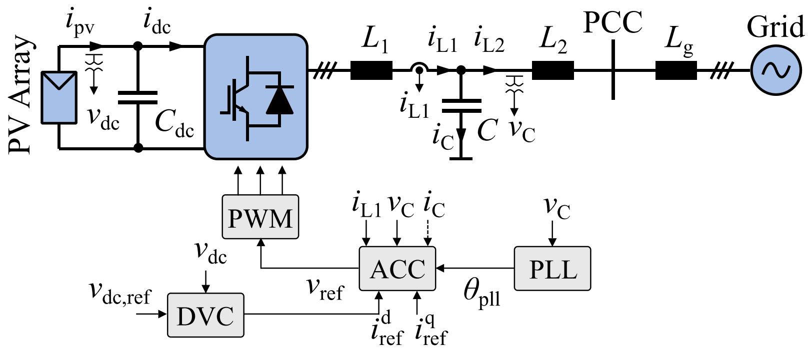
Small-Signal State-Space Modeling of Grid-Following Inverter



Established by

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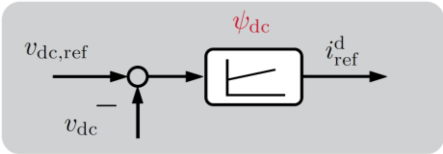
Aachen, on July 2021

Content

Direct-Voltage Control (DVC)



State equation

$$\frac{d\psi_{\mathrm{dc}}}{dt} = v_{\mathrm{dc,ref}} - v_{\mathrm{dc}}$$

Output equation

$$i_{\mathrm{ref}}^{\mathrm{d}} = K_{\mathrm{p}}^{\mathrm{DVC}}\left(v_{\mathrm{dc,ref}} - v_{\mathrm{dc}}\right) + K_{\mathrm{i}}^{\mathrm{DVC}}\psi_{\mathrm{dc}}$$

$$i_{\mathrm{ref}}^{\mathrm{q}} = 0$$

Linearization

$$\frac{d\Delta\psi_{\mathrm{dc}}}{dt} = \Delta v_{\mathrm{dc,ref}} - \Delta v_{\mathrm{dc}}$$

$$\Delta i_{\mathrm{ref}}^{\mathrm{d}} = K_{\mathrm{p}}^{\mathrm{DVC}}\left(\Delta v_{\mathrm{dc,ref}} - \Delta v_{\mathrm{dc}}\right) + K_{\mathrm{i}}^{\mathrm{DVC}}\Delta\psi_{\mathrm{dc}}$$

$$\Delta i_{\mathrm{ref}}^{\mathrm{q}} = 0$$

State-space model

$$\frac{d}{dt}\Delta\boldsymbol{x}_{\mathrm{DVC}} = \boldsymbol{A}_{\mathrm{DVC}}\Delta\boldsymbol{x}_{\mathrm{DVC}} + \boldsymbol{B}_{\mathrm{DVC}}\Delta\boldsymbol{u}_{\mathrm{DVC}}$$

$$\Delta\boldsymbol{y}_{\mathrm{DVC}} = \boldsymbol{C}_{\mathrm{DVC}}\Delta\boldsymbol{x}_{\mathrm{DVC}} + \boldsymbol{D}_{\mathrm{DVC}}\Delta\boldsymbol{u}_{\mathrm{DVC}}$$

$$\Delta\boldsymbol{x}_{\mathrm{DVC}}^{1\mathrm{x}1} = [\Delta\psi_{\mathrm{dc}}]^T$$

$$\Delta\boldsymbol{u}_{\mathrm{DVC}}^{2\mathrm{x}1} = [\Delta v_{\mathrm{dc,ref}} \quad \Delta v_{\mathrm{dc}}]^T$$

$$\Delta\boldsymbol{y}_{\mathrm{DVC}}^{1\mathrm{x}1} = \left[\Delta i_{\mathrm{ref}}^{\mathrm{d}}\right]^T$$

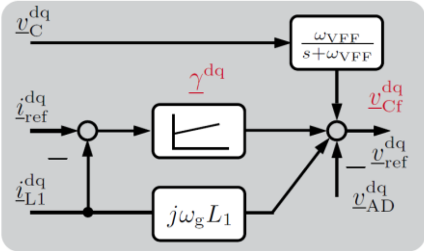
$$\boldsymbol{A}_{\mathrm{DVC}}^{1\mathrm{x}1} = [0]$$

$$\boldsymbol{B}_{\mathrm{DVC}}^{1\mathrm{x}2} = [1 \quad -1]$$

$$\boldsymbol{C}_{\mathrm{DVC}}^{1\mathrm{x}1} = \left[K_{\mathrm{i}}^{\mathrm{DVC}}\right]$$

$$\boldsymbol{D}_{\mathrm{DVC}}^{1\mathrm{x}2} = \left[K_{\mathrm{p}}^{\mathrm{DVC}} \quad -K_{\mathrm{p}}^{\mathrm{DVC}}\right]$$

Alternating-Current Control (ACC)



