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A mathematical model of "Gone with the Wind"

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HIGHLIGHTS

- This is the first serious modeling application in the field of love dynamics.
- The model and the analysis are in line with the tradition of classical physics.
- A mathematical model describing the evolution of a love story is presented.
- The love story considered is that portrayed in the famous film "Gone with the Wind".
- The story predicted by the model is shown to be in good agreement with the film.

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ABSTRACT

We develop a mathematical model for mimicking the love story between Scarlett and Rhett described in "Gone with the Wind". In line with tradition in classical physics, the model is composed of two Ordinary Differential Equations, one for Scarlett and one for Rhett, which encapsulate their main psycho-physical characteristics. The two lovers are described as so-called insecure individuals because they respond very strongly to small involvements of the partner but then attenuate their reaction when the pressure exerted by the partner becomes too high. These characteristics of Scarlett and Rhett clearly emerge during the first part of the film and are sufficient to develop a model that perfectly predicts the complex evolution and the dramatic end of the love story. Since the predicted evolution of the romantic relationship is a direct consequence of the characters of the two individuals, the agreement between the model and the film supports the high credibility of the story. Although credibility of a fictitious story is not necessary from a purely artistic point of view, in most cases it is very appreciated, at the point of being essential in making the film popular. In conclusion, we can say that we have explained with a scientific approach why "Gone with the Wind" has become one of the most successful films of all times.

1. Introduction

The consistency of the development of a love story with the characters of the involved individuals has been evaluated up to now on purely intuitive grounds. An alternative and potentially more powerful approach is conceivable if we admit to be able to encapsulate, as suggested by various authors [1–8], the main psychophysical characteristics of the individuals in a mathematical model. After the model has produced its own story, a comparison with the real story is possible. Obviously, the result will not always be satisfactory because we know that interpersonal relationships are often influenced (if not dominated) by unpredictable exogenous events.

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Of course, a mathematical model could also be developed for a fictional story like that described in a film. In such a case, the comparison between the described story and that predicted by the model allows one to give a motivated judgment on the credibility of the film. Although such credibility is not strictly necessary from a purely artistic point of view, in most cases it is appreciated by the audience, at the point of being essential in making the film popular. In conclusion, we can use a quantitative approach to predict the success of a film before realizing it or to explain why an already produced film has been successful.

This approach is here applied to "Gone with the Wind", released in Atlanta in December 1939, starring Vivien Leigh as Scarlett and Clark Gable as Rhett. The film, inspired by the 1936 bestseller of Margaret Mitchell, is one of the first films in color, has been awarded eight Oscars, has been seen at least once by 90% of Americans, and has been the most economically successful film until a few years ago. Undoubtedly, "Gone with the Wind" is one of the most popular films of the entire cinematographic history.

In the next section we shortly describe the love story between Scarlett O'Hara and Rhett Butler by dividing it in two phases, one before and one after the economic crash of Tara, the huge cotton plantation owned by the O'Hara's family. The analysis is based on a great number of details described in the film, but for simplicity reference is made here only to twelve short segments extracted from the film and available at www.home.dei.polimi.it/rinaldi/GoneWithTheWind/film.html. The first two segments are used to identify the characters of Scarlett and Rhett on which the model is based, while the remaining ones are used to show that the love story described in the film and that predicted by the model are in good agreement. The result is actually astonishing: It shows that the information available after only half an hour is sufficient to predict the evolution and the dramatic end of a story described in a very long film. This reveals why the impression one gets when watching the film, namely that the entire story is realistic, is fully motivated. In conclusion, we explain with a mathematical model why "Gone with the Wind" has become one of the most popular films of all times.

2. The love story between Scarlett and Rhett

"Gone with the Wind" describes the love story that develops between Scarlett and Rhett during the decade starting with The Civil War in 1861. In comparison with the film, the story is here extremely compressed, by eliminating a number of details concerning minor characters and historically important events. Fig. 1 reports the initial and final times of twelve short segments (S.1–S.12) of the film available at www.home.dei.polimi.it/rinaldi/GoneWithTheWind/film.html. and used for deriving and supporting the model.

The film starts with a barbecue party at Twelve Oaks where Scarlett, a beautiful young lady, flirts with a circle of admiring men (S.1). She responds promptly and positively even to small signs of interest of all her suitors, but then quickly changes her mood as soon as she has obtained a minimal success. This is the typical behavior of a conqueror, that first excites and then rebuffs her victim. This well identifiable tendency of Scarlett to avoid too heavy involvements is possibly due to her childish infatuation for Ashley (not described in S.1). In conclusion, her reaction is positive for small involvements of any potential partner but then declines sharply when the pressure exerted on her increases so much that the realization of her dream with Ashley is called into question.

