

# A NEW MATHEMATICAL MODEL OF “GONE WITH THE WIND”

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**ABSTRACT.** In the original paper, the authors extracted twelve scenes from the film “Gone with the Wind” to derive the mathematical model for analyzing the love dynamics between the two main characters of the movie, Scarlett O’Hara and Rhett Butler. Due to significant omissions in the movie plot and simplicity of the parameters set-ups, we believe that the original mathematical model developed by the authors is not accurate enough and can not represent the interactions between the main characters thoroughly. Therefore, we incorporate supplementary movie plots, add additional parameters, and adjust parameters in the original model to derive and support a new love mathematical model that is more comprehensive and fits the love evolution better.

## 1. INTRODUCTION

Our final project for Math 142 is based on one research paper with the name: A mathematical model of “Gone with the Wind” (*Rinaldi et al.*). Before presenting our old model, we’ll first briefly introduce the general idea and old model in the original paper.

To model the fluctuating relationship between Scarlett and Rhett, the original model consists of the following two ODEs, which represent the rate of change for the amount of love for Scarlett and Rhett respectively.

$$(1) \quad \frac{dx_1(t)}{dt} = -\alpha_1 x_1(t) + \rho_1 A_2 + R_1(x_2)$$

$$(2) \quad \frac{dx_2(t)}{dt} = -\alpha_2 x_2(t) + \rho_2 A_1 + R_2(x_1)$$

$$(3a) \quad A_i = \sum_h \lambda_j^h A_i^h \quad (3b) \quad R_i(x_j) = k_i x_j \exp\left(-\frac{x_j}{\beta_i}\right)$$

Specifically, the model consists of three parts:  $-\alpha_i x_i(t)$  represents the oblivion function ( $\alpha_i$  is the forgetting coefficient), the ‘-’ here is due to the fact that Scarlett and Rhett will forget each other gradually when they are separated (*Rinaldi et al.* 3233);

$\rho_j A_i$  denotes the flow of interest generated in individual  $j$  by the appeal of the partner  $i$ , where  $A_i$  is the appeal of individual  $i$  that consists of multiple components  $A_i^h$ , such as richness and beauty, with corresponding weights  $\lambda_j^h$  assigned by individual  $j$ ; and  $\rho_j$  is the sensitivity of individual  $j$  to  $i$ ’s appeal. For example, if  $A_1^1$  represents the richness, the more emphasis the person puts on money, the higher the  $\lambda$  will be. Conversely, if the person does not care if another one is rich or poor, his/her  $\lambda$  will be low (*Rinaldi et al.* 3234);

$R_i(x_j)$  is flow of interest generated in individual  $i$  by the love of the partner  $j$ . In the story context, Scarlett and Rhett are characterized as insecure lovers, which means their reaction to love will first increase with the love of the partner and then decrease once love reaches a certain threshold (*Rinaldi et al.* 3234).

By taking different initial values  $A_1$  and  $A_2$ , and assigning constant values for other parameters:  $\alpha_i = \beta_i = \rho_i = 1$ ,  $k_i = 15$ ,  $i = 1, 2$  (*Rinaldi et al.* 3235), the authors plot the following 3 figures in (A):

