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Structural Econometric Models

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Abstract In this tutorial, we discuss the concept of structural econometric models and their applications in business and management research. Structural models are constructed by grounding on economic and related theories, and they attempt to accurately reflect underlying data generating processes. One unique advantage of structural models is their capability to simulate user or firm strategic behaviors under new policies and measure the effectiveness of these proposed policies, and hence generate convincing managerial suggestions. We present three representative classes of structural models—demand estimation using aggregate data, dynamic choice models, and two-sided matching models—to demonstrate model specification, assumptions, and estimation procedures.
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Keywords structural models • demand estimation • dynamic choice models • matching model • policy simulation

1. Introduction

There has been a growing interest in applying structural models in business research. Numerous structural studies have appeared in digital communications (Erdem et al. [32], Fiebig et al. [34]), commodities (Erdem and Keane [31], Keane [58]), education (Arcidiacono [6], Keane and Wolpin [60]), auctions (Bajari and Hortaçsu [10], Guerre et al. [43]), personal life (Keane and Wolpin [61]), religion and charity (Bisin et al. [17], van Diepen [93]), healthcare (Fiebig et al. [34], Harris [46], Harris et al. [47], Mark and Swait [68], Yan and Tan [99]), entertainment (Roos and Shachar [78], Shachar and Emerson [88]), politics (Diermeier et al. [27], Shachar [87], and sports (Chiappori et al. [23], Walker and Wooders [94], Wiles [97]). Chintagunta et al. [24] lay out a comprehensive review of structural models. Despite the increasing interest in structural models in the literature, there is no clear consensus on what makes a "structural" model. In this tutorial, we focus on fundamentals of structural models, comparison with alternative approaches, the common features, various forms of applications, and, finally, discussions on challenges and opportunities.

The first question is, what constitutes a structural model? In the literature, structural models are often described as alternatives of reduced-form models. Guo [44], however, depicts this contrast as a continuum in which there are models completely ignoring the underlying process at one extreme. In this framework, there are models that are more structural than others in the continuum. This notion is helpful as it emphasizes the role of various factors in determining whether a model is structural: the role of data, underlying theory, and research questions.

Structural models, in general, describe the underlying decision-making process or economic theory. They rely on economic or related theories to derive model specification. Erdem et al. [34] contrast "theory-driven" with "data-driven" models. On one hand, a structural model is a model coming from econometric specification that is derived from theory. It is worth noting that the role of theory is to impose priori restrictions on the statistical relationships to be estimated. Reiss and Wolak (in Erdem et al. [33], p. 226) define a structural model as "any model that provides a behavioral interpretation for some or all of the parameters." This approach is especially suited to capture the strategic behaviors of rational individuals or firms that maximize their respective objectives, such as utility or profit, or psychological decision-making theories (Erdem et al. [33]). On the other hand, reduced-form models can be referred to as a data-driven approach, which are often used to fit specific predictions for data relationships that are derived by theory without referring to the full behavioral model. One can compare the structural/reduced-form models to the notion of generative/discriminative models in statistical learning. Generative models are the ones describing the joint distribution of observed data and outcome, whereas discriminative models estimate the conditional probability of outcome by observed data.

The second question is, why use structural models? Theory-driven models generate more accurate and useful insights to explain the choice process, the design of business strategy, and the design of public policy (Erdem et al. [33]). In addition, these approaches improve the in-sample and out-of-sample predictive performance relative to reduced-form models (Guo [44]). One can also study the effect of a policy change by structural models because they capture the underlying decision-making mechanism and primitives. Therefore, one advantage of structural models is their capability to conduct policy analyses and produce counterfactuals to answer some important "what-if" questions. However, any policy change that represents a significant change by altering the structure and elements of the decision process (e.g., beliefs, information set, or context) may not be estimated consistently (Lucas [65]).

Another advantage of structural models is that a researcher can sometimes resolve the potential endogeneity problem by explicitly modeling the relationship between endogenous variable and error term. For instance, firms observe all the shocks and demand, and they set their marketing activities to maximize their profit. As researchers, we often do not have access to all the information. One example is the relationship of price and the unobserved term. This relationship causes the estimated results to be biased. One approach is to use instrumental variables (IVs). Instruments are exogenous that are not correlated with the unobserved demand drivers but correlated with the endogenous variables (e.g., price). In practice, however, it is not an easy task to find the proper instruments satisfying these assumptions. In the structural approach, the endogeneity can be addressed by imposing the relationship between the observed and unobserved variables. For instance, Ebbes et al. [30] develop methods that circumvent the search for observable instruments. That requires an econometric model based on firms' optimizing equations and demand function. These assumptions distinguish structural from reduced-form models. In reduced-form models, the assumed relationships (such as exogeneity) are implicit in the model, whereas a researcher models such relationships explicitly in structural models. Structural methods may relax some assumptions in policy simulations by studying the primitives of how economic agents might behave (Guo [44]).

With more complex structural models, researchers can capture more complicated customer preferences. For example, one can even account for bounded rationality in structural models (Erdem et al. [33]). However, this practice does not come without cost. First, more complex models require more variations in data to enable its identification. No structural model can extract information that is absent in a data set (Shugan [89]). It is worth noting that if we have sufficient variability in the data, a comprehensive reduced-form model can approximate the decision-making process well. Second, the estimation of complex structural models is harder and computationally heavy. For example, some structural models require solving a dynamic programming (DP) problem numerous times during estimation (Rust [86]). To

add to the complexity, likelihood functions are usually nonlinear, making the maximization process more intensive. Nevertheless, with advances in computation power and techniques, one can afford the computational burden of such models while benefiting from its features.

Among all forms, the discrete choice models of McFadden (1974) have been the workhorse of structural models. In this framework, customers (or firms) are faced with a finite set of choices. The goal is to infer the underlying preference and utility that leads to behavior observed in data. The foundation of discrete choice models is the *random utility model* (RUM) framework, which assumes that the utility of options is independent of the other options in the choice set and their attributes. RUMs impose the assumption of stable preference orderings as well. That is, knowing choice probabilities from among all pairs of two options, one can derive the possible choice probabilities from the complete set of options (Block and Marschak [18]). Applying these approaches to learn about customers is characterized as demand-side analyses. We can formulate the underlying demand function in terms of product characteristics and marketing mix attributes. Berry et al. [16] use a discrete choice framework to estimate the demand for the car industry utilizing an aggregate data set. Nair et al. [72] develop an approach that allows for estimating individual discrete/continuous demand functions from aggregate data. Doosti and Tan [28] incorporate an aggregate demand model in reward-based crowdfunding with modeling the endogeneity using a set of statistical models. With individual-level data, one can achieve more efficient approaches (Bajari and Kahn [11]).

Structural models can be used to describe supply-side choices. Supply-side analyses can help us understand the nature of interactions among firms and competition and to identify endogeneity in demand-side parameters. One challenge to supply-side analyses is that researchers often do not observe some key variables such as costs. There are strategies developed by industrial organization economists to derive estimates of costs by first-order conditions for profit maximization.

There are many real-world settings in which agents maximize multiperiod utility instead of single-period utility. In such cases, static models with myopic users are not appropriate and cannot capture the true behavior. Examples of the cases with forward-looking agents include choosing a retirement plan, buying a durable good, and stockpiling storable goods. Ignoring dynamics in these settings will lead to bias in parameter estimates. A dynamic model allows describing behaviors such as forward-looking decision making. In dynamic models, agents face a trade-off between current utility and future utility. In a bus engine replacement problem, Rust [85] proposes a framework to account for forward-looking agents using a dynamic programming approach. Chintagunta et al. [24] describe the common characteristics of dynamic models: (1) time and uncertainty are explicitly treated; (2) agents have well-defined objective functions and make their decisions based on the current information available, their beliefs about nature, and, in the case of dynamic models of competition, their beliefs about the strategies of other players; and (3) agents maximize a multiperiod objective function—that is, they take the impact of their current choices on future expected utilities into consideration when making current decisions.

An extension to single-agent dynamic models is to account for interactions among multiple agents using a game-theory framework. The choice of an agent may affect others' utility (or profit). Like any other game-theory model, there could be several equilibria. The challenge is that the researcher does not know which equilibrium is played by agents. Additionally, the set of equilibria may not be finite. To overcome this issue, structural models assume that agents' decisions observed in the data are equilibrium outcomes for rival firms given a demand specification (Chintagunta et al. [24]) or assume that data are coming from a single equilibrium (Aguirregabiria and Mira [3]).

One of the challenges in estimating structural models is identification as in some cases it may not be clear whether the parameters are identified. This becomes more problematic in more complex models. Magnac and Thesmar [66], Bajari et al. (2007), and Hall and Zhou [45] lay out

approaches for identification in dynamic models. One key identification challenge is to identify the sources of persistence in choice behavior. That is, we need to disentangle customer heterogeneity in tastes and "structural" state dependence. Heckman [50] portrays the problem as to identify the effect of initial endowment (heterogeneity or serially correlated errors) and the past experiences and choices (state dependence). Uncovering whether state dependence exists is of great importance. An example of state dependence is that price discounts will affect not only current but also future demand. Heckman [49] (p. 92) discusses the "spurious state dependence" in which lagged choices will tend to be significant in a discrete choice model with serial correlation, even if there is no true state dependence. Chamberlain [21] shows a nonparametric test for the absence of state dependence. Like any other tests, this test relies on some assumptions.

There are opportunities to enhance structural models with more flexibility, stability, and prediction power. Recent advances in computer science and machine learning can be utilized to deal with some of the issues in structural modeling. For example, Manzanares et al. [67] perform a principal component analysis (PCA)–based dimension reduction to reduce the state space of the value functions to be estimated. Another front is to take advantage of the well-known bias/efficiency trade-off to increase the out-of-sample performance (Guo [44]).

In what follows, we present three representative structural models: demand estimation using aggregate data, dynamic discrete choice models including both single-agent models and dynamic games, and two-sided matching models. We will focus on the specification, assumptions, and estimation of each model.

2. Demand Estimation Using Aggregate Data

2.1. Introduction

Estimation of demand has been a key part of many recent studies. Armed with demand estimates, we can examine questions regarding market power, mergers, innovation, and valuation of new brands in differentiated-product industries. A common concern regarding demand estimation is that researchers often only have access to aggregate, instead of the individual consumer-level, data. There are two main challenges for demand estimation with aggregate-level data: (1) recovering consumers' heterogeneity without individual-level information and (2) accounting for potential endogeneity issues as a result of market unobservables.