At the same party, Scarlett and Rhett see each other for the first time (S.2). He is tall, attractive, very elegant, and looks like a professional Don Juan. This impression is also generated by the quite impertinent way he looks at her, and reinforced by the news that he has seduced a Charleston's girl and then refused to marry her.

One year later Scarlett and Rhett meet again just after she has lost her first husband (S.3). They meet at a dancing party organized for raising money for the Confederate Cause, and Rhett teases Scarlett about her widowhood, revealing that he knows that she is there not because of her devotion to the Cause but for her devotion to enjoyment.

One of the organizers causes a stir by announcing that to raise funds he is auctioning the right to dance with the ladies (S.4). Rhett bids a hundred and fifty dollars in gold for Scarlett, thus showing a definite interest for her. The organizer protests that Scarlett cannot dance because she is in mourning and asks him to change his bid to another lady, but Rhett refuses. To everyone's shock, Scarlett accepts and rushes onto the dance floor.

Rhett grows impatient as Scarlett continues to wear black mourning clothes even though she is taking part in all social activities (S.5). To tempt her to discard her black veil, he buys her a fashionable green bonnet in Paris. Scarlett cannot resist the bonnet but she warns him that she will not marry him in return. He replies that he is not the marrying type—though he kisses her, thus revealing once more his involvement.

In August 1864, when Atlanta is burning, Rhett rescues Scarlett from the Yankees and tells her that he is going to join the Confederate army (S.6). He says, in a light-hearted tone, that in spite of what he said to her previously, he does love her, because they are alike. He kisses her passionately, and she is surprised to feel herself responding. But she is still furious at him for deserting her, so she slaps him across the mouth and declares that he is not a gentleman.

In 1865, after the war is over, Scarlett is oppressed by financial problems related with the cotton production in Tara. She suddenly realizes that Rhett, who has become very rich in the meantime, could solve her problems (S.7). Scarlett puts on her new green dress and goes to visit Rhett in jail. She pretends to be distressed about his plight, claiming that she would suffer if he were hanged, while in reality she is mostly interested in his money.

Some time later, Rhett proposes marriage to Scarlett (S.8). Scarlett replies that she does not love him and does not want to marry again, but Rhett says that she married once for spite and once for money, and has never tried marrying for fun. He

(S.1) Scarlett flirting at a party
(S.2) The first encounter
(S.3) Rhett teases Scarlett about her widowhood
(S.4) Rhett bids to dance with Scarlett and she accepts
(S.5) The first kiss
(S.6) Rhett declares his love before leaving her
(S.7) Scarlett visits Rhett in jail
(S.8) Rhett proposes marriage to Scarlett
(S.9) Scarlett and Rhett just after marrying
(S.10) Strong signs of Rhett's disinvolvement
(S.11) Rhett offers Scarlett to get divorced
(S.12) Rhett quits Scarlett

Fig. 1. Snapshots of the twelve film segments (S.1–S.12) used in Section 2. The figures on each panel are the initial and final times of the segment. *Source:* See www.home.dei.polimi.it/rinaldi/GoneWithTheWind/film.html.

kisses her passionately, and she kisses him back. Feeling faint, she agrees to marry him. He asks her why she said yes, and she admits that it was partly because of his money.

After marrying, Rhett treats Scarlett with tenderness and allows her to spend as much money as she likes to restore Tara (S.9).

One day, during an argument, Scarlett threatens Rhett that she could stop having sexual relationship with him. He responds with indifference, saying that there are plenty of other women's beds in the world (S.10). Scarlett is mortified that he has taken her threat so lightly and regrets his indelicacy.

One night, after a party, Rhett, who has drunk too much, makes violent love to Scarlett. For the first time, Scarlett feels that she has met someone whom she cannot bully or break. The next morning, Scarlett awakes alone, in a state of wild excitement about the previous night (S.11). She feels passionate about Rhett, in spite of his cold attitude. Rhett makes light of the night he spent with Scarlett, apologizing for being drunk and offers her to get a divorce.