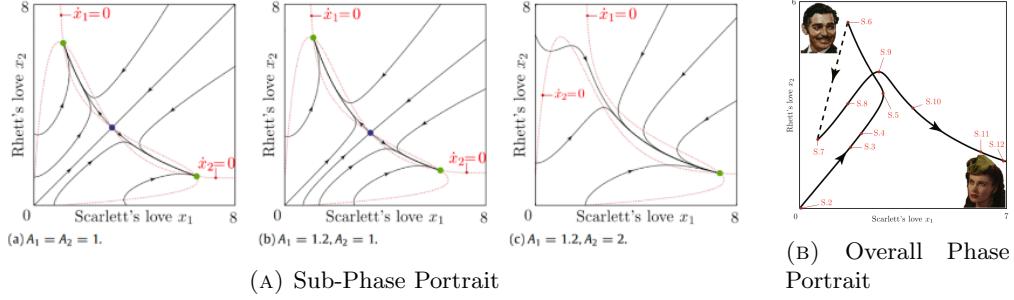


FIGURE 1. Original Phase Portraits

In Figure (b), the equilibrium points are  $(1.126, 6.678)$ ,  $(3.411, 2.889)$ , and  $(6.194, 1.39)$ , where  $(3.411, 2.889)$  is a saddle point and  $(1.126, 6.678)$ ,  $(6.194, 1.39)$  are stable equilibria. The phase portrait shows that Rhett would be much more involved than Scarlett in the long run (*Rinaldi et al.* 3235). In Figure (c), since Rhett is more appealing than Scarlett after the war, the authors set the value of  $A_2$  to be larger than  $A_1$ . The equilibrium point is  $(7.349, 1.271)$ , which is a stable equilibrium, and the model predicts that Scarlett would be much more involved than Rhett in the long term (*Rinaldi et al.* 3235).

These 3 figures are constructed intentionally based on the authors' interpretation of the evolution for Scarlett and Rhett's love. The authors assume that all coefficients in ODEs above are equal for two individuals except for  $A_1$  and  $A_2$ . And Figure (b) and (c) correspond to the two phases of this love story. Considering the appeal change between Scarlett and Rhett, the story should initially evolve along the trajectory starting from the origin in Fig. (b) and then end along a trajectory of Fig. (c) after the war. Finally, the authors present the overall evolution model, which is shown in Figure (B), and conclude that the model accurately predicts the evolution of Scarlett and Rhett's relationship in the film (*Rinaldi et al.* 3237).

## 2. BACKGROUND

Before presenting an analysis and adjustment of the model, it's necessary to briefly summarize the film's plot. In the original paper, the authors summarize the plot in 12 short segments, represented by the following 12 screenshots:

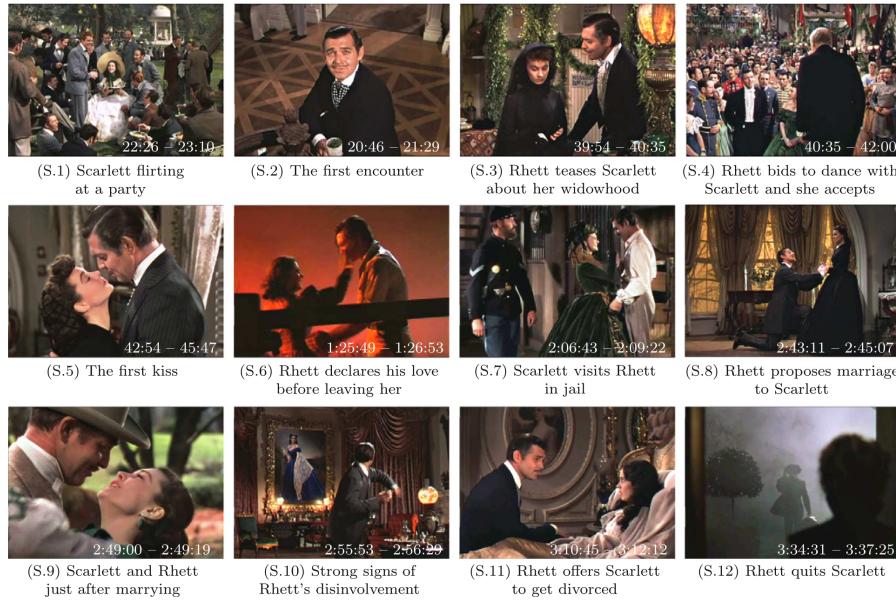


FIGURE 2. Original 12 Plot Segments

Source: See [www.home.dei.polimi.it/rinaldi/GoneWithTheWind/film.html](http://www.home.dei.polimi.it/rinaldi/GoneWithTheWind/film.html).

The film is set around the American Civil War. It follows Scarlett, a privileged Southern woman, and her unrequited love for Ashley, who ultimately marries Melanie. Scarlett later marries the charming but rebellious Rhett, but her heart still belongs to Ashley. The film depicts Scarlett's struggles during the war and her determination to save her family's plantation, Tara. Due to the burning of Atlanta and the changing times, Scarlett evolves from a privileged woman to a resilient survivor. The passionate relationship between Scarlett and Rhett is a focal point, but Scarlett's obsession with Ashley complicates their interactions. The movie ends with Scarlett realizing her love for Rhett, but he leaves her. Scarlett is left alone and returns to Tara, vowing to win Rhett back someday (*Rinaldi et al.* 3232-3233).