This section focuses on one of the main methods for estimating demand for differentiated products using aggregate data: the random-coefficients logit model (Berry [15], Berry et al. [16]) (henceforth referred to as BLP). This method has been applied to an increasingly broad range of markets and questions in economics and management disciplines (Ghose and Han, [40], Gordon and Hartmann [41], Wilbur [96]) because of its three major advantages: (1) the model can be estimated using only market-level price and quantity data, (2) it deals with the endogeneity of prices, and (3) it parsimoniously and flexibly captures more realistic substitution patterns between the products in a market (Nevo [73]).

The rest of this section is organized as follows. We start by defining the basics of the demand estimation problem and present the homogeneous logit demand estimation models in Section 2.2. The focus is on various modeling assumptions and their implications for estimation and the results. In Section 2.3, we describe the BLP method proposed by Berry et al. [16], with slight alternations to the homogeneous model. We then discuss the identification issues and demonstrate a step-by-step estimation algorithm in Section 2.4.

2.2. Homogeneous Logit Model

2.2.1. Model Setup. In a homogeneous logit model, all agents have the same parameters, and there is no heterogeneity of tastes. Assume that we observe $t = 1, \ldots, T$ markets, each with $i = 1, \ldots, I_t$ consumers. For each such market we observe aggregate quantities, average

prices, and product characteristics for J products. The indirect utility of consumer i from consuming product j in market t can be represented as

$$u_{itj} = x_{tj}\beta - \alpha p_{tj} + \xi_{tj} + \varepsilon_{itj}, \tag{1}$$

where p_{tj} is the price of product j in market t, x_{tj} is a K-dimensional vector of observable characteristics of product j, ξ_{tj} is the unobserved (by the researchers) product characteristic, and ε_{itj} is a mean-zero stochastic term. Finally, β is a K-dimensional vector of unknown coefficients on the observed characteristics, and α is a scalar of the unknown price coefficient. We can rewrite (1) in a more concise form:

$$u_{itj} = \delta_{tj} + \varepsilon_{itj}. \tag{2}$$

Here, $\delta_{tj} = x_{tj}\beta - \alpha p_{tj} + \xi_{tj}$ represents the deterministic part of the utility. The utility for outside good δ_{t0} is normalized to 0, so we have $u_{it0} = \varepsilon_{it0}$. If we assume that ε_{ijt} follows the Type I extreme value distribution across the population and independent and identically distributed (i.i.d.) across individuals and products, we can derive the market share of brand j in market t by integrating out individuals' errors (Train [91]):

$$s_{jt}(\delta_{t1}, \ldots, \delta_{tJ}) \equiv P_{jt} = \frac{\exp(\delta_{tj})}{1 + \sum_{k=1}^{J} \exp(\delta_{tk})}. \tag{3}$$

This is the aggregate logit demand model. Within a market t, we observed $J+1$ market shares $\{S_{t0}, \ldots, S_{tJ}\}$ and $J+1$ unknowns $\{\delta_{t0}, \ldots, \delta_{tJ}\}$. Taking logs of (3) yields a system of linear equations that allow the model parameters to be just identified. If there is no endogeneity issue, we can use ordinary least squares; otherwise, we need to instrument for price (Huang [54]).

2.2.2. Limitations of the Logit. Although the logit model is computationally convenient, it imposes some unrealistic restrictions on the data. Recall that the price elasticities of the market shares defined by (3) are

$$\eta_{tjk} = \frac{\partial s_{tj}}{\partial p_{tk}} \frac{p_{tk}}{s_{tj}} = \begin{cases} \alpha p_{tj}(1 - s_{tj}) & \text{if } j = k, \\ -\alpha p_{tk} s_{tk} & \text{otherwise.} \end{cases} \tag{4}$$

There are two problems with these elasticities. First, because in most cases the market shares are small, the factor $\alpha(1 - s_{tj})$is nearly constant; hence, own-price elasticities are proportional to own price, which implies that a standard pricing model predicts a higher markup for the lower-priced brands. This assumption is clearly not appropriate in many industries (Nevo [73]).

Second, these elasticities imply that the logit model has the independence of irrelevant alternatives (IIA) property. That is, it implies substitution toward brands in proportion to shares *regardless of characteristics.* For example, if $s_{j_1} = s_{j_2}$, then the cross-price elasticity of product j_1, j_2 with respect to (w.r.t.) product k will be identical regardless of differences in x_{j_1} and x_{j_2}. A famous example of the IIA assumption is the red bus/blue bus problem (McFadden [69]).

2.3. Random Coefficients Logit Model

2.3.1. Model Setup. Berry et al. [16] introduces heterogeneity into the model discussed above by adding random coefficients to (1):

$$u_{itj} = x_{tj}\beta_i - \alpha_i p_{tj} + \xi_{tj} + \varepsilon_{itj}, \tag{5}$$

where

$$\begin{bmatrix} \beta_i \\ \alpha_i \end{bmatrix} = \begin{bmatrix} \bar{\beta} \\ \bar{\alpha} \end{bmatrix} + \Pi D_i + \Sigma v_i. \tag{6}$$

Here, $[\bar{\beta}, \bar{\alpha}]'$ is the population mean; $D_i \sim \hat{P}_D^*(D)$ are consumer-level observable variables (e.g., demographics), and Π is a $(K+1) \times d$ matrix of coefficients that measure how the taste characteristics vary with demographics; $v_i \sim \hat{P}_v^*(v)$ are consumer-level unobservables, and Σ allows each component of v_i to have a different variance and allows for correlation between these characteristics. It is useful to decompose utility into two parts:

$$u_{itj} = \underbrace{x_{tj}\bar{\beta} - \bar{\alpha}p_{tj} + \xi_{tj} + \varepsilon_{itj}}_{\delta_{tj}} + \underbrace{[x_{tj}, p_{tj}](\Pi D_i + \Sigma v_i)}_{\mu_{itj}} + \varepsilon_{itj}. \tag{7}$$

The first part is a "mean" level of utility $\delta_{tj}(x_{tj}, p_{tj}, \xi_{tj}; \theta_1)$, and the second part is individual deviations $\mu_{itj}(x_{tj}, p_{tj}, v_i, d_i; \theta_2)$ that capture the effect of random tastes parameters, where $\theta_1 = (\bar{\beta}, \bar{\alpha})$ represents the coefficients that enter the linear component, and $\theta_2 = (\Pi, \Sigma)$ are the parameters that enter the utility function nonlinearly.

Consumers are assumed to purchase one unit of the good that gives the highest utility. Let the set $(D_i, v_i, \varepsilon_{it0}, \ldots \varepsilon_{itJ})$ be a vector of demographics and product-specific shocks that lead to the choice of product j. Formally, let the set be

$$A_{tj}(x_{t\cdot}, p_{t\cdot}, \delta_{t\cdot}; \theta_2) = \{(D_i, v_i, \varepsilon_{it0}, \ldots, \varepsilon_{itJ}) \mid u_{itj} \geq u_{itk} \ \forall k = 0, 1, \ldots, J\}, \tag{8}$$

where $x_{t\cdot} = (x_{t1}, \ldots, x_{tJ})'$, $p_{t\cdot} = (p_{t1}, \ldots, p_{tJ})'$, and $\delta_{t\cdot} = (\delta_{t1}, \ldots, \delta_{tJ})'$ are the observed characteristics, prices, and mean utilities of all brands, respectively. The set A_{jt} then defines the characteristics of individuals who choose product j in market t. Then the market share of the jth product is the integral over the mass of consumers in the region A_{jt}. Formally, this is given by

$$s_{tj}(x_{t\cdot}, p_{t\cdot}, \delta_{t\cdot}; \theta_2) = \int_{A_{tj}} dP^*(\varepsilon) dP_v^*(v) d\hat{P}_D^*(D). \tag{9}$$

Similar to the homogeneous logit model, we can get a logit form of the market share by assuming ε_{ijt} is Type I extreme value distributed:

$$s_{jt}(x_{t\cdot}, p_{t\cdot}, \delta_{t\cdot}; \theta_2) = \frac{\exp\big(\delta_{tj} + u_{itj}(v_i, D_i; \theta_2)\big)}{1 + \sum_{k=1}^{J} \exp\big(\delta_{tk} + u_{itk}(v_i, D_i; \theta_2)\big)}. \tag{10}$$

The price elasticities of the market shares, s_{tj}, defined by (9) are

$$\eta_{tjk} = \frac{\partial s_{tj}}{\partial p_{tk}} \frac{p_{tk}}{s_{tj}} = \begin{cases} -\dfrac{p_{tj}}{s_{tj}} \displaystyle\int \alpha_i s_{itj}\big(1 - s_{itj}\big) dP_v^*(v) d\hat{P}_D^*(D) & \text{if } j = k, \\ \dfrac{p_{tk}}{s_{tj}} \displaystyle\int \alpha_i s_{itj} s_{itk} dP_v^*(v) d\hat{P}_D^*(D) & \text{otherwise,} \end{cases} \tag{11}$$

where $s_{itj} = \exp(\delta_{tj} + \mu_{itj})/[1 + \sum_{k=1}^{K} \exp(\delta_{tk} + \mu_{itk})]$ is the probability of individual i purchasing product j. Note that these elasticities have a more flexible form. First, the own-price elasticity will not necessarily be driven by the functional form. The partial derivative of the market shares will no longer be determined by a single parameter, α. Instead, each individual will have a different price sensitivity, which will be averaged to a mean price sensitivity using the individual's specific probabilities of purchase as weights. Second, the substitution patterns are not constrained by the market share, and the composite random shock $\mu_{itj} + \varepsilon_{itj}$ is no longer independent of the product and consumer characteristics. Thus, if the price of a brand goes up, consumers are more likely to switch to brands with similar characteristics rather than to the most popular product.

