THE SHALLOW WATER MODEL
ASSIGNMENT REPORT

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Submitted on: 19/12/2019
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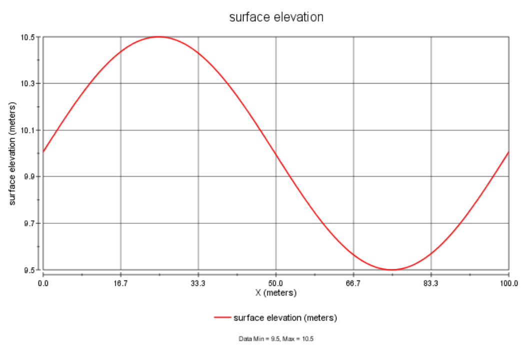
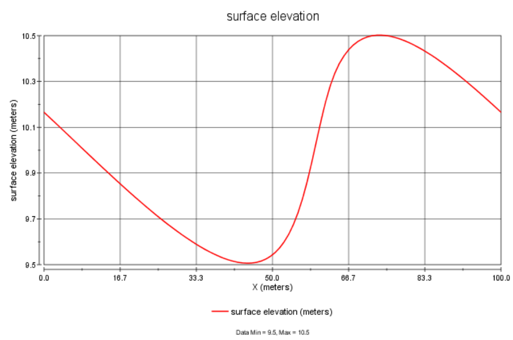
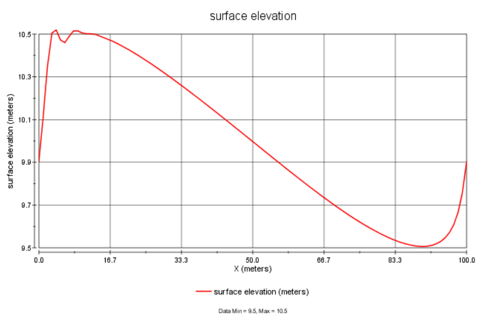
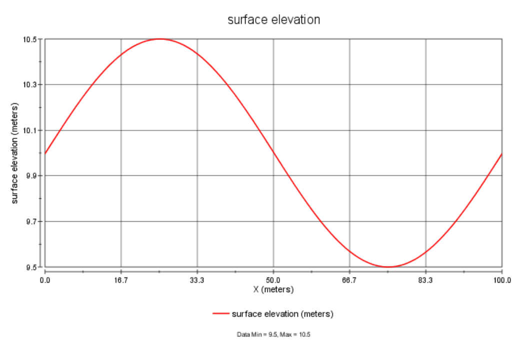
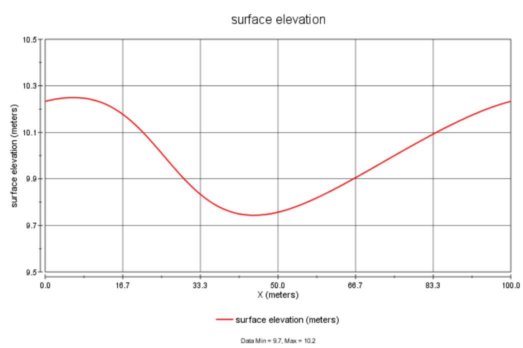
Assignment 1 A non-linear surface gravity wave propagation

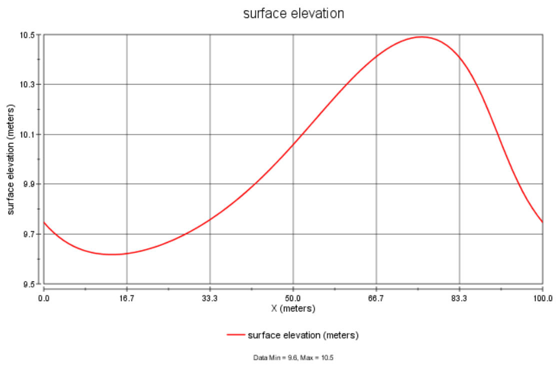
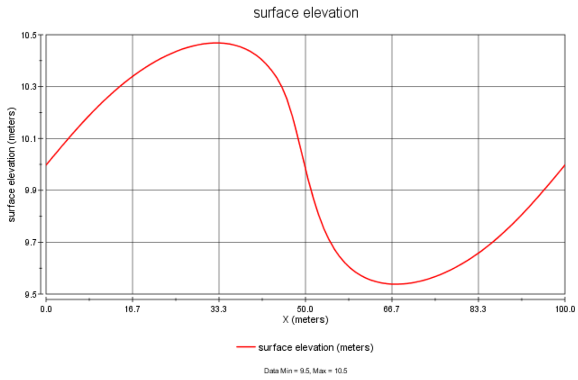
Applying a multiplication factor to the relationship between amplitudes HAMP and UAMP:

1): UAMP=np.sqrt(g*h0) *HAMP/h0*(-1)

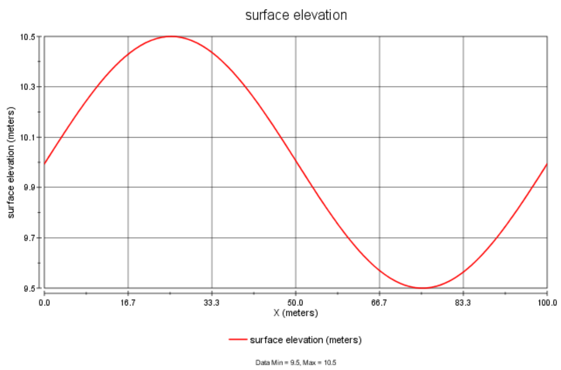
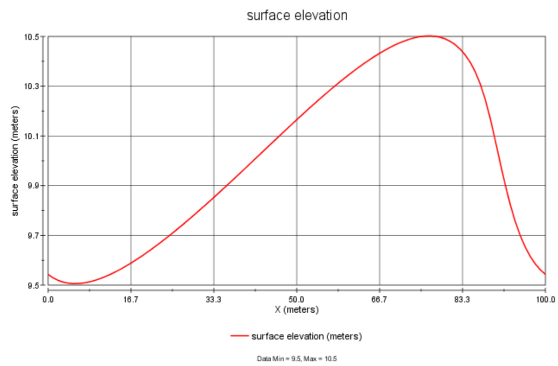
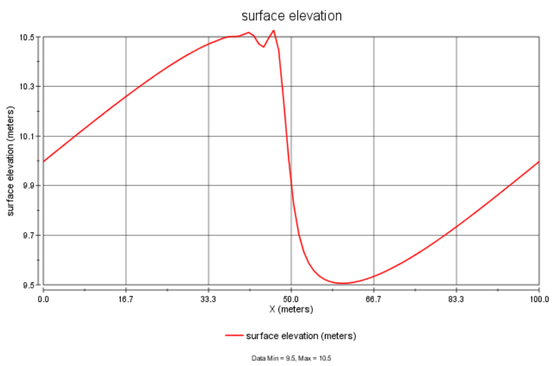
T=1	T=705
	
T=1000	
	

2): UAMP=np.sqrt(g*h0) *HAMP/h0*(0.5)

T=1	T=376
	

T=1000	
	

3): UAMP=np.sqrt(g*h0) *HAMP/h0*(1)

T=1	T=705
	
T=1000	
	

ANALYSIS:

In the above attempts, when the multiplication factor of the velocity amplitude is changed to different values, the output figure will be changed as well.

In the first set, the multiplication factor is -1. With the time passing, the peak of the surface elevation is moving from right to left, the shape of the graph tilts to the left. While at the end of the graph, the highest peak appeared and 2 peaks are so closed to each other.

In the second set, the multiplication factor is 0.5. with the time passing, the peak is moving from left to right. Meanwhile the amplitude of the shape becomes smaller firstly, then increases from time to time. What's more, the shape tilts to right gradually.

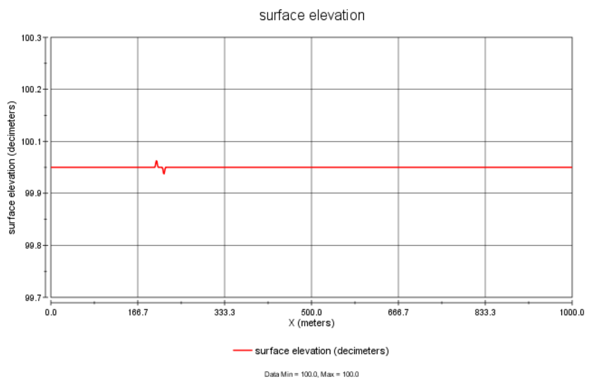
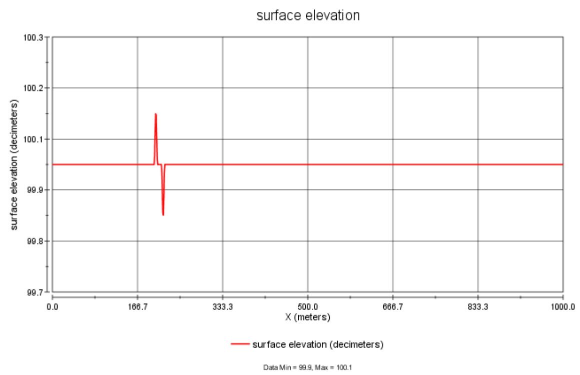
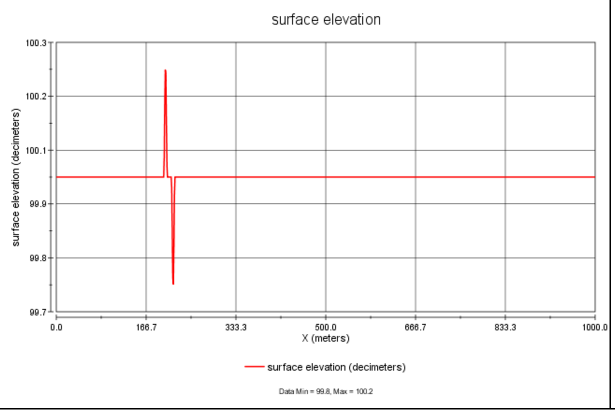
In the third set, the multiplication factor is changed to 1. Compared them with the first set, they are familiar to each other, however, the peak is moving left to right, and the shape of the graph tilts to right.

