Symbols index and Erratum for the book of GV[1]

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The following table is made to help readers find the location of symbols in the book[1] more efficiently.

Symbol	Definition	Page
V	a Hausdorff locally convex topological vector space	1
$\mathscr{B}(V)$	the algebra of all continuous linear operators on V	2
$M_c(G)$	the vector space of all complex-valued compactly supported Borel measures on G	3
$C_c(G)$	the space of continuous compactly supported complex functions on G	3
$\pi(\mu)$	$\int_G \pi(x)d\mu(x)$	3
$\pi(f)$	$\pi(fdx) = \int_G \pi(x)f(x)dx$	3
$\dagger$	denotes Hilbert space adjoint	4
$\tilde{f}$	$f(x^{-1})^{\mathrm{conj}}$	4
$U(\mathfrak{g}_c)$	the universal enveloping algebra of $\mathfrak{g}_c$	4
$\mathscr{E}'(G)$	the algebra of distributions on G with compact support	4
$f(x;a)$	$(af)(x) := \frac{\partial^r}{\partial t_1 \cdots \partial t_r}\|_0 f(x \exp t_1 X_1 \cdots \exp t_r X_r)$ where $a = X_1 \cdots X_r$	5
$f(a;x)$	$(fa)(x) := \frac{\partial^r}{\partial t_1 \cdots \partial t_r}\|_0 f(\exp t_1 X_1 \cdots \exp t_r X_r x)$ where $a = X_1 \cdots X_r$	5
$f_1(x,y)$	$f(xy)$	5
$f_{\pi,v}(x)$	$\pi(x)v$	5
$\pi(a)v$	$f_{\pi,v}(1;a)$	5
$\pi(X_1 \cdots X_r)$	$\frac{\partial^r}{\partial t_1 \cdots \partial t_r}\|_0 \pi(x \exp t_1 X_1 \cdots \exp t_r X_r)$	5
V^∞	the subspace of differentiable vectors	6
V^ω	the space of weakly analytic vectors	6
$\mathfrak{Z} = U(\mathfrak{g}_c)^G$	centralizer of G in $U(\mathfrak{g}_c)$	6
$\chi_\pi(\mathfrak{Z} \to \mathbb{C})$	a homomorphism such that $\pi(z)v = \chi_\pi(z)v, v \in V^\infty$	6
(G,K)	a pair where G is a second countable locally compact group, unimodular and K is a compact subgroup	9
$\mathscr{E}(K)$	the set of equivalence of the irreducible representations of K	9
$\mathrm{ch}_{\mathfrak{d}}$	the character of $\mathfrak{d} \in \mathscr{E}(K)$	9

(Continued)

Symbols	Definitions	Pages
$\xi_{\mathfrak{d}}(k)$	$\dim(\mathfrak{d})\,\mathrm{ch}_{\mathfrak{d}}(k^{-1}), k \in K$	9
$E_{\mathfrak{d}}$	$\pi(\xi_{\mathfrak{d}})$	9
$V_{\mathfrak{d}} = E_{\mathfrak{d}}V$	the isotypical subspace of V corresponding to $\mathfrak{d}$	10
$F \subset \mathscr{E}(K)$	finite set	10
E_F	$\sum_{\mathfrak{d}\in F} E_{\mathfrak{d}}$	10
V_F	$E_F V = \bigoplus_{\mathfrak{d}\in F} V_{\mathfrak{d}}$	10
$v_{\mathfrak{d}} = E_{\mathfrak{d}}v$	the Fourier coefficient of $v \in V$	10
$E_{l\times r,(\mathfrak{d}_1,\mathfrak{d}_2)}f$	$\xi_{\mathfrak{d}_1} * f * \xi_{\mathfrak{d}_2}$	11
$C_{\mathfrak{d}}(G)$	$E_{\mathfrak{d},\bar{\mathfrak{d}}}(C(G)) = \xi_{\mathfrak{d}} * C(G) * \xi_{\mathfrak{d}}$	11
$C_{c,\mathfrak{d}}(G)$	$C_c(G) \cap C_{\mathfrak{d}}(G) = \xi_{\mathfrak{d}} * C_c(G) * \xi_{\mathfrak{d}}$	11
$C(G//K)$	$C_1(G)$	11
$C_c(G//K)$	$C_{c,1}(G)$	11
$C_{c,F}(G)$	$\xi_F * C_c(G) * \xi_F$	11
$C^{\infty}_{c,F}(G)$	$\xi_F * C^{\infty}_c(G) * \xi_F$	11
ξ_F	$\sum_{\mathfrak{d}\in F} \xi_{\mathfrak{d}}$	11
$I_c(G)$	the subalgebra of $C_c(G)$ of elements invariant under inner automorphisms	11
$I_{c,F}(G)$	$\xi_F * I_c(G) * \xi_F$	11
$\pi_F(f)$	$\pi(f)\|_{V_F}, f \in C_{c,F}(G)$	12
V^0	$\sum_{\mathfrak{d}\in\mathscr{E}(K)} V_{\mathfrak{d}}$ (algebraic sum)	13
$\mathfrak{Q}$	$U(\mathfrak{g}_c)^K$ =centralizer of K in $U(\mathfrak{g}_c)$	13
$\Theta_\pi(C^{\infty}_c(G) \to \mathbb{C})$	$f \mapsto \mathrm{tr}(\pi(f))$ the character of π	13
$\Phi_{\pi,F}(x)$	$E_F\pi(x)E_F$ the spherical function of type $F \subset \mathscr{E}(K)$ associated with π	22
$\pi_F(f)$	$\langle f, \Phi_{\pi,F}\rangle = \int_G f(x)\Phi_{\pi,F}(x)dx$	22
γ	a representation of $C_{c,F}(G)$ in U where U is finite-dimensional	22
Ψ	$(G \to \mathrm{Hom}_{\mathbb{C}}(U,U))$ such that $\gamma(f) = \langle f, \Psi\rangle$ and $\Psi = \bar{\xi}_F * \Psi * \bar{\xi}_F$	22
$I_{c,\mathfrak{d}}(G)$	the subalgebra of elements invariant under the inner automorphisms	29
$\Phi^{\sharp}$	$G \to \mathrm{Hom}_{\mathbb{C}}(W,W)$	30
$\sigma_{\mathfrak{d}}$	an irreducible representations of $I_{c,\mathfrak{d}}(G)$ in some space $W_{\mathfrak{d}}$	30

(Continued)