2.3.2. Identification. Recall the market share defined in (9). Given $\{x_{t\cdot}, p_{t\cdot}, \delta_{t\cdot}(x_{t\cdot}, p_{t\cdot}, \xi_{t\cdot}, \theta_1), \theta_2\}$ and the market share actually observed in the data S_{tj}, we can write a system of J equations in J unknowns (the ξ_{tj}):

$$s_{tj} = s(x_{t\cdot}, p_{t\cdot}, \delta_{t\cdot}(x_{t\cdot}, p_{t\cdot}, \xi_{t\cdot}, \theta_1), \theta_2). \tag{12}$$

However, solving these equations directly would lead to biased estimates, as ξ_{tj} is potentially correlated with price p_{tj} and other marketing mix variables in x_{tj}. Thus, to consistently estimate the model parameters, we need to find a set of valid instruments correlated with the endogenous variable p_{tj} but uncorrelated with the unobserved attributes ξ_{tj}. There are some commonly used instruments:

1. *Cost shifters* (Nevo [73]): These are the variables that affect marginal cost (product, packaging, distribution costs) and cost proxies such as city density for storage cost, salary for labor cost, etc.
2. *Prices of products in other markets* (Hausman [48], Nevo [74]): If we assume demand shocks uncorrelated across markets, the prices of products in other markets will be correlated with p_{tj} because of the common marginal cost, but it will be uncorrelated with the market-specific valuation of the product.
3. *Characteristics of competing products* (Berry et al. [16]): A firm sets the price of its product based on characteristics of competing products from competitors, but these characteristics will not affect consumer valuation for the firm's own product.

2.3.3. Estimation Algorithm. In this subsection, we discuss the computational details for the BLP model estimation algorithm.

Step 0: *Initialization.* In this step, we need to first give initial guesses for θ_2^0. Then we draw v_i from $\hat{P}_v^*(v)$ to compute the market shares (and D_i if needed). Also, we need to set up initial values for δ_{tj}^0. A suggestion is to set it to the estimates from the homogeneous model so that the algorithm can reach the convergence faster (Nevo [73]).

Step 1: *Computing market shares based on the model.* Let h be the index of iteration. For a given value of θ_2^h and $\delta_{t\cdot}^h$, we can compute the market share implied by (10). For the full random coefficients model, this integral has to be computed by simulation:

$$s_{tj} = \frac{1}{NS}\sum_{i=1}^{NS} \frac{\exp\left(\delta_{tj}^h + u_{itj}(v_i, D_i; \theta_2^h)\right)}{1 + \sum_{k=1}^{J} \exp\left(\delta_{tk}^h + u_{itk}(v_i, D_i; \theta_2^h)\right)}, \tag{13}$$

where NS denotes the number of random consumers sampled.

Step 2: *Contraction mapping to get mean utility* $(\delta_{t\cdot})$. We choose our estimates for $(\delta_{t\cdot})$ by minimizing the distance of the predicted market shares s_{tj} to the observed market shares S_{tj}. That is, we want to compute the $J \times T$ dimensional vector of mean valuations δ_{tj} that equates the computed market share in Step 1 to the observed shares. This amounts to solving separately for each market the system of equations

$$s(\delta_{t\cdot}; \theta_2) = S_{t\cdot}. \tag{14}$$

The above system of equations can be solved by using the contraction mapping suggested by Berry et al. [16], which amounts to computing the series

$$\delta_{t\cdot}^{h+1} = \delta_{t\cdot}^h + \ln(S_{t\cdot}) - \ln\left(s_{t\cdot}\left(\delta_{t\cdot}^h\right)\right). \tag{15}$$

Step 3: *Compute the error term and GMM objective function.* Given $\delta(S; \theta_2)$, the error term $\xi(\theta_2)$ can be written as

$$\xi(\theta_2) = \delta_{tj}(s_{t\cdot}; \theta_2) - X_1\theta_1, \tag{16}$$

where X_1 contains the variables that enter the linear part of the estimation, including price p_{tj} and other characteristics x_{tj} common to all individuals. Given a set of valid instruments Z, we want to get the get the parameter estimators using the generalized method of moments (GMM) based on the assumption $E[\xi_{tj}(\theta_2)'Z] = 0$. The GMM objective function is then given by

$$Q(\theta) = \xi(\theta)' Z \Phi^{-1} Z' \xi(\theta), \tag{17}$$

where Φ is the weight matrix and is a consistent estimator of $E[Z'\xi(\theta_2)\xi(\theta_2)'Z]$.

Step 4: *Search for the value of* θ_2 *that minimizes the objective function.* The model parameters will be estimated in two steps. First, we express the linear estimator θ_1 as a function of θ_2; that is,

$$\hat{\theta}_1 = \left(X_1' Z \Phi^{-1} Z' X_1\right)^{-1} X_1' Z \Phi^{-1} Z' \delta_{tj}(s_{t\cdot};\theta_2). \tag{18}$$

Then, we can perform a nonlinear search over θ_2 that minimizes the objective function $Q(\theta_2)$.

Step 5: *Iterate among* Steps 2–4 *nested for every* θ_2 *trial until convergence.*

3. Dynamic Discrete Choice Models

3.1. Introduction

In dynamic discrete choice structural models, agents choose between two or more discrete alternatives and are assumed to be forward-looking and maximizing their expected intertemporal payoff. A dynamic structural model describes agents' preferences and beliefs, as well as how states evolve from period to period, often as a function of agents' decisions. Under the principle of revealed preference, parameters that govern these processes are estimated using data on agents' choices and outcomes (Aguirregabiria and Mira [3]). Given the dynamic nature of such models, typically, longitudinal data that track agents' behavior over time are required to estimate these models.

Dynamic structural models are more challenging to estimate than static models because in dynamic models, agents are assumed to make decisions based on the expected intertemporal utility, and consequently, when estimating such models, one needs to evaluate the expected discounted value of lifetime utility (or payoff) associated with each choice in each period, which is often a complex function of utility parameters. Miller [70], Pakes [76], Rust [85], and Wolpin [98] are among the earliest seminal papers that estimate a dynamic discrete choice model. The respective authors of these papers used different approaches to evaluate the lifetime value associated with each option (alternative), based on their specific research contexts and the structure of the models they constructed for these contexts. These seminal papers demonstrated that estimating dynamic discrete choice models is feasible and important for answering key research questions (Arcidiacono and Ellickson [7]). Among these different approaches, the framework proposed by Rust [85] is arguably the most widely adopted because of its simplicity and ease of interpretation. Therefore, in the following subsection, we use Rust's "bus engine replacement model" (Rust [85]) as an example of a (single-agent) dynamic discrete choice model, and we introduce the "nested fixed point algorithm" proposed in the same paper as the first estimation method. At the conceptual level, Rust's estimation algorithm is quite straightforward—a dynamic programming solution procedure that evaluates the lifetime value associated with each option is nested in the optimization of the estimation criterion (e.g., the maximization of the likelihood) (Aguirregabiria and Mira [3]). Rust's method is a "full solution" method for the estimation of dynamic discrete choice models, in that it requires solving the full dynamic programming problem. Because the inner dynamic programming solution procedure needs to be executed for each candidate set of parameter estimates (i.e., for each iteration of the outer optimization of the estimation criterion), the dynamic programming solution procedure is performed once for each agent. Full solution methods are typically computationally burdensome, and the computational complexity increases significantly as the size of the state space increases.

To overcome the computational challenges in estimating complex dynamic discrete choice models, starting from Hotz and Miller [52], researchers have developed a variety of *conditional choice probability* (CCP)–based estimation methods, which do not require repeated solution of the dynamic programming problem. In addition to simplifying the estimation of simple dynamic discrete choice models, CCP-based estimation methods also enable estimation of complex models and consideration of unobserved state variables. In the following subsection, we will also discuss the original CCP-based estimation method proposed in Hotz and Miller [52].

Depending on whether agents interact with each other in the model, dynamic discrete choice structural models can be classified into single-agent models where agents are independent of each other and dynamic game models where agents interact with each other. In dynamic games, an agent's payoff is affected by other agents' decisions, and therefore, rational agents take other agents' actions into consideration when making their own decisions. An agent's decisions depend on not only his or her own individual state but also other agents' states, which are sometimes summarized into an "industry/market"-level state. Because these models are "games," the concept of equilibrium is applied. An equilibrium in a dynamic game is a fixed point of a system of best-response operators, and each player's best response is itself the solution to a dynamic programming problem (Aguirregabiria and Mira [3]). Considering the need to solve for the equilibrium (equilibria) and the increased state space compared with single-agent dynamic models, estimating dynamic games is even more computationally burdensome. This makes the CCP-based estimation method more attractive in the estimation of dynamic games. In Section 3.3, we will first briefly describe how to use the full solution method to estimate a dynamic game model and move quickly on to the CCP-based methods proposed by Aguirregabiria and Mira [2] and Bajari et al. [12]. Until recently, the structural estimation of dynamic games was limited to games with only a few players (e.g., in an oligopoly industry organization setting). New estimation methods and equilibrium concepts have recently been proposed to deal with dynamic games with many players (e.g., Weintraub et al. [95]).

Dynamic discrete choice models have been used in the management disciplines to study a variety of research questions. For example, the single-agent dynamic discrete choice framework has been used to model consumer learning under quality uncertainty (Erdem and Keane [31]), salespeople's effort provision (Chung et al. [26]), callers' sensitivity to delay in a call center (Akşin et al. [5]) and agents' behavior in many other settings. Dynamic game models have been used to study the market structure of specific markets (e.g., Liu [62]), employees' blogging activities in the enterprise setting (Huang et al. [55]), physician prescription behavior under the influence of competitive detailing (Liu et al. [63]), knowledge sharing in customer support forum (Lu et al. [64]), hosts' choices of the image quality level on Airbnb (Zhang et al. [101]), and so forth.

In what follows, we will discuss the model setup and estimation strategies for single-agent discrete choice models and dynamic discrete games, with a focus on infinite-horizon dynamic models. Finite-horizon dynamic models can be set up in a similar way as infinite-horizon models. However, important differences between finite-horizon models include (1) in finite-horizon models, the value function (expected intertemporal utility) is period specific, and as a result, agents' strategies are also period specific; and (2) backward induction is the standard approach for solving and estimating finite-horizon models.

3.2. Single-Agent Models

3.2.1. Model Setup. For the ease of reference, we use the model Rust [85] proposed to capture the bus engine replacement problem as an example of single-agent dynamic discrete choice models (and use similar notations). The "bus engine" replacement model is set up as follows.

