University of Toronto Scarborough

CSC B36 | **Term Test 2** | **17 November 2017**

NAME: ______________________________
(circle your last name)

SIGNATURE: ____________________

Circle your tutorial section:

T01 MO 10 Bryan	T02 TU 4 Thomas	T03 WE 10 Jiali	T04 FR 12 Changyu	T05 FR 12 Eric

Do not begin until you are told to do so. In the meantime, put your name on this cover page and read the rest of this page.

Aids allowed: None.

Duration: 90 minutes.

There are 4 pages and each is numbered at the bottom. Make sure you have all of them.

Write legibly in the space provided. Use the backs of pages for rough work; they will not be graded.

If you write in pencil or erasable pen, or if your test shows any evidence that work has been erased, then you forfeit the right to have your test re-graded.

Question	Your mark	Out of
0.		1
1.		10
2.		10
3.		15
Total		36

0. **[1 mark]** Print your name and student number here: ....

1. **[10 marks total; 5 for each part]** Let $\Sigma = \{0, 1\}$. For a string $x \in \Sigma^*$, we define x to be *0-alternating* iff either all the symbols in odd positions within x are 0s, or all the symbols in even positions within x are 0s (or both). For example, if $x = b_1b_2b_3b_4b_5$, where each $b_i \in \Sigma$, then x is 0-alternating iff $b_1 = b_3 = b_5 = 0$ or $b_2 = b_4 = 0$. Let $L_1 = \{x \in \Sigma^* : x \text{ is 0-alternating}\}$.

   (a) Nick designed a state invariant for a DFSA that accepts L_1. Use it to complete the DFSA diagram below. Each pair of double parentheses indicates an accepting state. E.g., $((q_{E,O}))$.

$$\delta^*(q_\varepsilon, x) = \begin{cases} q_\varepsilon & \text{iff } x = \varepsilon; \\ q_0 & \text{iff } x = 0; \\ q_1 & \text{iff } x = 1; \\ q_{E,A} & \text{iff } |x| > 1,\ |x| \text{ is } \textbf{even},\ x \text{ contains 0s in } \textbf{all} \text{ positions;} \\ q_{E,O} & \text{iff } |x| > 1,\ |x| \text{ is } \textbf{even},\ x \text{ contains 0s in all } \textbf{odd} \text{ but not all even positions;} \\ q_{E,E} & \text{iff } |x| > 1,\ |x| \text{ is } \textbf{even},\ x \text{ contains 0s in all } \textbf{even} \text{ but not all odd positions;} \\ q_{O,A} & \text{iff } |x| > 1,\ |x| \text{ is } \textbf{odd},\ x \text{ contains 0s in } \textbf{all} \text{ positions;} \\ q_{O,O} & \text{iff } |x| > 1,\ |x| \text{ is } \textbf{odd},\ x \text{ contains 0s in all } \textbf{odd} \text{ but not all even positions;} \\ q_{O,E} & \text{iff } |x| > 1,\ |x| \text{ is } \textbf{odd},\ x \text{ contains 0s in all } \textbf{even} \text{ but not all odd positions.} \end{cases}$$

		$((q_{E,A}))$	$((q_{O,A}))$
	$((q_0))$	$((q_{E,O}))$	$((q_{O,O}))$
$\rightarrow((q_\varepsilon))$	$((q_1))$	$((q_{E,E}))$	$((q_{O,E}))$

   (b) Using as few states as possible, draw a DFSA that accepts L_1. No justification is required.
   Hint: There's a reason why so little space is provided here.

2. **[10 marks]** For a language L over alphabet Σ, we define the set of prefixes of L as follows.

$$\mathrm{Pre}(L) = \{x \in \Sigma^* : \text{there exists } y \in \Sigma^* \text{ such that } xy \in L\}.$$

Use structural induction to prove that for any language L, if L is regular, then so is $\mathrm{Pre}(L)$. Be sure to clearly define an appropriate predicate on regular expressions.

3. **[15 marks total; 5 for each part]** Let $L_3 = \{0^i 1^j : j \neq 2i\}$.

   (a) *There is nothing to do for this part.*
   Your mark for part (a) will be the maximum of your marks for parts (b) and (c).

   (b) Give a CFG that generates L_3 and briefly explain why your CFG is correct.

   (c) Give a PDA that accepts L_3 and briefly explain why your PDA is correct.