### 3. OUR MODEL

#### **Limitation 1:** Lack of Code for Phase Portraits and Trajectory

The original paper did not provide any code for plotting the phase portraits and trajectories.

To address this limitation, we built our own code from scratch to reproduce the figures presented in the original paper. Our code allowed us to visualize more clearly the dynamics of the system, including the identification of fixed points, nullclines, and vector fields. This enabled us to gain insights into the behavior of the model and the trajectories of the love evolution between Scarlett and Rhett.

#### **Limitation2:** Lack of necessary film plots

The second limitation of the original paper is the lack of important plots in the movie.

In addition to the 12 scenes used as a reference by the authors to describe the evolution of feelings between Scarlett and Rhett, we propose adding other scenes that we deem important and necessary for inclusion in our new model:

- **Scene 6.5**

S6 is oversimplified: in the paper, the authors only mention Scarlett's anger towards Rhett deserting her and joining the army. But actually from the movie we can perceive that Scarlett's really grateful about Rhett rescuing her from the Yankees and giving her support during that tough period of war, especially when we consider the hardship and struggle during this escape.

- **Scene 7.5**

After S.7, when Scarlett gets angry and upset after Rhett rejects to lend her money to save her Tara, on her way back, she meets her sister's fiancé, Frank. Frank, a successful man with a store and a logging business, is manipulated by Scarlett, leading to their marriage, and Scarlett becomes wealthy again. But Frank dies tragically in a subsequent incident, and Scarlett is again widowed.

- **Scene 10.5**

One day, Scarlett comforts Ashley after he expresses his desire to return to a simpler life. Ashley's sister, India, overhears this and spreads the rumor about Scarlett seducing a married man to dishonor her, which also leads to Rhett's embarrassment and anger towards Scarlett.

In the original paper, the authors only say that 'there is an argument' between Scarlett and Rhett, but we think the reason behind this argument matters a lot. We've incorporated this change into our new model, which will be explained in later parts.

- **Scene 11.5**

Later when Rhett returns home with Bonnie, Scarlett falls down the stairs during an argument with Rhett and has a miscarriage, which leads to Rhett's guilt and Scarlett's resentment.

#### **Limitation3:** Did not give the value of each $\lambda_i$ and not indicate how each $A_i$ is calculated.

- **Assumption Modification**

The authors state that Rhett is only interested in Scarlett's beauty, i.e., they assume A1 is constant throughout time. The authors also assume that Rhett's wealth is the only factor of his appeal to Scarlett, and they further claim that Scarlett doubles the weight of Rhett's wealth, i.e., A2 is doubled, in the entire latter phase of their model, starting from S.7. However, we think that Rhett is also attracted by Scarlett's other characteristics, and likewise Scarlett is also attracted by Rhett's other features. (see Parameter Modification below)

- **Parameter Modification**

Compared with the other parameters in the old model, such as  $\alpha_i$ ,  $k_i$ , and  $\beta_i$ , the parameter  $A_i$  seems to be easier to modify and does not require too much further background knowledge. For this reason, in our new model, we've only made changes to  $A_i$ . Recall its definition,  $A_i$  represents the appeal of individual  $i$  and consists of different factors denoted as  $A_i^h$ , such as beauty and wealth. In the original model and our new one, these components are assumed to be independent of the feeling  $x_i$  for the partner. Additionally,  $\lambda_j^h$  is introduced as the weight that individual  $j$  (where  $j \neq i$ ) assigns to the  $h$ -th component of the appeal of individual  $i$ . Combined, the multiplication of  $A_i^h$  and  $\lambda_j^h$  equals to  $A_i$ .

$$A_1 \text{ (Scarlett's appealing to Rhett)} = A_1^1 \cdot \lambda_2^1 + A_1^2 \cdot \lambda_2^2$$

Here we set  $A_1$  to have 2 components, i.e.,  $A_1^1$  and  $A_1^2$ .  $A_1^1$  is Scarlett's beauty, while  $A_1^2$  is Scarlett's feelings towards Ashley from Rhett's point of view.  $\lambda_2^1$  and  $\lambda_2^2$  are weights Rhett gives to them respectively.