In the last scene of the film (S.12) Scarlett insistently says that she loves him more than everything else. Rhett replies that he no longer loves her and all he feels for her now is pity. He is going away, maybe to Charleston to try to make peace with his family. Scarlett desperately asks him what she will do if he goes away. Leaving the house, he replies with the famous quote "Frankly, my dear, I do not give a damn".

3. The model

Following the minimal approach first proposed in Ref. [1] we describe the love story with two Ordinary Differential Equations, one for Scarlett and one for Rhett. In these equations $x_i(t)$, $i = 1, 2$ (where 1 is Scarlett and 2 is Rhett) represents the *feeling* at time t of individual i for the partner. In accordance with [9], positive values of the feelings range from sympathy to passion, while negative values are associated with antagonism and disdain. When Scarlett and Rhett meet for the first time at $t = 0$ (S.2), they are indifferent to each other, so that $x_1(0) = x_2(0) = 0$. Then the feelings evolve at a rate $\mathrm{d}x_i(t)/\mathrm{d}t$ dictated by the unbalance between the regeneration and consumption processes. Since only positive feelings are involved in this story, in the following they are often called *love*.

The consumption process is *oblivion*, which explains why individuals gradually lose memory of their partners after separating. As done in almost all fields of science, losses are assumed to occur at exponential rate, *i.e.* $x_i(t) = x_i(0)\exp(-\alpha_i t)$

or, equivalently, in terms of Ordinary Differential Equations, $\mathrm{d}x_i(t)/\mathrm{d}t = -\alpha_i x_i(t)$, where α_i is the so-called *forgetting coefficient*. In contrast, the regeneration processes are of two distinct kinds, namely *reaction to appeal* and *reaction to love*.

The appeal of individual i has various components A_i^h like physical attractiveness, intelligence, age, richness, as well as others, which are independent on the feeling x_i for the partner. If λ_j^h is the weight that individual j ($j \neq i$) gives to the h-th component of the appeal of his/her partner we can define the appeal of i (perceived by j) as

$$A_i = \sum_h \lambda_j^h A_i^h. \tag{1}$$

Thus, the appeal of an individual is not an absolute character of the individual, but rather a value perceived by his/her present or future partner.

The flow of interest generated in individual j by the appeal of the partner is obtained by multiplying A_i by a factor ρ_j identifying the *sensitivity* of individual j to appeal. During the whole period of interest, Rhett remains practically unchanged and substantially interested only in the beauty of Scarlett that does not fade over time. In contrast, the problem is not so simple for Scarlett because the war substantially modifies the economic status of both Scarlett and Rhett. In 1864, she almost loses Tara, her family property occupied by the Yankees, and needs a large amount of money to start producing cotton again. Under these circumstances, she is naturally induced to increase the weight she gives to money. At the same time, Rhett has become very rich, thanks to some successful though ambiguous financial operations performed during the war, so that the economic component of his appeal is now greater. As a result of these two independent facts, the flow of interest generated by the appeal of the partner remains constant in Rhett's equation and substantially increases in Scarlett's equation (this flow is doubled in our numerical analysis).

Finally, the second regeneration process is the reaction to the love of the partner. The most standard individuals, usually called *secure*, are those who like to be loved. An individual i belonging to this class is formally characterized by an increasing function $R_i(x_j)$ that identifies the flow of interest generated in individual i by the love x_j of the partner. In the first studies on love dynamics, reaction functions of secure individuals were linear [1,2], while, later, they have been assumed to be bounded [3,10]. The reason for this change is that unbounded functions do not capture the psychophysical limitations present in all individuals.

From the discussion in the previous section it is clear that Scarlett and Rhett do not belong to the class of secure individuals. Indeed, their reactions first increase with the love of the partner and then decrease. Individuals of this kind are called *insecure*, or, sometimes, *refusers*, because they react less and less strongly when the love of the partner exceeds a certain threshold. The simplest analytical form of the reaction of an insecure individual is

$$R_i(x_j) = k_i x_j \exp(-x_j/\beta_i). \tag{2}$$

The graph of this function is shown in Fig. 2 for two particular values of k_i and β_i. From a formal point of view the main property of an insecure individual is the change of sign of the derivative of the reaction function, a property which is satisfied by Eq. (2) for any value of the parameters (k_i, β_i). Other properties pointed out in Fig. 2, like the fact that the reaction vanishes when the love of the partner becomes very large, are not essential, as shown in the following, and are not characteristics of insecure individuals. In fact, an insecure individual can be so much disturbed by the pressure of the partner to invert the sign of his/her reaction [11].