State equation

$$\frac{d\gamma^{\mathrm{d}}}{dt} = i_{\mathrm{ref}}^{\mathrm{d}} - i_{\mathrm{L1}}^{\mathrm{d,c}}$$

$$\frac{d\gamma^{\mathrm{q}}}{dt} = i_{\mathrm{ref}}^{\mathrm{q}} - i_{\mathrm{L1}}^{\mathrm{q,c}}$$

$$\frac{dv_{\mathrm{Cf}}^{\mathrm{d,c}}}{dt} = -\omega_{\mathrm{VFF}} v_{\mathrm{Cf}}^{\mathrm{d,c}} + \omega_{\mathrm{VFF}} v_{\mathrm{C}}^{\mathrm{d,c}}$$

$$\frac{dv_{\mathrm{Cf}}^{\mathrm{q,c}}}{dt} = -\omega_{\mathrm{VFF}} v_{\mathrm{Cf}}^{\mathrm{q,c}} + \omega_{\mathrm{VFF}} v_{\mathrm{C}}^{\mathrm{q,c}}$$

Output equation

$$v_{\mathrm{ref}}^{\mathrm{d}} = K_{\mathrm{p}}^{\mathrm{ACC}}\left(i_{\mathrm{ref}}^{\mathrm{d}} - i_{\mathrm{L1}}^{\mathrm{d,c}}\right) + K_{\mathrm{i}}^{\mathrm{ACC}}\gamma^{\mathrm{d}} - \omega_{\mathrm{g}} L_1 i_{\mathrm{L1}}^{\mathrm{q,c}} + v_{\mathrm{Cf}}^{\mathrm{d,c}} - K_{\mathrm{AD}}\left(i_{\mathrm{L1}}^{\mathrm{d,c}} - i_{\mathrm{L2}}^{\mathrm{d,c}}\right)$$

$$v_{\mathrm{ref}}^{\mathrm{q}} = K_{\mathrm{p}}^{\mathrm{ACC}}\left(i_{\mathrm{ref}}^{\mathrm{q}} - i_{\mathrm{L1}}^{\mathrm{q,c}}\right) + K_{\mathrm{i}}^{\mathrm{ACC}}\gamma^{\mathrm{q}} + \omega_{\mathrm{g}} L_1 i_{\mathrm{L1}}^{\mathrm{d,c}} + v_{\mathrm{Cf}}^{\mathrm{q,c}} - K_{\mathrm{AD}}\left(i_{\mathrm{L1}}^{\mathrm{q,c}} - i_{\mathrm{L2}}^{\mathrm{q,c}}\right)$$

Linearization

$$\frac{d\Delta\gamma^{\mathrm{d}}}{dt} = \Delta i_{\mathrm{ref}}^{\mathrm{d}} - \Delta i_{\mathrm{L1}}^{\mathrm{d,c}}$$

$$\frac{d\Delta\gamma^{\mathrm{q}}}{dt} = \Delta i_{\mathrm{ref}}^{\mathrm{q}} - \Delta i_{\mathrm{L1}}^{\mathrm{q,c}}$$

$$\frac{d\Delta v_{\mathrm{Cf}}^{\mathrm{d,c}}}{dt} = -\omega_{\mathrm{VFF}} \Delta v_{\mathrm{Cf}}^{\mathrm{d,c}} + \omega_{\mathrm{VFF}} \Delta v_{\mathrm{C}}^{\mathrm{d,c}}$$

$$\frac{d\Delta v_{\mathrm{Cf}}^{\mathrm{q,c}}}{dt} = -\omega_{\mathrm{VFF}} \Delta v_{\mathrm{Cf}}^{\mathrm{q,c}} + \omega_{\mathrm{VFF}} \Delta v_{\mathrm{C}}^{\mathrm{q,c}}$$

$$\Delta v_{\mathrm{ref}}^{\mathrm{d}} = K_{\mathrm{p}}^{\mathrm{ACC}}\left(\Delta i_{\mathrm{ref}}^{\mathrm{d}} - \Delta i_{\mathrm{L1}}^{\mathrm{d,c}}\right) + K_{\mathrm{i}}^{\mathrm{ACC}}\Delta\gamma^{\mathrm{d}} - \omega_{\mathrm{g}} L_1 \Delta i_{\mathrm{L1}}^{\mathrm{q,c}} + \Delta v_{\mathrm{Cf}}^{\mathrm{d,c}} - K_{\mathrm{AD}}\left(\Delta i_{\mathrm{L1}}^{\mathrm{d,c}} - \Delta i_{\mathrm{L2}}^{\mathrm{d,c}}\right)$$

$$\Delta v_{\mathrm{ref}}^{\mathrm{q}} = K_{\mathrm{p}}^{\mathrm{ACC}}\left(\Delta i_{\mathrm{ref}}^{\mathrm{q}} - \Delta i_{\mathrm{L1}}^{\mathrm{q,c}}\right) + K_{\mathrm{i}}^{\mathrm{ACC}}\Delta\gamma^{\mathrm{q}} + \omega_{\mathrm{g}} L_1 \Delta i_{\mathrm{L1}}^{\mathrm{d,c}} + \Delta v_{\mathrm{Cf}}^{\mathrm{q,c}} - K_{\mathrm{AD}}\left(\Delta i_{\mathrm{L1}}^{\mathrm{q,c}} - \Delta i_{\mathrm{L2}}^{\mathrm{q,c}}\right)$$

