A set of Transfer-Matrix-Based MATLAB functions for simulating Fibre Bragg Grating Performance

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Abstract

Fibre Bragg Gratings (FBGs) stand as one of the most vital components in modern optical communication and sensing systems. These microscopic, periodic structures inscribed within an optical fibre possess remarkable properties that enable precise control over the propagation of light. By harnessing the principles of optical interference and wave modulation, FBGs have found wide-ranging applications in telecommunications, sensing technologies, and beyond. Different types of FBGs are fabricated with specific engineering considerations, catering to varying operational requirements and scientific objectives. While many papers have explored the science behind different FBGs, we observe the absence of a universal simulation tool capable of addressing the unique characteristics of all FBG variants.

A set of MATLAB functions that come with this report are developed to simulate different types of 1-dimentional FBG. These functions are based on transfer-matrix approach and allow interesting devices to be quickly simulated and optimised, reducing the need for 3-d models which are time-consuming to set up and require long simulation times. These MATLAB functions are capable of simulating uniformed-FBGs, chirped-FBGs, phase-shifted FBGs and even superstructure FBGs by adjusting the input parameters.

This report covers the theory as well as how the theory is applied computationally to simulate the performance of FBGs. It also compares the simulation results with those in the past literatures for validation purposes.

Theory

Fibre Bragg Gratings

A Fibre Bragg Grating is essentially a short section of optical fibre that has been modified in such a way that it reflects a specific wavelength of light while allowing all other wavelengths to pass through. The angles of the diffracted modes are given by the Bragg equation [1]:

$$2dsin\theta = m\lambda \tag{1}$$

In the situation of 1-dimensional FBG, the angle of incident light knowing θ is always 90°, distance d is equal to the perturbation period called Pitch Λ. And only the first mode of reflection is considered, m = 1. The Bragg Condition for the Fibre Grating is then:

$$\Lambda = \lambda/2n_{eff} \tag{2}$$

where n_{eff} is the effective index of the core mode, which for single mode fibre is very close to the refractive index of the glass. This expression relates the peak reflection wavelength of the grating - the Bragg wavelength, hereafter referred to as λ_B - to the period of the perturbations.

Coupled Mode Equation

The coupled mode equations are the common analytical treatment to the problem of periodic perturbations.[2] They could therefore be used to obtain the reflectivity of a uniform Fibre Bragg Grating, the fundamental building block to more complicated FBG structure.

Assume we have a uniform FBG with length L_g and coupling coefficient κ, the complex reflectivity ρ_{eff} is given by:

$$\rho_{eff} = \frac{i\kappa sinh(-\gamma L_g)}{-\Delta\beta sinh(\gamma L_g) + icosh(\gamma L_g)} \tag{3}$$

Where

$$\Delta\beta = 2\pi n_{eff}(\tfrac{1}{\lambda} - \tfrac{1}{\lambda_B}):$$

$$\gamma = \sqrt{\kappa^2 - \Delta\beta^2}$$

The reflective power is obtained by multiplying the complex reflectivity ρ_{eff} with its complex conjugate.

$$P_{re} = \left\|\rho_{eff}\right\|^2 \tag{4}$$

A plot of reflective power against a range of detuning parameters is re-created and shown below.[3]