In conclusion, the model we propose for Scarlett and Rhett is the following

$$\begin{aligned} \frac{\mathrm{d}x_1(t)}{\mathrm{d}t} &= -\alpha_1 x_1(t) + \rho_1 A_2 + R_1(x_2) \\ \frac{\mathrm{d}x_2(t)}{\mathrm{d}t} &= -\alpha_2 x_2(t) + \rho_2 A_1 + R_2(x_1) \end{aligned} \tag{3}$$

where A_i and $R_i(x_j)$ are as in (1) and (2). This model is very simple and enjoys a number of general properties. First, it is a positive model, namely $x_i(0) \geq 0$, $i = 1, 2$ implies $x_i(t) \geq 0$, $i = 1, 2$ for all $t > 0$, which means that the feelings of Scarlett and Rhett cannot become negative (this follows from the fact that $\mathrm{d}x_i(t)/\mathrm{d}t \geq 0$ for $x_i(t) = 0$, i.e. trajectories in state space cannot leave the positive quadrant). Second, the divergence of the model, which is $-(\alpha_1 + \alpha_2)$, does not change sign so that, in view of Bendixon's criterion [7], the model cannot have limit cycles, i.e. the feelings of Scarlett and Rhett cannot oscillate periodically. Finally, the model is bounded because $\mathrm{d}x_i(t)/\mathrm{d}t$ is negative when $x_i(t)$ is large, which means that the feelings of Scarlett and Rhett cannot grow indefinitely (this follows from the boundedness of the reaction functions). In conclusion, for any given initial condition in the first quadrant, the model can only evolve toward an equilibrium point, i.e. if all parameters remain unchanged Scarlett and Rhett must tend toward a stationary relationship.

The full catalog of dynamic behaviors of models describing insecure individuals has never been produced. For obtaining it a complete bifurcation analysis [7,12] with respect to all parameters appearing in the model (α_i, β_i, ρ_i, k_i, A_i, $i = 1, 2$ in the present case) should be performed. But this would take us quite far from our target, which is the study of a specific couple. For this reason, we focus here only on Scarlett and Rhett and we present our analysis in the most straightforward way, namely through simple geometric considerations involving the so-called null-clines [7], namely the curves in state space where $\mathrm{d}x_1(t)/\mathrm{d}t = 0$ and $\mathrm{d}x_2(t)/\mathrm{d}t = 0$. These curves are of interest because the equilibria of the model (characterized by

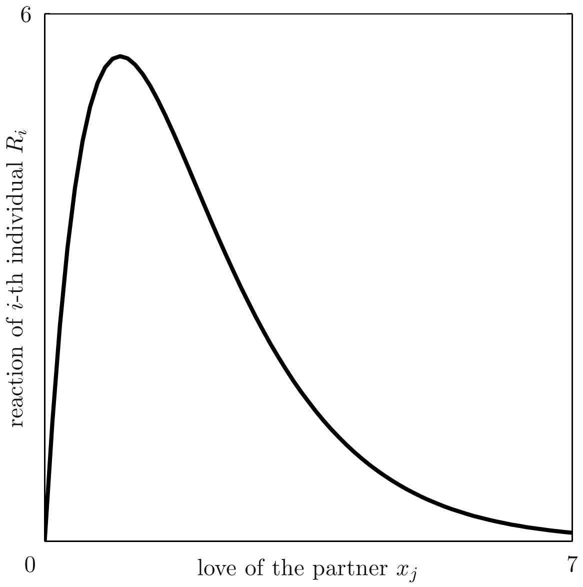
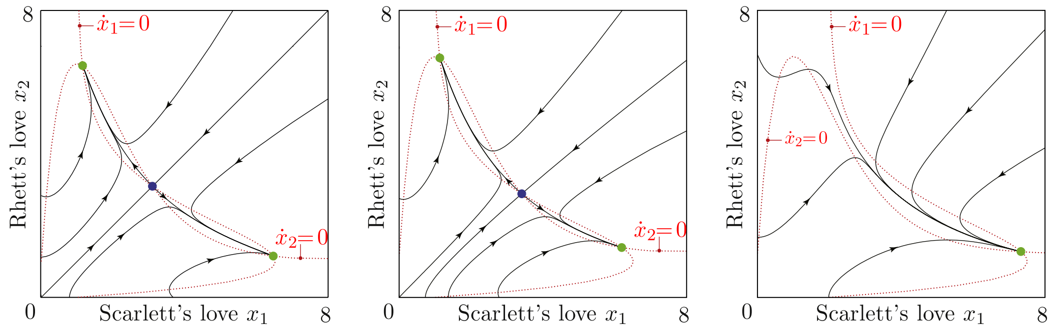


Fig. 2. Graph of the reaction function of an insecure individual. The reaction R_i is the amount of love generated in individual i by the love x_j of his/her partner in one unit of time. The graph shows that an insecure individual reacts strongly and positively to small amounts of interest of the partner but then inverts this reaction when the pressure of the partner becomes too high. Parameter values: $k_i = 15$ and $\beta_i = 1$ in (2).