State-space model

$$\frac{d}{dt}\Delta\boldsymbol{x}_{\mathrm{ACC}} = \boldsymbol{A}_{\mathrm{ACC}}\Delta\boldsymbol{x}_{\mathrm{ACC}} + \boldsymbol{B}_{\mathrm{ACC}}\Delta\boldsymbol{u}_{\mathrm{ACC}}$$

$$\Delta\boldsymbol{y}_{\mathrm{ACC}} = \boldsymbol{C}_{\mathrm{ACC}}\Delta\boldsymbol{x}_{\mathrm{ACC}} + \boldsymbol{D}_{\mathrm{ACC}}\Delta\boldsymbol{u}_{\mathrm{ACC}}$$

$$\Delta\boldsymbol{x}_{\mathrm{DVC}}^{4\mathrm{x}1} = \begin{bmatrix}\Delta\gamma^{\mathrm{d}} & \Delta\gamma^{\mathrm{q}} & \Delta v_{\mathrm{Cf}}^{\mathrm{d,c}} & \Delta v_{\mathrm{Cf}}^{\mathrm{q,c}}\end{bmatrix}^T$$

$$\Delta\boldsymbol{u}_{\mathrm{DVC}}^{8\mathrm{x}1} = \begin{bmatrix}\Delta i_{\mathrm{ref}}^{\mathrm{d}} & \Delta i_{\mathrm{ref}}^{\mathrm{q}} & \Delta i_{\mathrm{L1}}^{\mathrm{d,c}} & \Delta i_{\mathrm{L1}}^{\mathrm{q,c}} & \Delta v_{\mathrm{C}}^{\mathrm{d,c}} & \Delta v_{\mathrm{C}}^{\mathrm{q,c}} & \Delta i_{\mathrm{L2}}^{\mathrm{d,c}} & \Delta i_{\mathrm{L2}}^{\mathrm{q,c}}\end{bmatrix}^T$$

$$\Delta\boldsymbol{y}_{\mathrm{DVC}}^{2\mathrm{x}1} = \begin{bmatrix}\Delta v_{\mathrm{ref}}^{\mathrm{d}} & \Delta v_{\mathrm{ref}}^{\mathrm{q}}\end{bmatrix}^T$$

$$\boldsymbol{A}_{\mathrm{ACC}}^{4\mathrm{x}4} = \begin{bmatrix}0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & -\omega_{\mathrm{VFF}} & 0\\ 0 & 0 & 0 & -\omega_{\mathrm{VFF}}\end{bmatrix}$$

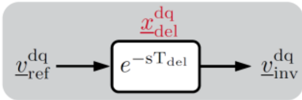
$$\boldsymbol{B}_{\mathrm{ACC}}^{4\mathrm{x}8} = \begin{bmatrix}1 & 0 & -1 & 0 & 0 & 0 & 0 & 0\\ 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & \omega_{\mathrm{VFF}} & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & \omega_{\mathrm{VFF}} & 0 & 0\end{bmatrix}$$

$$\boldsymbol{C}_{\mathrm{ACC}}^{2\mathrm{x}4} = \begin{bmatrix}K_{\mathrm{i}}^{\mathrm{ACC}} & 0 & 1 & 0\\ 0 & K_{\mathrm{i}}^{\mathrm{ACC}} & 0 & 1\end{bmatrix}$$

$$\boldsymbol{D}_{\mathrm{ACC}}^{2\mathrm{x}8} = \begin{bmatrix}K_{\mathrm{p}}^{\mathrm{ACC}} & 0 & -\left(K_{\mathrm{p}}^{\mathrm{ACC}} + K_{\mathrm{AD}}\right) & -\omega_{\mathrm{g}}L_1 & 0 & 0 & K_{\mathrm{AD}} & 0\\ 0 & K_{\mathrm{p}}^{\mathrm{ACC}} & \omega_{\mathrm{g}}L_1 & -\left(K_{\mathrm{p}}^{\mathrm{ACC}} + K_{\mathrm{AD}}\right) & 0 & 0 & 0 & K_{\mathrm{AD}}\end{bmatrix}$$

Modulation Delay

Refer to Padé approximation



State equation

$$\frac{dx_{\mathrm{del}}^{\mathrm{d}}}{dt} = A_{\mathrm{d}} x_{\mathrm{d}}^{\mathrm{d}} + B_{\mathrm{d}} v_{\mathrm{ref}}^{\mathrm{d}}$$

$$\frac{dx_{\mathrm{del}}^{\mathrm{q}}}{dt} = A_{\mathrm{d}} x_{\mathrm{d}}^{\mathrm{q}} + B_{\mathrm{d}} v_{\mathrm{ref}}^{\mathrm{q}}$$

Output equation

$$v_{\mathrm{inv}}^{\mathrm{d,c}} = C_{\mathrm{d}} x_{\mathrm{d}}^{\mathrm{d}} + D_{\mathrm{d}} v_{\mathrm{ref}}^{\mathrm{d}}$$

$$v_{\mathrm{inv}}^{\mathrm{q,c}} = C_{\mathrm{d}} x_{\mathrm{d}}^{\mathrm{q}} + D_{\mathrm{d}} v_{\mathrm{ref}}^{\mathrm{q}}$$

Linearization

$$\frac{d\Delta x_{\mathrm{del}}^{\mathrm{d}}}{dt} = A_{\mathrm{d}} \Delta x_{\mathrm{d}}^{\mathrm{d}} + B_{\mathrm{d}} \Delta v_{\mathrm{ref}}^{\mathrm{d}}$$

$$\frac{d\Delta x_{\mathrm{del}}^{\mathrm{q}}}{dt} = A_{\mathrm{d}} \Delta x_{\mathrm{d}}^{\mathrm{q}} + B_{\mathrm{d}} \Delta v_{\mathrm{ref}}^{\mathrm{q}}$$

$$\Delta v_{\mathrm{inv}}^{\mathrm{d,c}} = C_{\mathrm{d}} \Delta x_{\mathrm{d}}^{\mathrm{d}} + D_{\mathrm{d}} \Delta v_{\mathrm{ref}}^{\mathrm{d}}$$

$$\Delta v_{\mathrm{inv}}^{\mathrm{q,c}} = C_{\mathrm{d}} \Delta x_{\mathrm{d}}^{\mathrm{q}} + D_{\mathrm{d}} \Delta v_{\mathrm{ref}}^{\mathrm{q}}$$

State-space model

$$\frac{d}{dt}\Delta\boldsymbol{x}_{\mathrm{del}} = \boldsymbol{A}_{\mathrm{del}}\Delta\boldsymbol{x}_{\mathrm{del}} + \boldsymbol{B}_{\mathrm{del}}\Delta\boldsymbol{u}_{\mathrm{del}}$$

$$\Delta\boldsymbol{y}_{\mathrm{del}} = \boldsymbol{C}_{\mathrm{del}}\Delta\boldsymbol{x}_{\mathrm{del}} + \boldsymbol{D}_{\mathrm{del}}\Delta\boldsymbol{u}_{\mathrm{del}}$$

$$\Delta\boldsymbol{x}_{\mathrm{del}}^{2\mathrm{x}1} = \begin{bmatrix}\Delta x_{\mathrm{del}}^{\mathrm{d}} & \Delta x_{\mathrm{del}}^{\mathrm{q}}\end{bmatrix}^T$$

$$\Delta\boldsymbol{u}_{\mathrm{del}}^{2\mathrm{x}1} = \begin{bmatrix}\Delta v_{\mathrm{ref}}^{\mathrm{d}} & \Delta v_{\mathrm{ref}}^{\mathrm{q}}\end{bmatrix}^T$$

$$\Delta\boldsymbol{y}_{\mathrm{del}}^{2\mathrm{x}1} = \begin{bmatrix}\Delta v_{\mathrm{ref}}^{\mathrm{d,c}} & \Delta v_{\mathrm{ref}}^{\mathrm{q,c}}\end{bmatrix}^T$$

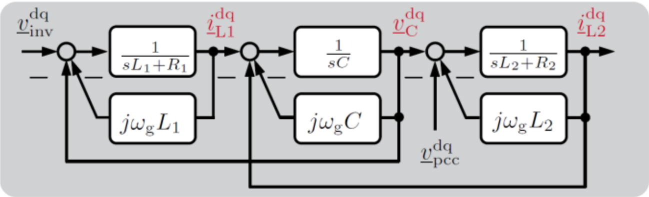
$$\boldsymbol{A}_{\mathrm{del}}^{2\mathrm{x}2} = \begin{bmatrix}A_{\mathrm{d}} & 0\\ 0 & A_{\mathrm{d}}\end{bmatrix}$$

$$\boldsymbol{B}_{\mathrm{del}}^{2\mathrm{x}2} = \begin{bmatrix}B_{\mathrm{d}} & 0\\ 0 & B_{\mathrm{d}}\end{bmatrix}$$

$$\boldsymbol{C}_{\mathrm{del}}^{2\mathrm{x}2} = \begin{bmatrix}C_{\mathrm{d}} & 0\\ 0 & C_{\mathrm{d}}\end{bmatrix}$$

$$\boldsymbol{D}_{\mathrm{del}}^{2\mathrm{x}2} = \begin{bmatrix}D_{\mathrm{d}} & 0\\ 0 & D_{\mathrm{d}}\end{bmatrix}$$

LCL Filter



State equation

$$\frac{di_{\mathrm{L1}}^{\mathrm{d,s}}}{dt} = \frac{1}{L_1}v_{\mathrm{inv}}^{\mathrm{d,s}} - \frac{1}{L_1}v_{\mathrm{C}}^{\mathrm{d,s}} - \frac{R_1}{L_1}i_{\mathrm{L1}}^{\mathrm{d,s}} + \omega_{\mathrm{g}}i_{\mathrm{L1}}^{\mathrm{q,s}}$$

$$\frac{di_{\mathrm{L1}}^{\mathrm{q,s}}}{dt} = \frac{1}{L_1}v_{\mathrm{inv}}^{\mathrm{q,s}} - \frac{1}{L_1}v_{\mathrm{C}}^{\mathrm{q,s}} - \frac{R_1}{L_1}i_{\mathrm{L1}}^{\mathrm{q,s}} - \omega_{\mathrm{g}}i_{\mathrm{L1}}^{\mathrm{d,s}}$$

$$\frac{dv_{\mathrm{C}}^{\mathrm{d,s}}}{dt} = \frac{1}{C}i_{\mathrm{L1}}^{\mathrm{d,s}} - \frac{1}{C}i_{\mathrm{L2}}^{\mathrm{d,s}} + \omega_{\mathrm{g}}v_{\mathrm{C}}^{\mathrm{q,s}}$$

$$\frac{dv_{\mathrm{C}}^{\mathrm{q,s}}}{dt} = \frac{1}{C}i_{\mathrm{L1}}^{\mathrm{q,s}} - \frac{1}{C}i_{\mathrm{L2}}^{\mathrm{q,s}} - \omega_{\mathrm{g}}v_{\mathrm{C}}^{\mathrm{d,s}}$$

$$\frac{di_{\mathrm{L2}}^{\mathrm{d,s}}}{dt} = \frac{1}{L_2}v_{\mathrm{C}}^{\mathrm{d,s}} - \frac{1}{L_2}v_{\mathrm{pcc}}^{\mathrm{d,s}} - \frac{R_2}{L_2}i_{\mathrm{L2}}^{\mathrm{d,s}} + \omega_{\mathrm{g}}i_{\mathrm{L2}}^{\mathrm{q,s}}$$

$$\frac{di_{\mathrm{L2}}^{\mathrm{q,s}}}{dt} = \frac{1}{L_2}v_{\mathrm{C}}^{\mathrm{q,s}} - \frac{1}{L_2}v_{\mathrm{pcc}}^{\mathrm{q,s}} - \frac{R_2}{L_2}i_{\mathrm{L2}}^{\mathrm{q,s}} - \omega_{\mathrm{g}}i_{\mathrm{L2}}^{\mathrm{d,s}}$$

Linearization

$$\frac{d\Delta i_{\mathrm{L1}}^{\mathrm{d,s}}}{dt} = \frac{1}{L_1}\Delta v_{\mathrm{inv}}^{\mathrm{d,s}} - \frac{1}{L_1}\Delta v_{\mathrm{C}}^{\mathrm{d,s}} - \frac{R_1}{L_1}\Delta i_{\mathrm{L1}}^{\mathrm{d,s}} + \omega_{\mathrm{g}}\Delta i_{\mathrm{L1}}^{\mathrm{q,s}}$$

$$\frac{d\Delta i_{\mathrm{L1}}^{\mathrm{q,s}}}{dt} = \frac{1}{L_1}\Delta v_{\mathrm{inv}}^{\mathrm{q,s}} - \frac{1}{L_1}\Delta v_{\mathrm{C}}^{\mathrm{q,s}} - \frac{R_1}{L_1}\Delta i_{\mathrm{L1}}^{\mathrm{q,s}} - \omega_{\mathrm{g}}\Delta i_{\mathrm{L1}}^{\mathrm{d,s}}$$

$$\frac{d\Delta v_{\mathrm{C}}^{\mathrm{d,s}}}{dt} = \frac{1}{C}\Delta i_{\mathrm{L1}}^{\mathrm{d,s}} - \frac{1}{C}\Delta i_{\mathrm{L2}}^{\mathrm{d,s}} + \omega_{\mathrm{g}}\Delta v_{\mathrm{C}}^{\mathrm{q,s}}$$

$$\frac{d\Delta v_{\mathrm{C}}^{\mathrm{q,s}}}{dt} = \frac{1}{C}\Delta i_{\mathrm{L1}}^{\mathrm{q,s}} - \frac{1}{C}\Delta i_{\mathrm{L2}}^{\mathrm{q,s}} - \omega_{\mathrm{g}}\Delta v_{\mathrm{C}}^{\mathrm{d,s}}$$

$$\frac{d\Delta i_{\mathrm{L2}}^{\mathrm{d,s}}}{dt} = \frac{1}{L_2}\Delta v_{\mathrm{C}}^{\mathrm{d,s}} - \frac{1}{L_2}\Delta v_{\mathrm{pcc}}^{\mathrm{d,s}} - \frac{R_2}{L_2}\Delta i_{\mathrm{L2}}^{\mathrm{d,s}} + \omega_{\mathrm{g}}\Delta i_{\mathrm{L2}}^{\mathrm{q,s}}$$

$$\frac{d\Delta i_{\mathrm{L2}}^{\mathrm{q,s}}}{dt} = \frac{1}{L_2}\Delta v_{\mathrm{C}}^{\mathrm{q,s}} - \frac{1}{L_2}\Delta v_{\mathrm{pcc}}^{\mathrm{q,s}} - \frac{R_2}{L_2}\Delta i_{\mathrm{L2}}^{\mathrm{q,s}} - \omega_{\mathrm{g}}\Delta i_{\mathrm{L2}}^{\mathrm{d,s}}$$

State-space model

$$\frac{d}{dt}\Delta\boldsymbol{x}_{\mathrm{LCL}} = \boldsymbol{A}_{\mathrm{ACC}}\Delta\boldsymbol{x}_{\mathrm{LCL}} + \boldsymbol{B}_{\mathrm{del}}\Delta\boldsymbol{u}_{\mathrm{LCL}}$$

$$\Delta\boldsymbol{y}_{\mathrm{LCL}} = \boldsymbol{C}_{\mathrm{LCL}}\Delta\boldsymbol{x}_{\mathrm{LCL}} + \boldsymbol{D}_{\mathrm{LCL}}\Delta\boldsymbol{u}_{\mathrm{LCL}}$$

$$\Delta\boldsymbol{x}_{\mathrm{LCL}}^{6\mathrm{x}1} = \begin{bmatrix} i_{\mathrm{L1}}^{\mathrm{d,s}} & i_{\mathrm{L1}}^{\mathrm{q,s}} & v_{\mathrm{C}}^{\mathrm{d,s}} & v_{\mathrm{C}}^{\mathrm{q,s}} & i_{\mathrm{L2}}^{\mathrm{d,s}} & i_{\mathrm{L2}}^{\mathrm{q,s}} \end{bmatrix}^{T}$$

$$\Delta\boldsymbol{u}_{\mathrm{LCL}}^{4\mathrm{x}1} = \begin{bmatrix} v_{\mathrm{inv}}^{\mathrm{d,s}} & v_{\mathrm{inv}}^{\mathrm{q,s}} & v_{\mathrm{pcc}}^{\mathrm{d,s}} & v_{\mathrm{pcc}}^{\mathrm{q,s}} \end{bmatrix}^{T}$$

$$\Delta\boldsymbol{y}_{\mathrm{LCL}}^{6\mathrm{x}1} = \begin{bmatrix} i_{\mathrm{L1}}^{\mathrm{d,s}} & i_{\mathrm{L1}}^{\mathrm{q,s}} & v_{\mathrm{C}}^{\mathrm{d,s}} & v_{\mathrm{C}}^{\mathrm{q,s}} & i_{\mathrm{L2}}^{\mathrm{d,s}} & i_{\mathrm{L2}}^{\mathrm{q,s}} \end{bmatrix}^{T}$$

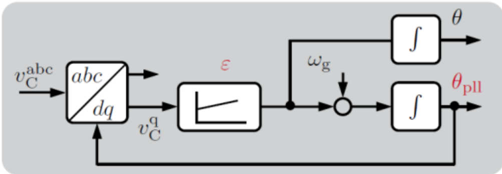
$$\boldsymbol{A}_{\mathrm{LCL}}^{6\mathrm{x}6} = \begin{bmatrix} -\frac{R_1}{L_1} & \omega_{\mathrm{g}} & -\frac{1}{L_1} & 0 & 0 & 0 \\ -\omega_{\mathrm{g}} & -\frac{R_1}{L_1} & 0 & -\frac{1}{L_1} & 0 & 0 \\ \frac{1}{C} & 0 & 0 & \omega_{\mathrm{g}} & -\frac{1}{C} & 0 \\ 0 & \frac{1}{C} & -\omega_{\mathrm{g}} & 0 & 0 & -\frac{1}{C} \\ 0 & 0 & \frac{1}{L_2} & 0 & -\frac{R_2}{L_2} & \omega_{\mathrm{g}} \\ 0 & 0 & 0 & \frac{1}{L_2} & -\omega_{\mathrm{g}} & -\frac{R_2}{L_2} \end{bmatrix}$$

$$\boldsymbol{B}_{\mathrm{LCL}}^{6\mathrm{x}4} = \begin{bmatrix} \frac{1}{L_1} & 0 & 0 & 0 \\ 0 & \frac{1}{L_1} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{L_2} & 0 \\ 0 & 0 & 0 & -\frac{1}{L_2} \end{bmatrix}$$

$$\boldsymbol{C}_{\mathrm{LCL}}^{6\mathrm{x}6} = \boldsymbol{I}^{6\mathrm{x}6}$$

$$\boldsymbol{D}_{\mathrm{LCL}}^{6\mathrm{x}4} = \boldsymbol{0}^{6\mathrm{x}4}$$

Phase-Locked Loop (PLL)



State equation

$$\frac{d\varepsilon}{dt} = v_{\mathrm{C}}^{\mathrm{q,c}}$$

$$\frac{d\theta}{dt} = K_{\mathrm{p}}^{\mathrm{PLL}} v_{\mathrm{C}}^{\mathrm{q,c}} + K_{\mathrm{i}}^{\mathrm{PLL}} \varepsilon + \omega_{\mathrm{g}}$$