Suppose that there are $i = 1, 2, ..., N$ buses owned by a bus company. In each period, for each bus (independently), the company needs to decide, based on the current state of the bus, whether to replace its engine. Engine replacement would generate cost c_r immediately, but after the engine replacement, the operating cost such as cost for engine maintenance would be lower in the following periods. Let the state variable x_{it} denote the cumulative mileage on bus i's engine at time t since the last replacement. Operating cost can then be assumed as an increasing, differentiable function of x_t : $c(x_t; \theta)$. The replacement decision for bus i in period t is denoted as d_{it}. Intuitively, the series of optimal replacement decisions should maximize the expected discounted utility (or equivalently, minimize the expected discounted costs):

$$E\left\{ \sum_{\tau=t}^{\infty} \beta^{\tau-t} u(d_{i\tau}, x_{i\tau}, \epsilon_{i\tau}; \theta_1) \mid x_{it}, \epsilon_{it} \right\}, \tag{19}$$

where the single-period utility function u is given by

$$u(d_{it}, x_{it}, \epsilon_{it}; \theta_1) = -I_{\{d_{it}=0\}} \cdot c(x_t; \theta) - I_{\{d_{it}=1\}} \cdot [c_r + c(0; \theta)], \tag{20}$$

and $\theta_1 = \{c_r, \theta\}$.

Equation (19) is the value function, representing the discounted sum of current and future utilities for being in state $\{x_{it}, \epsilon_{it}\}$ and taking optimal decisions in the future. The value function can be expressed in the following recursive form (the Bellman equation):

$$V(x_{it}, \epsilon_{it}) = \max_{j \in \{0,1\}} \left\{ u\big(d_{it} = j, x_{it}, \epsilon_{ijt}; \theta_1\big) + \beta \iint V(x_{it+1}, \epsilon_{it+1}) \cdot f(x_{it+1} \mid x_{it}, d_{it} = j; \theta_2) dx_{it+1} g(\epsilon_{it+1}) d\epsilon_{it+1} \right\}, \tag{21}$$

where f is the state-transition density when alternative j is selected, and θ_2 is the set of parameters governing the choice-specific state-transition process. Solving the above equation can provide us with the optimal policy for choice making.

Note that the above model can easily be generalized. To illustrate this, let's consider a more general framework in which agents select alternatives from a finite choice set $C(x_t)$ in each period $t\,(t = 1, 2, ..., T)$. The optimal decisions are determined sequentially by maximizing the discounted sum of utility flow in the future; that is, each agent is forward-looking. Two sets of state variables affect agents' choices: one is observed by both researchers and the agents and the other is only observed by the agents. Denote the former as x_t and the latter as ϵ_{it}. In each period, the state variables can transition into new values conditional on the choice made in the previous period. This unified framework demonstrates how single-agent models can be applied to a variety of settings. For example, customers decide whether and how much to purchase for some storable goods (e.g., detergent, ketchup) in each period, and their decisions should depend on current storage, prices, etc. In another example, individuals can make decisions about when to retire; retiring earlier would give them a lower yearly income but for a longer period—such an intertemporal trade-off lies at the core of individuals' choice making. In the following, we explain in detail how to estimate a single-agent model with necessary assumptions.

3.2.2. Assumptions.

Assumption 1 (Additive Separability). *Private information (error term) is assumed to be additively separable from the determined part of utility. That is,*

$$u\big(d_{it} = j, x_{it}, \epsilon_{ijt}; \theta_1\big) = \underline{u}(d_{it} = j, x_{it}; \theta_1) + \epsilon_{ijt}.$$

This is the standard additive separability assumption commonly known for discrete choice models.

Assumption 2 (First-Order Markov Process). *The state variables $\{x_{it}, \epsilon_{it}\}$ depend only on $\{x_{it-1}, \epsilon_{it-1}, d_{it-1}\}$ and not states/decisions from $t-2$ or earlier.*

This assumption implies that the state-transition process is a first-order Markov process. The advantage of this assumption is that we do not have to carry long memory of past information when computing the state-transition probabilities.

Assumption 3 (Conditional Independence [CI]). *The state-transition probability can be factored as*

$$P(x_{it}, \epsilon_{it} \mid x_{it-1}, \epsilon_{it-1}, d_{it-1}) = G(\epsilon_{it} \mid x_{it}) \cdot F(x_{it} \mid x_{it-1}, d_{it}). \tag{22}$$

This assumption implies two restrictions. First, x_{it} is a sufficient statistic for ϵ_{it}; in other words, any statistical dependence between ϵ_{it} and ϵ_{it-1} is transmitted entirely through x_{it}. Second, the probability density of x_{it} depends only on x_{it-1} and not ϵ_{it-1}. The CI assumption ensures that the derivation of the value function does not involve numerical integration over ϵ_{it}, which greatly alleviates computational burden in practice.

Assumption 4 (IID Errors). *The unobserved state variable ϵ_{it} is assumed to be independently and identically distributed as follows:*

$$G(\epsilon_{it} \mid x_{it}) = G(\epsilon_{it}). \tag{23}$$

This assumption states that the error terms have identical distribution over individuals and over time. More importantly, it rules out the possibility of having persistent unobservables that affect individuals' decisions. Persistent unobservables will enter into error terms and generate bias in model estimates. For example, in the bus engine problem, a persistent unobservable can be bus brand. Different brands may generate different operating costs; thus, bus brand can affect engine-replacement decisions. If researchers do not observe bus brand, it will generate serial correlation in error terms, leading to bias in the estimates for operating costs.

Assumption 5 (Discrete State Space). The observable state variables x_{it} are assumed to be discrete with finite dimensionality.

This assumption is made for practical purposes. In most of the cases, the value functions do not have a closed-form solution and need to be solved numerically. Having infinite values of observed state variables can be practically infeasible when deriving numerical estimates for the value functions. This point will be further illustrated in the estimation methods. Thus, in practice, continuous-valued state variables should be discretized into finite values.

3.2.3. Estimation. Parameter estimates for a single-agent model can be derived from likelihood maximization. With the assumptions defined above, the likelihood function for each individual i can be expressed as follows:

$$\begin{aligned} L_i(\theta_1, \theta_2 \mid d_{i1}, x_{i1}, d_{i2}, x_{i2}, ..., d_{iT}, x_{iT}) &= \prod_{t=1}^{T} P(d_{it}, x_{it} \mid d_{it-1}, x_{it-1}, ..., d_{i1}, x_{i1}; \theta_1, \theta_2) \\ &= \prod_{t=1}^{T} P(d_{it}, x_{it} | d_{it-1}, x_{it-1}; \theta_1, \theta_2) \\ &= \prod_{t=1}^{T} P(d_{it} | x_{it}; \theta_1) \cdot P(x_{it} | d_{it-1}, x_{it-1}; \theta_2). \end{aligned} \tag{24}$$

Thus, the log-likelihood for individual i is

$$LL_i(\theta_1, \theta_2 | d_{i1}, x_{i1}, d_{i2}, x_{i2}, ..., d_{iT}, x_{iT}) = \sum_{t=1}^{T} P(d_{it} | x_{it}; \theta_1) + \sum_{t=1}^{T} P(x_{it} | d_{it-1}, x_{it-1}; \theta_2). \tag{25}$$

Following the estimation procedure proposed by Rust [88], the log-likelihood can be estimated by two steps. In the first step, we estimate the state-transition parameters θ_2. In the second

step, we use the parameter estimates $\hat{\theta}_2$ derived from Step 1 to estimate the utility parameters θ_1.[1] The derivation of $\hat{\theta}_2$ is comparatively straightforward—researchers can estimate the state-transition probabilities nonparametrically in the following way:

$$\hat{f}(x'|x, d = j) = \frac{\sum_{i=1}^{N}\sum_{t=1}^{T-1} I(x_{it+1} = x', x_{it} = x, d_{it} = j)}{\sum_{i=1}^{N}\sum_{t=1}^{T-1} I(x_{it} = x, d_{it} = j)}. \tag{26}$$

Note that the state-transition probabilities can also be parametrically estimated based on certain model assumptions. Each method has pros and cons. Parametric estimation can avoid the sparsity issue caused by high dimensionality of the state space; however, the model assumptions may be inappropriately imposed. By contrast, nonparametric estimation is free of model assumptions and thus is truthful to the data; however, it suffers from the "curse of dimensionality."

In order to derive $\hat{\theta}_1$, researchers are required to obtain numerical estimates for the value function. Specifically, if the error terms follow i.i.d. the Type I extreme value distribution, the choice probabilities can then be written in the following way:

$$\begin{aligned} &P(d_{it} = j|x_{it}; \theta_1) \\ &= \frac{\exp\left[\underline{u}(d_{it} = j, x_{it}; \theta_1) + \beta \iint V(x_{it+1}, \epsilon_{it+1}) f\left(x_{it+1} \mid x_{it}, d_{it} = j; \hat{\theta}_2\right) dx_{it+1} g(\epsilon_{it+1}) d\epsilon_{it+1}\right]}{\sum_{j'} \exp\left[\underline{u}(d_{it} = j', x_{it}; \theta_1) + \beta \iint V(x_{it+1}, \epsilon_{it+1}) f\left(x_{it+1} \mid x_{it}, d_{it} = j'; \hat{\theta}_2\right) dx_{it+1} g(\epsilon_{it+1}) d\epsilon_{it+1}\right]} \\ &= \frac{\exp[\underline{u}(d_{it} = j, x_{it}; \theta_1) + \beta EV(x_{it}, j)]}{\sum_{j'} \exp[\underline{u}(d_{it} = j', x_{it}; \theta_1) + \beta EV(x_{it}, j')]}. \end{aligned} \tag{27}$$

In (27), $EV(x_{it}, j) = \iint V(x_{it+1}, \epsilon_{it+1}) f(x_{it+1} \mid x_{it}, d_{it} = j; \hat{\theta}_2) dx_{it+1} g(\epsilon_{it+1}) d\epsilon_{it+1}]$ and has the following property:

$$EV(x_{it}, j) = \int_{x_{it+1}} \log\left\{\sum_{j'} \exp(\underline{u}(d_{it} = j', x_{it}; \theta_1) + \beta EV(x_{it}, j'))\right\} f\left(x_{it+1}|x_{it}, d_{it} = j'; \hat{\theta}_2\right). \tag{28}$$

Note that the above choice probabilities are similar to what we estimate in a static discrete choice model—the only difference is the additional terms of value functions. As long as we can get the numerical estimates for the value functions, we can obtain the choice probabilities and proceed to estimate the whole likelihood function. In general, there are two types of methods for estimating the value functions. The first one deploys a nested fixed point algorithm. The second one is based on CCPs.

***Nested Fixed Point Algorithm* (Rust [85]).** The overall idea behind the nested fixed point algorithm is to iteratively search for θ_1 in the parameter space and numerically solve for the value functions until convergence. Specifically, the nested fixed point algorithm can be implemented by using the following procedure.

Step 1: Assume initial values for θ_1, denoted as θ_1^0.