Symbols	Definitions	Pages
$\pi_{\mathfrak{d}}$	$\theta \otimes \sigma_{\mathfrak{d}}$ the irreducible representation of $C_{c,\mathfrak{d}}(G)$ in $V_{\mathfrak{d}}$	30
$\Phi^{\sharp}_{\pi,\mathfrak{d}}$	the continuous map of G into $\mathrm{Hom}_{\mathbb{C}}(W_{\mathfrak{d}}, W_{\mathfrak{d}})$	31
$\varphi_{\pi,\mathfrak{d}}$	$\mathrm{tr}(\Phi^{\sharp}_{\pi,\mathfrak{d}})$	31
U	the algebra of endomorphisms of V_F	31
$\Phi = \Phi_{\pi,F}$	a function from G to U	32
a^k	$\mathrm{Ad}(k)a$ where $a \in U(\mathfrak{g}_c)$	33
$\Psi(x : y)$	$\int_K \Phi^{\sharp}(xkyk^{-1})dk$	34
(G, K)	Gelfand pair if $L^1(G//K)$ is commutative	36
θ	an involutive automorphism of G	36
G^{θ}	the subgroup of fixed points for θ	36
V^K	the space of vectors invariant under K	37
λ	$(G \to B(L^2(G/K))) : y \mapsto \lambda(y)$	39
$\mathfrak{H}$	a separable Hilbert space	40
$\mathfrak{U}$	a commutative algebra of bounded operators in $\mathfrak{H}$	40
$\mathfrak{U}_1$	a dense self-adjoint algebra of $\mathfrak{U}$	41
$\sharp(f)$	$f^{\sharp} = \int_{K\times K} l(k_1)r(k_2)f dk_1 dk_2$	43
$\Sigma(G//K)$	the spectrum of $L^1(G//K)$	43
φ_{τ}	uniquely determined elementary spherical function	43
π_{τ}	a completely irreducible uniformly bounded representation of class 1	43
$\Sigma_u(G//K)$	$\left\{\tau \in \Sigma(G//K) : \tau((f * \tilde{f})^{\sharp}) \geq 0, \forall f \in L^1(G)\right\}$	43
$\hat{G}$	the set of equivalence classes of irreducible unitary representations of G	43
$\hat{G}_1$	the set of classes of $\hat{G}$ corresponding to class 1 representations	43
$H \subset G$	a closed subgroup	49
$E_G(G/H)$	the algebra of all G-invariant continuous endomorphisms of $C^{\infty}(G/H)$	49
$\mathrm{Diff}_G(G/H)$	the subalgebra of G-invariant differential operators on G/K	49
G^0	the component contained identity	58
$[G : G^0]$	the index of subgroup of G^0	58
${}^0H$	$\bigcap_{X\in \mathrm{Hom}(H,\mathbb{R}^{\times}_{+})} \ker(X)$	59

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Symbols	Definitions	Pages
$C=\ker(\mathrm{Ad})$	the centralizer of $\mathfrak{g}$ in G	59
θ	the Cartan involution	60
G^θ	the set of fixed points of G	60
$\mathfrak{g}=\mathfrak{k}\oplus\mathfrak{s}$	the Cartan decomposition of $\mathfrak{g}$	60
$G=K\exp\mathfrak{s}$	the Cartan decomposition of G	60
B	the Cartan-Killing form	61
(X,Y)	$B_\theta(X,Y)=-B(X,\theta Y)$	61
$\|X\|^2$	$B_\theta(X,X)$	61
$\mathfrak{a}$	a maximal abelian subspaces of $\mathfrak{s}$	61
$\mathfrak{m}_1$	the centralizer of $\mathfrak{a}$ in $\mathfrak{g}$	62
$\mathfrak{m}$	$\mathfrak{m}_1\cap\mathfrak{k}$	62
M_1 $(\tilde{M}_1)$	the centralizer (normalizer) of $\mathfrak{a}$ in G	62
M	$M_1\cap K$: the centralizer of $\mathfrak{a}$ in K	62
$\tilde{M}$	$\tilde{M}_1\cap K$: the normalizer of $\mathfrak{a}$ in K	62
$\mathfrak{g}_\lambda$	$\{X\in\mathfrak{g}:[H,X]=\lambda(H)X,\forall H\in\mathfrak{a}\}$ for any $\lambda\in\mathfrak{a}^*$	62
$\Delta=\Delta(\mathfrak{g},\mathfrak{a})$	the set of all roots of $(\mathfrak{g},\mathfrak{a})$	62
Δ^+	the set of all positive roots	62
S	$\{\alpha_i:1\le i\le r\}$, the simple system	62
$\mathfrak{n}$	$\sum_{\lambda\in\Delta^+}\mathfrak{g}_\lambda$	62
$\mathfrak{k}\oplus\mathfrak{a}\oplus\mathfrak{n}$	(resp. $G=KAN$) the Iwasawa decomposition of $\mathfrak{g}$ (resp. G)	63
log	$A\to\mathfrak{a}$	63
s_λ	the reflection associated with λ	63
σ_λ	$\{\mu\in\mathfrak{a}_c^*:\langle\mu,\lambda\rangle=0\}$	63
$\mathfrak{w}$	the Weyl group of $(\mathfrak{g},\mathfrak{a})$	64
$W=\tilde{M}/M$	the Weyl group of (G,A)	64
$\mathfrak{a}^+$	the positive Weyl chamber of $\mathfrak{a}$	64
A^+	$\exp(\mathfrak{a}^+)$	65
$\mathfrak{h}_c\subset\mathfrak{g}_c$	a CSA(Cartan subalgebra)	66

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Symbols	Definitions	Pages
Q	a positive system of roots of $(\mathfrak{g}_c, \mathfrak{h}_c)$	66
$\mathfrak{g}_{c,\alpha}$	the root subspaces	66
$\mathfrak{b}_c$	$\mathfrak{h}_c + \sum_{\lambda\in Q} \mathfrak{g}_{c,\alpha}$, a Borel subalgebra of $\mathfrak{g}_c$	66
$S \subset Q$	the set of simple roots	66
$F \subset S$	finite subset	66
Q_F	positivelinear combinations of elements of F	66
$\mathfrak{q}_{c,F}$	$\mathfrak{b}_c + \sum_{\alpha\in -Q_F} \mathfrak{g}_{c,\alpha}$, a subalgebra of $\mathfrak{g}_c$ containing $\mathfrak{b}_c$	66
$\mathfrak{h}_{\mathfrak{m}}$	a CSA of $\mathfrak{m}$	66
$\mathfrak{h} := \mathfrak{h}_{\mathfrak{m}} \oplus \mathfrak{a}$	a CSA of $\mathfrak{m} \oplus \mathfrak{a}$	66
$Q_{\mathfrak{m}}$	a positive system of roots for pair $(\mathfrak{m}_c, (\mathfrak{h}\cap\mathfrak{k})_c)$	66
Q^+	the set of all roots of $(\mathfrak{g}_c, \mathfrak{a}_c)$ whose restrictions to $\mathfrak{a}$ lie in $\Delta^+(\mathfrak{g}, \mathfrak{a})$	66
Q	$Q_{\mathfrak{m}} \cup Q^+$	66
$\mathfrak{n}_c$	$\sum_{\alpha\in Q^+} \mathfrak{g}_{c,\alpha}$	66
$\mathfrak{m} + \mathfrak{a} + \mathfrak{n}$	a minimal parabolic subalgebra of $\mathfrak{g}$	66
Δ_F^+	posistive linear combinations of elements of F	66
$\mathfrak{p}_F$	$\mathfrak{p} + \sum_{\alpha\in -\Delta_F^+} \mathfrak{g}_\lambda$ the standard psalgebras with respect to $\mathfrak{p}$ or S	66
$\mathfrak{p}_0$	a psalgebra of $\mathfrak{g}$ (of course contains $\mathfrak{p}$)	67
$\mathfrak{n}_0$	the nilradical(nilpotent radical) of $\mathfrak{p}_0 \cap [\mathfrak{g}, \mathfrak{g}]$	67
$\mathfrak{m}_{10}$	$\mathfrak{p}_0 \cap \theta(\mathfrak{p}_0)$	67
$\mathfrak{p}_0 = \mathfrak{m}_{10} \oplus \mathfrak{n}_0$	the Levi decomposition of $\mathfrak{p}_0$	67
$\mathfrak{a}_0$	center$(\mathfrak{m}_{10}) \cap \mathfrak{s}$	67
$\mathfrak{m}_0$	$\mathfrak{m}_{10} \ominus \mathfrak{a}_0$	67
$\mathfrak{m}_0 \oplus \mathfrak{a}_0 \oplus \mathfrak{n}_0$	the Langlands decomposition of $\mathfrak{p}_0$	67
$\mathfrak{a}_F$	$\{H \in \mathfrak{a} : \alpha(H) = 0,\ \forall\alpha \in F\}$	67
Δ_F	$\Delta_F^+ \cup -\Delta_F^+$	67
$\mathfrak{m}_{1F}$	$\sum_{\lambda\in\Delta_F} \mathfrak{g}_\lambda \oplus \mathfrak{a} \oplus \mathfrak{m}$	67
$\mathfrak{n}_F$	$\sum_{\lambda\in\Delta^+\setminus\Delta_F^+} \mathfrak{g}_\lambda$, the nilradical of $\mathfrak{p}_F \cap [\mathfrak{g}, \mathfrak{g}]$	67
M_{10}	the centralizer in G of $\mathfrak{a}_0$ (the reductive component of P_0)	68