Compared with all these three sets figures, what can be concluded that if the multiplication factor is smaller than 0, the moving direction and the tilted direction will be opposite to that which is multiplied with the positive number. If the absolute values of multiplication factors are smaller than 1, the amplitude of the shape will be smaller than that with one as well. the velocity amplitude can determine the change of the surface elevation somehow. Due to velocity amplitude is consistent with surface elevation amplitude.

Assignment 2 Experiment the effect of the flow impingement upon an obstacle

1. First, simulate the influence of the high of the obstacle HOBST in the flow

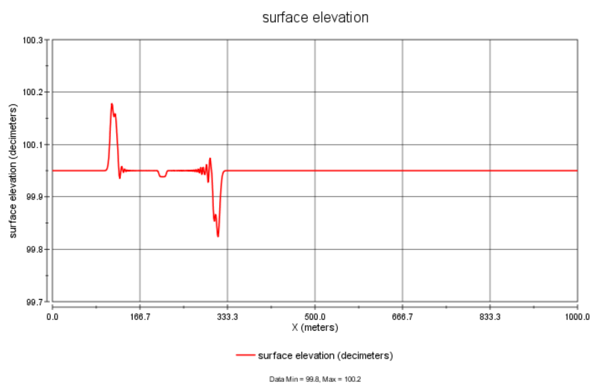
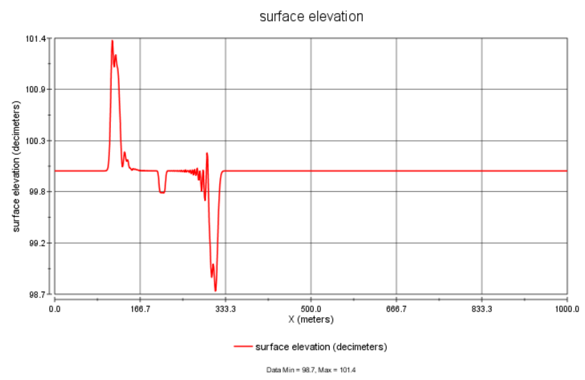
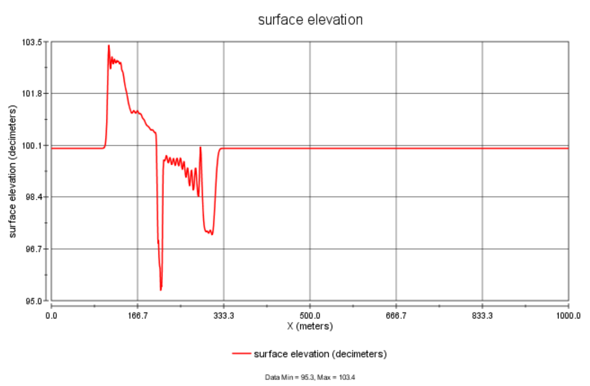
1) At the same time t=1 and same initial velocity u_0=0.5, compared with three different obstacle heights

HOBST= 0.5m	HOBST= 4m
	
HOBST=8m	
	

ANALYSIS:

Compared with these 3 figures, the height of these three obstacles is different. The range of the y axis is the same. From the comparison, with the increase of the obstacle height the surface elevation becomes bigger.

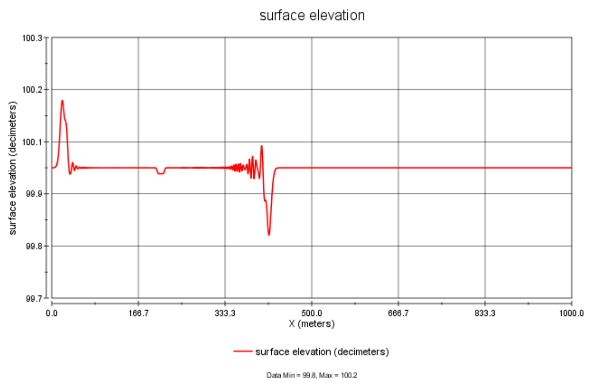
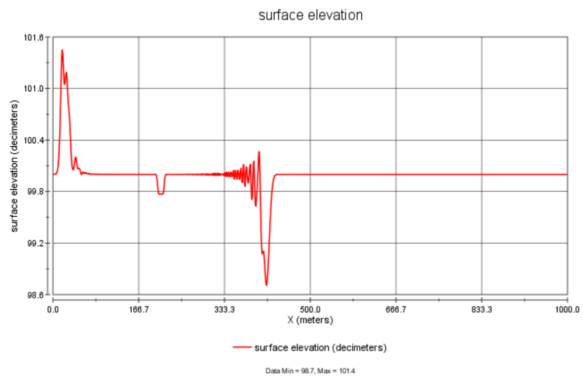
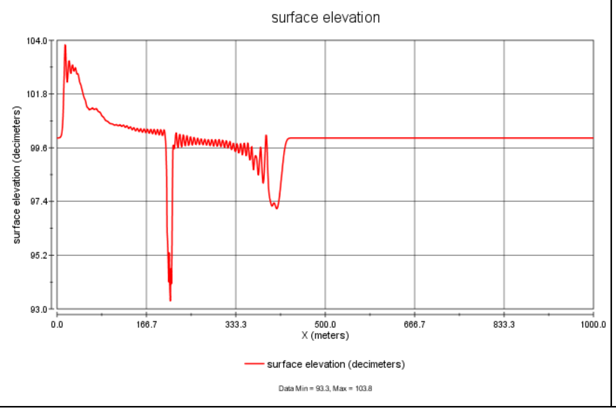
2) At the same time t=500 and same initial velocity u_0=0.5, compared with three different obstacle heights

HOBST= 0.5m	HOBST= 4m
	
HOBST=8m	
	

ANALYSIS:

It can be seen from the figure that at t = 500, the higher the obstacle is, the higher the surface height is. In addition, it can also be known that the lower the obstacle height, the sharper the waves it generates, and the faster the surrounding wave surface returns to calm.

3) At the same time, t=1000 and same initial velocity u_0=0.5, compared with three different obstacle height

HOBST= 0.5m	HOBST= 4m
	
HOBST=8m	
	

ANALYSIS:

So from all these figures, we can draw several conclusions in bellow:

a) When time and initial velocity are constant, the higher the obstacle is, the higher surface elevation is, which is the amplitude is greater.

b) When time and initial velocity are constant, the lower the obstacle height, the sharper the waves it generates, and the faster the surrounding wave surface returns to calm.

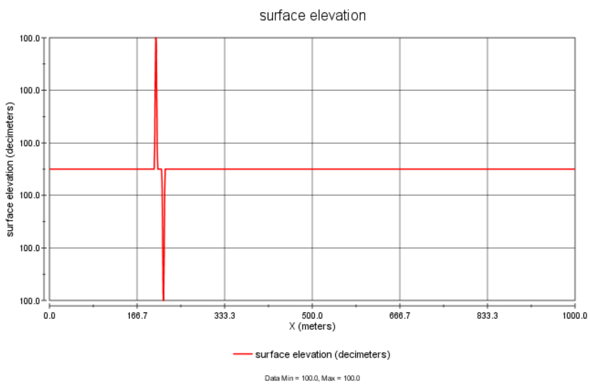
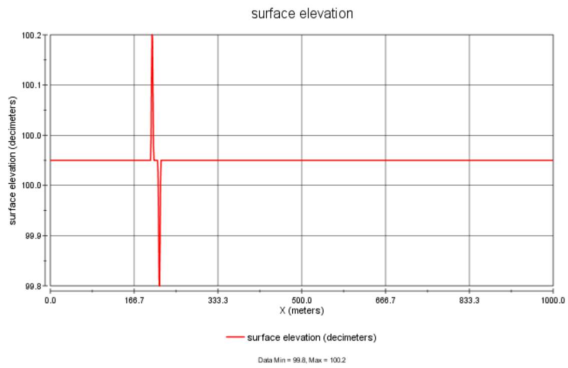
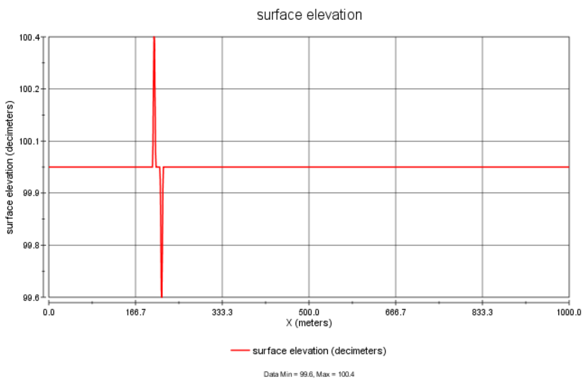
c) The higher the height of the obstacle, the lower the surface elevation at its location.

d) At the same time, the wave range of the flow is same even under the different obstacle heights.