Step 2: Let k be the iteration index. Start from $EV(x) = 0$ for $\forall x$. For a given value of θ_1^k and $\hat{\theta}_2$, iterate over (28) until $EV(x, j)$ converges. Denote the converged expected value function as $EV^k(x, j)$.

Step 3: Plug the converged values of $EV(x, j)$ into the likelihood function in (25). Then follow a standard likelihood-maximization procedure to update the value for θ_1. Denote the new value as θ_1^{k+1}.

Step 4: Repeat Steps 2 and 3 until the estimates for θ_1 converges.

Note that the standard errors derived from the above optimization procedure can be biased downward. This is because we are using $\hat{\theta}_2$ instead of the true value of θ_2 when computing the value functions. The standard approach to revamp the standard errors is to use the bootstrap method. If Assumption 4 (IID errors) holds, we can sample with replacement over the observations. It is worth noting that it is always better to sample over individuals (i.e., over i), because the IID assumption might fail under some circumstances and generate serial correlation among the observations for a same individual. The nested fixed point algorithm used in Rust [85] combines the successive approximation and Newton–Kantorovich iterations and is more efficient for an estimation problem with a large number of parameters (Iskhakov et al. [56]).

***CCP-Based Two-Step Method* (Hotz and Miller [52]).** As noted above, the nested fixed point algorithm is a full solution method for estimating single-agent models. In practice, this method can be computationally burdensome because it requires solving the expected value function for each state x and each draw of θ_1. As the dimensionality and the size of state x increase, value function iteration would become costly or even infeasible. To overcome this difficulty, a series of CCP-based two-step methods was invented by researchers. The main idea in CCP-based methods is to simulate the future value functions based on CCPs that are nonparametrically calculated for each state, state-transition probabilities that are parametrically or nonparametrically estimated, and the analytical relationship between value functions and CCPs. In the following, we focus on the CCP method proposed by Hotz and Miller [52].

Suppose that we are given the data sequence of (d_t, x_t) for each individual. The very first step of Hotz and Miller's [52] method is to nonparametrically estimate CCPs from the data. From a mathematical perspective, the CCP for choosing alternative j at state x can be estimated as follows:

$$\hat{p}(j|x) = \frac{\sum_{i=1}^{N}\sum_{t=1}^{T} I(d_{it} = j, x_{it} = x)}{\sum_{i=1}^{N}\sum_{t=1}^{T} I(x_{it} = x)}. \tag{29}$$

Similarly, the state-transition probabilities can be estimated nonparametrically using Equation (26). It is worth noting that the CCPs should always be nonparametrically estimated. This is because there is no way to express individuals' behavior scheme in a closed form, and imposing parametric formulations for CCPs would be conceptually fallacious. Yet, in the case of a high-dimensional state space, estimating CCPs would still face the sparsity issue. Fortunately, prior studies have proposed a handful of practical solutions to overcome this challenge, such as flexible logits and kernel smoothing, among other machine learning methods.

Given the estimated CCPs and state-transition probabilities, we can proceed to the next step: forward simulate the future value functions. Theoretically, the future value functions can be derived by summing up the discounted utility flows in the subsequent infinite periods. That is,

$$\begin{aligned}\hat{V}(d_{i\tau} = j, x_{i\tau} = x; \theta_1) &= u(d_{i\tau} = j, x_{i\tau} \\ &= x; \theta_1) + \sum_{t=1}^{T} \beta^t E_{x_{\tau+t}\,|\,x_{\tau+t-1},\, d_{\tau+t-1}} E_{d_{\tau+t}|x_{\tau+t}} E_{\epsilon_{\tau+t}|x_{\tau+t},\, d_{\tau+t}} [u(d_{\tau+t}, x_{\tau+t}; \theta_1) + \epsilon_{\tau+t}].\end{aligned} \tag{30}$$

For Type I extreme value errors, there exists an analytical relationship between the expectation of the errors and the CCPs. Specifically,

$$E[\epsilon \,|\, d = j, x] = \gamma - \log\hat{p}(j \,|\, x), \tag{31}$$

where γ is Euler's constant.

The expression of future value functions can be approached numerically as follows:

$$\hat{V}(d_{i\tau} = j, x_{i\tau} = x; \theta_1)$$

$$\approx \frac{1}{S}\sum_{s=1}^{S}\left\{u(d_{i\tau} = j, x_{i\tau} = x; \theta_1) + \sum_{t=1}^{T}\beta^t\left[u\left(d^s_{i\tau+t}, x^s_{i\tau+t}; \theta_1\right) + \gamma - \log\hat{p}\left(d^s_{i\tau+t} \mid x^s_{i\tau+t}\right)\right]\right\}, \quad (32)$$

where T denotes the number of periods that we simulate forward and S denotes the number of simulation paths. Two sources of errors exist in the above approximation process. One error source comes from the numerical estimation for the expectations: that is, the error generated when we use average of simulation paths to approximate the expectations. The second error source comes from the truncation of infinite time periods into T finite periods. These errors will result in less efficient estimates; however, it can be shown that the consistency of the estimation is always guaranteed. To eliminate the two error sources in the estimation, researchers need to choose sufficiently large S and T. With the simulated future value functions, we can use (30) to derive the likelihood function and compute optimal parameter estimates.

It is important to note that the CCPs estimated from the data can only be used for estimation, but *not* in counterfactual simulations. In counterfactual simulations, researchers have to resolve the dynamic program, find the new value functions, and calculate the corresponding CCPs. Given that no GMM or maximum likelihood estimation optimization is involved when resolving the dynamic program, it is not computationally cumbersome.

3.3. Dynamic Discrete Games

In this subsection, we discuss the construction and estimation of simple dynamic game models (imperfect information game). The model formulation and notations used in the example below are based primarily on Aguirregabiria and Mira [2].

3.3.1. Model Setup. Consider a game where there are N players (agents). Players are indexed by i, $i \in \{1, 2, \ldots, N\}$. In every discrete period t, all players simultaneously make their choices from a set of possible action A. We denote the choice made by player i in period t as a_{it}. For simplicity, we assume player actions are discrete and there are only two alternatives from which they can choose, denoted as 0 and 1, respectively (i.e., $A = \{0, 1\}$). At the beginning of period, each a player is characterized by two vectors of state variables: x_{it} and ϵ_{it}. Among them, x_{it} is a set of observable state variables, which are common knowledge among all players, whereas ϵ_{it} is a vector of choice-specific shocks, which is private information observable to player i herself but not to other players. The length of vector ϵ_{it} equals the number of alternatives. For simplicity, let us assume that there is only one common knowledge state variable; that is, x_{it} is a scalar.

To put this model into context, here is a potential setting to which we can apply the model. A researcher is interested in studying firms' technology investment decision in a particular industry with a small number of players (firms). The researcher can apply the model described above to capture firms' behavior. In the model, a period can be defined as a year, and in each period, each player is making a binary decision as to whether to invest in technology (a_{it}). If a firm decides to invest in technology, it incurs fixed costs. (For simplicity, the cost of technology investment is assumed to be a fixed parameter, as in Rust [88], but not a continuous decision variable.) The state variable of interest in this setting is each firm's technology capability (x_{it}), which is assumed to take a few discrete values (e.g., 1, 2, …, 5; the larger the value, the higher a firm's technology capability). There is a natural depreciation of technology, meaning that irrespective of whether a firm invests in technology in period t, its technology capability has a positive probability of dropping to a lower level. If a firm decides to invest in technology in a period t, there is a stochastic improvement of the firm's technology capability

compared with the "no investment" scenario. These two processes jointly determine the technology state-transition probabilities, which is obviously action specific. Each firm's technology capability at the beginning of a period affects the quality of the product it produces in that period. All firms compete against each other in the market, and a firm's periodic profit (utility) is a function of both its own and its competitors' technology capabilities (the state variable).

In dynamic games, a player's single-period utility can be affected by both her own action and other players' actions and her own state and other players' states. Let x_t and a_t denote the stack of the state variables of all players in the game (i.e., $x_t \equiv (x_{1t}, x_{2t}, ..., x_{Nt})$) and the stack of all actions taken by all players (i.e., $a_t \equiv (a_{1t}, a_{2t}, ..., a_{Nt})$), respectively, in period t. The private shocks other players receive do not directly affect the single-period utility of the focal player. (They can indirectly affect the single-period utility through affecting other players' actions, though. But controlling for other players' actions, their private shocks do not affect the single-period utility of the focal player.) Therefore, the utility that a player receives in period t is $U_i(a_t, x_t, \epsilon_{it})$. In the technology investment example, this is the periodic profit function of each firm, which is affected by the focal firm's investment decision, all firms' technology capability states, and the private shock to the focal firm.

As in single-agent dynamic discrete choice models, in a dynamic game model, a player's action in a period (t) will affect her future utility; otherwise, the model reduces to a static model as there is no intertemporal trade-off. Typically, it is assumed that the state-transition process is Markovian and that a player's action in period t affects the transition of her individual state from period t to period $t+1$. Under these assumptions, the state-transition probability (at the market level) can be characterized by a function $P(x_{t+1}, \epsilon_{t+1} \mid a_t, x_t, \epsilon_t)$. The state-transition process is common knowledge among all players as well. With this model setup, in period t, a rational forward-looking player i maximizes the expected discounted intertemporal utility, which can be expressed as

$$E\left\{\sum_{\tau=t}^{\infty} \beta^{\tau-t} U_i(a_\tau, x_\tau, \epsilon_{i\tau}) \mid x_t, \epsilon_{it}\right\}, \tag{33}$$

where β is the discount factor, which takes a value between 0 and 1 (e.g., 0.9). The expectation is taken over other players' current and future actions, the future values of the observable state variable, and player i's own future private information shocks.

As in the single-agent dynamic discrete choice models, we will make a few simplification assumptions.

3.3.2. Assumptions.

Assumption 6 (Additive Separability). *The single-period utility function is additively separable in common knowledge and private information components. That is,* $U_i(a_t, x_t, \epsilon_{it}) = u(a_t, x_t) + \epsilon_{it}(a_{it})$.