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Symbols	Definitions	Pages
M_0	${}^0(M_{10})$	68
$K_{M_{10}}$	$K \cap M_{10} = K \cap M_{10}$	68
G	$KP_0 = KM_0A_0N_0$	68
$\theta_{M_{10}}$	$\theta\|_{M_{10}}$	68
$B_{M_{10}}$	$B\|_{(\mathfrak{m}_{10})_c \times (\mathfrak{m}_{10})_c}$	68
$M_{0\mathfrak{s}}$	$\exp(\mathfrak{m}_0 \cap \mathfrak{s})$	69
$\Delta(\mathfrak{g}, \mathfrak{a}_0)$	$(= \Delta(P_0))$the set of all $\lambda \neq 0$ in $\mathfrak{a}_0^*$ for which $\mathfrak{g}_\lambda \neq 0$	70
$\mathfrak{a}_0^+$	a chamber of $\mathfrak{a}_0$	70
$\Delta(P_0 : \mathfrak{a}_0^+)$	the set of roots in $\Delta(P_0)$ which are > 0 in $\mathfrak{a}_0^+$	70
$\rho(H)$	$\frac{1}{2}\,\mathrm{tr}(\mathrm{ad}\, H)_{\mathfrak{n}} = \frac{1}{2}\sum_{\alpha \in \Delta^+} n(\alpha)\alpha$	71
$\rho_{P_0}(H)$	$\frac{1}{2}\,\mathrm{tr}(\mathrm{ad}\, H)_{\mathfrak{n}_0},\ H \in \mathfrak{a}_0$	71
d_P	$(MA \to \mathbb{R}_+^\times) : m_1 \mapsto \|\det \mathrm{Ad}(m_1)_{\mathfrak{n}}\|^{1/2}$	71
d_{P_0}	$(M_0A_0 \to \mathbb{R}_+^\times) : m_1 \mapsto \|\det \mathrm{Ad}(m_1)_{\mathfrak{n}_0}\|^{1/2}$	72
$J(a)$	$\prod_{\alpha \in \Delta^+} (\mathrm{e}^{\alpha(\log a)} - e^{-\alpha(\log a)})^{n(\alpha)}$ for all $a \in A$	73
G^*	G/A_0	74
a^{x^*}	xax^{-1} where $a \in A_0$ and $x \in G$	74
G/Q	flag manifolds of G where Q is a psgrp	76
G/P	the flag manifold of G if $P = MAN$ and $G = KAN$	76
$\mathrm{rk}(G)$	$\mathrm{rk}(G/K)$: the real rank of G	77
X	G/P	78
$\pi(x)$	$xP(x \in G)$	78
i	$(X \to K/M) : x \mapsto \overline{k(x)}$ the natural identification	78
x_s	a representative of $s \in \tilde{M}$	78
$\bigcup_{x \in \mathfrak{w}} Nx_sP$	the Bruhat decomposition of G	78
NsP	Nx_sP	78
$\pi(x_s)$	$\underline{s}(s \in \mathfrak{w})$	78
$\bigcup_{s \in \mathfrak{w}} N\underline{s}$	the Bruhat decomposition of X	78
Δ_s^+	$\{\alpha \in \Delta^+ : s^{-1}\alpha \in \Delta^+\}$	78

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Symbols	Definitions	Pages
${}_s\Delta^+$	$\{\alpha \in \Delta^+ : s^{-1}\alpha \in -\Delta^+\}$	78
$\mathfrak{n}_s$	$\sum_{\alpha \in \Delta_s^+} \mathfrak{g}_\alpha$	78
${}_s\,\mathfrak{n}$	$\sum_{\alpha \in {}_s\Delta^+} \mathfrak{g}_\alpha$	78
N_s	$\exp(\mathfrak{n}_s)$	79
${}_sN$	$\exp({}_s\mathfrak{n})$	79
$\mathfrak{w}_F$	the subgroup of $\mathfrak{w}$ generated by the reflexion s_α for all $\alpha \in F$	80
Ω_1	$\pi(\bar{N})$	80
Ω_s	$x_s \cdot \pi(\bar{N})$	80
$\gamma_s(\bar{n})$	$\pi(x_s\bar{n})$	80
$m(s)$	$x_s^{-1}mx_s$	80
$\beta_s(X)$	$\pi(x_s \cdot \exp X)$	81
$a(x:k)$	$a(xk)$	81
$H(x:k)$	$H(xk)$	81
L	real Lie group with its Lie algebra $\mathfrak{l}$	84
$W \subset L$	open set	84
$\mathrm{Diff}(W)$	the algebra of differential operators on W	84
$D_x \in U(\mathfrak{l}_c)$	the local expression of D at $x \in W$	84
$R_a(f)$	fa for all $a \in U(\mathfrak{l}_c)$	85
λ	$(S(\mathfrak{l}_c) \to U(\mathfrak{l}_c)) : X_1 \dots X_r \mapsto \frac{1}{r!}\sum_\sigma X_{\sigma(1)} \dots X_{\sigma(r)}$	86
I	$I(\mathfrak{g}_c) = S(\mathfrak{g}_c)^G$	86
$I_\mathfrak{w} = I_\mathfrak{w}(\mathfrak{h}_c)$	the algebra of $\mathfrak{w}$-invariant in $S(\mathfrak{h}_c)$	87
$\beta_\mathfrak{n}$	$U(\mathfrak{g}_c) \to U(\mathfrak{a}_c)$: the projection	91
$\gamma_\mathfrak{n}$	$(= \gamma = \gamma_{\mathfrak{g}\,/\,\mathfrak{a}}) : \mathfrak{Q} \to U(\mathfrak{a}_c)$ such that $\gamma_\mathfrak{n}(a)(\lambda) = \beta(a)(\lambda - \rho)$	92
$\mathfrak{k}_0$	$\mathfrak{k} \cap \mathfrak{m}_{10}$	95
$\mathfrak{s}_{10}$	$\mathfrak{s} \cap \mathfrak{m}_{10}$	95
$\mathfrak{s}_0$	$\mathfrak{s} \cap \mathfrak{m}_0$	95
${}^*\,\mathfrak{a}$	$\mathfrak{a} \cap \mathfrak{m}_0$	96
${}^*\,\mathfrak{n}$	$\mathfrak{n} \cap \mathfrak{m}_0$	96