**For  $A_1^1$ :** We follow the paper such that the level of Scarlett's beauty perceived by Rhett is unchanged throughout the film, so we set  $A_1^1 = 1.2$  and  $\lambda_2^1 = 1$  for all the time.

**For  $A_1^2$ :** We let  $A_1^2 = 1$  throughout the time and  $\lambda_2^2$ 's value varies. Before Scarlett's marriage with Rhett (S1-S8), we let  $\lambda_2^2 = -0.1$ ; after their marriage (S8-S10), we set  $\lambda_2^2 = -0.3$ , then  $\lambda_2^2 = -0.7$  when Rhett hears the rumor of Scarlett hugging Ashley (S10-end). It's reasonable for  $\lambda_2^2$  to have a negative value. Before their marriage, Rhett does not care that much about Scarlett's feelings towards Ashley, but afterwards he may worry that such feelings can affect their marriage and things get worse through later parts of the movie.

$$A_2 \text{ (Rhett's appealing to Scarlett)} = A_2^1 \cdot \lambda_1^1 + A_2^2 \cdot \lambda_1^2$$

Here we set  $A_2$  to have 2 components, i.e.,  $A_2^1$  and  $A_2^2$ .  $A_2^1$  is Rhett's richness, and  $A_2^2$  is Rhett's giving for Scarlett.  $\lambda_1^1$  and  $\lambda_1^2$  are weights Scarlett gives to them respectively.

**For  $A_2^1$ :** We can observe from the film that Rhett seems to be rich all the time, so we set  $A_2^1 = 1$  regardless of time. As for its weight, before the war (S1-S6), Scarlett's in a good financial situation, so we set  $\lambda_1^1 = 0.5$ . But during and after the war (S6-S6.5 and S7-S7.5), Scarlett really needs money to save her plantation Tara and she knows Rhett is really wealthy, so we set  $\lambda_1^1 = 1.5$ . Shortly after that, we set  $\lambda_1^1 = 1$  since Scarlett becomes the owner of a factory from her new husband Frank (S7.5-S8). Finally, we set  $\lambda_1^1 = 0.7$  after Scarlett marries Rhett (S8-end).

**For  $A_2^2$ :** We set  $A_2^2 = 1$  throughout the time due to Rhett's constant giving to Scarlett such as buying her gifts and saving her from the Yankees, etc. For its weight, we set  $\lambda_1^2 = 0.5$  before the war (S1-S6);  $\lambda_1^2 = 1.5$  after the war (S6-S6.5 and S7-S7.5) and  $\lambda_1^2 = 1$  (S7.5-S8) for the same reason as above; finally, we set  $\lambda_1^2 = 0.5$  again after their marriage.

Based on our setting and changes above, the new ODE system becomes:

$$(4) \quad \frac{dx_1}{dt} = -x_1(t) + \lambda_2^1 A_2^1 + \lambda_2^2 A_2^2 + 15x_2 \exp(-x_2)$$

$$(5) \quad \frac{dx_2}{dt} = -x_2(t) + \lambda_1^1 A_2^1 + \lambda_1^2 A_2^2 + 15x_1 \exp(-x_1)$$

The values of the new  $A_1$  and  $A_2$  we got for the above six different parts are listed below:

Part 1 (S.2 – S.6):  $A_1 = 1.1$ ,  $A_2 = 1$

Part 4 (S.7.5 – S.8):  $A_1 = 1.1$ ,  $A_2 = 2$

Part 2 (S.6 – S.6.5):  $A_1 = 1.1$ ,  $A_2 = 3$

Part 5 (S.8 – S.10):  $A_1 = 0.7$ ,  $A_2 = 1.2$

Part 3 (S.7 – S.7.5):  $A_1 = 1.1$ ,  $A_2 = 3$

Part 6 (S.10 – S.12):  $A_1 = 0.45$ ,  $A_2 = 1.2$

#### 4. RESULTS

In this section, the results of our new model will be presented, including the phase portraits and trajectory.