Linearization

$$\frac{d\Delta\varepsilon}{dt} = \Delta v_{\mathrm{C}}^{\mathrm{q,c}}$$

$$\frac{d\Delta\theta}{dt} = K_{\mathrm{p}}^{\mathrm{PLL}} \Delta v_{\mathrm{C}}^{\mathrm{q,c}} + K_{\mathrm{i}}^{\mathrm{PLL}} \Delta\varepsilon + \cancel{\Delta\omega_{\mathrm{g}}}$$

⇓ If directly convert $\Delta v_{\mathrm{C}}^{\mathrm{q,c}}$ from the control to system frame

Refer to power angle relationship

$$\frac{d\Delta\varepsilon}{dt} = \Delta v_{\mathrm{C}}^{\mathrm{q,c}} = \Delta v_{\mathrm{C}}^{\mathrm{q,s}} - V_{\mathrm{C}}^{\mathrm{d}} \Delta\theta$$

$$\frac{d\Delta\theta}{dt} = K_{\mathrm{p}}^{\mathrm{PLL}} \left(\Delta v_{\mathrm{C}}^{\mathrm{q,s}} - V_{\mathrm{C}}^{\mathrm{d}} \Delta\theta \right) + K_{\mathrm{i}}^{\mathrm{PLL}} \Delta\varepsilon$$

State-space model

$$\frac{d}{dt}\Delta\boldsymbol{x}_{\mathrm{PLL}} = \boldsymbol{A}_{\mathrm{PLL}}\Delta\boldsymbol{x}_{\mathrm{PLL}} + \boldsymbol{B}_{\mathrm{PLL}}\Delta\boldsymbol{u}_{\mathrm{PLL}}$$

$$\Delta\boldsymbol{y}_{\mathrm{PLL}} = \boldsymbol{C}_{\mathrm{PLL}}\Delta\boldsymbol{x}_{\mathrm{PLL}} + \boldsymbol{D}_{\mathrm{PLL}}\Delta\boldsymbol{u}_{\mathrm{PLL}}$$

$$\Delta\boldsymbol{x}_{\mathrm{PLL}}^{2\mathrm{x}1} = [\Delta\varepsilon \quad \Delta\theta]^T$$

$$\Delta\boldsymbol{u}_{\mathrm{PLL}}^{1\mathrm{x}1} = \left[\Delta v_{\mathrm{C}}^{\mathrm{q,c}}\right]^T$$

$$\Delta\boldsymbol{y}_{\mathrm{PLL}}^{1\mathrm{x}1} = [\Delta\theta]^T$$

$$\boldsymbol{A}_{\mathrm{PLL}}^{2\mathrm{x}2} = \begin{bmatrix} 0 & 0 \\ K_{\mathrm{i}}^{\mathrm{PLL}} & 0 \end{bmatrix}$$

$$\boldsymbol{B}_{\mathrm{PLL}}^{2\mathrm{x}1} = \begin{bmatrix} 1 \\ K_{\mathrm{p}}^{\mathrm{PLL}} \end{bmatrix}$$

$$\boldsymbol{C}_{\mathrm{PLL}}^{1\mathrm{x}2} = [0 \quad 1]$$

$$\boldsymbol{D}_{\mathrm{PLL}}^{1\mathrm{x}1} = [0]$$

If consider direct conversion from the control to system frame:

$$\Delta\boldsymbol{u}_{\mathrm{PLL}}^{1\mathrm{x}1} = \left[\Delta v_{\mathrm{C}}^{\mathrm{q,s}}\right]^T$$

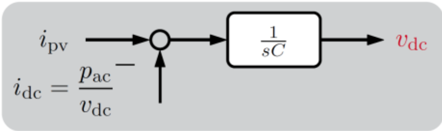
$$\boldsymbol{A}_{\mathrm{PLL}}^{2\mathrm{x}2} = \begin{bmatrix} 0 & -V_{\mathrm{C}}^{\mathrm{d}} \\ K_{\mathrm{i}}^{\mathrm{PLL}} & -V_{\mathrm{C}}^{\mathrm{d}} K_{\mathrm{p}}^{\mathrm{PLL}} \end{bmatrix}$$

$$\boldsymbol{B}_{\mathrm{PLL}}^{2\mathrm{x}2} = \begin{bmatrix} 0 & 1 \\ 0 & K_{\mathrm{p}}^{\mathrm{PLL}} \end{bmatrix}$$

$$\boldsymbol{C}_{\mathrm{PLL}}^{1\mathrm{x}2} = [0 \quad 1]$$

$$\boldsymbol{D}_{\mathrm{PLL}}^{1\mathrm{x}2} = [0 \quad 0]$$

DC-Link Capacitor



State equation

$$\frac{dv_{\mathrm{dc}}}{dt} = \frac{i_{\mathrm{pv}}}{C_{\mathrm{dc}}} - \frac{3}{2}\frac{\left(v_{\mathrm{inv}}^{\mathrm{d,s}} i_{\mathrm{L1}}^{\mathrm{d,s}} + v_{\mathrm{inv}}^{\mathrm{q,s}} i_{\mathrm{L1}}^{\mathrm{q,s}}\right)}{C_{\mathrm{dc}} v_{\mathrm{dc}}}$$