Assumption 7 (IID Errors). *Private information shocks* ϵ_{it} *are independently and identically distributed over players and over time. The cumulative distribution function of* ϵ_{it}*, denoted as* $G_\epsilon(\epsilon_{it})$*, has finite first moments and is continuous and twice differentiable in* ϵ_{it}*. (Note that* ϵ_{it} *is often assumed in the applied research to follow the Type I extreme value distribution, which will be discussed later.)*

Assumption 8 (Conditional Independence). *Conditional on* x_t *and* a_t*,* x_{t+1} *does not depend on current private information shocks* ϵ_t*.*

Assumptions 7 and 8 combined imply that we can rewrite $P(x_{t+1}, \epsilon_{t+1} \mid a_t, x_t, \epsilon_t)$ as $P(x_{t+1}, \epsilon_{t+1} \mid a_t, x_t, \epsilon_t) = G_\epsilon(\epsilon_{t+1})\, F(x_{t+1} \mid a_t, x_t)$. This has important implications for the estimation of the dynamic game model—the state-transition process of the observed state variable, characterized by $F(x_{t+1} \mid a_t, x_t)$, can be separately estimated without needing to solve the game.

The game has a Markov structure, in which players are assumed to play Markov strategies. That is, players' decisions depend only on current-period states. We further assume that the game is played for an infinite number of periods, and the Markov process is stationary. Under these assumptions, we can drop subscript t from all the notations. That is, as long as observable states (both the focal player's state and all the opponents' states) and the private shock are the same, players play the same strategy irrespective of t. Let $\sigma = \{\sigma_i(x, \epsilon_i)\}$ be a set of strategy functions (decision rules), one element for each player, which is a mapping from the common knowledge state and the focal player's private shock to her action. Given all other players except i are using strategies in σ, the decision problem of player i reduces to a single-agent dynamic discrete choice problem. The corresponding value function is

$$V_i^{\sigma}(x, \epsilon_i) = \max_{a_i \in A}\{v_i^{\sigma}(a_i, x, \epsilon_i)\}, \tag{34}$$

where $v_i^{\sigma}(a_i, x, \epsilon_i) = E_{\epsilon_{-i}} U_i(a_i, \sigma_{-i}(x, \epsilon_{-i}), x, \epsilon_i) + \beta E_{\epsilon_{-i}}[\int V_i^{\sigma}(x', \epsilon_i') dP(x', \epsilon_i' \mid a_i, \sigma_{-i}(x, \epsilon_{-i}), x, \epsilon_i)]$. The subscript $-i$ represent the collection of all players except player i. The expectation in both terms on the right-hand side of the equation is taken over all opponents' observed shocks ϵ_{-i}. The best-response function of player i when all other players behave according to strategy σ is then $b_i^{\sigma}(x, \epsilon_i, \sigma_{-i}) = argmax_{a_i \in A}\{v_i^{\sigma}(a_i, x, \epsilon_i)\}$ A Markov perfect equilibrium (MPE) for this game is a set of strategy functions σ^* in which for any i and for any combination of (x, ϵ_i), $\sigma_i^*(x, \epsilon_i) = b_i^{\sigma^*}(x_t, \epsilon_{it}, \sigma_{-i}^*) = \text{argmax}_{a_i \in A}\{v_i^{\sigma^*}(a_i, x, \epsilon_i)\}$ That is, the equilibrium strategy is a fixed point of a best-response mapping.

Under the three assumptions made above, we can use an *integrated value function*—$\tilde{V}^{\sigma}(x) \equiv \int V_i^{\sigma}(x, \epsilon_i) dG_{\epsilon}(\epsilon_i)$, which is only a function of x (ϵ_i is integrated out)—to characterize the single-agent dynamic programming problem faced by player i when all other players behave according to σ. Let $\tilde{v}_i^{\sigma}(a_i, x) = E_{\epsilon_{-i}} u(a_i, \sigma_{-i}(x, \epsilon_{-i}), x) + \beta E_{\epsilon_{-i}}[\sum_{x'} V_i^{\sigma}(x') dF(x' \mid a_i, \sigma_{-i}(x, \epsilon_{-i}), x)]$, which is often called the *choice-specific value function*. Then the integrated Bellman equation is $\tilde{V}_i^{\sigma}(x_t) = \int \max_{a_i \in A}\{\tilde{v}^{\sigma}(a_i, x) + \epsilon_i(a_i)\} dG_{\epsilon}(\epsilon_i)$. Here, $E_{\epsilon_{-i}} u(a_i, \sigma_{-i}(x, \epsilon_{-i}), x)$ is the expected single-period utility with the individual private shock ϵ_i excluded.

Define the set of CCPs corresponding to a given set of strategy σ as $P^{\sigma} = \{P_i^{\sigma}(a_i \mid x)\}$, with each element $P_i^{\sigma}(a_i \mid x)$ defined as follows:

$$P_i^{\sigma}(a_i \mid x) \equiv \Pr(\sigma_i(x, \epsilon_i) = a_i \mid x) = \int_{\epsilon_i} I\{\sigma_i(x, \epsilon_i) = a_i \mid x\} dG_{\epsilon}(\epsilon_i). \tag{35}$$

The best-response mapping in the probability space given that all players other than the focal player i behave according to an arbitrary P is then $\Gamma(P) = \{\Gamma_i(a_i \mid x, P_{-i})\}$ for all i, with $\Gamma_i(a_i \mid x, P_{-i})$ defined as

$$\Gamma_i(a_i | x, P_{-i}) = \int_{\varepsilon_i} I\Big\{a_i = \text{argmax}_{a_i' \in A}\big\{\tilde{v}_i^P(a_i', x) + \epsilon_i(a_i')\big\} dG_{\varepsilon}(\epsilon_i)\Big\}, \tag{36}$$

where $\tilde{v}_i^P(a_i', x)$ is the choice-specific value function that corresponds to P. If ϵ follows the i.i.d. Type I extreme value distribution, then $\Gamma_i(a_i \mid x, P_{-i}) = \exp(\tilde{v}_i^P(a_i', x)) / \sum_{a_i \in A} \exp(\tilde{v}_i^P(a_i', x))$. Let P^* be the set of CCPs that corresponds to the equilibrium strategy σ^*. An MPE can be represented in the probability space as a fixed point of the mapping of $P^* = \Gamma(P^*)$. That is, given a set of parameters, this is one way to find the MPE (in the probability space).

As a side note, Aguirregabiria and Mira [2] use a matrix inversion approach to directly solve for $\tilde{V}_i^{P^*}(x)$, which is the integrated value function under an equilibrium characterized by P^*. The corresponding integrated Bellman equation can be rewritten as

$$\tilde{V}_i^{P^*}(x) = \sum_{a_i \in A} P_i^*(a_i | x)\big[\tilde{u}_i^{P^*}(a_i, x) + e_i^{P^*}(a_i, x)\big] + \beta \sum_{x' \in X} \tilde{V}_i^{P^*}(x') f^{P^*}(x' \mid x), \tag{37}$$

where $\tilde{u}_i^{P^*}(a_i, x)$ is the expected single-period utility ($E_{\epsilon_{-i}} u(a_i, \sigma_{-i}^*(x, \epsilon_{-i}), x)$) and $f^{P^*}(x' \mid x)$ is the transition probability induced by the equilibrium characterized by P^*, which can be directly computed. The term $e_i^{P^*}(a_i, x)$ is the expectation of $\epsilon_i(a_i)$ conditional on x and action a_i being the optimal response for player i, which can be calculated as

$$e_i^{P^*}(a_i, x) = \frac{1}{P_i^*(a_i|x)} \times \int \epsilon_i(a_i) I\{\epsilon_i(a_i') - \epsilon_i(a_i) \le \tilde{v}_i^{P^*}(a_i', x) - \tilde{v}_i^{P^*}(a_i', x), \forall a_i' \ne a_i\} dG_\epsilon(\epsilon_i). \tag{38}$$

In a special case where ϵ follows the Type I extreme value distribution, i.i.d. w.r.t. individual, time, and action, $e_i^{P^*}(a_i, x)$ has a closed form: $e_i^{P^*}(a_i, x) = \gamma - \ln(P_i^*(a_i \mid x))$. Let $\tilde{V}_i^{P^*}$, $P_i^*(a_i)$, $\tilde{u}_i^{P^*}$, and $e_i^{P^*}(a_i)$ be the states of the corresponding state-specific elements for all possible values of x, and let F^{P^*} be the state-transition matrix with $f^{P^*}(x' \mid x)$ as its elements. Then $\tilde{V}_i^{P^*}$ can be computed as

$$\tilde{V}_i^{P^*} = \left(I - \beta F^{P^*}\right)^{-1} \left\{ \sum_{a_i \in A} P_i^*(a_i) \left[\tilde{u}_i^{P^*}(a_i) + e_i^{P^*}(a_i)\right] \right\}. \tag{39}$$

3.3.3. Estimation. Next, we will discuss two methods to estimate the dynamic game model described above. We need to first clarify what are observable and unobservable from the researchers' standpoint. In theory, both the common-knowledge states (x_t) and the private states (ϵ_t) could have observable and unobservable parts to researchers. In the discussion below, we consider the simplest case where *all common-knowledge states are observable and the private shocks are unobservable to researchers.* However, the distribution of the private shock is assumed to be known; that is, $G_\epsilon(\epsilon)$ is given, but the realization of ϵ_{it} for each (i, t) combination is unobservable. The action of each player in each period is also observable to researchers, and it is assumed that *data come from a single equilibrium.* We will discuss implications of relaxing these assumptions on data in Section 3.4.

Under the conditional independence assumption, we can separately estimate the parameters governing the state-transition process without needing to solve for the equilibrium of the game. Therefore, it is convenient to divide the entire parameter vector θ into θ_1, the parameters in the players' single-period utility function, and θ_2, the parameters describing the state-transition process. As in the single-agent dynamic discrete choice models, the discount factor is assumed constant across agents and is preset.

Among θ_1 and θ_2, θ_2 can be estimated by maximizing the partial likelihood function $\ln(\sum_{t=1}^{T-1} l(x_{t+1}|x_t, a_t; \theta_2))$, where T is the number of periods spanned by the data, and $l(x_{t+1} \mid x_t, a_t; \theta_2)$ is the likelihood of observing x_{t+1} in period $t+1$ given the stack of player state (x_t) and the stack of player actions in period t (a_t), conditional on a candidate value vector for θ_2. If the data contain multiple independent markets, the objective function of the maximum likelihood estimation can be the sum of $\log(\sum_{t=1}^{T-1} l(x_{t+1}|x_t, a_t; \theta_2))$ across different markets. If both state and action spaces are discrete as in our simple model, a frequency estimator can be used to estimate the state-transition probabilities.

We will focus the remaining discussion of the estimation methods on the estimation of the parameters in players' utility function (θ_1). One conceptually straightforward way to do this—which falls into the category of the "full solution methods"—is to solve the dynamic game for all MPEs in the probability space, denoted as P^*, which satisfies $P^* = \Gamma(P^*)$, for each candidate parameter vector (θ_1); the solution will then be used in the outer optimization of the estimation criterion, such as maximizing the likelihood. In mathematical terms,

following Aguirregabiria and Mira [2], we can define a "pseudo"-likelihood function for any arbitrary set of conditional choice probabilities P,[2] as

$$Q(\theta_1, \hat{\theta}_2, P) = \sum_{t=1}^{T} \sum_{i=1}^{N} \ln(\Gamma_i(a_{it}|x_t; P, \hat{\theta}_2, \theta_1), \tag{40}$$

and the maximum likelihood estimator of $\theta_1$can be defined as

$$\hat{\theta}_1^{\text{MLE}} = \text{argmax}_{\theta_1} Q(\theta_1, \hat{\theta}_2, P) \text{ subject to } P = \Gamma(\theta_1, \hat{\theta}_2, P). \tag{41}$$

This method works in theory, but in practice, it is difficult to implement because it requires finding all possible equilibria and selecting the one that gives the highest value of $Q(\theta_1, \hat{\theta}_2, P)$ for every candidate value of θ_1 and sometimes evaluating the gradient of the likelihood function, which is computationally challenging. Next, we will discuss two estimators; both are extensions of the Hotz–Miller CCP estimator introduced above, which simplifies the estimation of dynamic discrete games. The advantages of these CCP-based approaches are, again, that they avoid needing to solve the game and can deal with the issue of multiple equilibria.

***PML Estimator* (Aguirregabiria and Mira [2]).** The pseudo-maximum likelihood (PML) estimation method is proposed by Aguirregabiria and Mira [2]. Assume for a moment that the population CCPs (compared with the sampled CCPs observed in the data) is P^0. If P^0 are known (to the researcher), then $\hat{\theta} = \text{argmax}_\theta Q(\hat{\theta}, P^0)$ is a consistent estimator of $\hat{\theta}$.[3] In reality, P^0 is unknown, and therefore, the estimator defined above is not feasible. Aguirregabiria and Mira [2] define a two-step PML as $\hat{\theta}_1^{2S} \equiv \text{argmax}_{\theta_1} Q(\theta_1, \hat{\theta}_2, \hat{P})$, where Q is defined the same way as Equation (5). Note that in this definition of the two-step PML, parameters in players' utility function and those in the state-transition probabilities are separated, and the $\hat{\theta}_2$ can be estimated the same way as previously discussed. The term $\hat{P}$ is a consistent estimator of P^0, which, under the assumption that data are coming from a single MPE, can be obtained using a frequency estimator or a kernel estimator. Unlike in the full solution method, $\hat{P}$ is estimated directly from the data, as opposed to a solution to the dynamic game given a candidate θ_1 value. Aguirregabiria and Mira [2] show that this two-step PML estimator is consistent, but they also point out potential drawbacks of this estimator: (1) the estimator can be very inefficient when the variance of the estimator $\hat{P}$ is large; and (2) in cases where the state space is large, $\hat{P}$ can be imprecise. Therefore, the authors propose an extension to the two-step PML estimator, named the nested pseudo-likelihood (NPL) estimator, which will be discussed next.

Let $\hat{P}_0$ be an initial guess of the true CCPs. Given a $\hat{P}_0$, NPL generates a sequence of estimations $\{\hat{\theta}_1^K : K \geq 1\}$, where the Kth-stage estimator is defined as

$$\hat{\theta}_1^K = \text{argmax}_{\theta_1} Q(\theta_1, \hat{\theta}_2, \hat{P}_{K-1}), \tag{42}$$

and $\hat{P}_K$ is updated recursively as

$$\hat{P}_K = \Gamma(\hat{\theta}_1^K, \hat{\theta}_2, \hat{P}_{K-1}). \tag{43}$$

Note that if $\hat{P}_0$ is a consistent estimator, all elements of $\{\hat{\theta}_1^K : K \geq 1\}$ are consistent. Even if $\hat{P}_0$ is not consistent, if the sequence $\{\hat{\theta}_1^K, \hat{P}_K\}$ converges, we find a NPL fixed point. Aguirregabiria and Mira [2] (p. 19) establish that "a NPL fixed point exists in every sample and that if more than one exists, the one with the highest value of the pseudo likelihood is a consistent estimator". Because it is possible that the sequence$\{\hat{\theta}_1^K, \hat{P}_K\}$ converges to different fixed points, it is recommended that we try different initial points.

***BBL Estimator* (Bajari et al. [12]).** Bajari et al. [12] propose an estimator, the BBL method, that minimizes a set of moment inequalities. It is a simulation-based CCP estimator, which originated from Hotz et al. [53], commonly referred to as the HMSS estimator. The BBL

method consists of two steps. In the first step, state-transition probabilities as a function of individual player actions and the equilibrium policy function are directly estimated from the data. The estimation of the state-transition process is similar to how we obtained $\hat{\theta}_2$ in the PML estimator. Next, we discuss how to estimate the equilibrium policy.

When players' action is discrete, and the private shock follows the i.i.d. Type I extreme value distribution, Hotz and Miller [53] show that the first difference of the choice-specific value function for a given state (x) between any two alternatives $a_i, a_i' \in A$ can be expressed as

$$\tilde{v}_i^{\sigma}(a_i', x) - \tilde{v}_i^{\sigma}(a_i, x) = \ln\big(\Pr(a_i' \mid x; \sigma)\big) - \ln(Pr(a_i \mid x; \sigma)), \tag{44}$$

where each $\Pr(a_i \mid x; \sigma)$ is an element of P^{σ}, which has been previously defined. Under the assumption that data are coming from one equilibrium, as before, let $\hat{P}$ be a nonparametric estimator of P^0, so we have $\tilde{v}_i^{\hat{P}}(a_i', x) - \tilde{v}_i^{\hat{P}}(a_i, x) = \ln(\hat{P}(a_i' \mid x)) - \ln(\hat{P}(a_i \mid x))$. For a binary action a_i, player i optimally chooses action 1 at state x when $\tilde{v}_i^{\hat{P}}(1, x) + \epsilon_i(1) \geq \tilde{v}_i^{\hat{P}}(0, x) + \epsilon_i(0)$, or equivalently, $\hat{\sigma}_i^*(x, \epsilon_i) = I(\tilde{v}_i^{\hat{P}}(1, x) + \epsilon_i(1) \geq \tilde{v}_i^{\hat{P}}(0, x) + \epsilon_i(0))$, where I is an indicator function.

In the second step, a set of moment inequalities is constructed to estimate the utility parameters (θ_1). Forward simulation (instead of a fixed-point algorithm and matrix inversion) is used to approximate the integrated value function under any arbitrary strategy profile σ (denoted as $\tilde{V}_i^{\sigma}(x)$[4]). Specifically, each simulation path is generated as follows.

Step 1: Start at state $x_0 = x$, draw the private shock ϵ_0 for each player from $G_{\epsilon}(\epsilon_i)$.
Step 2: Under σ, each player's action in period 0 can be determined by $a_{i0} = \sigma_i(x_0, \epsilon_{i0})$. The current-period utility can be calculated as $U_i(a_0, x_0, \epsilon_{i0})$.
Step 3: Draw a new state x_1 from the state-transition probabilities $F(x_1 \mid x_0, a_0; \hat{\theta}_2)$.
Step 4: Repeat Steps 1–3 for a large number of period (T), such that β^T is sufficiently small or until each of the players reaches the terminal state.

The value function $\tilde{V}_i^{\sigma}(x)$ can be approximated by simulating many paths and taking the average of the discounted sum of single-period utility, and this estimate of $\tilde{V}_i^{\sigma}(x)$ is denoted as $\hat{V}_i^{\sigma}(x)$.

The BBL estimator makes use of the model's equilibrium conditions—a strategy profile σ^* is an MPE only if for all players (indexed by i), all possible state values, and all alternative Markov strategy profiles, $\tilde{V}_i^{\sigma_i^*, \sigma_{-i}^*}(x; \theta_1, \hat{\theta}_2) \geq \tilde{V}_i^{\sigma_i', \sigma_{-i}^*}(x; \theta_1, \hat{\theta}_2)$. Note that in the inequality, we divide σ into σ_i and σ_{-i}, and bring both θ_1 and $\hat{\theta}_2$ back; each combination of $\{i, x, \sigma_i'\}$ corresponds to one equilibrium condition (Aguirregabiria and Mira [3]). Define

$$Q(\theta_1, \hat{\theta}_2) \equiv \int_{\{i,x,\sigma_i'\} \in H} (\min\{0, \tilde{V}_i^{\sigma_i^*, \sigma_{-i}^*}(x; \theta_1, \hat{\theta}_2) - \tilde{V}_i^{\sigma_i', \sigma_{-i}^*}(x; \theta_1, \hat{\theta}_2)\})^2 dH(i, x, \sigma_i'). \tag{45}$$

Then the BBL estimator of θ_1 minimizes a (simulated) sample counterpart of $Q(\theta_1, \hat{\theta}_2)$. That is,

$$\hat{\theta}_1^{\text{BBL}} = \text{argmin}_{\theta_1} 1/n_I \sum_{\{i,x,\sigma_i'\} \in H} \Big(\min\{0, \hat{V}_i^{\hat{\sigma}_i^*, \hat{\sigma}_{-i}^*}(x; \theta_1, \hat{\theta}_2) - \hat{V}_i^{\sigma_i', \hat{\sigma}_{-i}^*}(x; \theta_1, \hat{\theta}_2)\}\Big)^2, \tag{46}$$

where n_I is the number of alternative equilibrium conditions drawn from the distribution H. The combination $\{i, x, , \sigma_i'\}$ might be selected in the following ways. One possibility is to draw individuals (i) and states (x) randomly and then introduce slight perturbations to the $\hat{\sigma}_i^*(x, \epsilon_i)$ to generate alternative policies. For example, a $\sigma_i'(x, \epsilon_i)$ can be generated by $\sigma_i'(x, \epsilon_i) = I(\tilde{v}_i^{\hat{P}}(1, x) + \epsilon_i(1) + \varsigma \geq \tilde{v}_i^{\hat{P}}(0, x) + \epsilon_i(0))$, where ς is a random perturbation drawn from a known distribution. Here, $\hat{\sigma}^*$ is the estimate of the equilibrium policies obtained in the first stage. As both the number of sample observations and the number of simulations approach infinity, this estimator converges to the true parameter value.