On the one hand, having new  $A_1$  and  $A_2$ , we obtain the following results of **phase portraits** for the six different parts using our own codes.

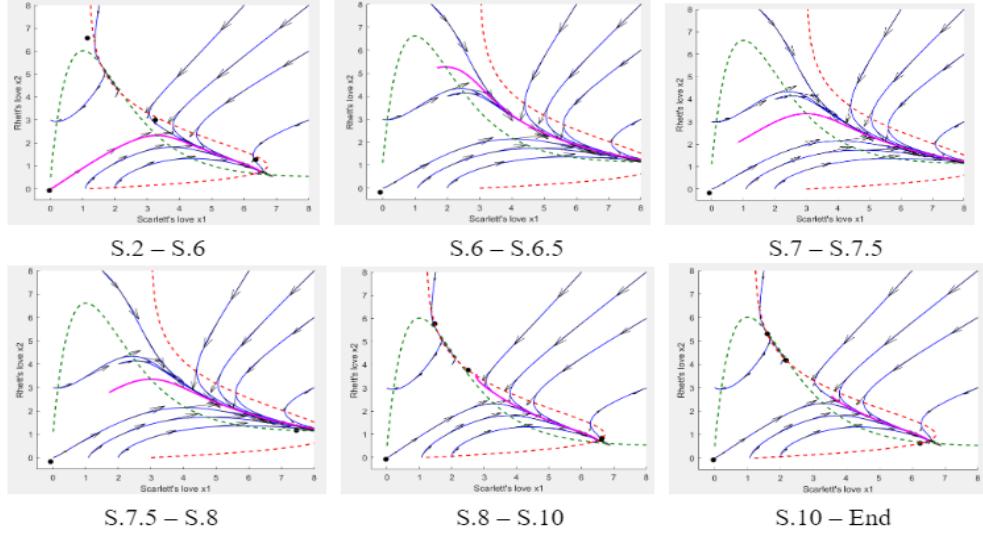


FIGURE 3. New Phase Portrait

On the other hand, we've plotted the comparison for the **trajectory** of love's evolution for Scarlett and Rhett between the old and new model:

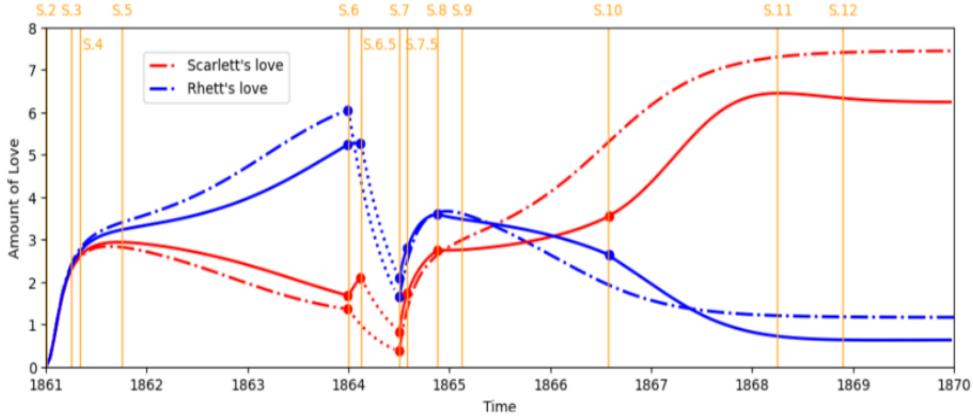


FIGURE 4. Old vs. New Trajectory Comparison

It might be overwhelming at first glance to see this picture, so I'll provide some necessary explanations for you to better understand the plot. For corresponding trajectories of these separate time intervals, please refer to Appendix C for details.

Firstly, let's take a look at the legend in Fig.4. Here the red line represents Scarlett's love while the blue line is for Rhett's. You can also notice there are two kinds of lines, the dotted ones, which correspond to trajectory from the old model, and the solid/continuous line represents the trajectory of our new model. Now let's go through each part one by one.

