Warsaw University of Technology

FACULTY OF BUILDING SERVICES,
HYDRO AND ENVIRONMENTAL ENGINEERING

Assignment 05 - Numerical differentiation and integration

Computational Methods in Environmental Engineering

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5.1 Newtonian fluids

y (m)	0	0.002	0.004	0.006	0.008
c (m/s)	0	0.00618	0.011756	0.01618	0.019021

First of all, I bring all the given data into the MATLAB and came out the spline interpolation. The graph shows in below.

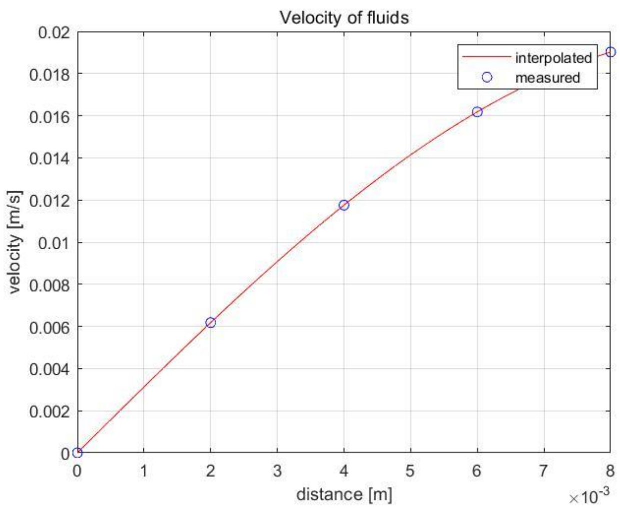

Fig 1 – The interpolation curve of the velocity of fluid

I then performed a quadratic and cubic fit on this plot to get two fitting formulas and show the corresponding residual norms.

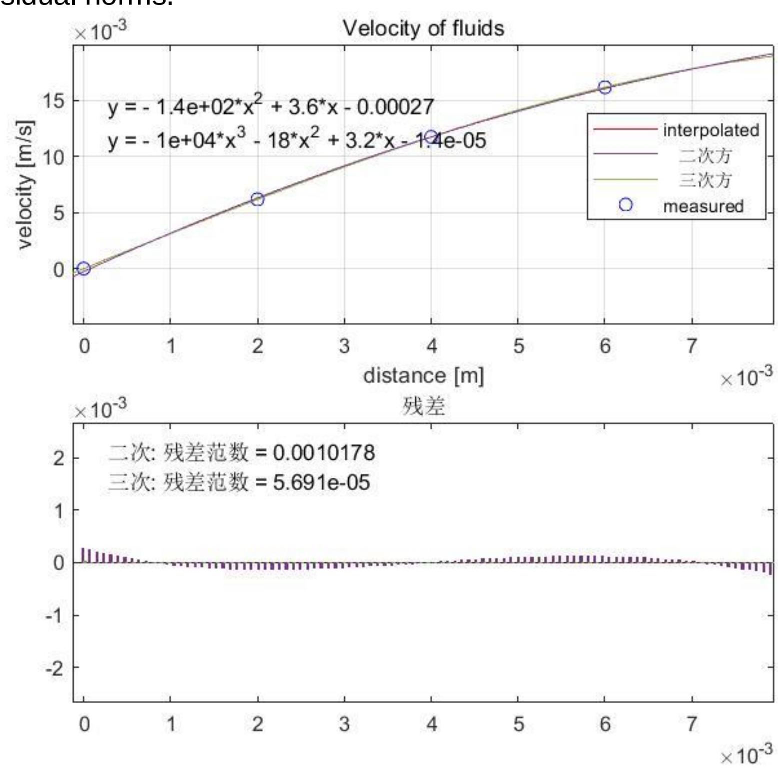

Fig 2 – The quadratic and cubic fit of the given data

The formula for the quadratic and cubic fitting curve of this data from Figure 2 is as follows:

$$y = -1.4 \times 10^2 \cdot x^2 + 3.6 \cdot x - 0.00027 \quad 1)$$

$$y = -1 \times 10^4 \cdot x^3 - 18 \cdot x^2 + 3.2 \cdot x - 1.4 \times 10^{-5} \quad 2)$$

a) Calculate the sharing stress at y=0, when the coefficient of dynamic viscosity u= 0.002 Ns/m²

