Quantum-like Bayesian Networks for Modeling Decision Making

Real probability numbers are replaced by quantum probability amplitudes

$$\Pr(X_1, \ldots, X_n) = \left| \prod_{i=1}^{n} \psi(X_i | Parents(X_i)) \right|^2$$

Example of a Quantum-Like Bayesian Network

X1	$\varphi_{x2} = T$	$\varphi_{x2} = F$
T	$\varphi_{1=T}\varphi_{2=T}$	$\varphi_{1=T}\varphi_{2=F}$
F	$\varphi_{1=F}\varphi_{2=T}$	$\varphi_{1=F}\varphi_{2=F}$

X2	$\varphi_{x3} = T$	$\varphi_{x3} = F$
T	$\varphi_{2=T}\varphi_{3=T}$	$\varphi_{2=T}\varphi_{3=F}$
F	$\varphi_{2=F}\varphi_{3=T}$	$\varphi_{2=F}\varphi_{3=F}$

The quantum marginal probability distribution equation

$$\Pr(X_1|e) = \alpha|\sum_{y}\prod_{x=1}^{n}\psi(X_x|Parents(X_x), e, y)\;|^2$$

$$= \alpha\sum_{\mathrm{i}=1}^{|Y|}\left|\prod_{x}^{N}\psi(X_x|Parents(X_x), e, y) = i\right|^2 + 2Interference$$

$$Interference$$

$$= \sum_{i=1}^{|Y|-1}\sum_{j=i+1}^{|y|}|\prod_{x}^{N}\psi(X_x|parents(X_x), e, y = i)\,|\cdot|\prod_{x}^{N}\psi(X_x|parents(X_x), e, y = i)|$$

$$\cdot\cos(\Theta_{\mathrm{i}} - \Theta_j)$$

Representation of Beliefs/Actions

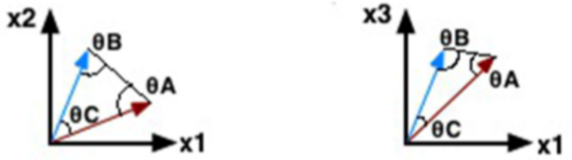
For each pair of

$$\sum_{i=1}^{N-1} \sum_{J=i+1}^{N} |\psi_i|\,|\psi_j| cos(\theta_i - \theta_j) \big)$$

It can be represent as a 2-dimensional vector like:

$$a(X = T) = \begin{bmatrix} |\psi_i \cdot e^{i\theta_i}|^2 \\ |\psi_j \cdot e^{i\theta_j}|^2 \end{bmatrix}$$

$$a(X = F) = \begin{bmatrix} |\psi_i \cdot e^{i\theta_i}|^2 \\ |\psi_j \cdot e^{i\theta_j}|^2 \end{bmatrix}$$



$$Pr(B) = \alpha\,[\sum_{i=1}^{N} |\psi_i|^2 + 2\cdot|\psi_1|\cdot|\psi_2|\cdot\cos(\theta_1 - \theta_2) + 2\cdot|\psi_1|\cdot|\psi_3|\cdot\cos(\theta_1 - \theta_3) + \cdots$$

FIGURE 3 | Illustration of the different 2-dimensional vectors that will be generated for each step of iteration during the computation of the quantum interference term.

Definition of the Similarity Heuristic

$$h(a,b) = \begin{cases} \pi & if \phi < 0 \\ \pi - \theta_C/2 & if \phi > 0.2 \\ \pi - \theta_C & otherwise \end{cases}$$