For the first two parts (S2-S6-6.5), due to our change of coefficients, Rhett's love is lower in the new model while Scarlett's love is higher, but the overall trend is similar. What's different is that we've added a short segment from S6 to S6.5 about 1-2 months, during which time Scarlett's condition changes from a rich and loved girl to a lost woman in the war. Starting from this point, we increase Scarlett's weight assigned to both Rhett's wealth and giving due to Scarlett's lack of both care and money. But Scarlett's appeal to Rhett does not change in this interval.

After S6.5, Scarlett and Rhett separate for several months, and during this period only forgetting coefficients play a role in our model, until S7, where part 3 begins. At this time, the Civil War ends, and Scarlett

is still in bad financial condition related to her plantation Tara, so she wants Rhett to lend her money to save Tara. Although Rhett doesn't lend the money, at the beginning of part 4 (S7.5), as explained previously, Scarlett marries Frank who runs a successful business, and she becomes wealthy again. As a result, the weights Scarlett assigns to both Rhett's richness and giving for part 3 are still high and then we decrease them in part 4, so A2 is 3 in part 3 and 2 in part 4. But Scarlett's appeal to Rhett is still the same as in part 2 and does not change in both parts 3 and 4. The trajectory of our new model does not change a lot compared with the old one, so it seems that the old model is quite valid in parts 3 and 4.

As for the part S8-S10, which continues from the previous part, Scarlett's new husband Frank dies later in an incident, which leads to Scarlett's marriage with Rhett, which has a big impact on our new model. Specifically, we increase the magnitude of Rhett's weight on Scarlett's feelings to Ashley from 0.1 to 0.5, since it's reasonable for a husband to care about whether there is 'the other man' in his marriage. Now A1 becomes 0.7 rather than 1.1 in previous parts. Also, since now Scarlett has both plenty of money and love from Rhett, so we decrease the weight she assigns to Rhett's richness and giving. Now A2 changes from 2 to 1.2. We can observe that the trajectory of our new model is smoother during this time interval, which is more accurate since there are not many ups and downs for Scarlett and Rhett's feelings towards each other at this time.

Finally, in the last part S10-end, we decrease the value of Scarlett's appeal to Rhett (A1) again due to the plot that Rhett finds some evidence that Scarlett is still in love with Ashley. Now A1 decreases from 0.7 to 0.45. While Rhett's appeal to Scarlett does not change. The overall pattern of the new model is similar to the old one, except that there is a slight decrease in Scarlett's love for Rhett around S11, which can not be recognized in the old trajectory. Such a slight decrease makes sense since at that time Scarlett falls down the stairs and has a miscarriage during an argument with Rhett, which decreases her love for Rhett, and shows that our model is more accurate. Even though we've not deliberately changed coefficients of our model at this specific time, it turns out that our trajectory still matches the original plots much better.

## 5. CONCLUSIONS AND DISCUSSION

To sum up, in the new model, we've incorporated more necessary details from the original movie plots, reassigned different parts of our new ODE system based on the plots, and introduced more reasonable values for the parameters, more specifically, for A1 and A2, into our system. Based on the phase portraits as well as the overall trajectory generated from our code, it shows that our new model is more accurate and comprehensive, and it fits much better with the film's plots compared to the old model.

Due to the time constraints, we think there are also other possible improvements to our model.

Firstly, even though we've already modified the components of A1 and A2, we would still like to conduct further investigation into other potential factors that can affect A1 and A2, such as the impact of the social environment at that time or the influence from other characters in the movie. Besides this point, we can also include the modification of parameters other than  $A_i$  in the model, such as  $\alpha_i$ ,  $k_i$ , and  $\beta_i$ .

In the original paper, the authors assume that  $\alpha_i$ , which is the forgetting coefficient, is constant throughout the entire movie. However, if we plug in different values of  $\alpha_i$  on different time intervals, we might acquire results that are more satisfying and align better with the storyline. For example, it's reasonable to suppose that the forgetting coefficient may be different during the time when Scarlett and Rhett are still in contact and when they are actually separated.

In addition to the modification of the value of  $\alpha_i$ , we would also like to consider the values of  $k_i$  and  $\beta_i$  more carefully because the authors only give their values in the original paper but did not explain the exact meaning for these two parameters and how to set the values of them. Modification on them may require knowledge and skills outside of the scope of this class. Similarly, when we've fully investigated the meanings behind  $k_i$  and  $\beta_i$ , we may further assign different values for them during different intervals to make our model more accurate.