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Symbols	Definitions	Pages
$\mathscr{F}$	$\mathfrak{a}_{\mathbb{C}}^*$	101
$\mathscr{F}_R$	$\mathfrak{a}^*$	101
$\mathscr{F}_I$	$(-1)^{1/2}\,\mathscr{F}_R$	101
$\xi_\beta(a)$	$\mathrm{e}^{\beta \log a}(\beta \in \mathscr{F}, a \in A)$ the quasicharacter of A	101
σ	a f.d. unitary reps. of the compact group M in a Hilbert space $W(\sigma)$	101
$(\sigma, \lambda)(man)$	$\sigma(m)\xi_{\lambda+\rho}(a)$: the representation of $P = MAN$	101
$\mathfrak{B}(\sigma, \lambda)$	the space of Borel functions $f(G \to W(\sigma))$	102
f_K	$f\|_K$	102
$\mathfrak{B}(\sigma)$	the space of Borel functions $g(K \to W(\sigma))$	102
$\mathfrak{H}(\sigma)$	the Hilbert space of functions $g \in \mathfrak{B}(\sigma)$	102
$\mathfrak{H}(\sigma, \lambda)$	the Hilbert space of functions $f \in \mathfrak{B}(\sigma, \lambda)$	102
$\pi_{\sigma,\lambda}$	$(G \to B(\mathfrak{H}(\sigma))) : (\pi_{\sigma,\lambda}(x)g)(k) = \mathrm{e}^{-(\lambda+\rho)(H(x^{-1}k))}g(x^{-1}[k])$	102
π_λ	$\pi_{1,\lambda}$ where 1 = trivial representation of M	103
φ_λ	the matrix coefficient of π_λ	103
$\varphi_\lambda(x)$	$\int_K \mathrm{e}^{-(\lambda+\rho)(H(x^{-1}k))}dk$	104
$(\mathscr{H} f)(\lambda)$	the Harish-Chandra transform of $f \in C_c(G//K)$	106
$(\mathscr{A} f)(a)$	the Abel transform of $f \in C_c(G//K)$	107
$\hat{g}(\lambda)$	the Fourier transform of $g \in C_c(A)$	107
$\mathscr{P}(\mathscr{F})$	the space of all entire functions on $\mathscr{F}$ of Paley-Weiner type	107
$\mathscr{P}(\mathscr{F})^{\mathfrak{w}}$	the subspace of $\mathfrak{w}$-invariant elements of $\mathscr{P}(\mathscr{F})$	107
	$\begin{tikzcd} C_c^\infty(G//K) \arrow[r, hook, "\mathscr{H}"] \arrow[d, "\mathscr{A}"] & \mathscr{P}(\mathscr{F})^{\mathfrak{w}} \\ C_c^\infty(A)^{\mathfrak{w}} \arrow[ur, "\wedge"] & \end{tikzcd}$	107
$V(\pi_\Lambda)_{\mathfrak{d}}$	the corresponding isotypical space for any $\mathfrak{d} \in \mathscr{E}(K)$	111
$\Lambda \in \mathfrak{h}_c^*$		111
$\mathfrak{u} = \mathfrak{k} + i\,\mathfrak{s}$	a compact form of $\mathfrak{g}_c$	111
$\mathscr{D}$	the set of all $\Lambda \in \mathfrak{h}_c^*$ which are dominant(relative to Q) and integral	112
$\mathscr{D}_1$	the set of all $\Lambda \in \mathfrak{h}_c^*$ for which π_Λ are of class 1	112

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Symbols	Definitions	Pages
L	the semilattice generated by $S \subset \Delta^+$	112
$Q_\alpha(X)$	$\frac{4\langle X,\theta X\rangle}{\langle \bar{H}_\alpha,\theta\bar{X}_\alpha\rangle} = \|\alpha\|^2\,\|X\|^2$	117
H_1	the restriction of H to $\bar{N}_1$	117
$^+\mathfrak{a}$	$\{H \in \mathfrak{a} : \langle H, H'\rangle \geq 0 \text{ for all } H' \in \mathfrak{a}^+\}$	119
a^*	$(a^{s_0})^{-1} = u_0 a^{-1} u_0^{-1}$	120
$\mathrm{Co}(H, \mathfrak{w})$	the convex hull of the elements H^s	121
$\tilde{D}$	the radial component of differential operator D	124
$\tilde{\varphi}_\lambda$	the restriction of φ_λ on A^+	124
$\tilde{q}/\tilde{\varphi}_\lambda = \gamma(q)(\lambda)\tilde{\varphi}_\lambda$	the differential equations where $q \in \mathfrak{Q}, \gamma = \gamma_{\mathfrak{g}/\mathfrak{a}}$	124
$\Phi(\lambda : \cdot)$	$= \mathrm{e}^{\lambda-\rho} + \sum_{\mu\in L_+} a_\mu(\lambda)\mathrm{e}^{\lambda-\rho-\mu}$: the solutions of the above	124
ψ	$(K \times A \times K \to G) : (k_1, h, k_2) \mapsto k_1 h k_2$	125
G^+	KA^+K	125
A'	$= \bigcup_{s\in\mathfrak{w}} sA^+$: the regular subset of A	125
b	$(U(\mathfrak{g}_c) \to U(\mathfrak{a}_{\mathbb{C}})) : g \mapsto b(g)$	127
ξ_λ	$\mathrm{e}^{\lambda\circ\log}$	128
f_α	$(\xi_\alpha - \xi_{-\alpha})^{-1}$	128
g_α	$\xi_{-\alpha}(\xi_\alpha - \xi_{-\alpha})^{-1}$	128
$\mathscr{R}_0$	the algebra with unit generated over $\mathbb{C}$ by the f_α and $g_\alpha(\alpha > 0)$	128
$\mathscr{R}_{0,d}$	the span of monomials in these generators of degree d	128
$\mathscr{R}_0^+$	$\sum_{d\geq 1} \mathscr{R}_{0,d}$	128
$\mathscr{R}_0^{(d)}$	$\sum_{1\leq e\leq d} \mathscr{R}_{0,e}$	128
$\delta'(g)$	$b(g) + \sum_{1\leq \imath\leq n} \psi_i u_i$	129
χ	$\mathfrak{Q} \to \mathbb{C}$	130
$A(U : \chi)$	$\{g \in C^\infty(U) : \delta'(q)f = \chi(q)f \text{ for all } q \in \mathfrak{Q}\}$	130
ω	Casimir operator	132
$J(h)$	$\prod_{\alpha>0}(\mathrm{e}^{\alpha(\log h)} - \mathrm{e}^{-\alpha(\log h)})^{n(\alpha)}$	133
$g_1(t)$	$\mathrm{e}^{-2t}(1 - \mathrm{e}^{-2t})^{-1}$	135

(Continued)

Symbols	Definitions	Pages
$g_2(t)$	$\mathrm{e}^{-4t}(1-\mathrm{e}^{-4t})^{-1}$	135
$\Xi=\varphi_0$	the basic spherical function	200