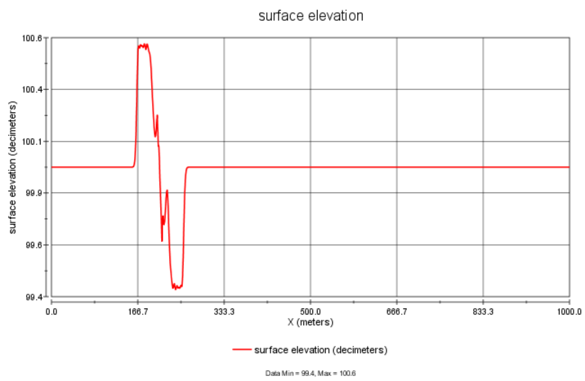
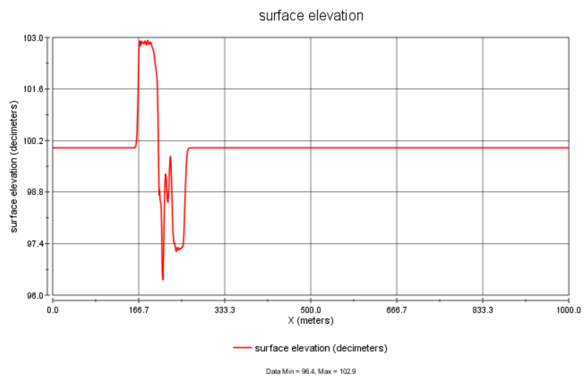
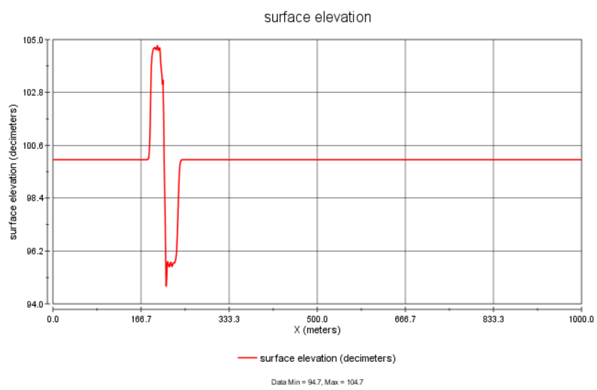
e) When the obstacles in the same height, the longer the elapsed time, the larger the range of the flow fluctuation.

2. Secondly, simulate the influence of the initial velocity in the flow

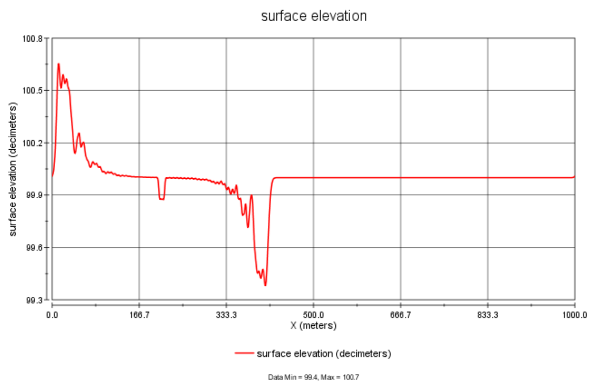
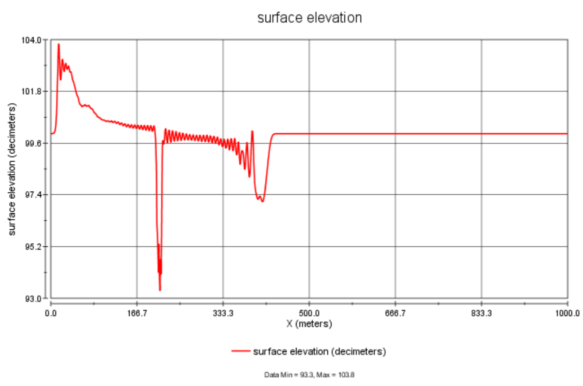
4) At the same time, t=1 and same obstacle height HOBST=8m, compared with three different initial velocity

<table>
<tr><td>U_0= 0.1 m/s</td><td>U_0= 0.5 m/s</td></tr>
<tr><td>
</td><td>
</td></tr>
<tr><td>U_0= 0.8 m/s</td><td></td></tr>
<tr><td>
</td><td></td></tr>
</table>

5) At the same time, t=200 and same obstacle height HOBST=8m, compared with three different initial velocity

U_0= 0.1 m/s	U_0= 0.5 m/s
	
U_0= 0.8 m/s	
	

6) At the same time, t=500 and same obstacle height HOBST=8m, compared with three different initial velocity

<table>
<tr><td>U_0= 0.1 m/s</td><td>U_0= 0.5 m/s</td></tr>
<tr><td>
</td><td>
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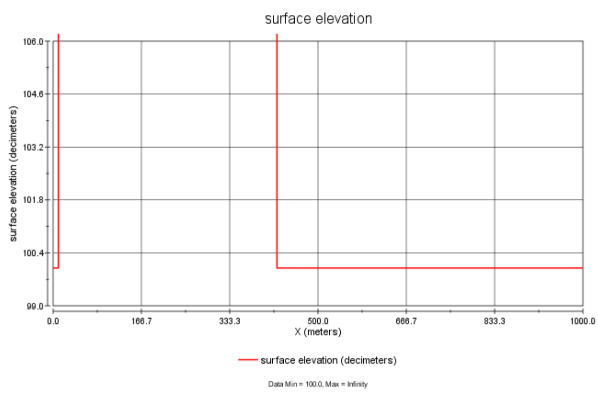
<tr><td>U_0= 0.8 m/s</td><td></td></tr>
<tr><td>
</td><td></td></tr>
</table>

ANALYSIS:

So from all these figures, we can draw several conclusions in bellow:

a) Under the same height obstacle, the higher the fluid velocity, the higher the surface elevation, which is the amplitude is greater.

b) When time and the height of obstacles are constant, the lower the initial velocity, the sharper the waves it generates, and the faster the surrounding wave surface returns to calm.

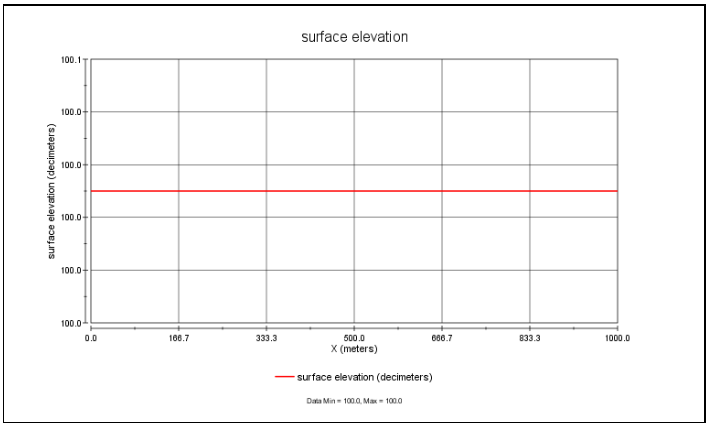
c) When the initial velocity is the same, the longer the elapsed time, the larger the range of the flow fluctuation.

d) The waves are fluctuate outward from the obstacles, the more peripheral waves, the larger the amplitude, and the more dull the wave.

e) When T = 500, HOBST = 0.8m, U0 = 0.8m / s, the surface elevation seems unusual, perhaps this exceeds the maximum value that the flow can withstand, resulting in abnormal simulation results.

Assignment 3 modify the obstacle height and geometry

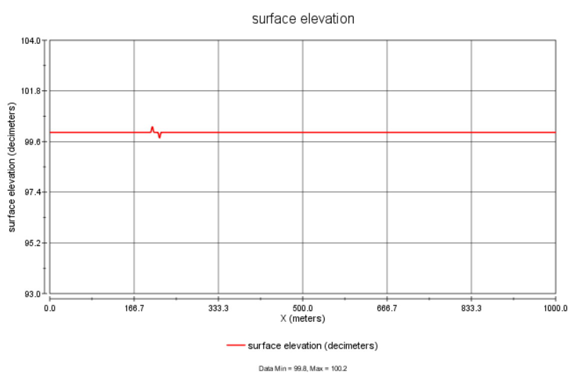
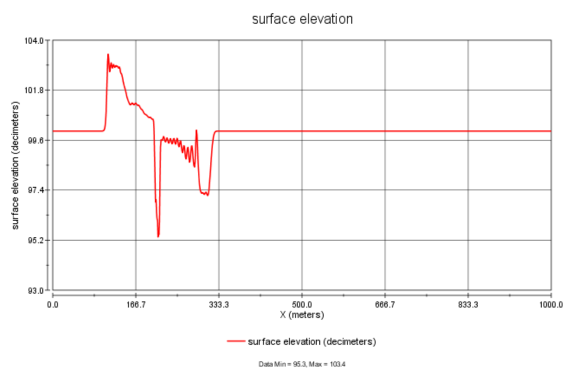
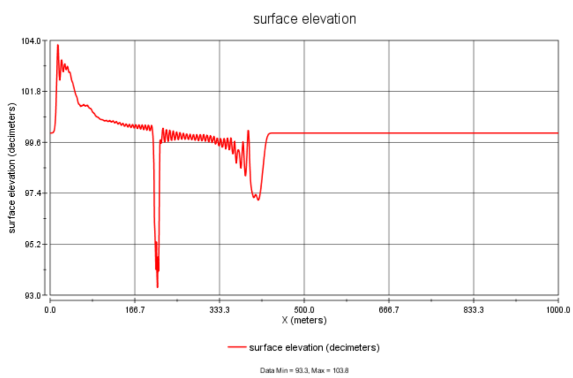
1) When the shape of the obstacle is width 2m and the height is 8m



ANALYSIS:

No matter how time changes, the surface elevation of the obstacle with this shape is same.