Finally, we would like to include more plots in our model. For example, at the beginning of the movie, shortly after Scarlett's first meeting with Rhett, Rhett happens to oversee Ashley rejecting Scarlett's advances. Then it's reasonable to assume that Scarlett may have a bad first impression on Rhett, which may lead to a negative initial value for Scarlett's love for Rhett and will produce a huge impact on the later evolution of their love.

## REFERENCES

Rinaldi, Sergio, et al. "A mathematical model of 'gone with the wind.'" *Physica A: Statistical Mechanics and Its Applications*, vol. 392, no. 15, 2013, pp. 3231–3239, <https://doi.org/10.1016/j.physa.2013.03.034>.

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## APPENDIX A. GROUP CONTRIBUTIONS

**Zhiqi Ma:** Wrote Python code for solving our ODE system and plotting trajectory together with Yue Yang; Adjusted parameters, i.e. A1 & A2, and incorporated additional movie plots for our new model based on limitation of the old model; Contributed to the main content of our final presentation and project report.

**Zhihan Chen:** Analyzed the parameters of the model; Modified the MATLAB code to generate the line from the given starting point to the fixed point; Helped find the new fixed point and plot phase portrait; Wrote up part of the final report.

**Yue Yang:** Wrote the MATLAB and Python code for solving the equations, and generating the phase portraits and trajectories of both old and new models; Found limitations for the old model; Improved the old model by adding parameters and giving values; Wrote the content for presentation.

**Hung-Yu Chen:** Analyzed the old model from the original paper with Zhihan Chen; participated in the discussion to design a new model; helped find the new fixed point and plot phase portraits; did the old model part of the presentation; write up the partial final report.

## APPENDIX B. CODE REPOSITORY

The Python and MATLAB code we wrote while working on this project can be found here: [Click here](#) to visit the GitHub repository.

## APPENDIX C. SUPPLEMENTARY PLOTS

For further reference, supplementary plots are presented here due to space constraints, and they are all generated by codes we've written ourselves:

- The first plot is the new trajectory of our model:

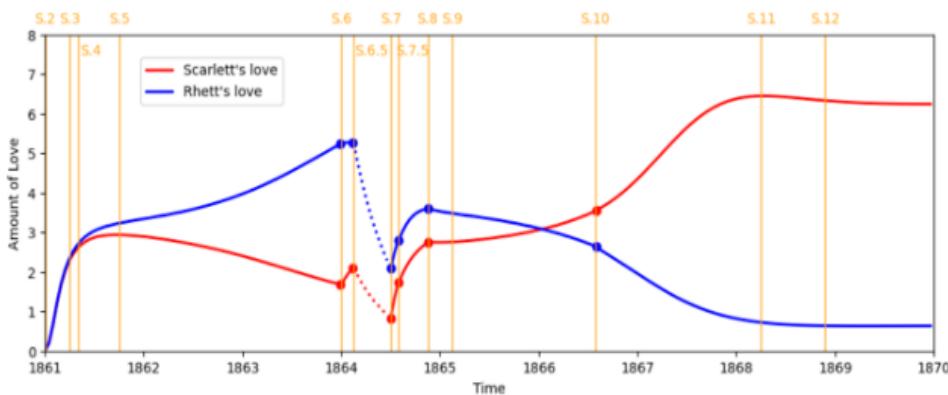


FIGURE 5. New Trajectory

- We've also computed the exact values for the amount of love for Scarlett and Rhett at each specific time points, and the values were collected into the table below for more direct and quantitative comparison between the old model and our new model. Here x1 old/new denotes the amount of Scarlett's love for Rhett in the old/new model; and similarly x2 old/new denotes the amount of

Rhett's love for Scarlett in the old/new model;  $ts_i$  denotes specific movie scene corresponding to the point in the trajectory above.

Scenes	x1 old	x1 new	x2 old	x2 new
ts2	0	0	0	0
ts3	2.26	2.27	2.38	2.33
ts4	2.56	2.59	2.75	2.68
ts5	2.82	2.94	3.39	3.22
ts6	1.36	1.68	6.05	5.23
ts6.5		2.1		5.26
ts7	0.37	0.83	1.65	2.08
ts7.5		1.72		2.79
ts8	1.62	2.76	2.78	3.6
ts9	2.94	3.07	3.73	3.53
ts10	4.72	3.55	2.29	2.64
ts11	7.05	6.39	1.35	1.02
ts12	7.26	6.63	1.29	0.87

FIGURE 6. Results Table for Amount of Love

- Then plots below correspond to each segment of comparison between the old trajectory and the new one:

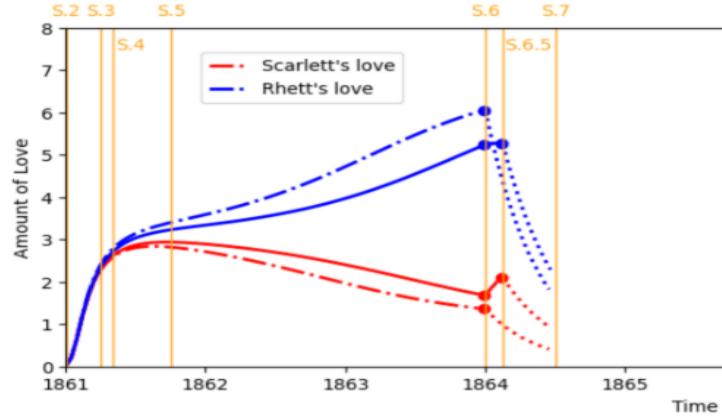


FIGURE 7. Part 1 and 2: S2-S6-S6.5



FIGURE 8. Part 3 and 4: S7-S7.5-S8

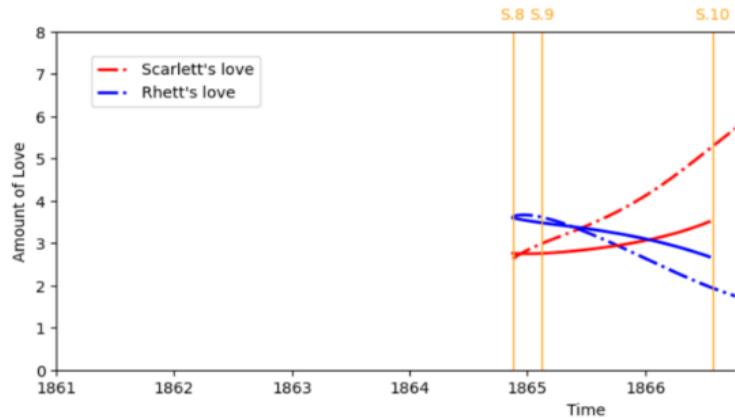


FIGURE 9. Part 5: S8-S10

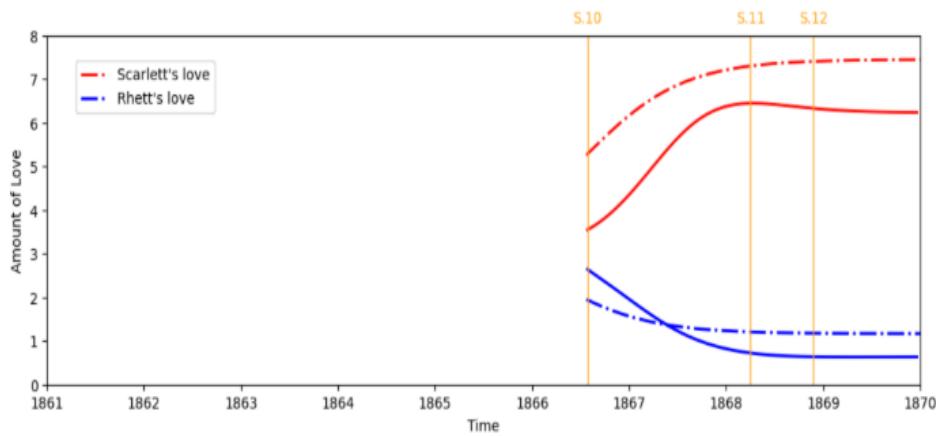


FIGURE 10. Part 6: S10-End