- $\beta(q)=\mathrm{e}^{-\rho}\circ\gamma(q)\circ\mathrm{e}^{\rho}$

0.1 Construction of the eigenfunctions on G^+

- $\psi(\lambda:t)=\mathrm{e}^{t\rho}\varphi_\lambda(\exp tH_0),\ t\in\mathbb{R}$
- $\psi_\lambda(h)=\mathrm{e}^{\rho(\log h)}\varphi_\lambda(h)$ for $h\in A^+$
- $L=\left\{\sum_{1\leq i\leq r}m_i\alpha_i : m_1,\cdots,m_r \text{ integers} \geq 0\right\}$
- $L^+=L\setminus\{0\}$
- $\mu\prec\mu'$ if $\mu'-\mu\in L^+$
- $m(\mu)=m_1+\cdots+m_r$ if $\mu=m_1\alpha_1+\cdots+m_r\alpha_r$ the level of μ
- b a complex-valued function on L
- $f_b(H)=\sum_{\mu\in L}b(\mu)\mathrm{e}^{-\mu(H)},\ H_R\in\mathfrak{a}^+$
- $\mathscr{R}_{00}$ the algebrta of all functions f_b
- $\mathscr{R}_{00}^+$ the ideal in $\mathscr{R}_{00}$ of all functions with $b(0)=0$
- $\tilde{f}(h)=f(\log h),\ h\in A^+$
- $\widetilde{\mathscr{R}}_{00}^+$ coressponding function in $\mathscr{R}_{00}^+$ defined on A^+
- $a_\mu(\lambda)$ the rational functions of λ
- $\sigma_\mu=\left\{\lambda\in\mathscr{F}:\langle\mu,\lambda\rangle=\frac{1}{2}\langle\mu,\mu\rangle\right\},\ \mu\in L^+$ the hyperplane in $\mathscr{F}$
- $\mathscr{F}^\vee=\mathscr{F}\setminus\bigcup_{\mu\in L^+}\sigma_\mu$
- $\mathscr{F}_\eta=\{\lambda\in\mathscr{F}:\langle\lambda_\mathbb{R},\alpha_i\rangle<\eta, 1\leq i\leq r\}\subset\mathscr{F}^\vee$
- $\mathscr{F}_I(\varepsilon)=\{\lambda\in\mathscr{F}:\|\lambda_R\|\leq\varepsilon\}\subset\mathscr{F}^\vee$
- $\mathscr{F}_\mu^\vee=\mathscr{F}\setminus\bigcup_{\nu\in L^+,\nu\preceq\mu}\sigma_\nu$

0.2 The Harish-Chandra series for φ_λ and the c-fucnction

- $\tau_\nu(s,t) = \{\lambda \in \mathfrak{a}^*_{\mathbb{C}} : s\lambda - t\lambda = \nu\}$
- ${}^*\mathscr{F} = \mathscr{F} \setminus \bigcup_{s\in\mathfrak{w},\mu\in L^+} s\sigma_\mu \cup \bigcup_{s,t\in\mathfrak{w},\nu\in L^+} \tau_\nu(s,t)$
- $\mathfrak{w}_\lambda$: the stabilizer of λ in $\mathfrak{w}$
- $c_s(\lambda) = \mathbf{c}(s\lambda)$

0.3 Estimates for the Harish-Chandra series when λ becomes unbounded

- $\Psi(\lambda : h) := J(h)^{1/2}\Phi(\lambda : h)$
- $\sigma(G \to \mathbb{R}) : x \mapsto d(K, Kx)$ where $d(\cdot,\cdot)$ is the geodesic distance on S.
- $c_\mu(\lambda)$ page 154, is of at most polynomial growth in μ

0.4 Estimates for the elementary spherical functions. The functions Ξ and σ

- λ_R: the components in $\mathscr{F}_R$ for $\lambda \in \mathscr{F}$
- λ_I: the components in $\mathscr{F}_I$ for $\lambda \in \mathscr{F}$
- $\mathscr{F}^+_R$: the open chamber of elements $\nu \in \mathfrak{a}^*$ such that $H_\nu \in \mathfrak{a}^+$
- $\beta(\log a) = \min_{1\le i\le r} \alpha_i(\log a)$ for $a \in A^+$
- $\pi(\lambda) = \prod_{\alpha\in\Delta^{++}} \langle \lambda, \alpha\rangle$
- $\mathbf{b} = \pi \cdot \mathbf{c}$
- $\mathscr{F}_{R,\varepsilon} = C_0(\varepsilon\rho : \mathfrak{w})$: the convex hull of the points $\varepsilon s\rho(\varepsilon > 0, s \in \mathfrak{w})$
- $\sigma(x) := d(K, Kx)$ where $S = K\backslash G$, and $d(\cdot,\cdot)$ is the geodesic distance on S

0.5 The c-function

- $\hat{T}(g) = T(\hat{g})$: the Fourier transform of tempered distribution T on A where $g \in \mathscr{S}(\mathscr{F}_I)$
- R: a set of roots having the following properties: (i) R is contained in some positive system of roots, (ii) $\alpha, \beta \in R, \alpha + \beta \in \Delta \Rightarrow \alpha + \beta \in R$
- $\mathfrak{g}_{-R} = \sum_{\alpha\in -R} \mathfrak{g}_\alpha$
- $\bar{\mathfrak{n}}_{-R} = \sum_{\alpha\in -R\cap -\Delta^+} \mathfrak{g}_\alpha$
- $\mathfrak{n}_{-R} = \sum_{\alpha\in -R\cap\Delta^+} \mathfrak{g}_\alpha$
- $J_R(\lambda) = \int_{\bar{N}_{-R}} \mathrm{e}^{-(\lambda+\rho)(H(\bar{n}))} d\bar{N}_{-R}$ for all $\lambda \in \mathscr{F}$

- $\Omega = \left\{\lambda \in \mathscr{F} : \frac{\langle \lambda_R, \alpha\rangle}{\langle \alpha,\alpha\rangle} > -\min(a, \frac{1}{2}n(\alpha)) \text{ for all } \alpha \in \Delta^{++}\right\}$
- $\mathscr{D}_\lambda = \left\{f \in C^\infty(G) : f(xan) = \mathrm{e}^{-(\lambda+\rho)(\log a)} f(x) \text{ for all } x \in G, a \in A, n \in N\right\}$
- $\mathscr{S}(\mathscr{F}_I)$: the Schwartz space of $\mathscr{F}_I$

1 Asymptotic behavior of elementary spherical functions

1.1 The case when $\operatorname{rank}(G/K) = 1$

- $\psi(\lambda : t) = \mathrm{e}^{t\rho_0}\varphi(\lambda : \exp tH_0)$ where $\alpha(H_0) = 1$ and $\rho_0 = \rho(H_0) = \frac{1}{2}(p + 2q)$
- $f = 2(pg_1 + 2qg_2)$ where g_1, g_2 is defined as before
- $\Psi(\lambda : t) = \begin{bmatrix} \psi(\lambda : t) \\ \frac{d}{dt}\psi(\lambda : t) \end{bmatrix}$
- $\Gamma(\lambda) = \begin{bmatrix} 0 & 1 \\ \lambda^2 & 0 \end{bmatrix}$
- $M(t) = \begin{bmatrix} 0 & 0 \\ \rho_0 f & f \end{bmatrix}$
- $\Theta(\lambda : t) = \Theta(-\lambda : t) = \exp(-t\Gamma(\lambda))\Psi(\lambda : t)$
- $M(\lambda; t) = \exp(-t\Gamma(\lambda))M(t)\exp(t\Gamma(\lambda))$
- $\Theta(\lambda) = \lim_{t\to\infty} \Theta(\lambda : t)$
- $E(\lambda) = \begin{bmatrix} \frac{1}{2} & \frac{1}{2\lambda} \\ \frac{\lambda}{2} & \frac{1}{2} \end{bmatrix}$