2) When the shape of the obstacle is width 20m and the height is 8m

T=1	T=500
 	
T=1000	
 	

3) When the shape of the obstacle is width 8m and the height is 8m

<table>
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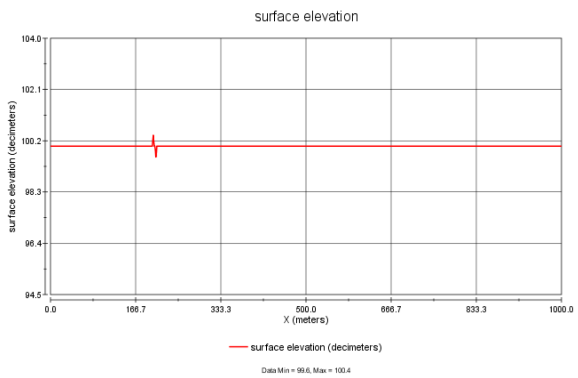

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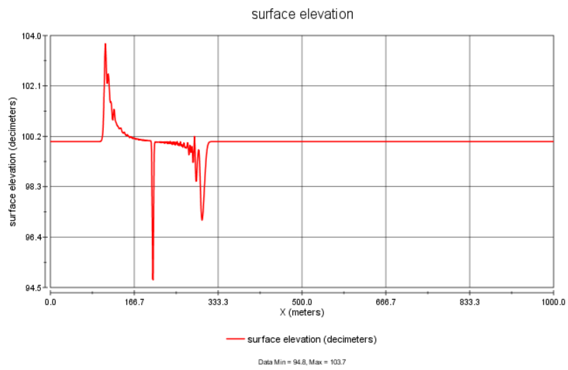

</td></tr>
<tr><th>T=1000</th><th></th></tr>
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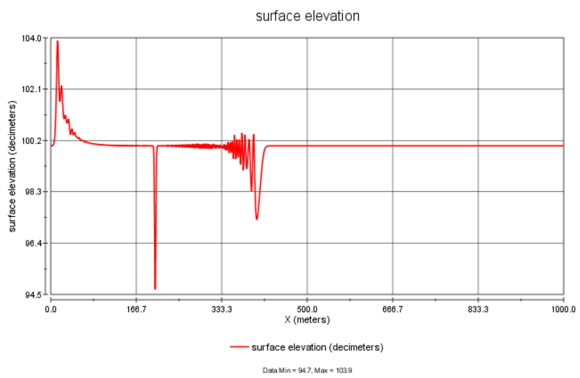


</td><td></td></tr>
</table>

ANALYSIS:

Comparing the influence of the obstacle shape to the surface elevation, clear influence can be observed. Firstly, the shape is changed to a stick, its width is 2m and the height is 8m. Probably its width is too small, there is no influence on the surface elevation.

While widening the width of the obstacle to 20m and keep the height on 8m, there is a big effect on the surface elevation on the water system, compare to the first try, the result shows that the width of the obstacles have a great influence on the surface elevation.

The third obstacle is in a square shape, compared with the other two shapes, we can easily draw a conclusion that the changes of this it is sharper.

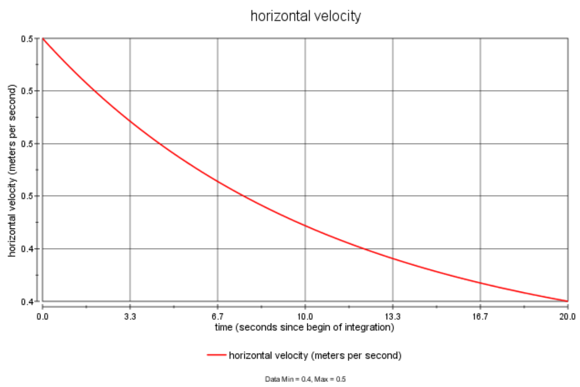
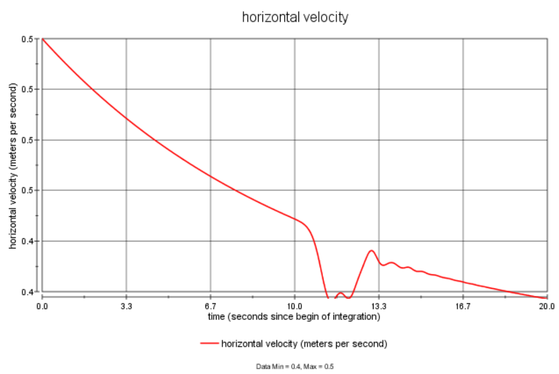
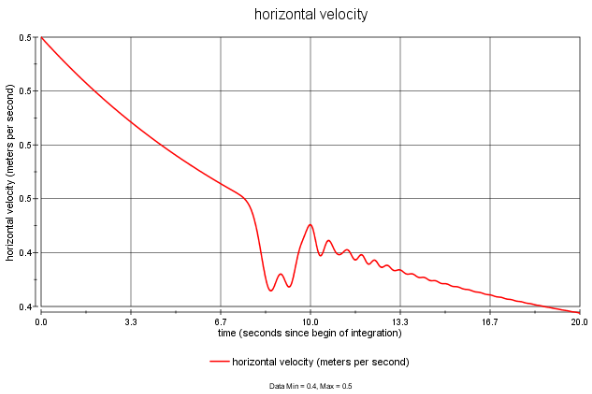
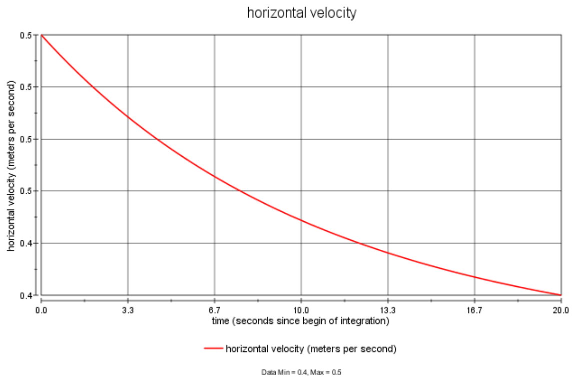
From the latter two obstacles, we can also draw, the longer the time, the wider the range of its fluctuations. Besides, they all fluctuate outward from the obstacles. At the beginning, both sides advance steadily outward, then the amplitude increases sharply, and then immediately stabilizes at the maximum

Assignment 4 Find the critical values beyond which the integration scheme begins to exhibit instabilities

In this assignment, our purpose is to find the critical value of the water flow before it is unstable by changing the simulation parameters, so I specify the variable parameters as initial velocity U_0, angle alpha of inclination and the time T.

1. First of all, I assume initial velocity is constant, U_0=0.5 m/s, and change the angle alpha to find the critical value.

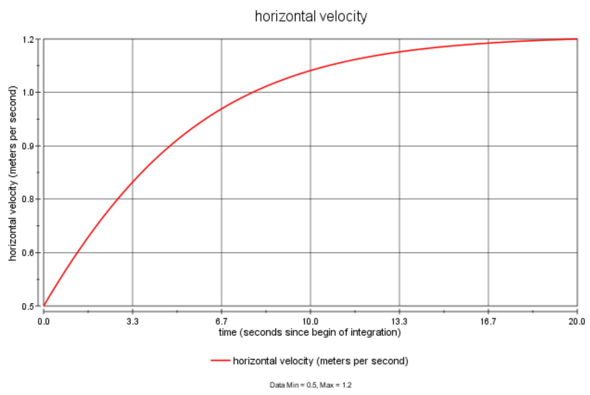
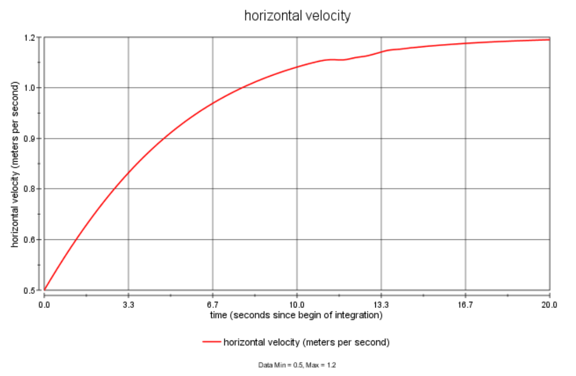
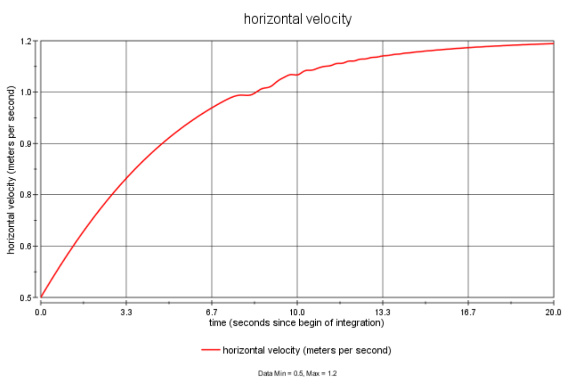
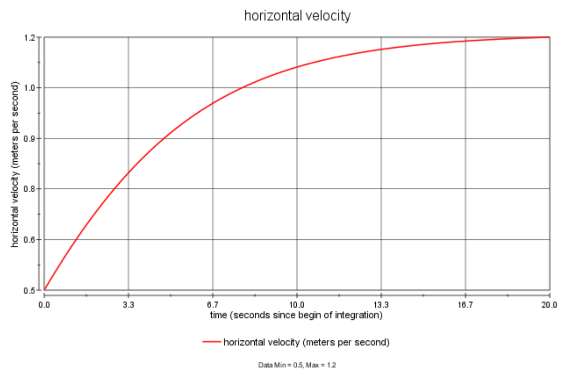
1) U_0=0.5 m/s, alpha=0.1

T=1	T=100
	
T=300	**T=500**
	

ANALYSIS:

So from changing the time, I got these figures, It can be clearly seen that as time changes, the horizontal velocity of the flow is decreasing, which means that when the inclination alpha is 0.1, it has a certain slowing effect on the water flow.