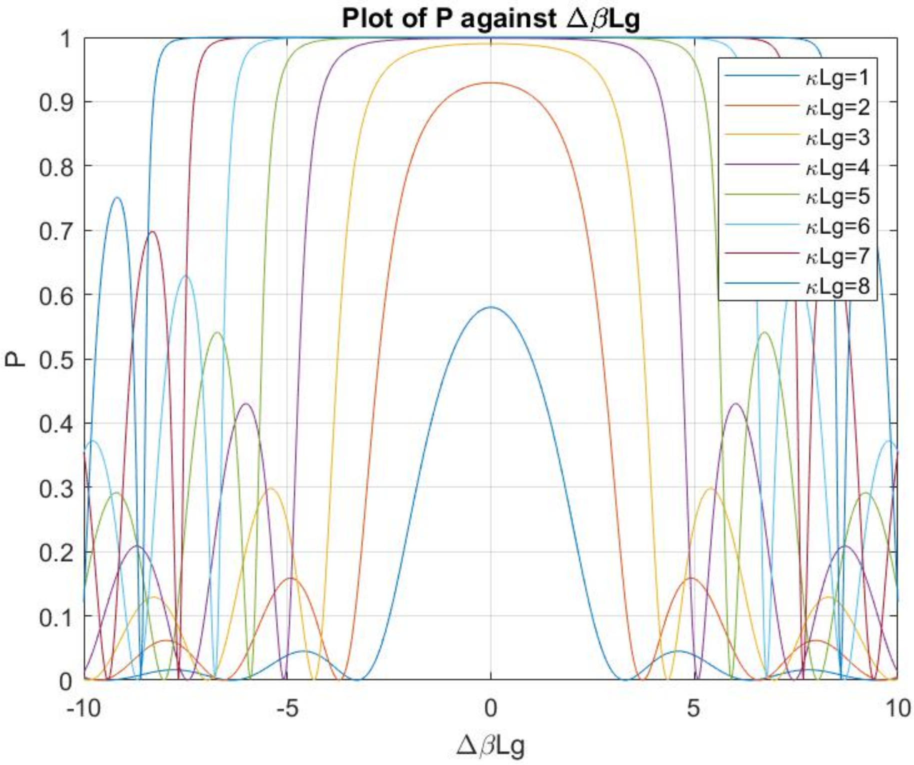

Figure 1 Reflected Power of a Bragg Reflector as a function of detuning parameters

While the coupled mode equation is an effective analytical solution to a uniform FBGs. It does not give researchers the flexibility to test other types of FBGs. Some popular FBG variants include Chirped Fibre Bragg Gratings (CFBG), phase shifted Fibre Bragg Gratings (PSFBG) and Fabry-Perot, etc. In response to this, we implement a numerical approach called Transfer Matrix Approach to simulate FBG variants.

Transfer Matrix Approach

The Transfer Matrix Approach simply works by splitting a non-uniformed grating into N sections. Each section is a uniformed Bragg reflector of length Δl.

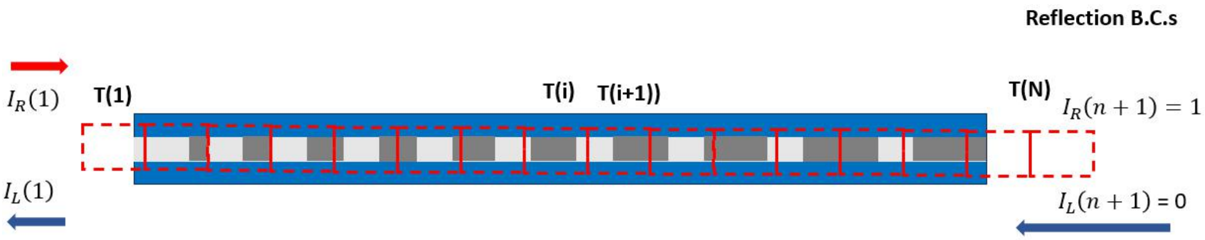

Figure 2 Segmenting a non-uniform Bragg Reflector into many uniformed Bragg reflector

Zooming into the i-th section of the grating. A transfer matrix T(i) is constructed to completely describes the relationship between the optical electric fields entering and leaving the element.

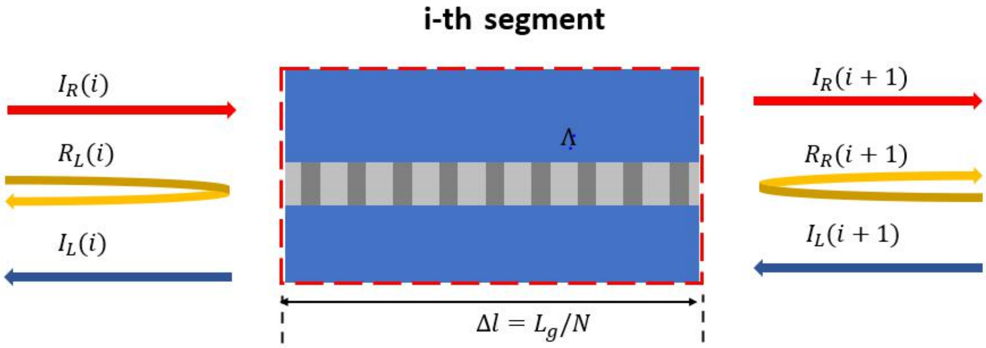

Figure 3 Section-i of the non-uniformed Fibre Bragg Grating defined by its parameters

$$\begin{bmatrix} I_R(i+1) \\ I_L(i+1) \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} I_R(i) \\ I_L(i) \end{bmatrix} \tag{5}$$

Values of $T_{11}, T_{12}, T_{21,} T_{22}$ could be obtained again from the coupled mode equation because we assume each section to be a uniformed FBG. The reflection coefficient in given in equation (3), the transmission coefficient in given in the equation below.

$$t_{eff} = \frac{i\gamma exp(-i\beta\Delta l)}{-\Delta\beta sinh(\gamma\Delta l) + icosh(\gamma\Delta l)} \tag{6}$$

One thing which we must be aware of when applying such a differential approach is to account for the discontinuity of pitch at the edge of the segments. The coupled mode equations always assume the pitch phase at the start of the section to be 1. A phase correction must be implemented at the end to ensure continuity.