1.2 The basic differential equations viewed as a perturbation of a linear system: the regular case

- $I = I_{\mathfrak{w}}(\mathfrak{a}_c)$: the subalgebra of $\mathfrak{w}$-invariant elements of $U(\mathfrak{a}_c)$
- $u_1 = 1, u_2, \ldots, u_w (w = |\mathfrak{w}|)$ are homogeneous and such that $U(\mathfrak{a}_c) = \bigoplus_{1\le i\le w} Iu_i$
- $p_{u:ij} \in I$ such that, for given $u \in U(\mathfrak{a}_c)$, $uu_j = \sum_{1\le i\le w} p_{u,ij}u_i,\ i \le j \le w$
- $B(u) = (p_{u:ij})_{1\le i,j\le w}$
- $\Gamma(u) = B(u)^t$ where $t =$ transpose

- $\Gamma(\lambda : u) = \Gamma(u)(\lambda) = (p_{u:ij}(\lambda))_{1\le i,j\le w}$
- $\Phi_0(\lambda : h) = \begin{bmatrix} \varphi(\lambda : h_j; u_1 \circ \mathrm{e}^{\rho}) \\ \vdots \\ \varphi(\lambda : h_j; u_w \circ \mathrm{e}^{\rho}) \end{bmatrix}$
- $\delta'(q)$: the radial component of $q \in \mathfrak{Q}$ on A^+
- $\delta(q) = \mathrm{e}^{\rho} \circ \delta'(q) \circ \mathrm{e}^{-\rho}$
- $g_{u:ijr} \in \mathscr{R}^+,\ q_{u:ijr} \in \mathfrak{Q}(1 \le i, \le w,\ 1 \le r \le w)$ such that for $i \le j \le w$,

$$uu_j = \sum_{1\le i\le w} u_i\delta(q_{u:ij}) + \sum_{1\le i\le w}\sum_{1\le r\le m} g_{u:ijr}u_i\delta(q_{u:ijr})$$

- $\tau(H) = \min_{\alpha\in S}\alpha(H) \quad (H \in \mathfrak{a})$
- $(E_u(\lambda : h))_{jk} = \sum_{1\le r\le m}\gamma(q_{u:kjr})(\lambda)g_{u:kjr}(h)$
- $S_\eta = \{h \in A^+ : |\tau(\log h)| \ge \eta \, \|\log h\|\}$
- $H_0 \in \mathrm{Cl}(\mathfrak{a}^+)$ such that $H_0 \notin Z_{\mathfrak{g}}$
- G_{H_0}: the centralizer of H_0 in G
- $\mathfrak{Q}_{H_0}$: the analogue of $\mathfrak{Q}$ for G_{H_0}

1.3 Radial components on M_{10}' and M_{10}^+

- F: the set of simple roots vanishing at H_0
- $P_0 = M_{10}N_0 = M_0A_0N_0$: standard psgrp associated to F
- Δ_0^+: the set of roots in Δ^+ vanishing at H_0
- $\mathfrak{q} = \mathfrak{g} \ominus \mathfrak{m}_{10}$
- $\mathfrak{q}_{\mathfrak{k}} = \mathfrak{q} \cap \mathfrak{k}$
- $\mathfrak{q}_{\mathfrak{s}} = \mathfrak{q} \cap \mathfrak{s}$
- $K_0 = K \cap M_0 = K \cap M_{10}$: the maximal compact subgroup of M_{10}
- $M_{10}' = \left\{m \in M_{10} : (\mathrm{Ad}\, m - \mathrm{Ad}(\theta(m)))_{\mathfrak{n}_0} \text{ is invertible}\right\}$
- $\mathfrak{w}_0 = \mathfrak{w}(\mathfrak{m}_{10}, \mathfrak{a})$
- $G' = KM_{10}'K$
- $\mathfrak{Q}_0 = U(\mathfrak{s}_{10})^{K_0}$

- $\varepsilon_0(a) = \exp\left(-\min_{\alpha\in\Delta^+\backslash\Delta_0^+}\alpha(\log a)\right),\ \forall a \in A$
- $M_{10}^+ = \{m \in M_{10} : \varepsilon_0(m) < 1\}$
- $A_{H_0}^+ = \left\{a \in \mathrm{Cl}(A^+) : \alpha(\log a) > 0, \forall\alpha \in \Delta^+\backslash\Delta_0^+\right\}$
- $\mathfrak{b}$: a linesr subspace of $\mathfrak{g}$
- $U(\mathfrak{b}) = U(\mathfrak{b}_c) = \lambda(S(\mathfrak{b}))$ where $S(\mathfrak{b}) = S(\mathfrak{b}_c)$ is the symmetric algebra over $\mathfrak{b}_c$
- $S_d(\mathfrak{b})$: the homogeneous subspaces of $S(\mathfrak{g})$
- $U_d(\mathfrak{b}) = \lambda(S_d(\mathfrak{b}))$
- $U^+(\mathfrak{b}) = \bigoplus_{d\geq 1} U_d(\mathfrak{b})$
- $\beta_0 : U(\mathfrak{g}) \to U(\mathfrak{s}_{10})$: the projection such that $\beta_0(g^k) = \beta_0(g)^k$ for all $g \in U(\mathfrak{g}), k \in K_0$

1.4 The basic differential equations viewed as a perturbation of a linear system: the general case

- $\mathfrak{H} = \bigoplus_{d\geq 0}(\mathfrak{H} \cap U_d(\mathfrak{a}))$
- $\mathfrak{H}_{\mathfrak{w}_0}$: the subspace of $\mathfrak{w}_0$ invariant elements on $\mathfrak{H}$
- $\dim \mathfrak{H}_{\mathfrak{w}_0} = [\mathfrak{w}, \mathfrak{w}_0] := k$
- $\Phi_0(\lambda : m : v) = \Gamma_0(\lambda : v)\Phi_0(\lambda : m) + \Phi_0(\lambda : m : E_v)$ and $v \in \mathfrak{Q}_0, \lambda \in \mathscr{F}, m \in M'_{10}$
- $\Phi_0(\lambda : m) = \begin{bmatrix} \varphi(\lambda : m; u_1 \circ \mathrm{e}^{\rho}) \\ \vdots \\ \varphi(\lambda : m; u_w \circ \mathrm{e}^{\rho}) \end{bmatrix}$
- $E_v = \begin{bmatrix} \sum_{1\leq p\leq p_v} \psi_{v:1p}\mu_{v:1p},\ 0 \\ \vdots \\ \sum_{1\leq p\leq p_v} \psi_{v:jp}\mu_{v:jp},\ 0 \\ \vdots \\ \sum_{1\leq p\leq p_v} \psi_{v:kp}\mu_{v:kp},\ 0 \end{bmatrix}$: a $k \times k$ matrix of differential operators on M'_{10} with coefficients $\mathscr{R}^+$
- $\Gamma_0(\lambda : v) := \Gamma_0(v)(\lambda) = (\gamma_{\mathfrak{g}\,/\,\mathfrak{a}}(q_{v:ij})(\lambda))_{1\leq i,j\leq k}$

