structure of induction proofs

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⋆ Introduction

Suppose $P(n)$ is a predicate on natural numbers and we want to prove that $P(n)$ holds for all $n \geq b$, where b is some natural number. We present the structure of a simple induction proof and also of that a complete induction proof.

While there are other ways of presenting such proofs, please keep in mind that proof structure is an important part of this course (for details, see additional notes on what you should know about proofs). However you present your proofs, your proof structure should always be clearly evident.

∘ Proof structure for simple induction

Basis:

Let $n = b$.

[Proof of $P(n)$ — i.e., proof of $P(b)$]

Induction Step:

Let $n \geq b$.

Suppose $P(n)$. [IH]

[Proof of $P(n+1)$ — under assumption of $P(n)$]

Therefore, by induction, $P(n)$ holds for all $n \geq b$. □

∘ Proof structure for complete induction (with k base cases)

Base Cases:

Let $n = b$.

[Proof of $P(n)$ — i.e., proof of $P(b)$]

Let $n = b + 1$.

[Proof of $P(n)$ — i.e., proof of $P(b+1)$]

⋮

Let $n = b + k - 1$.

[Proof of $P(n)$ — i.e., proof of $P(b+k-1)$]

Induction Step:

Let $n \geq b + k$. [Alternatively, let $n > b + k - 1$.]

Suppose $P(j)$ holds whenever $b \leq j < n$. [IH]

[Proof of $P(n)$ — under assumption of IH]

Therefore, by induction, $P(n)$ holds for all $n \geq b$. □