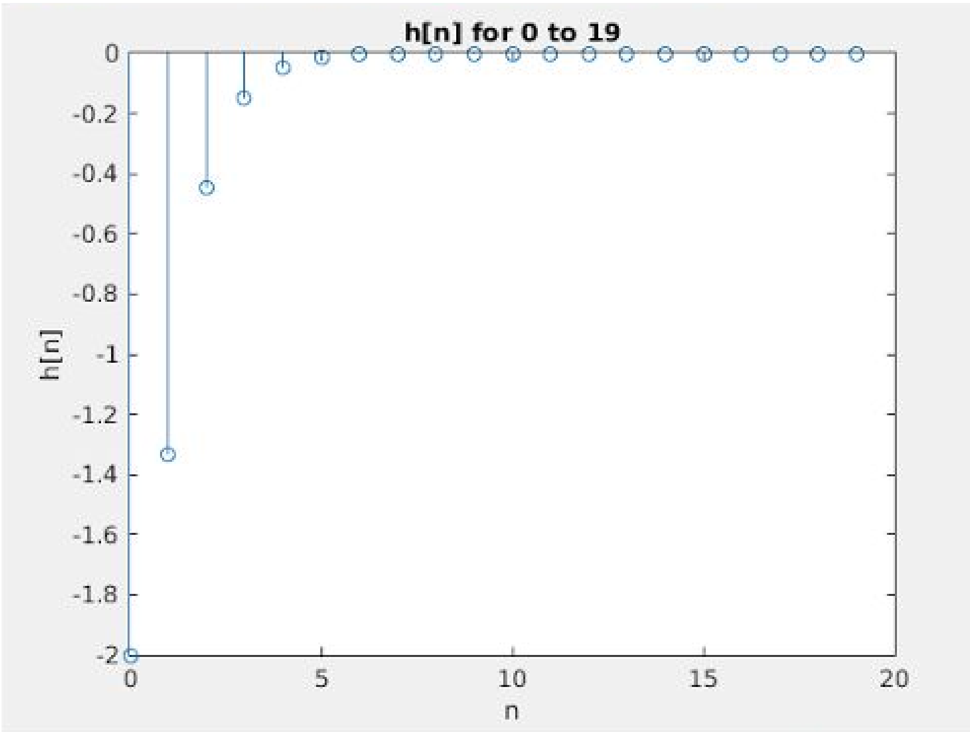
$$= 2-4\,\frac{1}{3z-1}$$

$$= 2-\frac{4}{3}\,\frac{z^{-1}}{1-z^{-1}}$$

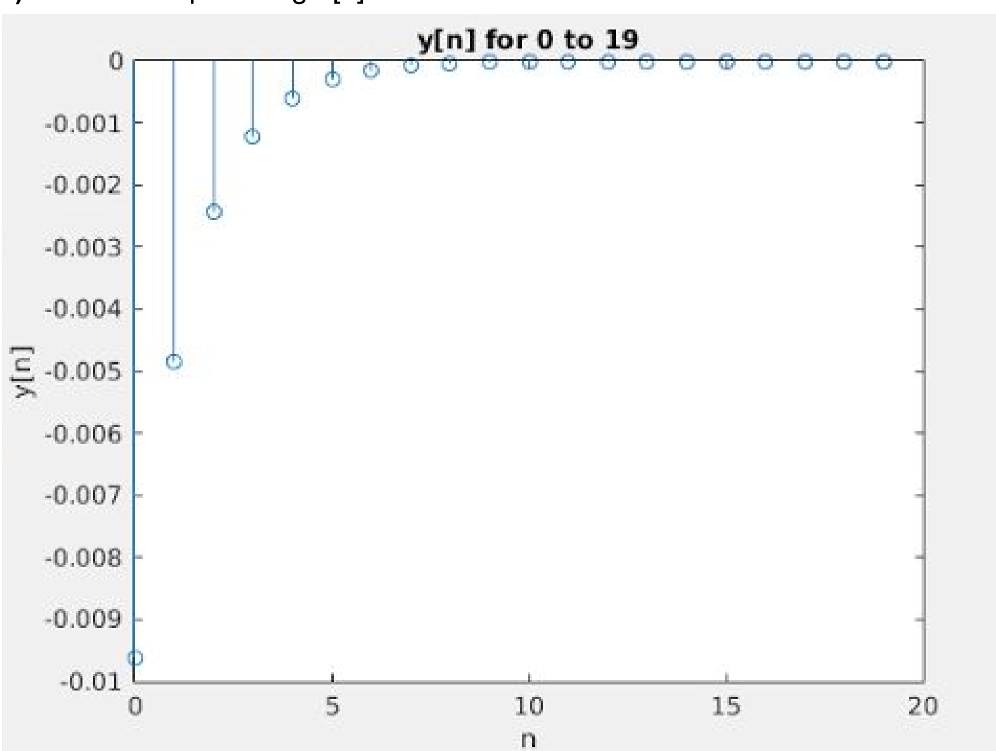
$$\therefore h[n] = 2\delta[n] - \frac{4}{3}\left(\tfrac{1}{3}\right)^{n-1} u[n]$$

$$h[n] = 2\delta[n] - 4\left(\tfrac{1}{3}\right)^{n} u[n]$$

b). Use Stem to plot h[n] from 0 to 19



c). Plot the output using h[n] in b.