1.5 Spectral theory of representations of polynomial rings associated to finite reflexion groups

- $F_{\mathbb{R}}$: a real vector space of finite dimension
- F: complexification of $F_{\mathbb{R}}$
- W: a finite reflexion group on $F_{\mathbb{R}}$
- P: the algebra of polynomials on F
- $P_d(d \geq 0)$: the homogeneous components
- $H \subset P$ is homogeneous if $H = \bigoplus_{d \geq 0}(H \cap P_d)$
- $I = I_W$: the algebra of W-invariant elements of P
- $w = |W|$
- $\lambda_\sigma \in F_{\mathbb{R}}^* \subset F^*$: associated to $\sigma \in W$
- $\pi = \prod_\sigma \lambda_\sigma$
- W_0: arbitrary reflexion subgroup of W
- $I_0 = I_{W_0}$
- $\pi = \prod_\sigma^0 \lambda_\sigma$ where the superfix 0 means the product is only over the reflexions in W_0
- $w_0 = |W_0|$
- $e(\lambda) = (u_1(\lambda) \cdots u_w(\lambda))^T$
- $e_s(\lambda) = e(s^{-1}\lambda)$
- $E(\lambda) := (e_{js}(\lambda))_{1 \leq j \leq w, s \in W}$

1.6 The initial estimates

- $\rho_0 = \frac{1}{2} \sum_{\alpha \in \Delta_0^+} m_\alpha \alpha$
- $d_{P_0}(h) = \mathrm{e}^{(\rho - \rho_0)(\log h)}$ for all $h \in A$
- $H_1, \cdots, H_p$: a specific basis of $\mathfrak{a}_0$ such that: $\mathfrak{a}_0 = (\mathfrak{a}_0 \cap [\mathfrak{g}, \mathfrak{g}]) \oplus \mathfrak{v}$. $\alpha_1, \cdots \alpha_q$ is the set of simple roots not vanishing at H_0, we have $q = \dim(\mathfrak{a}_0 \cap [\mathfrak{g}, \mathfrak{g}])$ and $\alpha_i = 0$ on $\mathfrak{v}$ for all i. Choose H_i so that
    1. $H_1, \cdots H_q$ the basis of $(\mathfrak{a}_0 \cap [\mathfrak{g}, \mathfrak{g}])$ dual to $\alpha_1, \cdots, \alpha_q$
    2. $H_{q+1}, \cdots, H_p$ the basis of $\mathfrak{v}$
- $\mathfrak{a}_0^+ = \left\{H \in \mathfrak{a}_0^+ : \alpha_i(H) > 0, 1 \leq i \leq q\right\}$: a conical open set in $\mathfrak{a}_0^+$ when η is small enough.
- $\tau_0(H) = \min_{1 \leq i \leq q} |\alpha_i(H)|, H \in \mathfrak{a}_0$

1.7 Asymptotics of $\Phi_0(\lambda:\cdot)$ on M_{10}^+. The function Θ

- $\mathfrak{a}_0^+(\eta) := \left\{H \in \mathfrak{a}_0^+ : \tau_0(H) > \eta \|H\|\right\}$ for any $\eta > 0$
- $H \xrightarrow{P_0} \infty$ if for some $\eta > 0$, $H \in \mathfrak{a}_0^+(\eta)$ and $\|H\| \to \infty$
- $\mathscr{F}_I(\kappa) = \{\lambda \in \mathscr{F} : \|\lambda_R\| < \kappa\}$ for some $\kappa > 0$
- $\Theta(\lambda : m) = \lim_{\mathfrak{a}_0^+ \ni H \xrightarrow{P_0} \infty} \exp(-\Gamma_0(\lambda : H))\Phi_0(\lambda : m \exp H)$
- $\Theta(\lambda : m; \mu) = \lim_{\mathfrak{a}_0^+ \ni H \xrightarrow{P_0} \infty} \exp(-\Gamma_0(\lambda : H))\Phi_0(\lambda : m \exp H; \mu)$ where $\mu \in U(\mathfrak{m}_{10})$
- $\Theta(\lambda : m; g) = \Gamma_0(\lambda : g)\Theta(\lambda : m)$
- $\theta(\lambda : m) = \int_{K_0} \mathrm{e}^{(\lambda-\rho_0)(H(mk_0))} dk_0$
- $\theta(\lambda : ma) = \mathrm{e}^{\lambda(\log a)}\theta(\lambda : m)$ for $m \in M, a \in A$ if H_0 is regular
- $\psi(\lambda : m) = \Theta_1(\lambda : m)$ where $\Theta_j (1 \le j \le k)$ are the components of Θ
- $\mathbf{c}_0$: the Harish-Chandra $\mathbf{c}$-function on M_{10}
- $f_1 \sim f_2$ if $f_1(\exp tX) - f_2(\exp tX) \to 0$ as $t \to +\infty$
- $S(\mathscr{F})$: the symmetric algebra over $\mathscr{F}$
- $\psi(\lambda : m) = |\mathfrak{w}_0|^{-1} \sum_{s\in\mathfrak{w}} (\mathbf{c}(s\lambda)/\, \mathbf{c}_0(s\lambda))\theta(s\lambda : m)$ for all $m \in M_{10}$
- $\tau_0(H) = \min_{\alpha\in\Delta^+\setminus\Delta_0^+} \alpha(H),\ H \in \mathfrak{a}$

1.8 Asymptotics of $\varphi(\lambda:\cdot)$

- $\gamma_0(b) = d_{P_0} \circ \beta_0(b) \circ d_{P_0}^{-1}$
- $A^+(H_0 : \zeta) := \{a \in \mathrm{Cl}(A^+) : \tau_0(\log a) > \zeta \|\log a\|\}$ for any $\zeta > 0$

2 The L_2 theory. The Harish-Chandra transform on the Schwartz space of G/K

2.1 The Schwartz spaces $\mathscr{C}(G)$ and $\mathscr{C}(G/K)$

- $\mathscr{C}(G)$: the space of all C^∞ functions f on G such that, for each $a, b \in U(\mathfrak{g})$, the two-sided derivative afb satisfies the strong inequality
- $\mathscr{C}(G//K)$: the spherical Schwartz space which is the subspace of $\mathscr{C}(G)$ consisting of spherical functions

2.2 The Harish-Chandra transform on $\mathscr{C}(G//K)$

- $\mathscr{S}(A)$: the Schwartz space of the vector group A
- $$\begin{tikzcd} \mathscr{C}_c^\infty(G//K) \arrow[r, hook, "\mathscr{H}"] \arrow[d, hook, "\mathscr{A}"] & \mathscr{P}(\mathscr{F}_I)^{\mathfrak{w}} \\ \mathscr{S}(A)^{\mathfrak{w}} \arrow[ur, "\wedge"] & \end{tikzcd}$$