3.4. Extensions and Further Discussions

We have introduced several commonly used methods for estimating single-agent dynamic models and dynamic games. One thing to note is that the methods we introduced in the dynamic games subsection can also be used to estimate single-agent dynamic discrete models. Also note that the models we have been working with are simple models that impose a number of assumptions such as additive separability, i.i.d. errors, conditional independence, etc. In the interest of space, below we provide a few examples of solutions to situations where some of the assumptions do not hold. For example, Keane and Wolpin [59] do not require that unobservables be additive separable, serially correlated unobservables are considered in Keane [57], and permanent unobserved heterogeneity is allowed for in Arcidiacono and Jones [8] and Arcidiacono and Miller [9]. See Aguirregabiria and Mira [3] for a more detailed review of these possibilities.

4. Two-Sided Matching

As researchers, we often observe data on relationships. For example, we observe which student attends which university, which employer works for which company, who is married to whom, who dates whom, which ride-sharing rider is matched with which driver, which company merges with which, and so on. A matching model is a framework that allows researchers to model how such relationships, or matches, are formulated. A two-sided matching model refers to the problem of matching the preferences of agents on two disjoint sides of a transaction. The existing literature on matching is substantial and extensive (e.g., Gale and Shapley [38], Niederle et al. [75], Roth, [79, 82], Roth and Sotomayor [84]). Roth and Sotomayor [84] provide a comprehensive survey of the two-sided matching model. In this section, we cover matching models' key terminologies and theorems and provide a literature survey on matching.

4.1. One-to-One Matching

The simplest two-sided matching model is a one-to-one model, also known as the marriage problem (Gale and Shapley [37]). The model consists of two disjoint sets of agents denoted as $M = \{m_1, \ldots, m_k\}$ for men and $W = \{w_1, \ldots, w_p\}$ for women. The market is two-sided; each m_i has preferences over the set of $W \cup m_i$, and each w_i has preferences over the set of $M \cup w_i$. The preferences are complete and transitive. The preference of m is denoted as $P(m)$, which is an ordered list. For example, $P(m) = \{w_1, w_2, m, \ldots, w_p\}$ indicates that m's first choice is to be matched with w_1, second choice is to be matched with w_2, and third choice is to be matched to himself (or to remain unmatched). The notation $w >_m w'$ is used to mean that m prefers w to w', and the notation $w \geq_m w'$ is used to mean that m prefers w at least as well as w'. Then w is acceptable to m if m likes w at least as well as remaining unmatched (i.e., if $w \geq_m m$). A matching is a function $\mu : M \cup W \to M \cup W$ that satisfies the following three conditions: $\mu(w) \in M \cup w$ (i.e., w is matched to some m or remains unmatched), $\mu(m) \in W \cup m$ (i.e., m is matched to some w or remains unmatched), and $w = \mu(m)$ if and only if $\mu(w) = m$ (i.e., the matching is mutual).

An important equilibrium concept in the matching model is *stability*. A matching is stable if there are no individual players or pairs of players who can profitability deviate from their match. In other words, a matching is stable if a match is not blocked by any individual or a pair of agents. A matching μ is blocked by an individual k if k prefers being single to being matched with $\mu(k)$ (i.e., $k >_k \mu(k)$). A matching is blocked by a pair of agents (m, w) if they each prefer each other to their current outcome (i.e., $w >_m \mu(m)$ and $m >_w \mu(w)$).

An important key theorem in matching is the Gale–Shapley theorem (theorem 1 in Gale and Shapley [38]), which states that a stable matching exists for every marriage market. To prove the theorem, Gale and Shapley [38] propose a deferred acceptance algorithm. In the deferred acceptance algorithm where men propose, each man m, after arbitrarily breaking ties in

preferences, proposes to his first choice if he has any acceptable choice. Each woman rejects any unacceptable proposals, and if she receives more than one acceptable proposal, she holds the most preferred man and rejects all others. Then, any man who was rejected in the first step makes a new proposal to his most preferred acceptable woman who has not yet rejected him. The man does not make any proposal if there is no remaining acceptable choice. Each woman holds her most preferred acceptable offer to date and rejects the rest. Such steps are repeated until no more proposals are made. At the end of the algorithm, each woman is matched to a man whose proposal she is holding. The matching produced by the deferred acceptance algorithm is a stable matching because the match will not be blocked by any individual or a pair of agents. There is also a woman-proposing version where the algorithm begins by women proposing to their first choice.

4.2. Many-to-One Matching

The many-to-one matching model, often referred to as the college admission model, is another common matching scenario. Colleges admit many students, but students can choose only one college to attend. Firms hire multiple workers, but workers can choose only one company to work for. In a many-to-one matching, the two players of the market are denoted by colleges $C = \{c_1, \ldots, c_n\}$ with the number of positions, $q_1, \ldots, q_n$, and students, $S = \{s_1, \ldots, s_p\}$. Each college has preferences over students, and each student has preferences over colleges. A key difference of the college admission model from the marriage model is that for each college c, there is a quota q_c, which refers to the number of positions the college can offer.

A matching is a correspondence that satisfies the following conditions: $\mu(s) \in C \cup s, \mu(c) \subseteq S$ (i.e., each college is matched to a group of students), $|\mu(c)| \leq q_c$ (i.e., no college quota is exceeded), and $\mu(s) = c$ if and only if $s \in \mu(c)$ for every student $s \in S$ and college $c \in C$. Stability of a matching is defined similarly as in the one-to-one matching model, and a matching is stable if it is not blocked by any individual agent or any college-student pair. The Gale–Shapley theorem (Gale and Shapley [38]) also holds in many-to-one matchings, and a stable matching always exists in many-to-one matchings. The theorem can again be proved by using the deferred acceptance algorithm.

4.3. Stable Matchings and an Impossibility Theorem

Although stability is a theoretically appealing concept, some researchers may question its applicability in the world. Roth [81] studies the British medical match where different regions use different matching mechanism. Roth [81] finds that stable mechanisms are successfully used—and in some cases still in use—but most unstable mechanisms were abandoned after a short period of time. The deferred acceptance algorithm has been used in various settings, including the New York City and Boston school choice systems and medical clearinghouses (Roth [80]). Additionally, many stable clearinghouses use the Roth–Peranson match algorithm, which is based on the deferred acceptance algorithm (Roth and Peranson [83]).

The prior literature examines whether there is any procedure that can yield stable matching for all preferences (see Roth and Sotomayor [84] for further references). To apply a centralized matching procedure to actual markets, each agent's list of preferences needs to be elicited. In designing a centralized clearinghouse, a question arises as to whether agents revealing their true preferences can be a dominant strategy for agents. Roth's [79] impossibility theorem states that no stable matching mechanism exists for which stating the true preferences form a dominant strategy for every agent. However, it is possible to design a mechanism such that one side of the market can never do any better than to state its true preferences (Dubins and Freedman [29], Roth [79]).

4.4. Empirical Research on Matching Models

Empirical research on matching models is not as extensive and developed as the theoretical literature. However, a growing empirical literature employs matching models (e.g., see Fox [35] and Graham [42] for reviews). Empirical studies utilize the observed data on matching outcomes and employ pairwise stability as an equilibrium concept to estimate the model.

Fox [35] provides a comprehensive survey of structural empirical work using matching models. A structural approach to matching models allows researchers to estimate parameters of the model that cannot be directly observed and conduct counterfactual analysis. The estimation of matching models can be related to the literature on the estimation of Nash games and entry models (i.e., Berry [14], Bresnahan and Reiss [20], Fox [35]). In matching models, the concept of stability, instead of Nash equilibrium, is employed, but similar estimation techniques and challenges are applicable. A key empirical challenge in estimating matching models is the large number of agents who are involved in the matching, which can make the estimation computationally burdensome. For example, in the medical residency market, thousands of residents need to be matched to a program every year (Agarwal [1]).

In estimating match models, a common straightforward approach is to use simulated maximum likelihood or simulated method of moments. Although simulation estimators are conceptually straightforward, it can be computationally burdensome. Boyd et al. [19] use simulated method of moments to match public school teachers to schools, and Agarwal [1] uses simulated method of moments for matching in medical residency market. Another estimation method is inequality methods. In order to break the computational curse of dimensionality, Fox [36] proposes an inequality method where the maximum score estimator maximizes the number of inequalities that are implied by pairwise stability that hold true. Fox [36] and Fox and Bajari [37] also use the inequality method for matching car parts to automotive suppliers and for the Federal Communications Commission spectrum auction, respectively.

Matching models have been applied in examining marriage markets (Chiappori and Oreffice [22], Choo and Siow [25], Galichon and Salanié [39]), online dating (Hitsch et al. [51]), bank mergers (Akkus et al. [4], Uetake and Watanabe [92]), mutual fund mergers (Park [77]), matching of professional athletes to sports teams (Yang et al. [100]), university–firm research collaboration (Mindruta [71]), the teacher labor market (Boyd et al. [19]), and the medical residency market (Agarwal [1]). Sørensen [90] uses a matching model to correct for selection in venture capital investment. The recent advent of technology-enabled online platforms, including online dating platforms, ride-sharing platforms, and online labor market platforms, opens avenues for exciting future research on matching models.

5. Concluding Remarks

In this tutorial, we present some representative structural econometric models that have been increasingly applied in business and management research. These models rely on economic or behavioral theories to guide model specification and, consequently, more accurately reflect the underlying data generating process. This approach can capture the strategic behaviors of rational users or firms in a competitive setting, and dynamic models can describe agents' forward-looking decision making. Compared with reduced-form models, structural models can proactively address the potential endogeneity problems, allow policy analyses based on simulations, and produce counterfactuals that offer meaningful managerial suggestions. Of course, like any other model, structural models do not come without limitations. We need to diligently ground our model construction on sound theoretical foundations and convincingly justify the assumptions introduced.

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Endnotes

[1]In most applications the discount factor is poorly identified.

[2]It is called "pseudo"-likelihood because here P is arbitrary and is not necessarily the equilibrium CCPs.

[3]Here, instead of solving for the equilibrium CCPs given a parameter value, we assume P^0 is given.

[4]Note that $\tilde{V}_i^\sigma(x)$ is a function of θ_1 and $\hat{\theta}_2$. We suppress the dependence of $\tilde{V}_i^\sigma(x)$ on θ_1 for now and will bring it back later.

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