4. Poles and Zeros

a). Determine H(z) and plot poles and zeros

$$y[n] = \frac{1}{2}y[n-1] + x[n] - \frac{1}{1024}x[n-10]$$

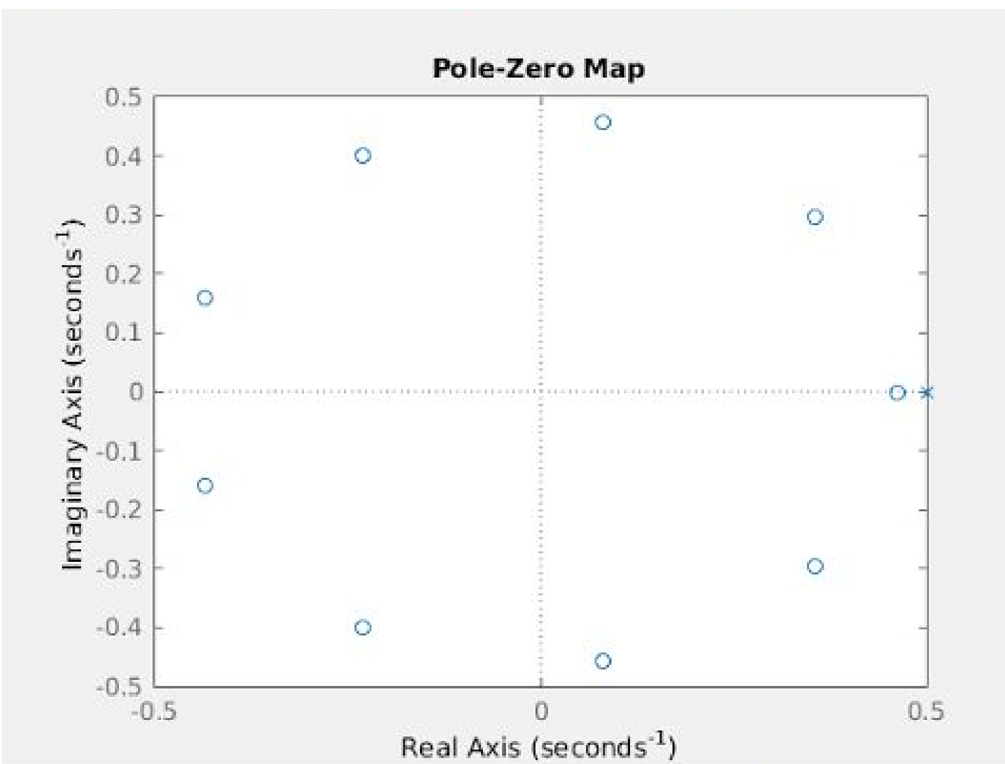
$$y[n] = \frac{1}{2}y[n-1] + x[n] - \left(\frac{1}{2}\right)^{10}x[n-10]$$

$$a = \frac{1}{2},\ D = 4$$

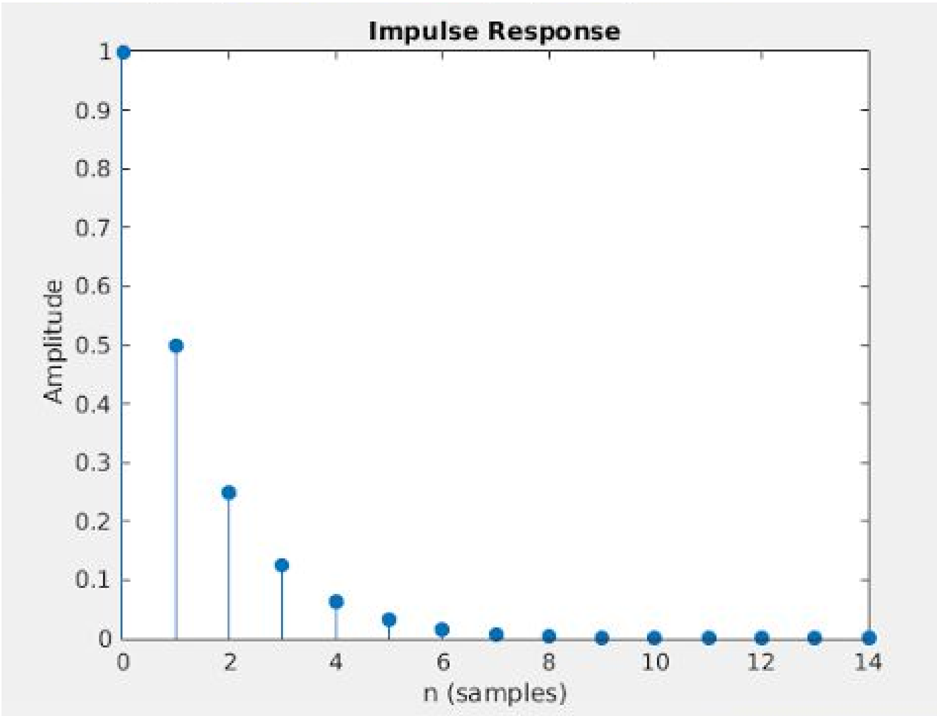
$$\therefore h[n] = a^n\{u[n] - u[n-D]\}$$