2.3 Wave packets in $\mathscr{C}(G//K)$

- $\varphi'_a(x) = \varphi'(a:x) = \int_{\mathscr{F}_I} a(\lambda)\varphi(\lambda:x)d\lambda$ for any $a \in L^1(\mathscr{F}_I)$
- $\varphi_a(x) = \varphi(a:x) := \int_{\mathscr{F}_I} a(\lambda)\pi(\lambda)\varphi(\lambda:x)d\lambda$ for any $a \in \mathscr{S}(\mathscr{F}_I)$
- $\varphi_a(x;b) = \varphi(a:x;b) := \int_{\mathscr{F}_I} a(\lambda)\pi(\lambda)\varphi(\lambda:x;b)d\lambda$ for any $a \in \mathscr{S}(\mathscr{F}_I)$ and $b \in U(\mathfrak{g})$
- $\hat{a}(\lambda:y) = \hat{a}_y(\lambda) = \int_{L_I^*} a(\lambda:\nu)\mathrm{e}^{\langle y,\nu\rangle}d\nu$: the partial Fourier transform of a, where $a \in \mathscr{S}(\mathscr{F}_I \times L_I^*)$ and L_I^* being the imaginary dual $(-1)^{1/2}L^*$ of L
- $E(\lambda:h)$: the "error term" is estimated by $|E(\lambda:h)| \leq C(1+\|\lambda\|)^{m_0}(1+\sigma(h))^{m_0}\mathrm{e}^{-2\tau_0(\log h)}$
- $E(\pi a:h) = \int_{\mathscr{F}_I} \pi(\lambda)a(\lambda)E(\lambda:h)d\lambda$
- $A^+(H_0:\zeta) = \{h \in \mathrm{Cl}(A^+) : \tau_0(\log h) > \zeta\sigma(h)\}$ where $H_0 \neq 0$
- $\psi_a(x) := \psi(a:x) = \int_{\mathscr{F}_I} a(\lambda)\varphi(\lambda:x)\,|\mathbf{c}(\lambda)|^{-2}\,d\lambda$

2.4 The method of Harish-Chandra

- $\theta^{(v)}(\lambda:x:h) = \sum_{s\in\mathfrak{w}} \mathbf{c}(s\lambda)v(s\lambda)\mathrm{e}^{(s\lambda-\rho)(H(x)+(s\lambda)(\log h))}$ for any $v \in U(\mathfrak{a}),\ h \in A,\ \lambda \in \mathscr{F}'_I$
- $\theta^{(v)}(a:x:h) = \int_{\mathscr{F}_I} a(\lambda)\theta^{(v)}(\lambda:x:h)\,\mathbf{c}(\lambda^{-1})\,\mathbf{c}(-\lambda)^{-1}d\lambda$ for any $v \in U(\mathfrak{a}),\ h \in A,\ a \in C_c^\infty(\mathscr{F}'_I)^{\mathfrak{w}}$
- $a^{(v)}(\lambda) = a(\lambda)\,\mathbf{c}(-\lambda)^{-1}v(\lambda)$
- $\widehat{a^{(v)}}(h) = \int_{\mathscr{F}_I} \mathrm{e}^{\lambda(\log h)}a^{(v)}(\lambda)d\lambda$
- $F(\bar{n}:h) = F(\bar{n}h),\ (\bar{n} \in \bar{N}, h \in A)$ for any function F on G
- $\mathscr{J}(\mathscr{S}(\mathscr{F}_I)^{\mathfrak{w}} \to \mathscr{C}(G//K)) : \mathscr{J}\,a = |\mathfrak{w}|^{-1}\,\psi_{a^\vee}$
- $a^\vee(\lambda) = a(-\lambda)(\lambda \in \mathscr{F}_I)$

2.5 The method of Gangolli-Helgason-Rosenberg

- $A(r) = \{a \in A : \sigma(a) = \|\log a\| \leq r\}$ for any $r > 0$
- $G(r) = \{x \in G : \sigma(x) = \leq r\}$
- $\mathscr{P}_r(\mathscr{F})$: the space of all entire functions f on $\mathscr{F}$ with the following property: for any $N \geq 0$ there is a constant $C_N > 0$ such that $|f(\lambda)| \leq C_N(a + \|\lambda\|)^{-N} \mathrm{e}^{r\|\lambda_R\|}$

Appendix A Errata in [1]

$\text{Page}^{\text{line}\downarrow}_{\text{line}\uparrow}$	instead of	Read
67_{10}	$\mathfrak{m}_{1F} = \sum_{\lambda \in \Delta_F} \mathfrak{g}_\lambda$	$\mathfrak{m}_{1F} = \mathfrak{m} \oplus \mathfrak{a} \oplus \sum_{\lambda \in \Delta_F} \mathfrak{g}_\lambda$
95^7	$-2\sum_{1 \leq i \leq a} \rho(H_i)$	$-2\sum_{1 \leq i \leq a} \rho(H_i) H_i$
103_{10}	$\mathrm{e}^{(\lambda+\rho)(H(x^{-1}k))} f(x^{-1}[k])$	$\mathrm{e}^{(\lambda+\rho)(H(x^{-1}k))} g(x^{-1}[k])$
152^{10}	$\sum_{s \in \mathfrak{w}} \mathbf{c}_s(\lambda)(s\lambda : h)$	$\sum_{s \in \mathfrak{w}} \mathbf{c}_s(\lambda)\Phi(s\lambda : h)$
156^1	$\mathscr{F}_n$	$\mathscr{F}_\eta$
183^{13}	$1/2(n-r)$	$(1/2)(n-r)$
198_{10}	$\Phi_0(\lambda : h) = \begin{bmatrix} \varphi(\lambda : h_j; u_1 \circ \mathrm{e}^\rho) \\ \varphi(\lambda : h_j; u_w \circ \mathrm{e}^\rho) \end{bmatrix}$	$\Phi_0(\lambda : h) = \begin{bmatrix} \varphi(\lambda : h_j; u_1 \circ \mathrm{e}^\rho) \\ \vdots \\ \varphi(\lambda : h_j; u_w \circ \mathrm{e}^\rho) \end{bmatrix}$
200_{18}	next n^0	next section.
201^{10}	$K_0 = K_0 \cap M_0$	$K_0 = K \cap M_0$
204^{11}	(5.2.23)	(5.3.23)
212_7	$E_v = \begin{bmatrix} \sum_{1 \leq p \leq p_v} \psi_{v:1p}\mu_{v:1p}, & 0 \\ \sum_{1 \leq p \leq p_v} \psi_{v:jp}\mu_{v:jp}, & 0 \\ \sum_{1 \leq p \leq p_v} \psi_{v:kp}\mu_{v:kp}, & 0 \end{bmatrix}$	$E_v = \begin{bmatrix} \sum_{1 \leq p \leq p_v} \psi_{v:1p}\mu_{v:1p}, & 0 \\ \vdots & \\ \sum_{1 \leq p \leq p_v} \psi_{v:jp}\mu_{v:jp}, & 0 \\ \vdots & \\ \sum_{1 \leq p \leq p_v} \psi_{v:kp}\mu_{v:kp}, & 0 \end{bmatrix}$
243_{11}	$\varphi(\lambda :; b)$	$\varphi(\lambda : a; b)$
229_7	$-t_1 T_1$	$t_1 T_1$
227_3	$f(\mathbf{t})$	$\mathbf{f}(\mathbf{t})$
243_2	$s = s(b, \zeta) > 0$	$s = s(b, \zeta) \geq 0$

References

[1] R. Gangolli and V. S. Varadarajan, *Harmonic analysis of spherical functions on real reductive groups*, Ergebnisse der Mathematik und ihrer Grenzgebiete, vol. 101, Springer-Verlag, Berlin, 1988. MR 954385