The newton's equation is:

$$\tau_{yx} = \mu \frac{\partial c}{\partial y}$$

As we know the $\frac{\partial c}{\partial y}$ can be present as the derivative:

$$\frac{\partial c}{\partial y} = \frac{\partial y}{\partial x} = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$\Delta x = \frac{(0 + 0.002 + 0.004 + 0.006 + 0.008)}{5} = 0.004 \text{ [m]}$$

So the $\frac{\partial c}{\partial y}$ at y=0 can be seen as:

$$\frac{\partial c}{\partial y} = \lim_{\Delta x \to 0} \frac{f(0 + 0.004) - f(0)}{0.004} = \lim_{\Delta x \to 0} \frac{f(0.004)}{0.004}$$

Bring the quadratic fitting formula and cubic fitting formula into this derivative function:

$$\frac{\partial c}{\partial y_1} = \frac{-1.4 \times 10^2 \cdot 0.004^2 + 3.6 \cdot 0.004 - 0.00027}{0.004} = 2.9725$$

$$\frac{\partial c}{\partial y_2} = \frac{-1 \times 10^4 \cdot 0.004^3 - 18 \cdot 0.004^2 + 3.2 \cdot 0.004 - 1.4 \times 10^{-5}}{0.004} = 2.9645$$

Finally bring them back to the Newton's equation and calculate the shearing stress separately:

$$\tau_1 = 0.002 \times 2.9725 = 0.005945 \text{ [N/m}^2\text{]}$$

$$\tau_2 = 0.002 \times 2.9645 = 0.005929 \text{ [N/m}^2\text{]}$$

b) calculate the derivative using two-points forward formula and improve the accuracy of the result with Richardson Extrapolation

When I calculate the derivative by two-points forward method, I choose two different step size, one is 0.002 another one is 0.004, and the function i choose is the cubic fitting equation (equation 2) for this series of data, then I performed them individually in the MATLAB, and received the following results. In additional, I performed Richardson Extrapolation for this calculation to improve the accuracy of my derivative result:

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>> assignment05_1b

The value of derivative with 0.002 step size is = 3.124        2.812        2.26        1.468

The value of derivative with 0.004 step size is = 2.968        2.536        1.864

The Derivative value of Richardson Extrapolation   =3.2
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I also obtained the following graph to visually show the difference in results accroding to different step size:

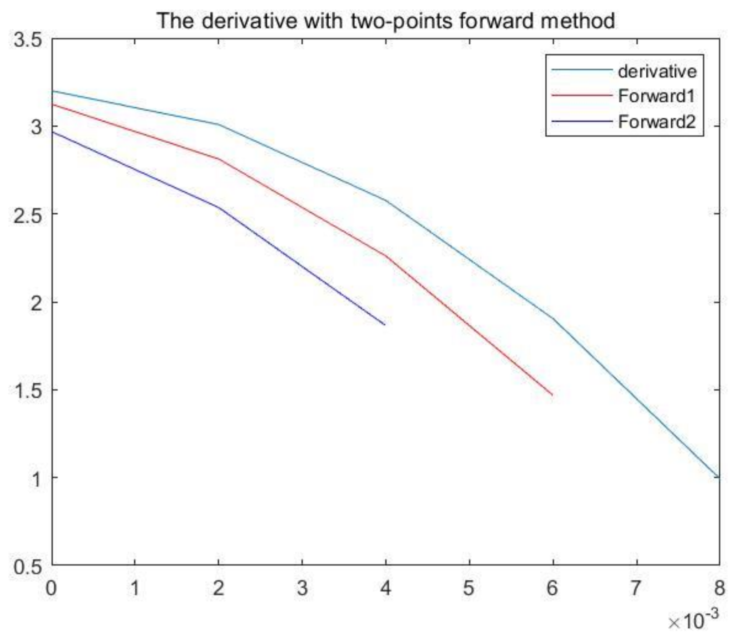

Fig 3 – The derivative results of two-points forward method with two different step size

From Fig 3, we can see three derivative curves. The top blue curve is derived from the cubic fitting equation of the given data. The middle red curve is derived when step size =0.002 under applying the two-point forward method, and the bottom purple curve is obtained when the step size is 0.004.