$$= \left(\frac{1}{2}\right)^n u[n] - \left(\frac{1}{2}\right)^n u[n-10]$$

$$H(z) = \frac{1}{1-\left(\frac{1}{2}\right)z^{-1}} - \frac{\frac{1}{1024}z^{-10}}{1-\left(\frac{1}{2}\right)z^{-1}} = \frac{1-\frac{1}{1024}z^{-10}}{1-\left(\frac{1}{2}\right)z^{-1}}$$

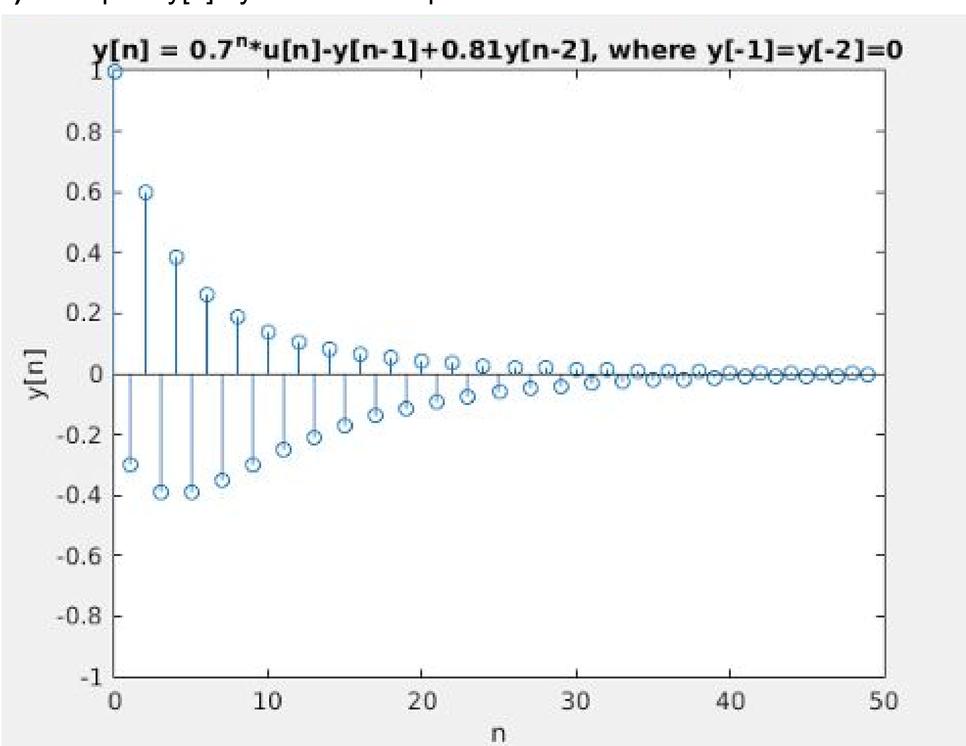


b). Compute and plot impulse response h[n]