$$where\ \phi = \frac{\theta_C}{\theta_A} - \frac{\theta_B}{\theta_A}$$

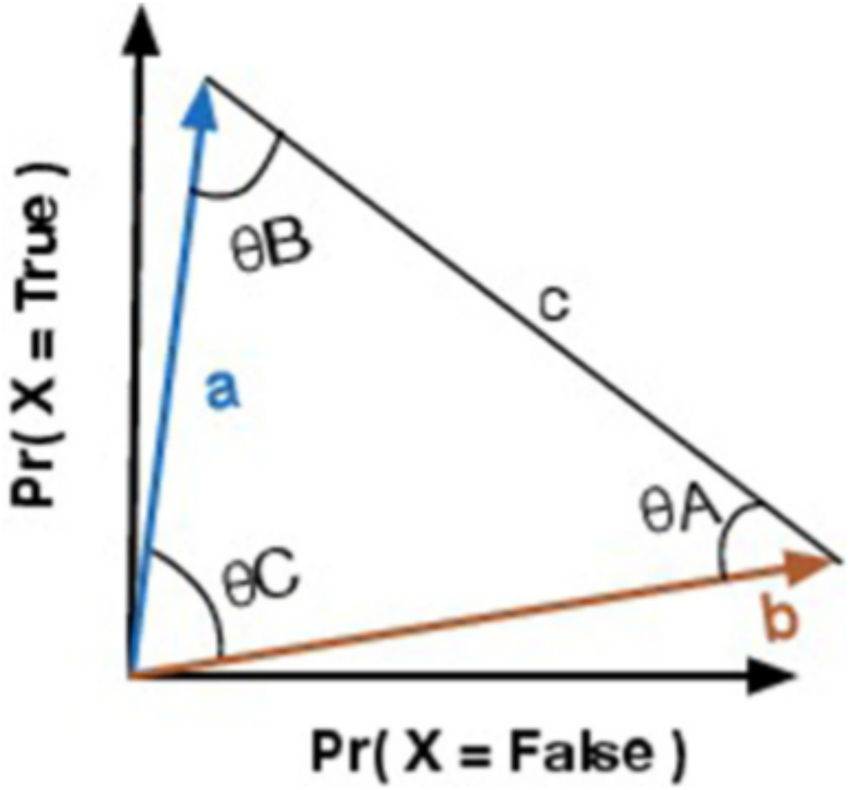


FIGURE 2 | Vector representation of two events representing a certain state.

Example

Step 1: Create a Bayesian Network Representation of Problem

Literature	Known to Defect	Known to Collabrate	Unknown	Classical Probability
Shafir and Tversky	0.9700	0.8400	0.6300	0.9050
Croson	0.6700	0.3200	0.3000	0.4950
Li and Taplin	0.8200	0.7700	0.7200	0.7950
Busemeyer et al	0.9100	0.8400	0.6600	0.8750
Hristova and Grinberg	0.9700	0.9300	0.8800	0.9500
Average	0.8700	0.7400	0.6400	0.8050

$\psi_{P1 = \text{defect}} = \sqrt{0.5}e^{i\theta_1}$

$\psi_{P1 = \text{cooperate}} = \sqrt{0.5}e^{i\theta_2}$

P1	ψ P2 = defect	ψ P2 = cooperate
defect	$\sqrt{0.87}e^{i\theta_3}$	$\sqrt{0.13}e^{i\theta_4}$
cooperate	$\sqrt{0.74}e^{i\theta_5}$	$\sqrt{0.26}e^{i\theta_6}$

Step 2: Compute the Vectors associated to each action

$$a(X=T)=\begin{bmatrix}|0.6595\cdot e^{i\theta_A}|^2\\|0.6083\cdot e^{i\theta_C}|^2\end{bmatrix}=\begin{bmatrix}0.435\\0.370\end{bmatrix}$$

$$a(X=T)=\begin{bmatrix}|0.2550\cdot e^{i\theta_A}|^2\\|0.3606\cdot e^{i\theta_C}|^2\end{bmatrix}=\begin{bmatrix}0.065\\0.130\end{bmatrix}$$