(a) $A_1 = A_2 = 1$. (b) $A_1 = 1.2, A_2 = 1$. (c) $A_1 = 1.2, A_2 = 2$.

Fig. 3. Evolution of the love story between Scarlett and Rhett for three different assumptions on their appeals A_1 and A_2. Black curves are trajectories of model (3) in love space, red dotted curves are null-clines, blue points are saddles and green points are stable equilibria. In (a) Scarlett and Rhett have the same appeal ($A_1 = A_2 = 1$). The figure shows that the trajectory starting from the origin tends toward the saddle equilibrium with $x_1 = x_2$. In (b) Scarlett is slightly more appealing than Rhett ($A_1 = 1.2, A_2 = 1$) and the trajectory starting from the origin tends to the stable equilibrium where Rhett is much more involved than Scarlett. In (c) the appeal of Rhett is definitely greater than that of Scarlett ($A_1 = 1.2, A_2 = 2$) and there is only one globally stable equilibrium where Scarlett is much more involved than Rhett. Other parameter values: $\alpha_i = \beta_i = \rho_i = 1, k_i = 15, i = 1, 2$. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

$\mathrm{d}x_i(t)/\mathrm{d}t = 0\; i = 1, 2)$ are the points of intersections of the two null-clines. From (3) it follows that the two null-clines are

$$x_1 = \frac{1}{\alpha_1}\left(R_1(x_2) + \rho_1 A_2\right)$$
$$x_2 = \frac{1}{\alpha_2}\left(R_2(x_1) + \rho_2 A_1\right),$$

namely the graphs of the two reaction functions shifted and rescaled in the space (x_2, x_1) and (x_1, x_2) respectively.

Fig. 3-(a) shows the two null-clines (red dotted curves) and the trajectories for the singular case of two identical individuals (parameters are specified in the caption). Fig. 3-(a) points out three equilibria: Two are stable (green points in the figure), while the central one is a saddle (blue point in the figure)—a proof can be given through linearization. Since the two individuals are identical they must be characterized by $x_1(t) = x_2(t)$ for $t > 0$ if $x_1(0) = x_2(0)$. This explains why the trajectory starting from the origin is a straight line tending toward the saddle. This particular trajectory, known as the stable manifold of the saddle, is the boundary of the basins of attraction of the two stable equilibria.

Of course, if the two individuals are not identical, the null-clines and the trajectories differ from those in Fig. 3-(a). If only the appeals of the individuals are different the null-clines can be immediately obtained from those in Fig. 3-(a) by shifting

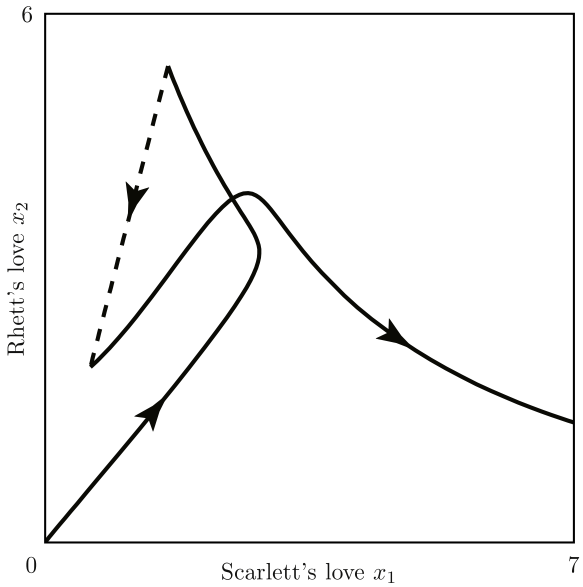


Fig. 4. The love story between Scarlett and Rhett predicted by the model. The story starts from the origin when Scarlett and Rhett see each other for the first time. In this phase Scarlett is slightly more appealing than Rhett. Then a period of separation due to the Civil War follows (dotted trajectory). Finally, a last phase where Rhett is definitely more appealing than Scarlett closes the story.