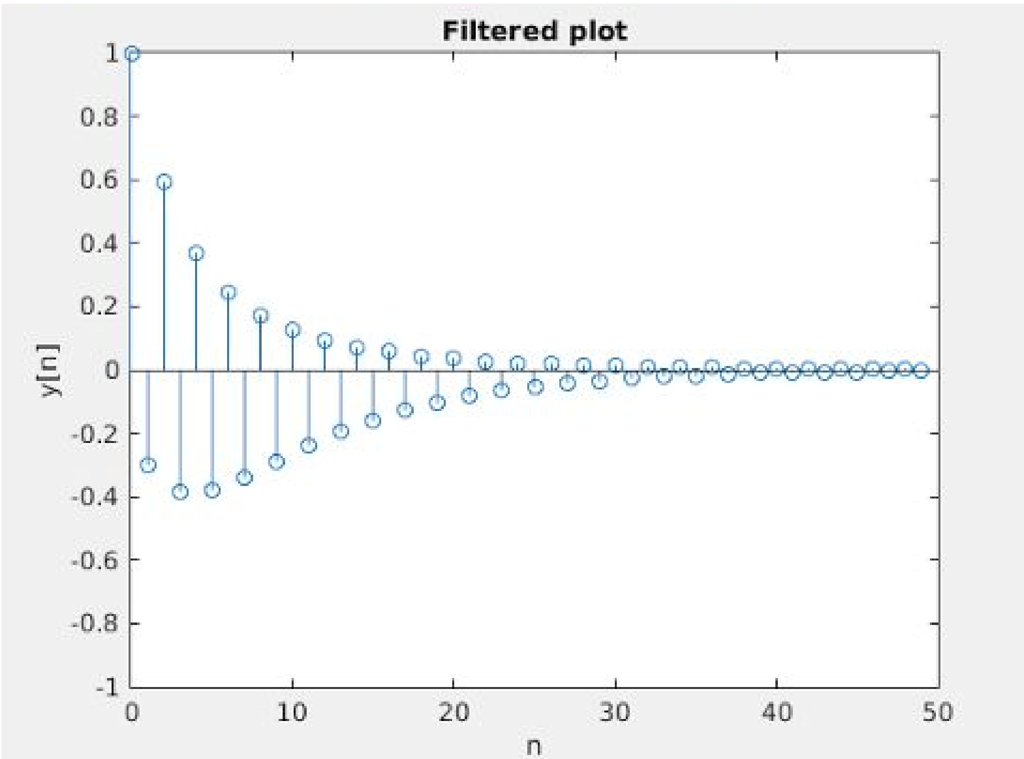


5. **Filter Function vs Recursive For Loop**

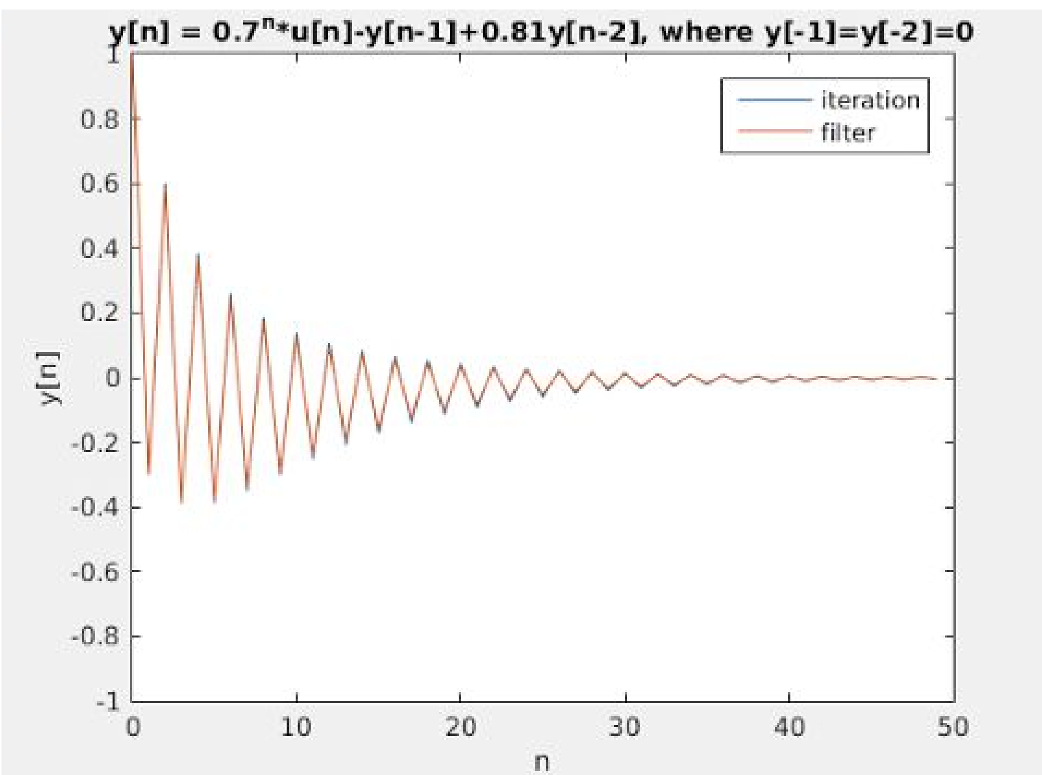
a). Compute y[n] by recursive loop



b). Compute y[n] using filter function



c). Plot y[n] from a and y[n] from b on the same graph



When zooming in: