Digesting Anomalies: An Investment Approach

Kewei Hou Chen Xue Lu Zhang

Presenter: Zhiming Mei

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Motivation

- Fama-French 3-factor models summarize the cross-section of average stock returns as of the mid-1990s.
- Over the past two decades, the Fama-French model cannot explain a wide range of anomalies.
- Q-model is in part inspired by investment-based asset pricing, which is in turn built on the neoclassical q-theory of investment.

The Q-Model

$$E\left[r^i\right]-r^f=\beta^i_{\mathrm{MKT}}E[\mathrm{MKT}]+\beta^i_{\mathrm{ME}}E\left[r_{\mathrm{ME}}\right]+\beta^i_{\mathrm{I/A}}E\left[r_{\mathrm{I/A}}\right]+\beta^i_{\mathrm{ROE}}E\left[r_{\mathrm{ROE}}\right]$$

- Q-model: market factor, size factor, investment factor, and profitability factor.
- $E[\cdot]$ are expected factor premiums and $\beta^i_{\cdot}$ are factor loadings

The Q-Factors

	$\overline{R}$	α	β_{MKT}	β_{SMB}	β_{HML}	β_{UMD}
R_{ME}	0.31	0.23	0.17			
	(2.12)	0.04	0.02	0.99	0.17	
		0.01	0.02	0.99	0.19	0.03
$R_{I/A}$	0.45	0.52	−0.15			
	(4.95)	0.33	−0.06	−0.02	0.39	
		0.28	−0.05	−0.02	0.41	0.05
R_{ROE}	0.58	0.63	−0.11			
	(4.81)	0.77	−0.09	−0.33	−0.20	
		0.50	−0.03	−0.33	−0.10	0.28

	$R_{I/A}$	R_{ROE}	MKT	SMB	HML	UMD
R_{ME}	−0.11	−0.31	0.25	0.95	−0.07	0.01
$R_{I/A}$		0.06	−0.36	−0.22	0.69	0.05
R_{ROE}			−0.19	−0.38	−0.09	0.50

Figure: Empirical properties of the q-factors: 01/1972-12/2012 CRSP

Results Overivew

- About one-half of nearly 80 anomalies are insignificant with NYSE breakpoints and value-weighted decile returns.
- In explaining 35 significant anomalies, the q-factor model **outperforms** the Fama-French 3-factor and Carhart 4-factor models:
  1. The average magnitude of HML alphas: 0.20% in q, 0.33% in Carhart, and 0.55% in Fama-French
  2. The number of significant HML alphas: 5 in q, 19 in Carhart, and 27 in Fama-French
  3. The number of rejections by the GRS test: 20 in q, 24 in Carhart, and 28 in Fama-French

Results Overview

- The q-model outperforms the Fama-French and Carhart models in capturing momentum (HML earnings momentum and HML price momentum)
- The q-model performs similarly to the other 2 models in fitting the 25 size and book-to-market portfolios.
- The q-model underperforms the Fama-French and Carhart models in capturing the operating accrual anomaly and the R&D-to-market anomaly.

Contribution

- Traditional approach is to look for common factors from the consumption side of the economy.
  - This paper: A link between stock returns and firm characteristics from the production side.
- A new factor model: More flexible in practice (simplicity and high-frequency returns) to explain the cross-section of expected stock returns.

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Q-Theory: Investment and Returns

Firm i's value-maximization problem:

$$P_{i0} + D_{i0} \equiv \max_{\{I_0\}} \Pi_{i0} A_{i0} - I_{i0} - \frac{a}{2}\left(\frac{I_{i0}}{A_{i0}}\right)^2 A_{i0} + E_0\left[M_1 \Pi_{i1} A_{i1}\right]$$

The first principle for investment:

$$1 + a\frac{I_{i0}}{A_{i0}} = E_0\left[M_1 \Pi_{i1}\right]$$

A more familiar form from the corporate finance perspective:

$$r_{i1}^S = \frac{P_{i1} + D_{i1}}{P_{i0}} = \frac{\Pi_{i1} A_{i1}}{E_0\left[M_1 \Pi_{i1} A_{i1}\right]} = \frac{\Pi_{i1}}{E_0\left[M_1 \Pi_{i1}\right]} = \frac{\Pi_{i1}}{1 + a\left(I_{i0}/A_{i0}\right)}$$

Implications

$$E_0\left[r_{i1}^S\right] = \frac{E_0\left[\Pi_{i1}\right]}{1 + a\left(I_{i0}/A_{i0}\right)}$$

- **Investment factor**: All else equal, high-investment stocks should earn lower expected returns than low-investment stocks.
- **Profitability factor**: All else equal, high expected profitability stocks should earn higher expected returns than low expected profitability stocks

The Investment Factor

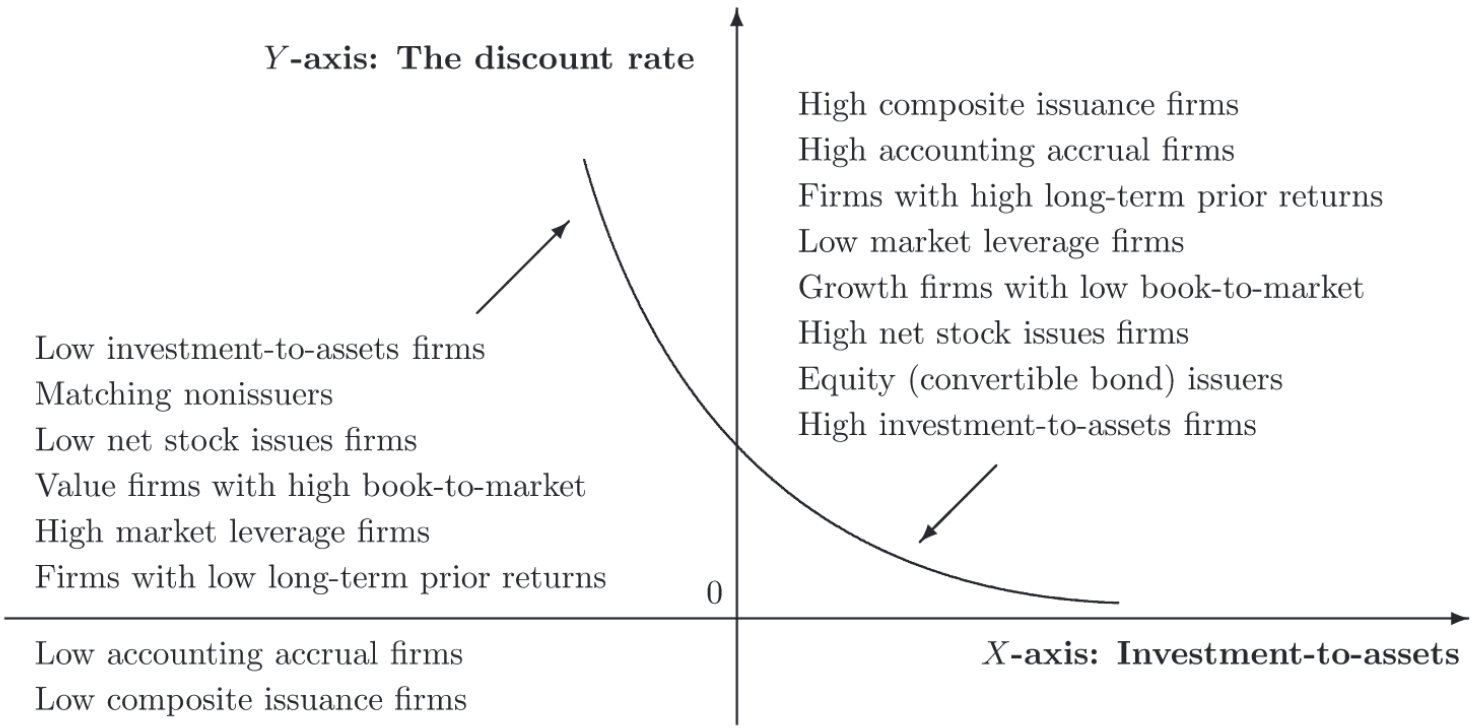

Figure: The investment channel

Investment Factor: Intuition

- Firm invests more when its q (the NPV of FCF generated from an additional unit of asset) is high.
- Low discount rates (project/firm cost of capital) imply high marginal q.
- The negative investment-return relation is a **conditional relation**.
  - Firms with high profitability will invest more.
  - Natural portfolio interpretation: sorting on net stock issues, composite issuance, book-to-market, and other valuation factors.
  - These sorts provide wider cross-sectional expected return spreads associated with investment than those associated with profitability.

The Profitability Factor: Intuition

- High profitability firms will have high expected returns.
  - From a capital budgeting perspective, high expected profitability relative to low investment must mean a high discount rate (expected return).
  - If the discount rates were not high enough to counteract the high expected profitability, then the firm would invest more
  - Discount rate must be high to counteract high ROE to induce low investment.
- Common implied sort on expected profitability: momentum, distress, and earnings surprises.

Limitations

- Factor model delivers better empirical performance.
  - Stock return data are less subject to measurement errors than accounting variables.
  - Structural estimation of the economic model involves specification errors in the production and capital adjustment technologies.
- Factor model assumes that returns of stocks with similar investments comove together, i.e., q-model is a linear approximation of non-linear investment.

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Factors

Construct $R_{\mathrm{ME},t}$, $R_{\mathrm{I/A,t}}$, and $R_{\mathrm{ROE},t}$ with a triple 2-by-3-by-3 sort on size, investment, and ROE

Variable definition:

- Size: Stock price times shares outstanding from CRSP
- Investment, I/A: Annual changes in total assets (item AT) divided by lagged total assets
- ROE: Income before extraordinary items (item IBQ) divided by one-quarter-lagged book equity

NYSE breakpoints: 50-50 for size, 30-40-30 for investment, and 30-40-30 for ROE

Timing

- Annual sort in June on the market equity at the June end
- Annual sort in June of year t on I/A for the fiscal year ending in calendar year $t-1$
- Monthly sort at the beginning of each month on ROE with the most recently announced quarterly earnings

Why:

$$E_0\left[r_{i1}^S\right] = \frac{E_0\left[\Pi_{i1}\right]}{1 + a\left(I_{i0}/A_{i0}\right)}$$

Testing Portfolios

- Use an extensive array of anomaly portfolios (nearly 80).
- List of anomalies: momentum, value-versus-growth, investment, profitability, intangibles, and trading frictions (12 out of 13 are insignificant)

Insignificant Anomalies

- Many claims in anomalies literature are likely exaggregated.

	R6-1	A/ME	Rev	EF/P	D/P	O/P	SG	LTG	ACI	NXF
$\overline{R}$	0.48	0.43	−0.39	0.45	0.27	0.35	−0.27	0.01	−0.27	−0.30
$t_{\overline{R}}$	1.43	1.82	−1.57	1.73	0.94	1.53	−1.34	0.02	−1.70	−1.55
	TA	RNA	PM	ATO	CTO	*F*	TES	TI/BI	RS	*O*
$\overline{R}$	−0.19	0.13	0.10	0.22	0.20	0.37	0.32	0.13	0.29	−0.08
$t_{\overline{R}}$	−1.31	0.61	0.40	1.11	1.11	1.28	1.92	0.86	1.82	−0.37
	BC/A	RD/S	RC/A	H/N	*G*	AccQ	ME	Ivol	Tvol	MDR
$\overline{R}$	0.18	0.01	0.32	−0.25	0.03	−0.18	−0.24	−0.54	−0.37	−0.31
$t_{\overline{R}}$	0.73	0.06	1.27	−1.47	0.09	−0.79	−0.90	−1.56	−0.95	−0.94
	β	D-β	S-Rev	Disp	Turn	1/P	Dvol	Illiq		
$\overline{R}$	−0.13	0.07	−0.31	−0.33	−0.12	−0.00	−0.26	0.27		
$t_{\overline{R}}$	−0.36	0.30	−1.39	−1.24	−0.43	−0.01	−1.30	1.14		

Testing Portfolios: Momentum Anomalies

Panel A: Momentum (plus six momentum-reversal variables)

SUE-1, earnings surprise (1-month holding period), Foster, Olsen, and Shevlin (1984)	SUE-6, earnings surprise (6-month holding period), Foster, Olsen, and Shevlin (1984)
Abr-1, cumulative abnormal stock returns around earnings announcements (1-month holding period), Chan, Jegadeesh, and Lakonishok (1996)	Abr-6, cumulative abnormal stock returns around earnings announcements (6-month holding period), Chan, Jegadeesh, and Lakonishok (1996)
RE-1, revisions in analysts' earnings forecasts (1-month holding period), Chan, Jegadeesh, and Lakonishok (1996)	RE-6, revisions in analysts' earnings forecasts (6-month holding period), Chan, Jegadeesh, and Lakonishok (1996)
R6-1, price momentum (6-month prior returns, 1-month holding period), Jegadeesh and Titman (1993)	R6-6, price momentum (6-month prior returns, 6-month holding period), Jegadeesh and Titman (1993)
R11-1, price momentum, (11-month prior returns, 1-month holding period), Fama and French (1996)	I-Mom, industry momentum, Moskowitz and Grinblatt (1999)

Factor Regressions: Alphas

	SUE-1	SUE-6	Abr-1	Abr-6	RE-1	RE-6	R6-6	R11-1	I-Mom	$\lvert\overline{\alpha}\rvert$
$\overline{R}$	0.45	0.24	0.73	0.30	0.89	0.60	0.85	1.18	0.51	
α_{FF}	0.55	0.39	0.84	0.38	1.20	0.94	1.12	1.52	0.68	0.85
α_C	0.34	0.18	0.62	0.19	0.56	0.37	0.06	0.09	−0.18	0.29
α_q	0.16	0.02	0.64	0.26	0.12	0.03	0.24	0.24	0.00	0.19
$t_{\overline{R}}$	3.59	2.17	5.50	3.11	3.43	2.58	3.17	3.52	2.33	
t_{FF}	4.50	3.62	5.93	3.89	4.81	4.52	4.47	4.99	3.25	
t_C	2.62	1.69	4.37	2.06	2.56	2.15	0.51	0.67	−1.11	
t_q	1.12	0.18	4.07	2.18	0.43	0.14	0.71	0.54	0.01	
$\overline{\lvert\alpha_{FF}\rvert}$	0.17	0.13	0.16	0.11	0.27	0.23	0.19	0.26	0.15	0.19
$\overline{\lvert\alpha_C\rvert}$	0.11	0.09	0.12	0.08	0.11	0.09	0.10	0.13	0.06	0.10
$\overline{\lvert\alpha_q\rvert}$	0.05	0.07	0.13	0.07	0.10	0.11	0.08	0.13	0.13	0.10
p_{FF}	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.09	
p_C	0.00	0.00	0.00	0.01	0.16	0.12	0.00	0.00	0.45	
p_q	0.42	0.04	0.00	0.02	0.46	0.08	0.00	0.01	0.03	

Factor Regression: Factor Loadings (β)

	SUE-1	SUE-6	Abr-1	Abr-6	RE-1	RE-6	R6-6	R11-1	I-Mom
β_{MKT}	−0.08	−0.06	−0.06	−0.03	−0.05	−0.07	−0.09	−0.14	−0.11
β_{ME}	0.10	0.09	0.07	0.09	−0.15	−0.19	0.27	0.40	0.31
$\beta_{I/A}$	0.02	−0.11	−0.13	−0.16	0.04	−0.12	−0.07	0.04	−0.03
β_{ROE}	0.48	0.45	0.28	0.18	1.33	1.12	1.02	1.48	0.82
$t_{\beta_{MKT}}$	−1.82	−1.53	−1.31	−1.20	−0.76	−1.24	−1.17	−1.43	−1.72
$t_{\beta_{ME}}$	1.94	1.27	0.67	1.82	−1.42	−1.98	1.43	1.74	1.86
$t_{\beta_{I/A}}$	0.18	−0.97	−1.25	−2.24	0.25	−0.82	−0.27	0.12	−0.13
$t_{\beta_{ROE}}$	5.75	5.95	3.26	2.94	10.09	9.96	5.31	5.67	4.90
ME	0.69	0.75	−0.01	0.03	0.77	0.87	0.40	0.52	0.62
I/A	−1.46	−0.96	−1.37	−1.13	−0.80	0.72	−4.07	−3.83	−1.18
ROE	5.80	3.38	1.59	1.49	6.58	6.47	4.14	5.34	1.61
t_{ME}	4.91	5.38	−0.29	1.31	8.75	9.65	4.92	4.95	3.67
$t_{I/A}$	−3.30	−2.57	−2.36	−2.58	−1.22	1.13	−5.54	−4.66	−1.79
t_{ROE}	16.46	19.07	13.38	15.47	29.77	27.86	16.00	17.06	10.24

Digesting Momentum: SUE-1

Panel A: SUE-1

	Low	2	3	4	5	6	7	8	9	High
m	0.36	0.34	0.35	0.28	0.44	0.43	0.64	0.64	0.64	0.80
α	−0.13	−0.15	−0.15	−0.20	−0.01	−0.03	0.20	0.18	0.19	0.38
α_{FF}	−0.12	−0.15	−0.15	−0.18	−0.01	−0.02	0.21	0.24	0.21	0.43
α_C	0.00	−0.06	−0.07	−0.10	0.01	0.02	0.21	0.16	0.13	0.34
t_m	1.47	1.41	1.36	1.17	2.00	1.86	3.01	3.06	2.96	3.89
t	−1.39	−1.92	−1.88	−2.38	−0.18	−0.35	2.58	2.54	2.64	5.14
t_{FF}	−1.29	−1.73	−1.75	−2.30	−0.08	−0.28	2.81	3.43	2.87	5.86
t_C	0.01	−0.69	−0.80	−1.18	0.07	0.29	2.63	2.13	1.69	4.55
The q-factor model regressions										
α_q	0.05	0.00	0.04	0.05	0.00	−0.03	0.09	0.02	0.04	0.21
β_{MKT}	1.03	1.00	1.02	0.94	0.96	0.98	0.98	1.01	0.97	0.95
β_{ME}	−0.16	0.04	0.00	0.02	0.00	−0.04	−0.05	−0.03	0.01	−0.05
$\beta_{\text{I/A}}$	0.00	−0.16	−0.12	−0.25	0.06	0.03	0.06	0.07	0.06	0.02
β_{ROE}	−0.22	−0.12	−0.20	−0.19	−0.06	−0.01	0.13	0.21	0.20	0.26
t_q	0.42	−0.04	0.37	0.53	−0.04	−0.32	1.21	0.31	0.44	2.63
$t_{\beta_{\text{MKT}}}$	32.75	39.64	39.24	34.70	43.64	41.42	51.90	55.30	42.68	37.45
$t_{\beta_{\text{ME}}}$	−3.61	0.97	0.09	0.55	0.09	−0.85	−1.37	−1.22	0.18	−1.50
$t_{\beta_{\text{I/A}}}$	−0.01	−2.39	−1.99	−3.30	1.04	0.48	1.12	1.23	0.99	0.36

Short-Lived Momentum Profits

	Panel A: Earnings momentum			Panel B: Price momentum		
	SUE-12	SUE-13-36	SUE-37-60	R6-12	R6-13-36	R6-37-60
m	0.16	−0.11	0.13	0.57	−0.24	−0.03
α	0.19	−0.08	0.16	0.60	−0.25	−0.05
α_{FF}	0.31	−0.04	0.14	0.86	−0.04	0.07
α_C	0.10	0.01	0.14	0.09	−0.02	0.06
α_q	−0.01	0.04	0.07	0.17	−0.09	0.08
t_m	1.57	−1.58	1.69	2.67	−1.84	−0.29
t	2.09	−1.09	2.09	2.86	−1.91	−0.56
t_{FF}	3.31	−0.56	1.90	4.41	−0.37	0.85
t_C	1.17	0.11	1.83	0.83	−0.18	0.76
t_q	−0.06	0.51	0.80	0.71	−0.70	0.86
$\overline{\lvert\alpha\rvert}$	0.09	0.07	0.07	0.13	0.11	0.08
$\overline{\lvert\alpha_{FF}\rvert}$	0.11	0.09	0.09	0.17	0.09	0.08
$\overline{\lvert\alpha_C\rvert}$	0.08	0.09	0.09	0.07	0.09	0.09
$\overline{\lvert\alpha_q\rvert}$	0.07	0.07	0.07	0.06	0.08	0.08

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Key Takeaway

- One-half of anomalies earn insignificant average returns for the high-minus-low deciles formed with NYSE breakpoints and value-weighted returns.
  - The exaggerated "anomalies" are likely driven by excessive weighting on microcaps.
- q-factor model outperforms Fama-French and Carhart models in capturing most (not all) anomalies.