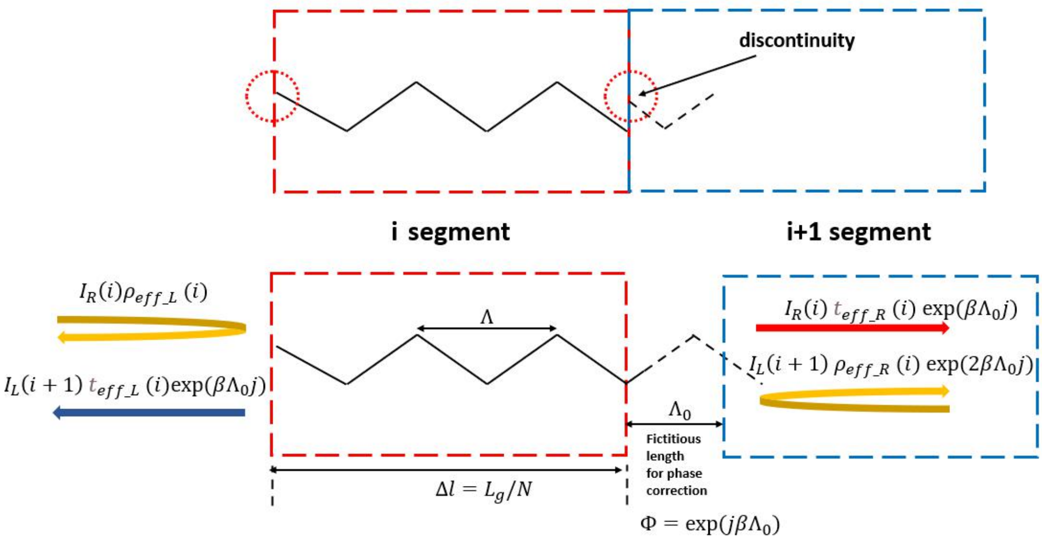

Figure 4 The figure below shows how each segment's pitch starts at same phase (pointing downward) by introducing a phase correction constant

We modify the phase angle by imagining a fictitious length of fibre $\Lambda_0(i)$ that causes a phase delay to the light before the start of the i+1 segment. The magnitude must be 1 and the phase contributed by this fibre must be such that the phase at the start of i+1 segment is identical to the phase at the start of i segment. Phase correction contributed by this length of wire is therefore:

$$\Phi(\mathrm{i}) = exp(j\beta(\mathrm{i})\Lambda_0(i)) \tag{7}$$

This, however, would impose a change to our transmission and reflection coefficients. Assuming field transmission characteristics is isotropic. Fielding entering from the left and right experiences same transmission and thus is delayed by $\Lambda_0(i)$. The corresponding transmission coefficients, combining equation (6) and (7) are:

$$t_{eff\_L} = t_{eff\_R} = t_{eff}exp(j\beta\Lambda_0) \tag{8}$$

Reflection characteristics, however, is dependent on the direction of the field. While the leftward reflection does not enter the segment and is not delayed. The rightward reflection travels twice the length of the fibre. The reflection coefficient, combining equation (3) and (7), is therefore:

$$\rho_{eff\_L} = \rho_{eff} \tag{9}$$

$$\rho_{eff\_R} = \rho_{eff}\ exp(2j\beta\Lambda_0) \tag{10}$$

Assembling all information, we could finally construct our transfer matrix T(i) as follows:

$$\begin{bmatrix} I_R(i+1) \\ I_L(i+1) \end{bmatrix} = \begin{bmatrix} \dfrac{1}{t_{eff}exp(j\beta\Lambda_0)} & -\dfrac{\rho_{eff}\ exp(2j\beta\Lambda_0)}{t_{eff}exp(j\beta\Lambda_0)} \\ \dfrac{\rho_{eff}}{t_{eff}exp(j\beta\Lambda_0)} & t_{eff}exp(j\beta\Lambda_0) - \dfrac{\rho_{eff}^2\ exp(2j\beta\Lambda_0)}{t_{eff}exp(j\beta\Lambda_0)} \end{bmatrix} \begin{bmatrix} I_R(i) \\ I_L(i) \end{bmatrix} \tag{11}$$

Phase-matrix

We design the simulation functions that gives researchers the ability to manipulate phase in order to create a desired phase shifted FBGs. A pi-phase shift in an FBG refers to a change in the grating structure that introduces a phase change of π radians (180 degrees) to the reflected light at a specific wavelength.

$$\begin{bmatrix} I_R(i+1) \\ I_L(i+1) \end{bmatrix} = [T(i)] \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} I_R(i) \\ I_L(i) \end{bmatrix} \tag{12}$$

This phase shift can be achieved by introducing a discontinuity or a defect in the grating structure. In the MATLAB function, we apply a phase shift matrix to each section to account for this feature as an add-on to equation 5.

$$\begin{bmatrix} I_R(i+1) \\ I_L(i+1) \end{bmatrix} = [T(i)][P(i)] \begin{bmatrix} I_R(i) \\ I_L(i) \end{bmatrix} \tag{13}$$

As the phase shift is relative, we fixed the phase shift to the transmitted light to 0 while varying the phase shift to the reflected light. We can immediately see that the phase matrix will be an identify matrix when there is no phase shift ($\varphi = 0$) and the complete out-of-phase shift is obtained by introducing a π -phase shift.

$$[P(i)] = \begin{bmatrix} 1 & 0 \\ 0 & exp(j\varphi) \end{bmatrix} \tag{14}$$

Assembly all matrices

Multiplying all transfer matrices (11) and phases matrices (14) together, we are able to obtain the total transfer matrix for the non-uniformed Fibre Bragg Gratings, that is:

$$[T_{tot}] = \prod_{i=1}^{N} [T(i)][P(i)] \tag{15}$$

The boundary conditions are set as shown in figure 2. Looking in the x direction (along the axis of the fibre), there is no leftward travelling wave at x = Lg and we normalise the rightward travelling wave to 1.

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} = [T_{tot}] \begin{bmatrix} I_R(1) \\ \mathrm{I}_{\mathrm{L}}(1) \end{bmatrix} \tag{16}$$

The overall reflection performance is characterized by the reflection coefficient obtained by:

$$\rho_{eff} = \frac{I_L(1)}{I_R(1)} = - \frac{T_{tot21}}{T_{tot22}} \tag{17}$$

Validation Test:

A MATLAB script is implemented to simulate based on both the coupled mode equations and transfer matrices. The reflection results are plotted shown in the figure below. It validates that the transfer matrix provides consistent results with the analytical coupled mode equations.

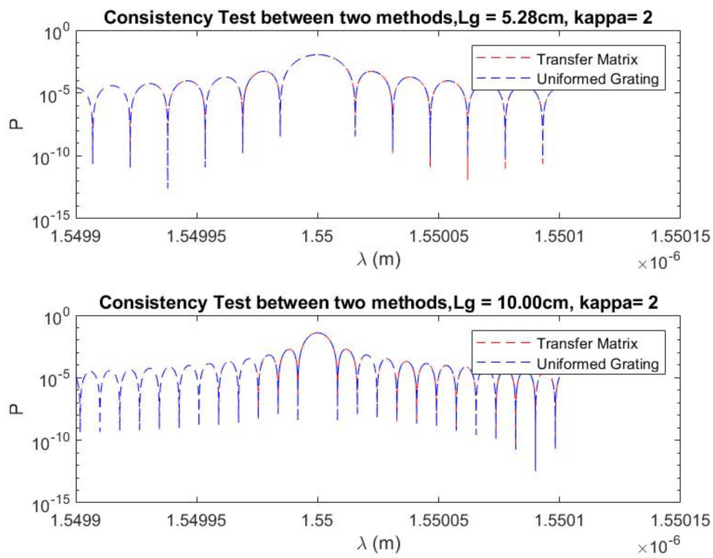

Figure 5 Simulation results based on analytical and numerical methods are consistent

MATLAB implementation of the transfer matrix approach

The aforementioned theories are the physics fundamentals to our MATLAB tools. This section describes various scripts and demonstrates how to implement the code via following steps.

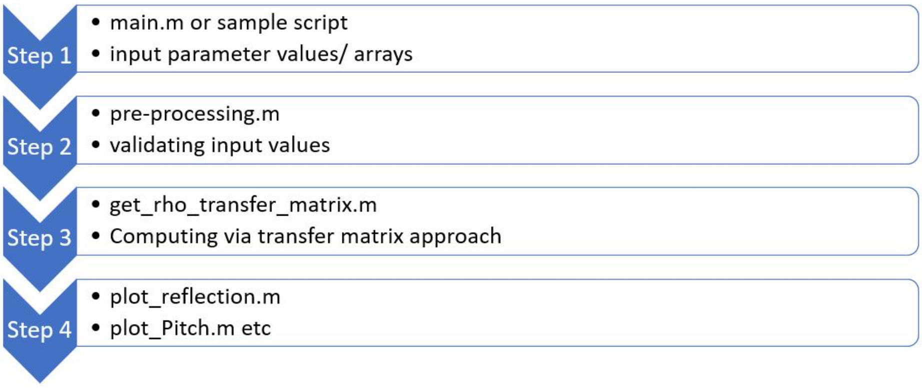

Figure 6 User procedures

`main.m`

Some sample scripts are included for users to get familiar with the codes. The main.m script is the one which users specify desired FBG parameters. Users need to input values regarding the grating length, effective index and speed of light.

```
% This is the main function to implement the simulation of Fibre Bragg Gratings

% Specify FBG Properties
Lg = ;                              % length of the FBG grating in meters
n_eff = ;                           % effective index of the grating
c = ;                               % Speed of light

% Specify parameters along the Fibre Bragg Gratings:
Pitch = ;
% Pitch = pitch*linspace(0.9975,1.0025,1000);
% Pitch = pitch*ones([1,1000]);

Kappa = ;
% Kappa = [linspace(0,7.8,100) linspace(7.8,10,750) linspace(10,0,150)];
% Kappa = 10*ones([1,1000]);
% window_func = 'rectangular';
% Kappa = Kappa.*select_wdw(window_func,1000);

% Phase
Phase = ;
```

Figure 7 A screenshot of the main.m script. Some examples are commented out.

`pre_processing.m`

This function processes the inputs providing by the users before delivering them to the transfer matrix processing scripts. It checks for errors, evaluates relevant parameters, and determines the range of incident wavelengths to scan. Users can change the range of incident wavelengths (the detuning parameters) to fit their needs in the following section of the code.

```
    % Determine range of lambda/detuing parameters for plotting
if numel(unique(Pitch)) == 1
    lambda = linspace(pi*lambda_B/(pi+5000*lambda_B),pi*lambda_B/(pi-lambda_B.*5000),1000);
    fprintf('Plotting reflection spectra for UNCHIRPED grating');

else
    lambda = linspace(pi*lambda_B/(pi+10000*lambda_B),pi*lambda_B/(pi-lambda_B.*10000),1000);
    disp('Plotting reflection spectra for CHIRPED grating');
end
```

Figure 8 Different ranges are used for chirped and un-chirped gratings as unchirped grating tends to have a wider reflective spectra

`get_rho_transfer_matrix.m`

This is where the theory is implemented. The transfer matrix and phase matrix are initialised for a particular section, and they are all multiplied together. A for loop is used to compute the complex reflective coefficient in response of a range of incident wavelengths. Most of the parameters are stored in initialised array to improve the codes' time efficiency.

```
for j = 1:N
    dBeta = 2*pi*n_eff/lambda(i)-Beta(j);
    gamma=sqrt(Kappa(j)^2-dBeta^2);
    % Obtaining corrected r_eff and t_eff for each segment
    r_eff = 1i*Kappa(j)*sinh(-gamma*Delta1)/(-dBeta*sinh(gamma*Delta1)+1i*gamma*cosh(gamma*Delta1));
    t_eff = 1i*gamma*exp(-1i*Beta(j)*Delta1)/(-dBeta*sinh(gamma*Delta1)+1i*gamma*cosh(gamma*Delta1));
    t = t_eff*exp(Phi(j));
    rl = r_eff;
    rr = r_eff*exp(2*Phi(j));
    % Assembling the transfer matrix
    T_11 = 1/t;
    T_12 = -rr/t;
    T_21 = rl/t;
    T_22 = t-rr*rl/t;
    T_new = [T_11 T_12; T_21 T_22];
    T = T*T_new;
    % Assembly Phase Matrix;
    P_new = [1 0; 0 exp(1i*Phase(j))];
```

Figure 9 The numerical transfer-matrix approach in MATLAB scripts.

`plotting functions`

Plotting graphs based on need bespoke by users. Plotting of reflection spectra, pitch, kappa, phase and group velocity profile are offered.

Validation of results

Results obtained based on these MATLAB functions are compared against past literature results as shown below.

The graph on the left are simulation results based on `sample_unchirped_FBG_comparison.m` script. The graph on the right is referred from literature: Fibre Bragg Gratings and Their Applications [3]. Consistent results for uniformed and chirped FBG are obtained.

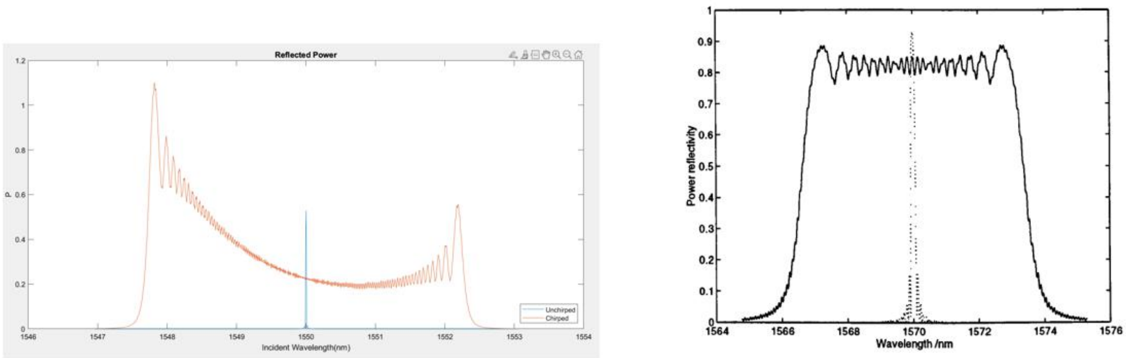

Figure 2.11. Comparison between a 1cm chirped grating with 7nm bandwidth (solid) and a 1cm unchirped grating with κL_g=2 (dotted).

Figure 10. Chirped & Unchirped FBG comparison.

The graph on the left are simulation results based on ` sample_unapodised_linear_chirped_FBG.m ` script. The graph on the right is referred from literature: Fibre Bragg Gratings and Their Applications Figure 2.12.[3]. We observe consistent reflection spectra and group delay profile.

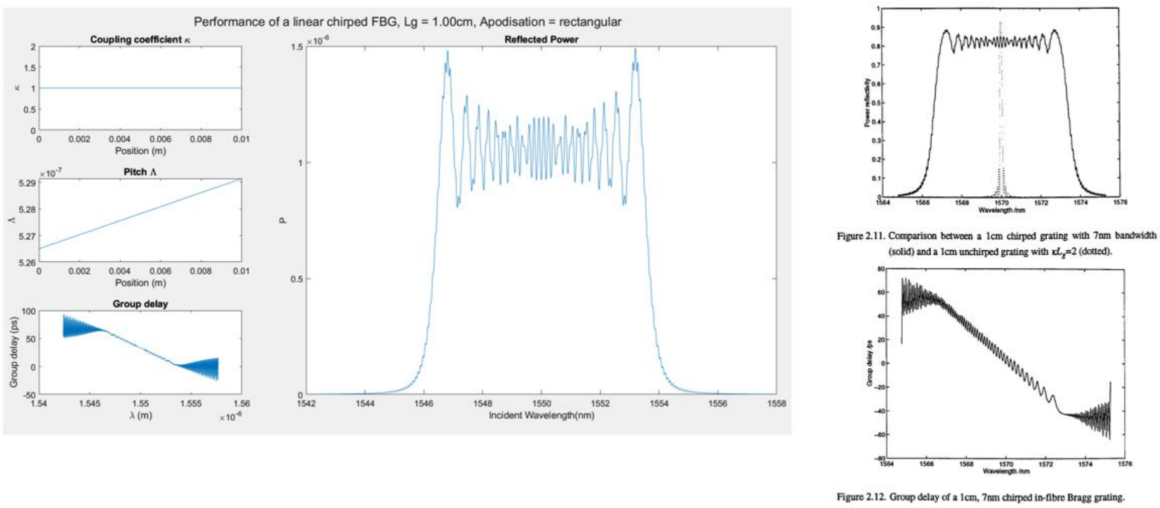

Figure 11. Reflective spectra and Group Delay Profile Comparison

The graph on the left are simulation results based on ` sample_phase_shifted_FBG.m` script. The graph on the right is referred from literature: Effects of the phase shift split on phase-shifted fibre Bragg Gratings, Figure 3 [4]. We observe consistent reflection spectra profile for π-phase shifted FBG.

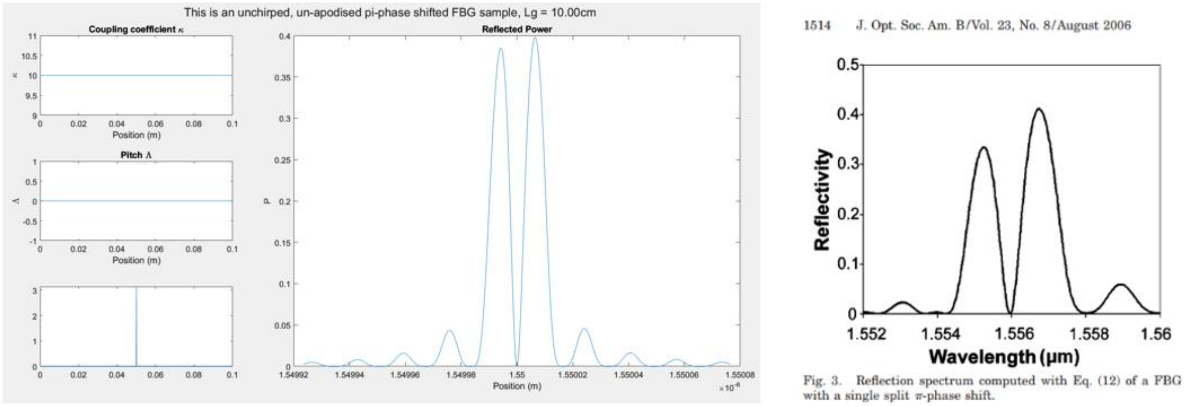

Figure 12. π-phase shifted FBG reflection profile comparison

The graph on the left are simulation results based on ` sample_different_phase_shifted_FBG.m` script. The graph on the right is referred from literature: Experimental and Theoretical Investigation of Phase Shifted Fiber Bragg Grating for Temperature Measurement, figure 3 [5]. Transmission profile is plotted instead to have more straight forward comparison against the literature. We observe the MATLAB function is able to simulate arbitrary phase-shifted FBG as well.

