In the lecture of VAE, we've talked about VAE by introducing it as a tweak of Autoencoder. In this Problem set for VAE, we are going to explore the probalility intuition of VAE as a generative model.

1. Assume the observed variable x is random sample from a distribution $p^*(x)$ that's unknown, our VAE is going to approximate a model $p_\theta(x) \approx p^*(x)$

$$\log p_\theta(x) = \mathbb{E}_{q_\phi(z|x)}[\log q_\theta(x)]$$

rewrite it to seperate it into 2 models: inferene model and generative model;

Answer:

$$= \mathbb{E}_{q_\phi(z|x)}\left[\log\left[\frac{p_\theta(x,z)}{p_\theta(z|x)}\right]\right]$$

$$= \mathbb{E}_{q_\phi(z|x)}\left[\log\left[\frac{p_\theta(x,z)}{q_\phi(z|x)} \; \frac{q_\phi(z|x)}{p_\theta(z|x)}\right]\right]$$

$$= \underbrace{\mathbb{E}_{q_\phi(z|x)}\left[\log\left[\frac{p_\theta(x,z)}{q_\phi(z|x)}\right]\right]}_{\substack{= L_{\theta,\phi}(x) \\ (\text{ELBO})}} + \underbrace{\mathbb{E}_{q_\phi(z|x)}\left[\log\left[\frac{q_\phi(z|x)}{p_\theta(z|x)}\right]\right]}_{\substack{= D_{KL}(q_\phi(z|x) \| p_\phi(z|x)) \\ \text{KL Divergence}}}$$