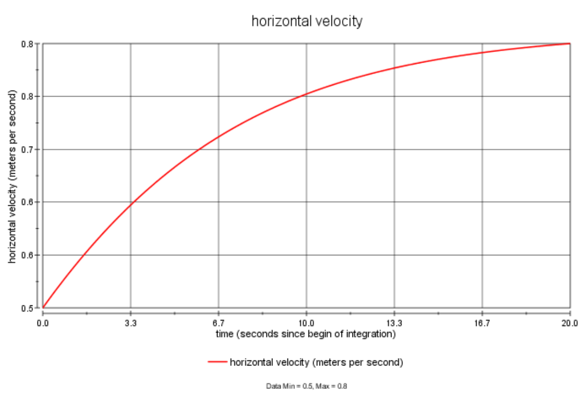
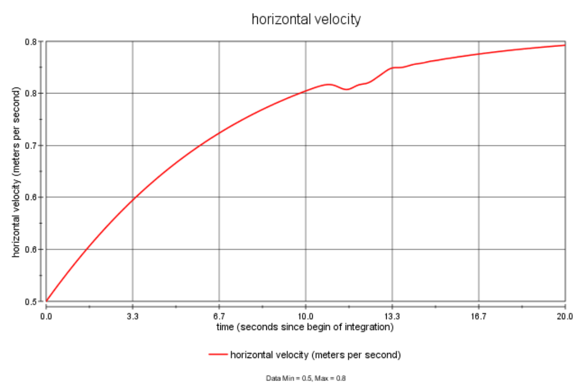
2) U_0=0.5 m/s, alpha=0.8

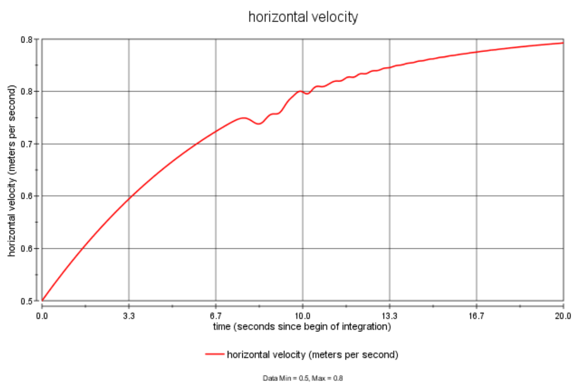
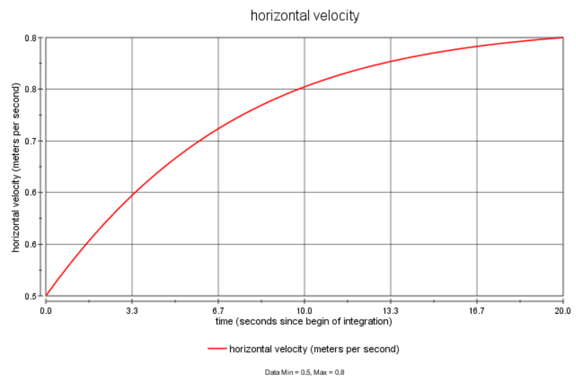
T=1	T=100
	
T=300	**T=500**
	

ANALYSIS:

As we changed the angle alpha to 0.8, from changing the time, we can seen that the horizontal velocity of the flow is increasing, which means that when the inclination alpha is 0.8, it makes the speed of the water increase rapidly.

3) U_0=0.5 m/s, alpha=0.4

T=1	T=100
	
T=300	**T=500**

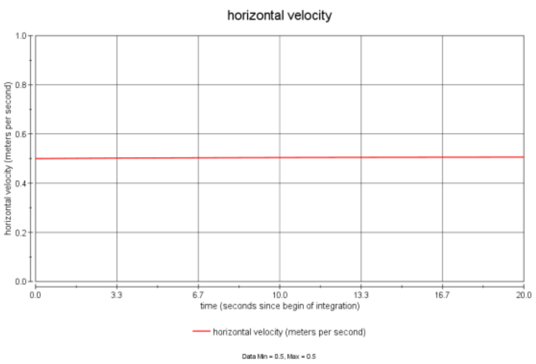
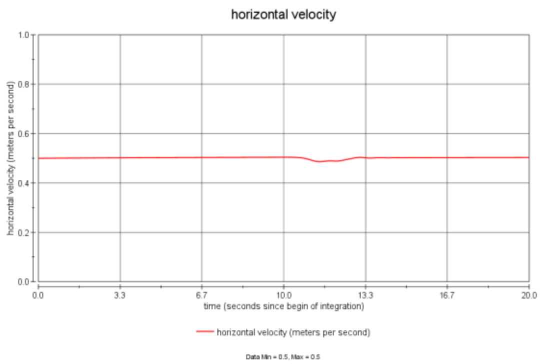
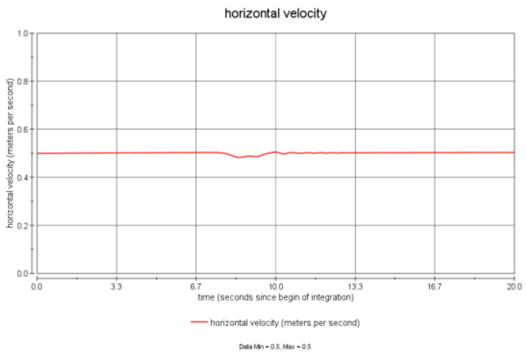
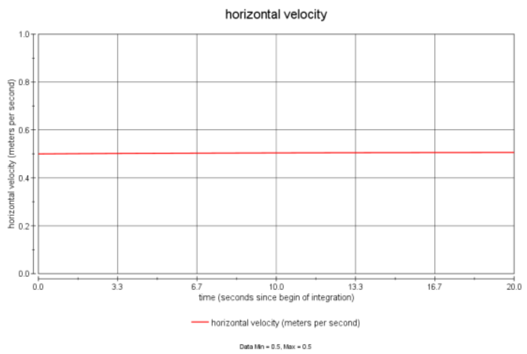
ANALYSIS:

Since the angle alpha 0.8 make the flow velocity change too fast, I change the value of it from 0.8 to 0.4 to approach the critical value we are looking for.

As time changing, that the horizontal velocity of the flow is still increasing, but the growth rate has slowed down, which means that when the inclination alpha is 0.4, still can't meet the threshold value, but at least can be speculated that we can find the critical value when the value of alpha is between 0.1 to 0.4.

By analogy, I gradually approximated the result by changing the alpha value. Finally, at the initial velocity U_0 = 0.5 m/s, the angle alpha = 0.15 can meet the critical condition, the figures of results show in below:

4) U_0=0.5 m/s, alpha=0.15

T=1	T=100
	
T=300	T=500
	

ANALYSIS:

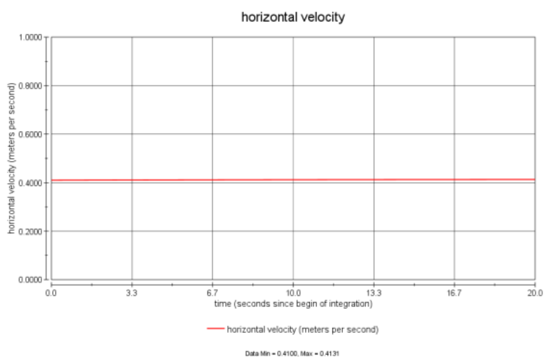
The first critical value I found is when the initial velocity U_0 = 0.5 m/s, the angle alpha = 0.15, It can be seen from the figure that, as time goes by, the fluctuation rate of the horizontal velocity under this condition is very small, which can be basically ignored.

When we analyze the horizontal velocity in different time periods, it can be roughly summarized that the fluctuation of velocity starts from right to left, and the overall horizontal velocity increases slightly after reaching the leftmost, then immediately drops to the initial velocity, and then advances from left to right, until the time around T=420 to 430, there is no fluctuation, and the overall velocity is stable at u=0.5 m/s.

2. Then, I assume the inclination alpha is constant where alpha = 0.1, and change the initial velocity to find the critical value.