rightward (upward) the first (second) null-cline if A_2 [A_1] increases. For example, in Fig. 3-(b) A_1 is increased of 20% and the only relevant consequence is that the origin is now in the basin of attraction of the equilibrium with $x_1 < x_2$ (actually $x_1 \ll x_2$). This means that the most appealing individual is penalized at the end of the story. This result is intuitively understandable: An increase of appeal of individual 1 has the consequence of increasing the love of individual 2 at the equilibrium with $x_1 < x_2$ (this is a consequence of the upward shift of the second null-cline). This in turn has, in general, a negative impact on the involvement of an insecure individual like 1. Notice that all this remains true also if the reaction functions (and hence the null-clines) are slightly different from those proposed in (2) even if they are negative for very high pressures of the partner.

In Fig. 3-(c) the effect of a large increase of the appeal (A_2 is doubled with respect to Fig. 3-(b)) is shown. In this case, the first null-cline is shifted to the right of such a large amount that it intersects the other null-cline only at a single point. Again, the most appealing individual is the one who is less involved at the end of the story.

Fig. 3 has not been produced by chance, but intentionally with the aim of formally interpreting the love story between Scarlett and Rhett. In fact, since the film hardly allows one to distinguish between the 4 parameters α, β, ρ and k identifying Scarlett and Rhett, we have assumed them to be equal, i.e. $\alpha_1 = \alpha_2$, $\beta_1 = \beta_2$, $\rho_1 = \rho_2$ and $k_1 = k_2$. In contrast, the appeals of Scarlett and Rhett are different. In the initial phase of the story, when Scarlett is not so much interested in money, the two individuals have a comparable appeal. However, noticing (see (3)) that

$$\left.\frac{\mathrm{d}x_i(t)}{\mathrm{d}t}\right|_{t=0} = \rho_i A_j$$

and comparing the reaction of Scarlett and Rhett (S.2) when they see each other for the first time (she says: "He looks as if he knows what I look like without my shimmy!"), it is reasonable to imagine that A_1 is marginally bigger than A_2. For this reason, we have assumed that $A_1 = 1.2$ and $A_2 = 1$, as in Fig. 3-(b).

In the final phase of the story, namely when the war is over and Scarlett is oppressed by financial problems, the appeal of Rhett becomes much bigger as repeatedly confirmed by Scarlett (S.7, S.8). For this reason, we have assumed that in this phase $A_1 = 1.2$ and $A_2 = 2$, as in Fig. 3-(c). Under these assumptions, the story should initially evolve along the trajectory starting from the origin in Fig. 3-(b) and then end, after the war, along a trajectory of Fig. 3-(c). The only thing that remains to be done is the concatenation of the initial and final phases, which are interrupted by a relatively long (6 months) period of separation. Assuming that during the separation period only the oblivion process is active, we can simply derive the initial conditions of the final phase by integrating the equations

$$\frac{\mathrm{d}x_1(t)}{\mathrm{d}t} = -\alpha_1 x_1(t)$$
$$\frac{\mathrm{d}x_2(t)}{\mathrm{d}t} = -\alpha_2 x_2(t)$$

starting from the final conditions of the initial phase (since $\alpha_1 = \alpha_2$ the oblivion phase is represented by a straight trajectory pointing toward the origin). The result of this analysis is shown in Fig. 4 where the dotted trajectory corresponds to the

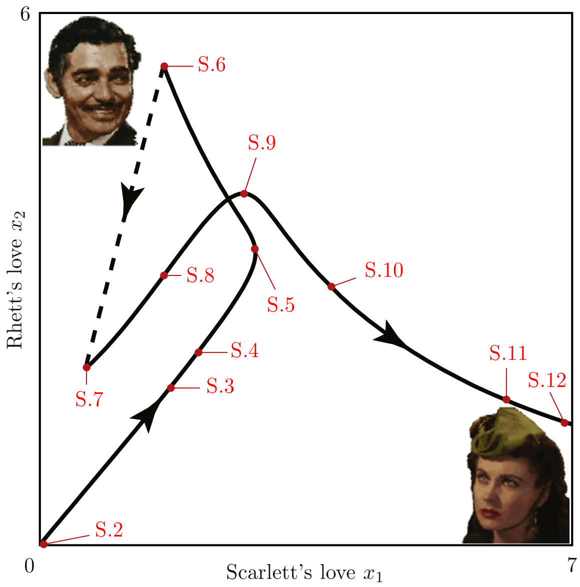


Fig. 5. Proposed allocation of the film segments along the trajectory of Fig. 4. The eleven segments S.2, . . . , S.12 extracted from the film are allocated along the trajectory describing the love story between Scarlett and Rhett following a subjective judgment of their involvements. S.2 marks the beginning of the story, S.6 and S.7 the beginning and the end of the separation period, and S.12 the end of the story when Rhett quits Scarlett.