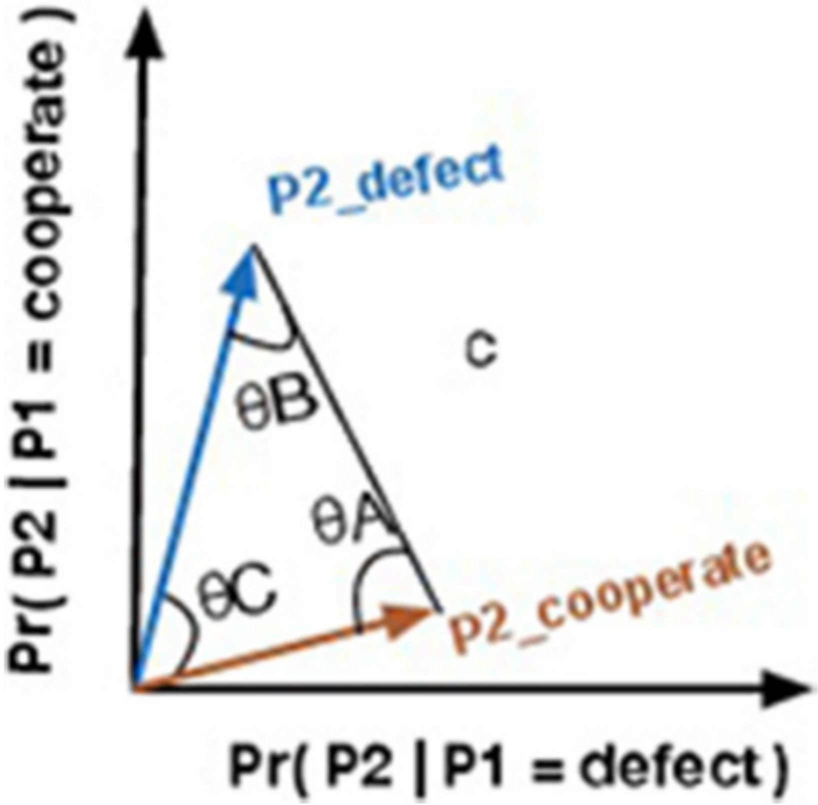


FIGURE 5 | Vector representation of events $P2_{Defect}$ and $P2_{Cooperate}$ plus the euclidean distance vector *c*.

Step 3: Determine the quantum parameters using the proposed similarity heuristic

$\theta_A = 2.6102$
$\theta_B = 0.1294$ Therefore,
$\theta_C = 0.4023$

$\phi = \frac{\theta_C}{\theta_A} - \frac{\theta_B}{\theta_A}$=0.1046

$$h(a,b) = \begin{cases} \pi & if \phi < 0 \\ \pi - \theta_C/2 & if \phi > 0.2 \\ \pi - \theta_C & otherwise \end{cases}$$

$\theta = h = \pi - \theta_C = 2.7393$

Step 4: Perform the Probable Inference

$$Pr(P2 = Defect) = \alpha \Big[\left|\psi_{P2=D|P1=D}\right|^2 + \left|\psi_{P2=D|P1=D}\right|^2 + 2 \cdot \left|\psi_{P2=D|P1=D}\right| \cdot \left|\psi_{P2=D|P1=C}\right| \cdot \cos(\theta) \Big] \quad (42)$$

$$Pr(P2 = Defect) = \alpha \Big[0.5 \times 0.87 + 0.5 \times 0.74 + 2 \times \sqrt{0.5 \times 0.87} \times \sqrt{0.5 \times 0.74} \cos(2.7393) \Big] \quad (43)$$

Computing the probability of $Pr(P2 = Cooperate)$ in the same way, we obtain:

$$Pr(P2 = Defect) = \alpha \cdot 0.0667$$
$$Pr(Cooperate) = \alpha \cdot 0.0258 \quad (44)$$

Step 5: Compute Normalization Factor and Final Probabilities

$$\alpha = \frac{1}{0.0667 + 0.0258} = \frac{1}{0.0925} = 10.8108$$

$$\Pr(P2 = Defect) = 0.7208$$

$$\Pr(P2 = Cooperate) = 0.2792$$