To solve this situation, I use the same way as before to gradually approximated the result by changing the value initial velocity. Finally, I found the value whitch can meet the critical condition at the initial velocity U_0 = 0.4 m/s, the angle alpha = 0.1, the figures of results show in below:

5) U_0=0.4 m/s, alpha=0.1

<table>
<tr><td>T=1</td><td>T=100</td></tr>
<tr><td>


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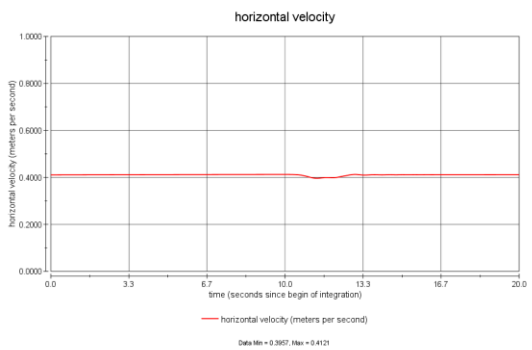
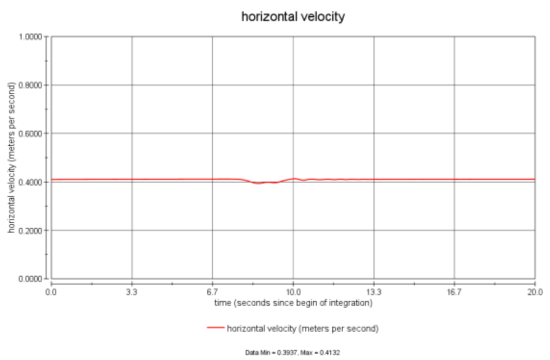


</td></tr>
<tr><td>T=300</td><td>T=500</td></tr>
<tr><td>


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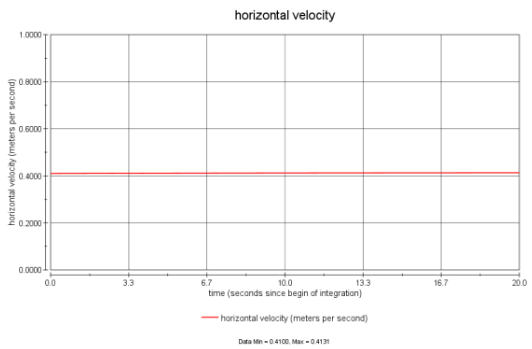


</td></tr>
</table>

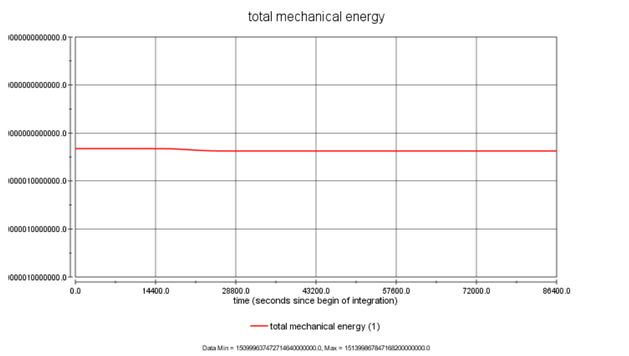
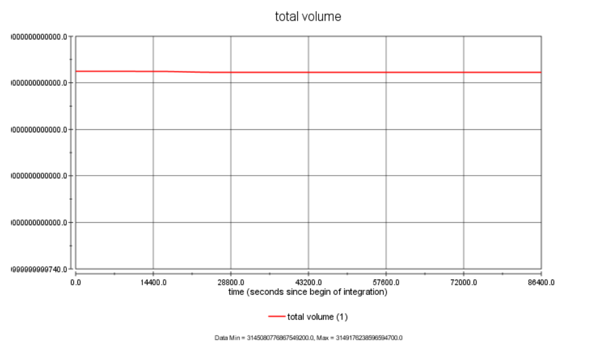
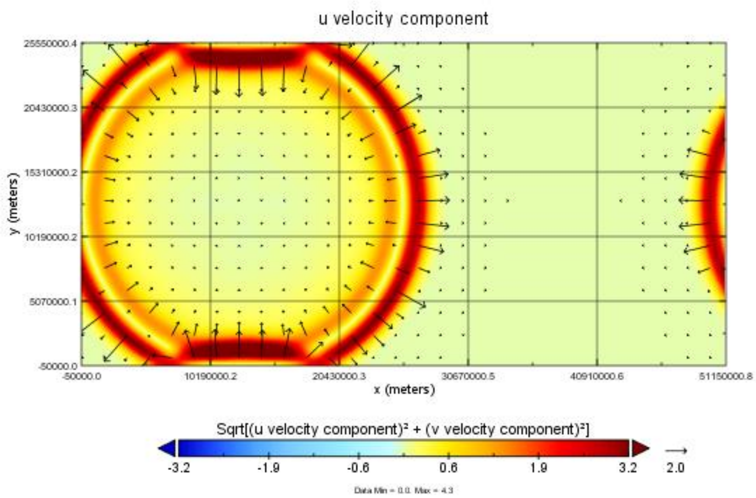
ANALYSIS:

The second critical value I found is when the initial velocity U_0 = 0.4 m/s, the angle alpha = 0.1, It can be seen from the figure that, as time goes by, the fluctuation rate of the horizontal velocity under this condition is very small, which can be basically ignored.

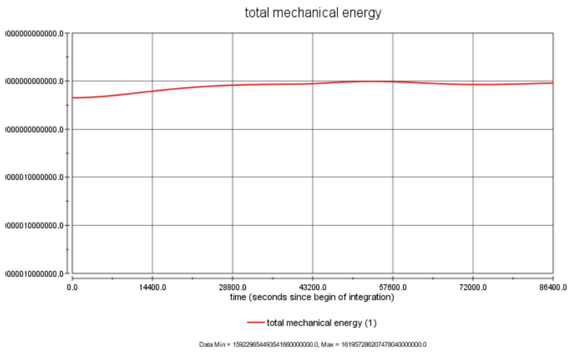
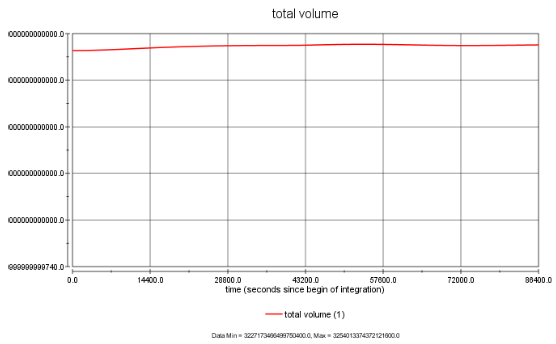
The analysis of the relationship between flow velocity and time is the same as the previous analysis.

Assignment 5 Experiments with the propagation of the wave excited by the “Gaussian blob”

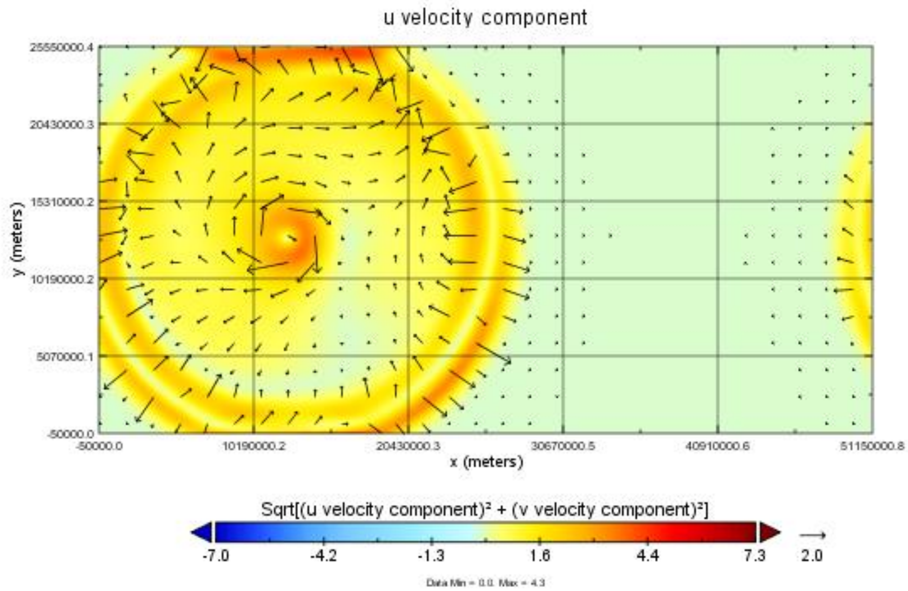
1) Flat bottom, no rotation of the coordinate frame

<table>
<tr><td>Total mechanical energy</td><td>Total volume</td></tr>
<tr><td>
</td><td>
</td></tr>
<tr><td colspan="2">Velocity component</td></tr>
<tr><td colspan="2">
</td></tr>
</table>