separation phase. The figure shows that in the initial phase Rhett is more involved than Scarlett while after the war the story evolves in the opposite direction with Rhett finally becoming less and less involved. The story predicted by the model tends toward an equilibrium with $x_2 < x_1$. Since this equilibrium strongly penalizes Rhett, one can easily imagine that a possible consequence in a real context is that Rhett quits Scarlett when approaching the equilibrium.

4. Agreement between model and film

We show in this section that the love story predicted by the model (synthetically described in Fig. 4) is, qualitatively speaking, in good agreement with the story described in the film.

First we place the segments of the film reported in Fig. 1 along the trajectory as shown in Fig. 5. This is done by giving a subjective judgment of the involvements of Scarlett and Rhett emerging from each segment. Thus, (S.2) is placed at the origin, because $x_1(0) = x_2(0) = 0$ when Scarlett and Rhett see each other for the first time, while (S.6) is placed at the end of the first part of the trajectory because it corresponds to the beginning of the separation period. Then, (S.3) and (S.4), which are almost identical points on the trajectory (they refer to almost contemporary events) are placed in the part where the feelings of both Scarlett and Rhett are increasing. Finally, the segment (S.5) (where they kiss each other for the first time) is roughly placed at the turning point of the trajectory because Scarlett involvement x_1 at that point is greater than in (S.4) but also than in (S.6) where she slaps him after a kiss.

(S.7) is placed at the end of the separation period, while (S.11) and (S.12) are at the end of the trajectory because they close the love story. The remaining three segments (S.8)–(S.10) are placed in such a way that (S.9), where they both look happy, corresponds to the point of highest involvement of Rhett during the last phase.

In conclusion, we have verified that the sequence (S.2), . . . , (S.12) can be allocated on the trajectory of Fig. 4 without clashing with the spontaneous impression one gets when watching the film. Since all this is highly subjective, the reader is invited to go through the same exercise to check if he agrees with our proposed allocation of the segments or if he finds better solutions.

But we can do even more and perform a second check of the agreement between model and film. For this, we first use the model to plot the time evolution of the feelings, obtaining the two graphs of Fig. 6, and then determine the chronology of (S.2), . . . , (S.12) consistent with the allocation proposed in Fig. 5. This allows one to check that the chronology predicted by the model is in quite good agreement with that roughly indicated in the film and the novel.

5. Discussion and conclusions

We have shown in this paper how a love story can be described with a mathematical model composed of two Ordinary Differential Equations, one for each partner. This is not a new idea: It has been proposed in a 1988 pioneering paper of Strogatz [1] and then followed by a number of other authors [2,3,5,6,8,10,11]. In all these contributions the rate of change of the love of each individual for the partner is determined by the unbalance between two regeneration processes (reaction to appeal and to love of the partner) and a consumption process (oblivion).

