A SIGNATURE FOR THE LORENZ INSTABILITY IN QUANTUM OPTICS

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We have numerically examined the time averaged output power of a Lorenz laser in the self-pulsing and chaotic regime. The results indicate that the change in average power when unstable behavior occurs are of the order of 1–5%. This negligible change could serve as an additional signature of the Lorenz instability in quantum optics.

1. Introduction

The early work of Risken and Nummedal showed that the single mode homogeneously broadened laser could exhibit a second instability in the mean-field limit which can result in either chaotic or self-pulsing behavior [1–3]. The equations of Risken and Nummedal were shown to be isomorphic to the celebrated Lorenz equations by Haken [4]. This one-to-one correspondence results for the perfectly tuned laser.

Motivated by these results, experimentalists have sought to unambiguously observe the Lorenz instability in quantum optics. Observations of self-pulsing behavior in optically pumped far infrared lasers were reported as early as 1980 by Lawandy [5] and Koepf and recently measurements on unidirectional systems have been reported by Weiss et al. [6,7]. In all of these measurements the unambiguous interpretation of the instability has been complicated by the possibility of other physical phenomena [8].

The early work on the homogeneously broadened $^{13}CH_3F$ laser suffered from the presence of standing wave effects [9] while the observations in Doppler broadened systems observed only self-pulsing for the resonantly tuned laser. The latter observations are difficult to accept in view of the stringent requirements on relaxation rates ($\gamma_\parallel/\gamma_\perp < 0.2$) and the anomalously low excitations at which the instability occurred [10]. Moreover, all of the experiments reported to date have been for low pressure molecular gases where pump coherence effects indicate that two level models may be inadequate [9,11]. In addition these lasers are optically pumped and are plagued by inhomogeneous gain distributions and possible spatial mode effects.

The primary signature of the Lorenz instability in laser physics is the absence of a "route" to chaos. The true Lorenz instability should exhibit no scenario to chaos, period doubling or otherwise if the isomorphism shown by Haken is valid. In this paper we suggest an additional test for Lorenz chaos based on the expected change in average output power when the laser becomes unstable. Typically lasers may become unstable by multimode effects (transverse modes in molecular laser experiments) and inhomogeneous broadening effects. Previously, sizable changes in output power have been reported in inhomogeneously broadened systems such as the xenon laser [12].

Similarly when new modes reach threshold in systems with spatially nonuniform excitation, the output power of the system usually jumps due to the increased population from which energy can be extracted.

We have numerically integrated the Lorenz model for the laser parameter space and calculated the average power an averaging detector would register. Such experiments are easy to perform and seem to have been ignored as a possible signature of the true single

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mode Lorenz instability. We present results on the average power of the unstable Lorenz system for the chaotic and self-pulsing regimes. The Lorenz equations are given below

$$\dot{X}=\sigma(Y-X)\,,\quad \dot{Y}=rX-Y-XZ\,,$$
$$\dot{Z}=XY-bZ\,. \tag{1}$$

The variable X is proportional to the electric field envelope, $\sigma=\kappa/\gamma_\perp$, $b=\gamma_\parallel/\gamma_\perp$ and r is the ratio of unsaturated gain to threshold gain. κ is the cavity decay rate, $\gamma_\parallel$ and $\gamma_\perp$ are the longitudinal and transverse relaxation rates respectively. The single mode instability occurs when $\sigma>b+1$ and $r>r_c$ where

$$r_c=\sigma(\sigma+b+3)/(\sigma-b-1)\,. \tag{2}$$

The self-pulsing regime occurs for $b\le 0.21$ and the chaotic regime occurs for $b>0.21$.

2. Average power

The average power of the single mode laser is given by

$$\langle x^2\rangle=\lim_{\Delta\to\infty}\frac{1}{\Delta}\int\limits_{t_1}^{t_1+\Delta} x^2(t)\,\mathrm{d}t\,, \tag{3}$$

when the laser is stable ($r<r_c$), the average power is given by the fixed points of the Lorenz equations

$$\langle x^2\rangle=b(r-1)\,. \tag{4}$$

The average power of the self-pushing and chaotic laser output was calculated using eq. (3) and the $x(t)$ values obtained by numerical integration of the Lorenz equations. The initial values are chosen to be 98% of the fixed point values and the numerical integration is carried out on a VAX 730 computer using a fifth order Runge-Kutta algorithm. The time t_1 was chosen to correspond to three times the time it took for initial transients to decay and Δ was chosen large enough so that the average was insensitive to an increase in the average time.

The average power as a function of excitation for the self-pulsing laser instability ($b=0.15$, $\sigma=4$) is shown in fig. 1a. The results show that near the value $r=r_c$ the average power of the self-pulsing laser exhibits the largest deviation from the fixed point

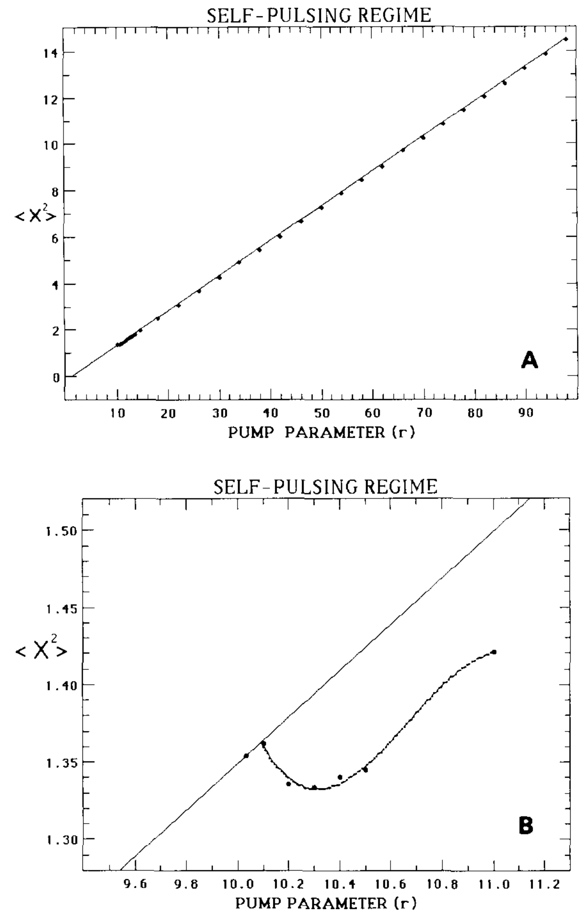

Fig. 1. (a) Average power of the single mode laser as a function of the excitation parameter (r) in the self-pulsing regime ($b=0.5$, $\sigma=4$). The solid line is the fixed point value and the dots are averages computed from the numerical solution of the Lorenz equations. (b) Same as (a) focused around the regime of excitation near the instability threshold (r_c). The solid line through the dots is the polynomial fit given in eq. (5).

value. Fig. 1b shows the behavior of $\langle x^2\rangle$ near $r=r_c$. The maximum drop in power was found to be of the order of 5.0%. The behavior of $\langle x^2\rangle$ as a function of r in the region $10.1<r<11.0$ was fitted by a polynomial given by

$$\langle x^2\rangle^{\mathrm{sp}}=565.05-158.76r$$
$$+14.88r^2-0.465r^3\,. \tag{5}$$

The variation of the average power in the chaotic regime ($b=1.0$, $\sigma=4$) showed similar behavior as the

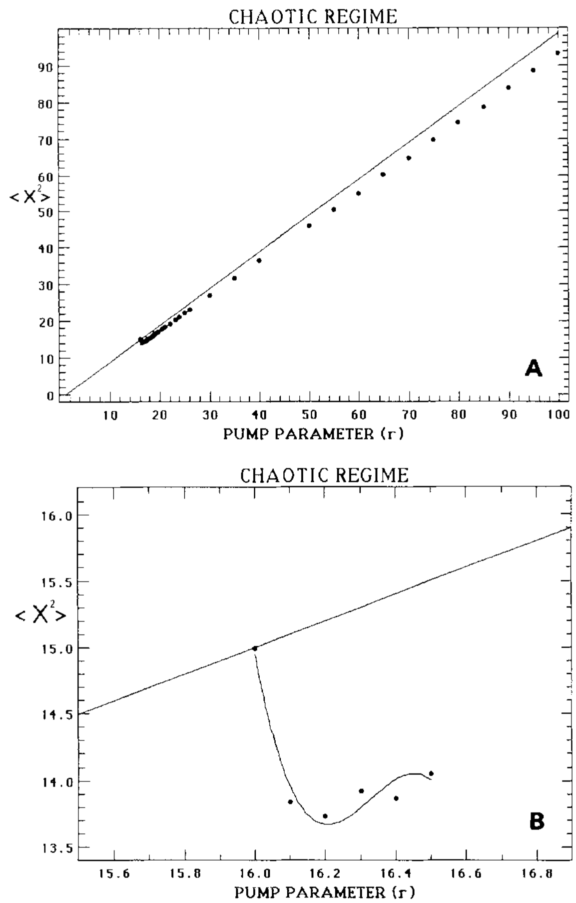

Fig. 2. (a) Same as fig. 1(a) but for the chaotic regime ($b=1$, $\sigma=4$). (b) Same as (a) focused around the instability threshold r_c. The solid line through the dots is the polynomial fit given in eq. (6).

self-pulsing regime. The average power was always below the steady value and showed the largest change just beyond $r=r_c$. Figs. 2a and b show the behavior of $\langle x^2 \rangle$ versus r. The behavior of $\langle x^2 \rangle$, approximated by a polynomial r in the chaotic regime for the range $16.0<r<16.50$ is given by

$$\langle x^2 \rangle^c = 228944.0 - 42065.9r + 2576.4r^2 - 52.6r^3 . \quad (6)$$

Studies of the effect of σ on the average power indicate that as the laser goes further into the bad cavity limit ($\sigma \to \infty$) the average power of the self-pulsing and chaotic outputs approches the steady fixed point value for $r>r_c$.

3. Conclusions

In conclusion, we have shown that the average power of the unstable laser above its second threshold is essentially unchanged for $r>r_c$ and only changes slope in a small region around r_c. We hope that experimentalists will use these results as another signature of the Lorenz instability. It is expected that multimode effects which plague most open resonator experiments will result in significantly altered output powers as these may be effectively spatially inhomogeneously broadened unstable systems.

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