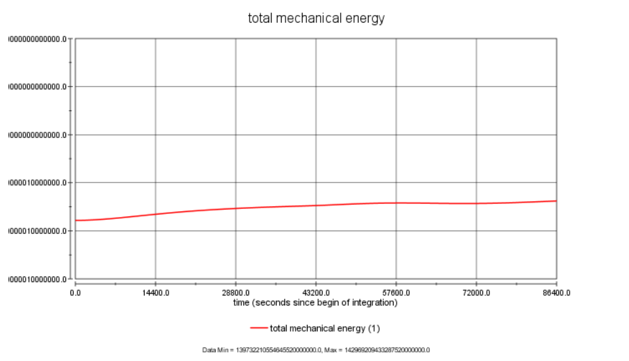
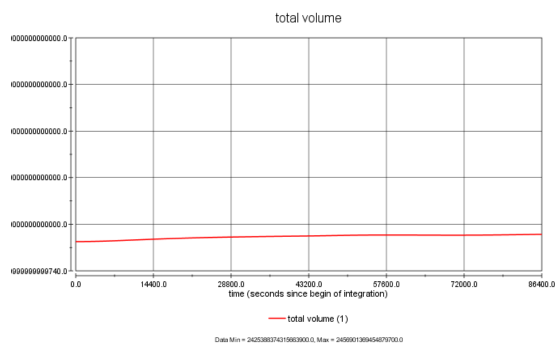
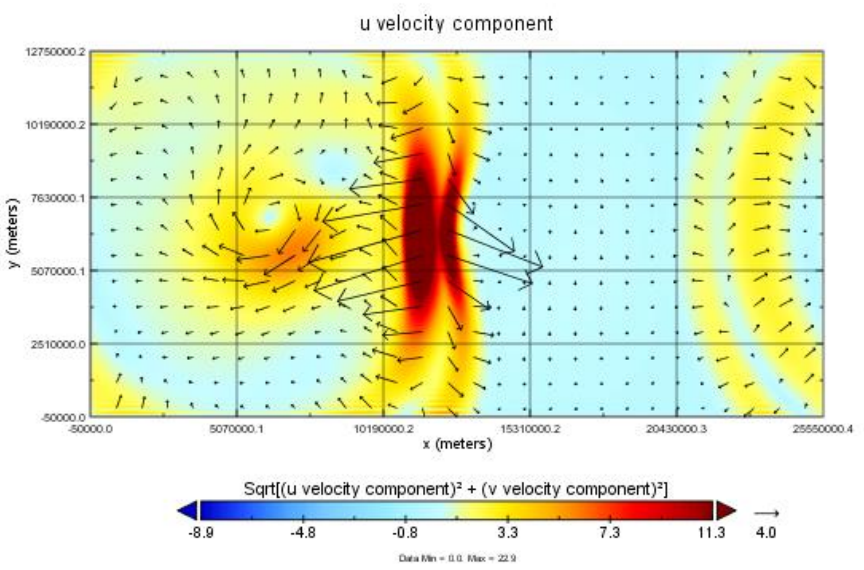
2) Flat bottom, assume Earth's angular velocity (7.27), f-plane at 50°N

Total mechanical energy	Total volume
	

Velocity component



3) As above, with a sea mount at the middle of the domain, 9000 m high, sigma = 4000 km

<table>
<tr><td>Total mechanical energy</td><td>Total volume</td></tr>
<tr><td>
</td><td>
</td></tr>
<tr><td colspan="2">Velocity component</td></tr>
<tr><td colspan="2">
</td></tr>
</table>

ANALYSIS:

As is shown in these 9 figures above, it compared three situations. Firstly, in the term of total mechanical energy, it has the maximum total mechanical energy while in the case of the system with flat bottom and Earth's angular velocity, and it increases gradually. While if there is no rotation, the total mechanical energy is less than in the second case which with rotation and it decreases a little bit with the time passing. However, if there is a seamount, it has the lowest total mechanical energy. What is found, in each case with the rotation, the mechanical energy always increases.

Secondly, in the term of the total volume, it looks the similar situation with the total mechanical energy, with the flat bottom and rotation, water system has the maximum total volume and it keeps increasing with the time passing. If the bottom is not flat and has obstacle but it has angular velocity, the total volume is the minimum but it increases still. If the situation without the rotation, the total volume will decrease.

Finally, in the term of the velocity component of the water system, totally it has the same features, the vector always points to the lower elevation area and this direction can represent the water moving direction. To compare the first two cases, the difference of the initial situation is the existence of the rotation. It can cause the different results. With the rotation of the coordination, there will be a high velocity center in the circle. What's more, with the rotation the average velocity in the circle increases compared with the one without rotation. When it comes to the vector direction, it seems that there will be a swirl in the circle. Without the rotation, the water velocity gradient is much more big than the one with rotation. Their velocity distribution are symmetrical.

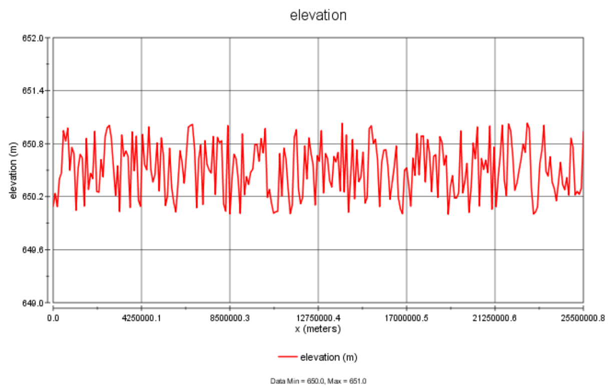
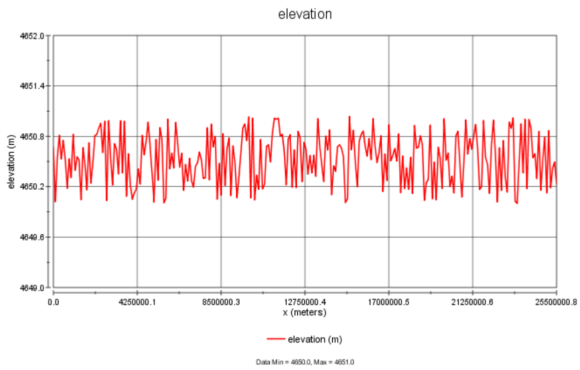
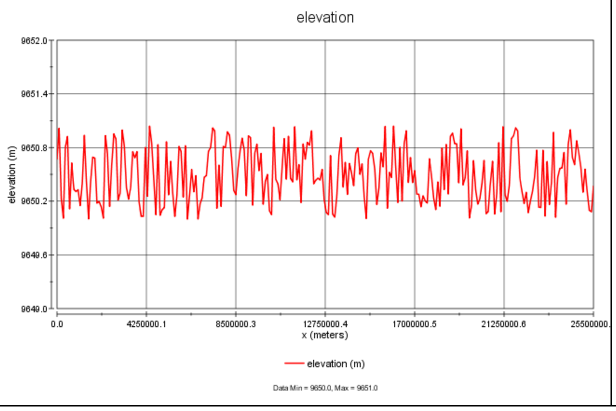
If they are compared with the case 3, the one with the rotation and seamount, more differences can be found. From the figure, there is a region with has very high speed. It is on the right side of circle. From this phenomenon, it can be deduced that the seamount could be there which can obstruct the water flow and cause the high velocity. Compared with the case 2, there is a high velocity as well. Combine with the vector direction, it could be a swirl.

To combine with all these three case, the conclusion can be drawn, all the velocity distributions are based in the circle pattern. If the rotation is added into the system, there will be a high velocity centre in the circle. Gradually, if an obstacle is added like seamount, the circle pattern will be disturbed and a very high velocity region will show up near the obstacle. With the time passing, this region moves from right to left as well.

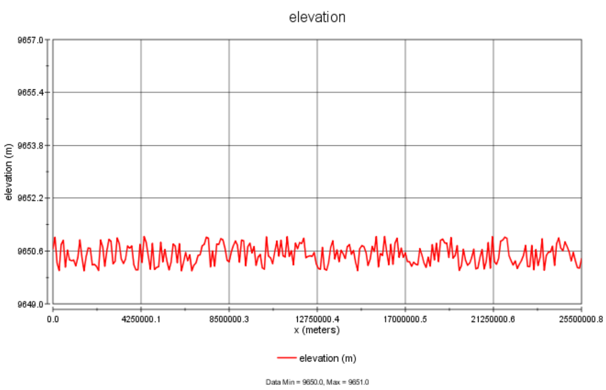
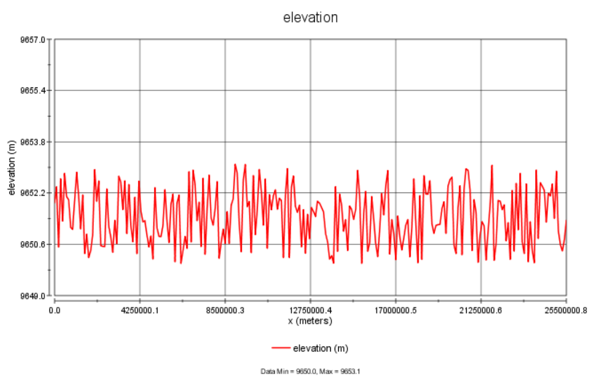
Assignment 6 Inspect the surface elevation disturbances and the vorticity fields

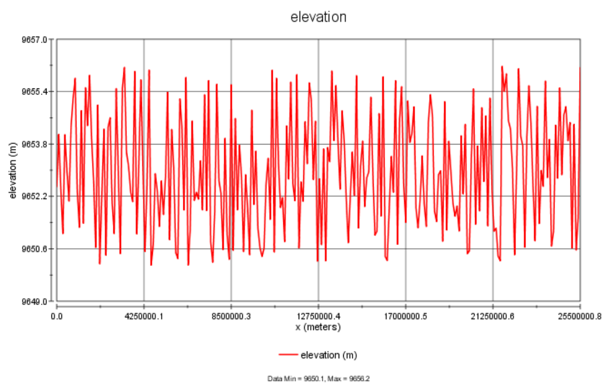
1) Surface elevation

Fluid depth:

<table>
<tr><td>Depth=1000</td><td>Depth=5000</td></tr>
<tr><td>
</td><td>
</td></tr>
<tr><td>Depth=10000</td><td></td></tr>
<tr><td>
</td><td></td></tr>
</table>

Earth's angular velocity:

<table>
<tr><td>Velocity=10</td><td>Velocity=30</td></tr>
<tr><td>
</td><td>
</td></tr>
<tr><td>Velocity=60</td><td></td></tr>
</table>