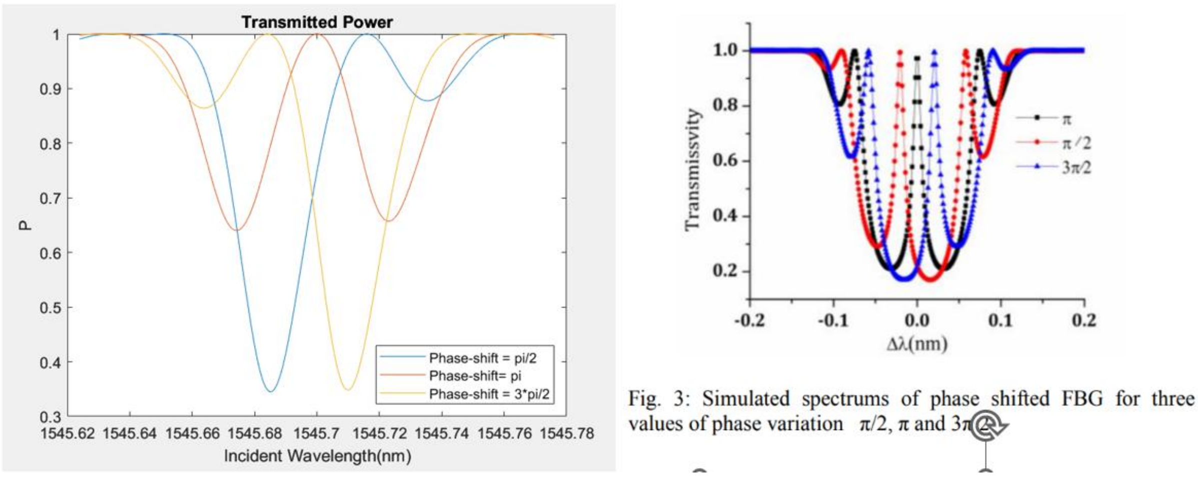

Figure 13. Arbitrary Phase Shifted FBG's Reflection Profile comparison

The graph on the left are simulation results based on ` sample_quadrature_chirped_FBG.m ` script. The graph on the right is referred from literature: Twin Fiber Grating Tunable Dispersion Compensator, figure 3a [6]. We observe consistent results and we could manipulate the coupling coefficients of a arbitrary chirped FBG to obtain a top-hat reflection spectra. The MATLAB functions are capable of delivering rapid simulation results.

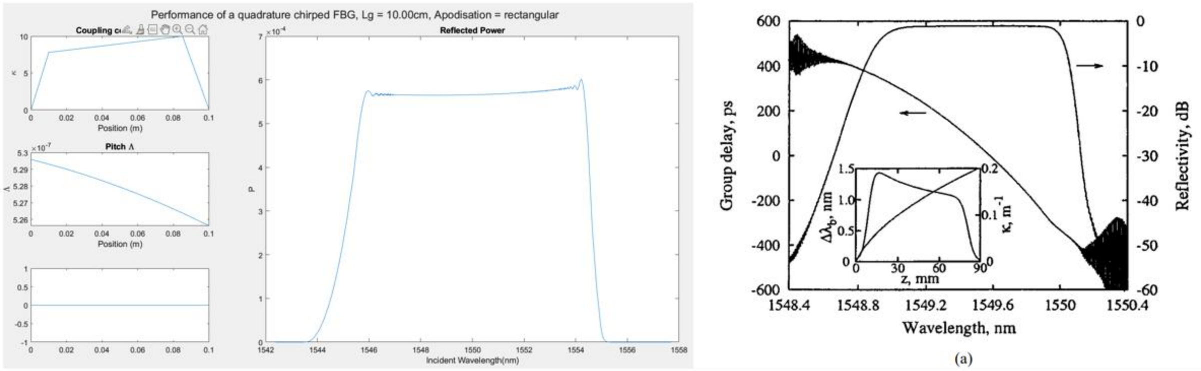

Figure 14. Quadrature Chirped FBG gratings with top-hat reflection profile comparison