It can be clearly seen from the figure that these three curves become shorter in sequence. The larger the step size, the larger the range of data obtained and the less accurate it is. After Richardson Extrapolation makes the results more accurate, comparing these three groups of results, It can be concluded that the size of the step size affects the accuracy of the result. The smaller the step size, the higher the accuracy.

5.2 Mass of the Earth

The density of Earth according to the different radius

r (km)	0	800	1200	1400	2000	3000	3400	3600	4000	5000	5500	6370
ρ(kg/m3)	13000	12900	12700	1200	11650	10600	9900	5500	5300	4750	4500	3300

The mass of the Earth is:

$$M = \int_0^{6370} \rho 4\pi r^2 dr$$

1) **Interpolate the density**

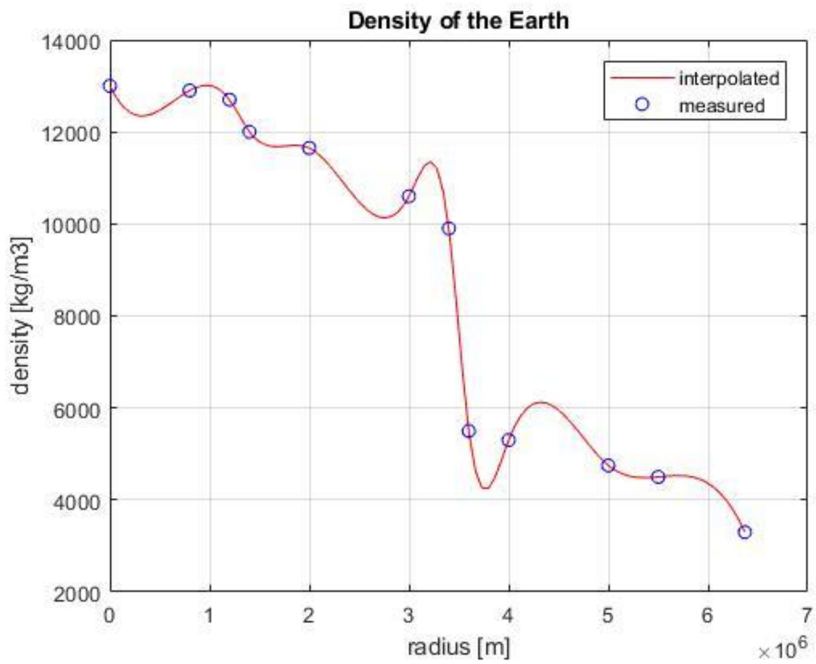

Fig 4 – The interpolated curve of the density of the Earth

2) **Calculate the mass of the Earth using the interpolated data and trapezoidal method, find the density of Earth at the radius equals to 3200km, using spline and linear interpolation**

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>> asignment05_2

Mass of the Earth using interpolation is =6.234992749119933e+24kg

Density at 3200 km using spline interpolation is =11350.3635kg/m3

Density at 3200 km using linear interpolation is =10250kg/m3
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5.3 The flow rate

The flow rate Q in the pipe is:

$$Q = \int_0^R 2\pi r v dr$$

The relationship between the flow rate and the pipe radius:

r [in]	0.0	0.25	0.5	0.75	1.0	1.25	1.5	1.75	2.0
v [in/s]	38.0	37.6	36.2	33.6	29.7	24.5	17.8	9.6	0

1) Interpolate the data

First of all, I use spline method to interpolate the data then obtained the following graph. Besides, I fitted this curve and got a cubic formula:

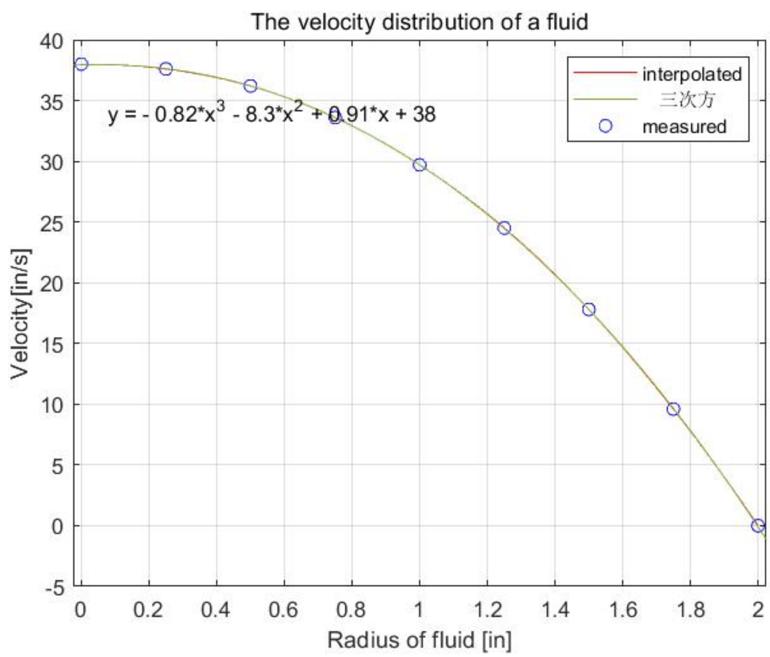

Fig 5 – The interpolated curve of the velocity of the fluid

The cubic formula of velocity I got is：

$$y = -0.82 \cdot x^3 - 8.3 \cdot x^2 + 0.91x + 38$$

Bring this formula into the flow rate Q equation, then I can obtain the function of Q:

$$y = 2\pi x(-0.82 \cdot x^3 - 8.3 \cdot x^2 + 0.91x + 38)$$

2) calculate Q by using interpolated data and Simpson method and use trapezoid method

I applied the Simpson's method and trapezoid method to calculate Q individually in MATLAB, the obtained figure shows below：

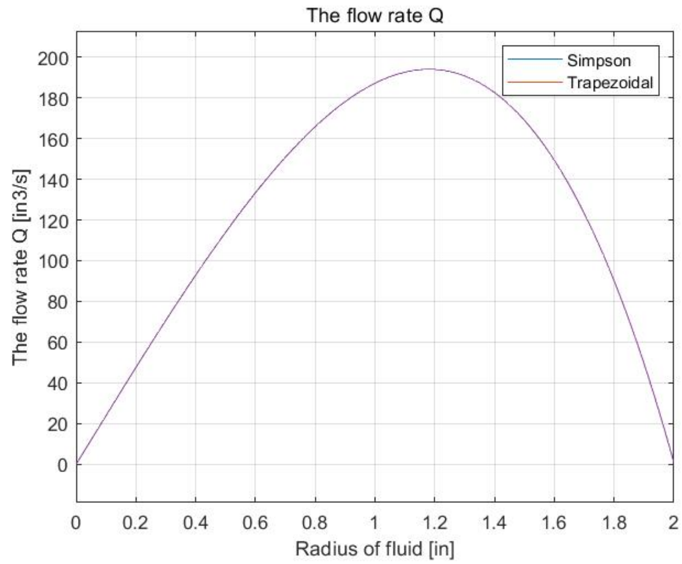

Fig 6 – The result of flow rate calculated by Simpson and Trapezoidal method

When we observe the calculated curve, we find that the result curves calculated by the two methods basically coincide, so it can be said that the accuracy of the results of the two methods is basically the same.

However, because the different sub-intervals draw the different results, so I simulated two times by change the sub-intervals n to see the difference:

1st when n=100:

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>> assignment05_3

Enter the number of sub-intervals (even value):100

value of integral (Simpson's method): 251.1934

Enter the number of sub-intervals: 100

value of integral (trapezoidal method): 251.1678
```

2nd when n=10000:

```
>> assignment05_3

Enter the number of sub-intervals (even value):10000

value of integral (Simpson's method): 251.1934

Enter the number of sub-intervals: 10000

value of integral (trapezoidal method): 251.1934
```

Comparing the simulation results of these two methods, I can draw the following conclusions: the more sub-intervals are calculated, the more accurate the calculation results are; Because the Simpson's method uses the fewer sub-intervals to obtain the final accurate results, so the Simpson method is more accurate than the trapezoidal method.