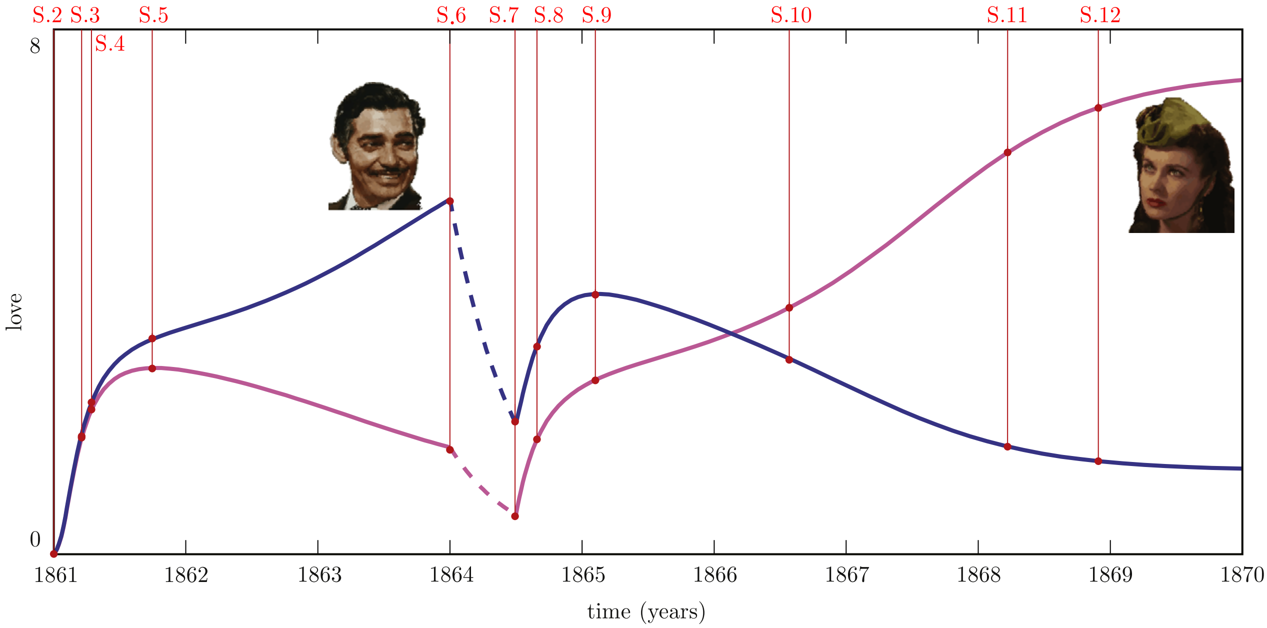


Fig. 6. Time evolution of Scarlett's and Rhett's involvements during their love story. Pink and blue lines represent the evolution of Scarlett's and Rhett's love, respectively. The figure points out the chronology of the proposed allocation of the segments S.2, ..., S.12, which is consistent with the information available in the film. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

The class of models that has been mainly investigated until now [1–3,5–7,10] is that of the so-called secure individuals, who positively react to any increase of the partner's love. In contrast, in this paper we consider for the first time a couple composed of two insecure individuals, who reduce their reaction when the love of the partner exceeds a certain threshold. The particular case we analyze is that of Scarlett and Rhett described in "Gone with the Wind", one of the most popular films of the entire cinematographic history. The mathematical model of couples of this kind predicts that the love story tends toward a stationary regime in which the involvement of the two partners can be quite unbalanced. More precisely, we show that in the long run the individual who is less involved is the most appealing one. This is why the end of "Gone with the Wind", where Rhett quits Scarlett after a long relationship, can be perfectly predicted. In fact, the two lovers are both physically attractive but during the war Rhett becomes very rich and has therefore a relevant extra-component in his appeal.

Another original aspect of this paper is the discussion of the agreement between model and film. Since in a context like this quantitative data are not available, the comparison is not done in the standard technical way (e.g., by comparing the mean square error between predictions and available measures), but rather through a much softer approach. Indeed, in the specific application, after the model has produced a trajectory in the love space (see Fig. 4) representing the predicted time evolution of the involvement of Scarlett and Rhett, eleven chronologically ordered short segments of the film (representing the data!) are analyzed and compared pairwise. This is done with the aim of deriving a clear (though subjective) impression on the fact that Scarlett's and Rhett's loves increase or decrease from one segment to the next. After this has been done, it is possible to check if the eleven segments can be allocated along the predicted trajectory in a way consistent with the subjective impressions. In the case of "Gone with the Wind" a very satisfactory allocation is easily obtained so that the agreement of the model and the film is somehow verified (see Fig. 5). However, all this is questionable because it is based on our subjective judgments. For this reason, the reader can go through the same exercise to check if he agrees or not with our proposed allocation. Actually, even more can be done to check the agreement between model and film. In fact, it is possible to derive from the model the predicted chronology of the proposed allocation and then compare it with the chronology approximately indicated in the film (and in the novel). To our great surprise this comparison also turns out to be satisfactory.

Finally, by looking at our work from a slightly different angle, we can say that we have shown for the first time in a very technical way that a love story described in a film is highly realistic because it is consistent with the characters of the involved individuals. Although realism is not necessary from a purely artistic point of view, in most cases it is very much appreciated by the audience, at the point of being strategically important in making the film popular. Thus, we have understood, with a mathematical approach, why "Gone with the Wind" has been one of the most successful films of all times.

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