The graph on the left are simulation results based on ` sample_quadrature_chirped_FBG.m ` script. The graph on the right is referred from literature: Investigation of Performance for a Two Regions Superstructure Fiber Bragg Grating. 137 Figure 8 [7]. Both figures are plotted in logarithmic scale and we observe consistent results

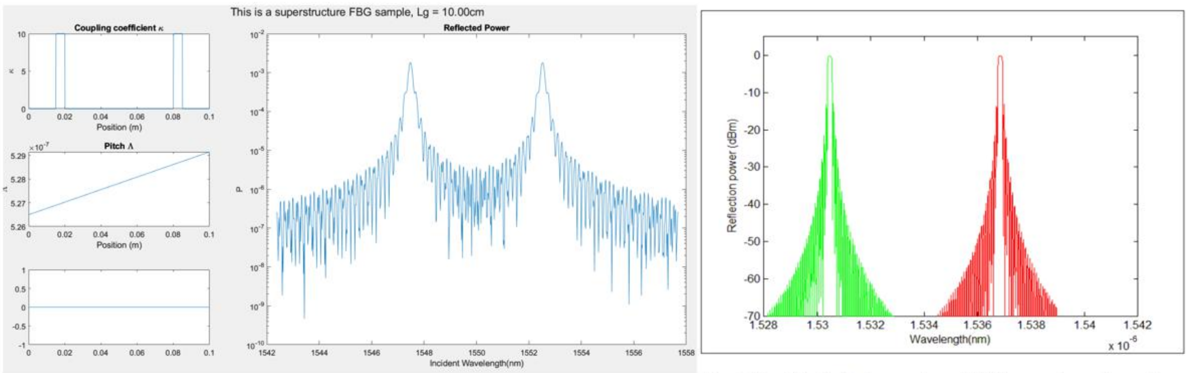

Fig. 8. Simulation Reflection spectrum of SFBG sensor two grating regions.

Figure 15. Superstructure Fibre Bragg Gratings comparison

Discussion

Validation tests are implemented and illustrated in the section above. The MATLAB functions have successfully recreated graphs in past literatures to a extent of extreme accuracy and are capable of simulating many fibre Bragg Gratings variants. It allows engineers to quickly test and refine FBG designs without the need for physical prototypes. This saves time and resources in the development process. In addition, various parameters (e.g., grating length, refractive index modulation, chirp rate) could be quickly adjusted to optimize the FBG's performance for specific applications based on research and engineering needs. The MATLAB codes could also be a powerful education tool to university students for teaching for university students and professionals about FBG principles and performance characteristics.

During the code developments, some ideas for new features are surfaced and areas of further development are discovered. For example, A MATLAB GUI could be designed for even-more visualised simulation process, although this will come at the expense of less flexibility in adjusting parameters. In addition, parameters could be further processed and evaluated to become the inputs to FBG fabrication machines to speed up the prototype manufacturing process.

Research-wise, they are three areas of potential development.

Auto-tuning Kappa based on a FBG's Pitch Profile

The concept of auto-tuning Kappa based on the pitch profile represents a significant advancement in the optimization of FBG. By dynamically adjusting the grating pitch in response to real-time environmental conditions or input parameters, FBGs can adapt to changing operational requirements. This approach has promising implications for applications such as dynamic sensing and adaptive signal processing. Future investigations could delve deeper into refining the auto-tuning algorithms and exploring potential applications in areas where FBG performance must dynamically adapt to varying conditions.

Development of Inverse Scattering Algorithms for Arbitrary FBG

The development of inverse scattering algorithms represents a critical step towards the realization of arbitrary FBGs. By reverse engineering the grating structure from its spectral response, researchers can engineer FBGs with tailored characteristics for specific applications. This capability opens new avenues in areas such as spectral shaping and dispersion engineering. Future investigations should focus on refining and extending these algorithms to handle more complex grating structures, enabling the creation of FBGs with even more intricate spectral responses.

Machine Learning in FBG Technology:

Integrating machine learning techniques with FBG technology represents a compelling frontier in optical sensing and signal processing. By leveraging the power of data-driven algorithms, FBGs can be employed in novel ways for tasks such as anomaly detection, pattern recognition, and predictive maintenance. Future investigations should explore the potential synergies between machine learning and FBGs, developing algorithms that can learn and adapt to complex and dynamic sensing environments.

Reference

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2. A. Yariv, "Coupled-mode theory for guided-wave optics," in IEEE Journal of Quantum Electronics, vol. 9, no. 9, pp. 919-933, September 1973, doi: 10.1109/JQE.1973.1077767.

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7. Khorsheed, Suha & Hemed, Ayser & Fdhala, Mayyada. (2019). Investigation of Performance for a Two Regions Superstructure Fiber Bragg Grating. 137 Figure 8. 42-57.