

# VAE Problem Set

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In the lecture of VAE, we've learned that the variational autoencoder (VAE) as a tweak of autoencoder with given objective. In this problem set we will explore how ELBO is the objective function of VAE.

## 1 Problem 1

Assume the observed variable  $x$  is random sampled from a distribution  $p^*(x)$  that's unknown, our VAE is going to approximate a model  $p_\theta(x) \approx p^*(x)$ .

Given the log marginal likelihood of  $x$ :

$$\log p_\theta(x) = \mathbb{E}_{q_\phi(z|x)}[\log q_\theta(x)]$$

Rewrite it to separate it into two terms:

$$\log p_\theta(x) = \mathbb{E}_{q_\phi(z|x)}[\log[\frac{p_\theta(x, z)}{q_\phi(z|x)}]] - KL(q_\theta(z|x)||p_\theta(z))$$

## 2 Problem 2

Rewrite the equation you get from question 1 to explain why the term  $\log[\frac{p_\theta(x, z)}{q_\phi(z|x)}]$  would be called evidence lower bound (ELBO).