<table>
<tr><td>
</td><td></td></tr>
</table>

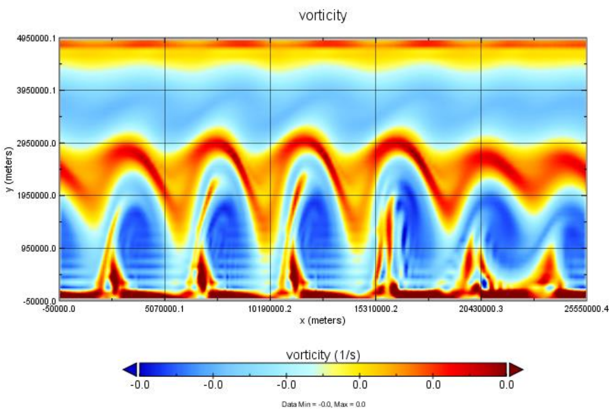
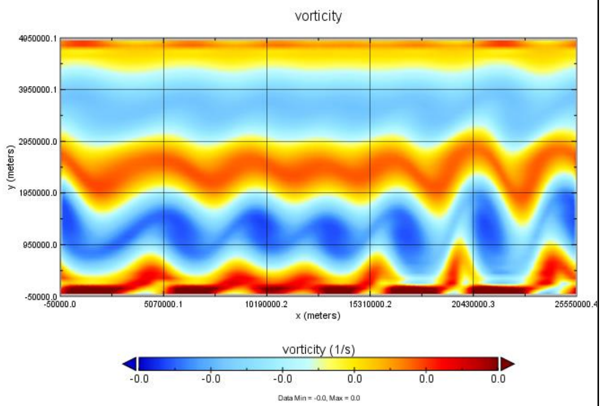
ANALYSIS:

In the terms of the surface elevation, if the fluid depth increases, the surface elevation of the water system increases as well. However, the amplitude is the same no matter how the fluid depth changes. What's more, with the increase of the fluid depth, the fluctuation is more frequent.

For the surface elevation with different Earth's angular velocity, the y axis range is set in the same values, it is clearly to compare both these three figures. The higher the velocity is, the higher the fluctuation amplitude is. However, the average surface elevation is also increasing with the increase of angular velocity.

2）**Vorticity**

a) In the same time t1= 97, change the fluid depth:

<table>
<tr><td>Depth=1000</td><td>Depth=5000</td></tr>
<tr><td>
</td><td>
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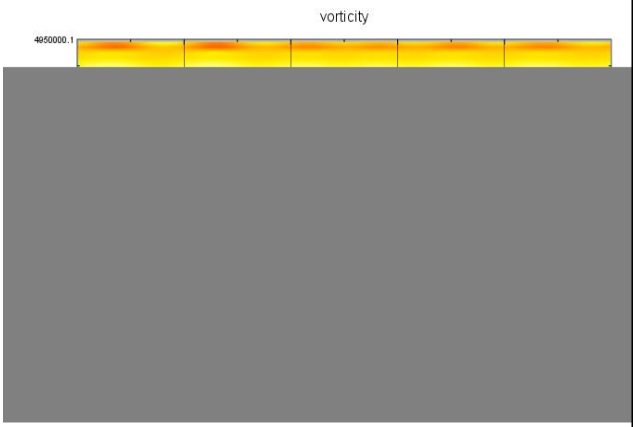
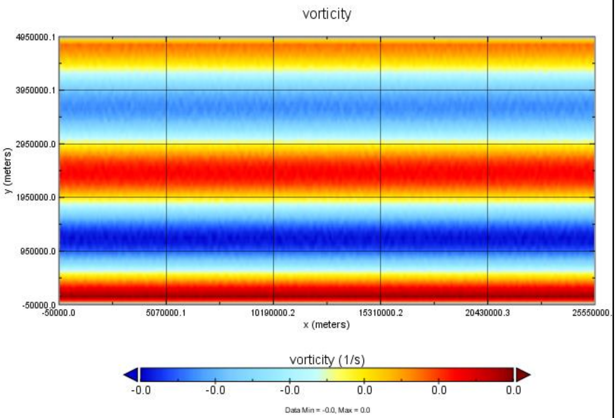
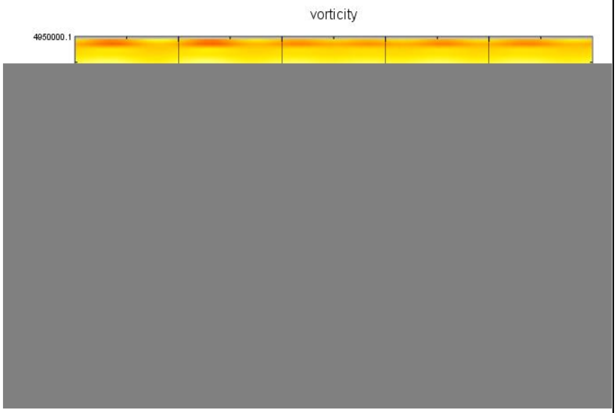
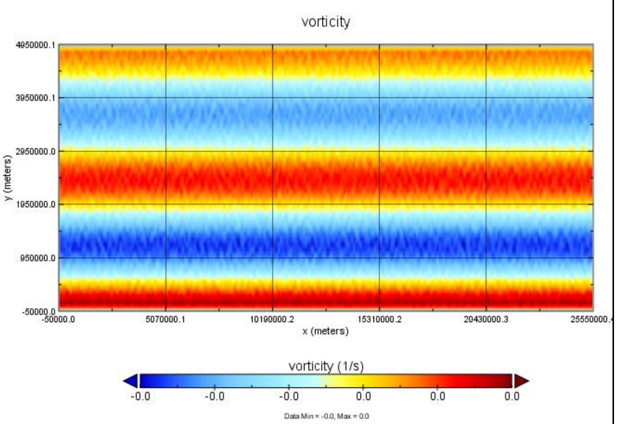
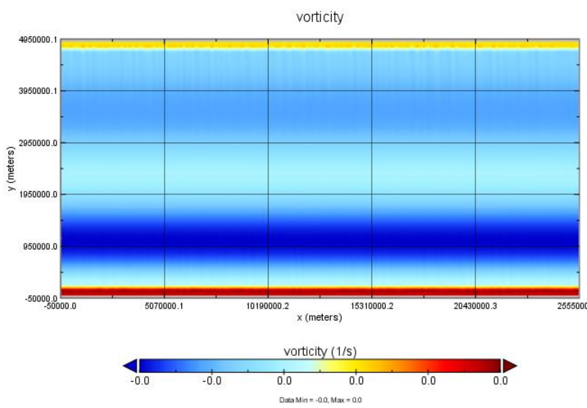
	

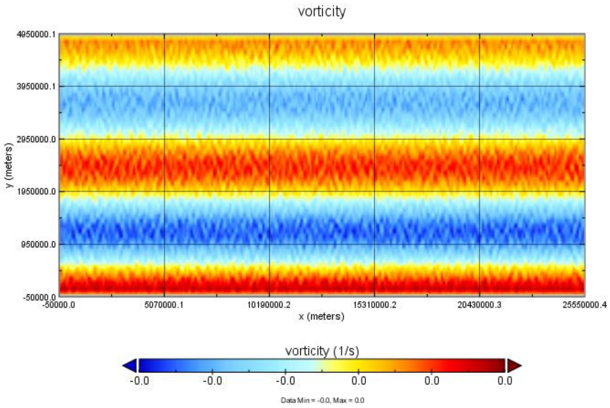
b) In the same velocity, v1= 10, change the time:

T=1	T=97
	

c) In the same velocity, v2= 30, change the time:

T=1	T=97
	

d) In the same velocity, v3= 60, change the time:

<table>
<tr><th>T=1</th><th>T=97</th></tr>
<tr><td>


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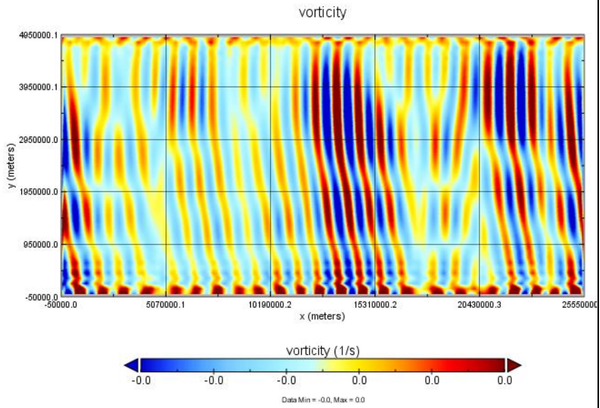

</td></tr>
</table>

<u>ANALYSIS:</u>

In the situation a, the fluid depth is changed, the deeper the water system is, the smaller the vorticity is which means the amplitude is smaller. And the amplitude is different in the same figure, while when the depth is deep enough, this kind of the difference is not so obvious.

When the velocity is changed but keep the fluid depth stable, from the above 6 figures, with the increasing of the velocity, at the initial of the process, the vorticity is denser. And the vorticity distributes horizontally.

However, when at the end of the process, the vorticity distribution is not horizontal any more. In the case of 10 velocity, it starts to flux. While in the case of 30 velocity, at the end of the process, the vorticity becomes more stable and difference becomes smaller. When the velocity becomes 60, the vorticity doesn’t distribute horizontally any more, it distributes almost vertically.

To conclude, the earth Earth’s angular velocity has stronger effect on both vorticity and surface elevation. Because vorticity is the square of the angular velocity.