Linearization

$$\frac{d\Delta v_{\mathrm{dc}}}{dt} = \frac{\Delta i_{\mathrm{pv}}}{C_{\mathrm{dc}}} + \frac{3}{2}\frac{\left(V_{\mathrm{inv}}^{\mathrm{d}} I_{\mathrm{L1}}^{\mathrm{d}} + V_{\mathrm{inv}}^{\mathrm{q}} I_{\mathrm{L1}}^{\mathrm{q}}\right)}{\mathrm{C}_{\mathrm{dc}} V_{\mathrm{dc}}^{2}}\Delta v_{\mathrm{dc}} - \frac{3}{2}\frac{I_{\mathrm{L1}}^{\mathrm{d}}}{C_{\mathrm{dc}} V_{\mathrm{dc}}}\Delta v_{\mathrm{inv}}^{\mathrm{d,s}} - \frac{3}{2}\frac{I_{\mathrm{L1}}^{\mathrm{q}}}{C_{\mathrm{dc}} V_{\mathrm{dc}}}\Delta v_{\mathrm{inv}}^{\mathrm{q,s}} - \frac{3}{2}\frac{V_{\mathrm{C}}^{\mathrm{d}}}{C_{\mathrm{dc}} V_{\mathrm{dc}}}\Delta i_{\mathrm{L1}}^{\mathrm{d,s}} - \frac{3}{2}\frac{V_{\mathrm{C}}^{\mathrm{q}}}{C_{\mathrm{dc}} V_{\mathrm{dc}}}\Delta i_{\mathrm{L1}}^{\mathrm{q,s}}$$

State-space model

$$\frac{d}{dt}\Delta\boldsymbol{x}_{\mathrm{DC}} = \boldsymbol{A}_{\mathrm{DC}}\Delta\boldsymbol{x}_{\mathrm{DC}} + \boldsymbol{B}_{\mathrm{DC}}\Delta\boldsymbol{u}_{\mathrm{DC}}$$

$$\Delta\boldsymbol{y}_{\mathrm{DC}} = \boldsymbol{C}_{\mathrm{DC}}\Delta\boldsymbol{x}_{\mathrm{DC}} + \boldsymbol{D}_{\mathrm{DC}}\Delta\boldsymbol{u}_{\mathrm{DC}}$$

$$\Delta\boldsymbol{x}_{\mathrm{DC}}^{1\mathrm{x}1} = [\Delta v_{\mathrm{dc}}]^T$$

$$\Delta\boldsymbol{u}_{\mathrm{DC}}^{5\mathrm{x}1} = \begin{bmatrix}\Delta v_{\mathrm{inv}}^{\mathrm{d,s}} & \Delta v_{\mathrm{inv}}^{\mathrm{q,s}} & \Delta i_{\mathrm{L1}}^{\mathrm{d,s}} & \Delta i_{\mathrm{L1}}^{\mathrm{q,s}} & \Delta i_{\mathrm{pv}}\end{bmatrix}^T$$

$$\Delta\boldsymbol{y}_{\mathrm{DC}}^{1\mathrm{x}1} = [\Delta v_{\mathrm{dc}}]^T$$

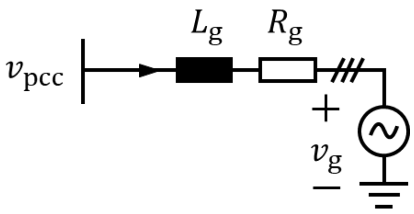
$$\boldsymbol{A}_{\mathrm{DC}}^{1\mathrm{x}1} = \left[\frac{3}{2}\frac{\left(V_{\mathrm{inv}}^{\mathrm{d}} I_{\mathrm{L1}}^{\mathrm{d}} + V_{\mathrm{inv}}^{\mathrm{q}} I_{\mathrm{L1}}^{\mathrm{q}}\right)}{\mathrm{C}_{\mathrm{dc}} V_{\mathrm{dc}}^2}\right]$$

$$\boldsymbol{B}_{\mathrm{DC}}^{1\mathrm{x}5} = \begin{bmatrix}-\frac{3}{2}\frac{I_{\mathrm{L1}}^{\mathrm{d}}}{C_{\mathrm{dc}}V_{\mathrm{dc}}} & -\frac{3}{2}\frac{I_{\mathrm{L1}}^{\mathrm{q}}}{C_{\mathrm{dc}}V_{\mathrm{dc}}} & -\frac{3}{2}\frac{V_{\mathrm{inv}}^{\mathrm{d}}}{C_{\mathrm{dc}}V_{\mathrm{dc}}} & -\frac{3}{2}\frac{V_{\mathrm{inv}}^{\mathrm{q}}}{C_{\mathrm{dc}}V_{\mathrm{dc}}} & \frac{1}{C_{\mathrm{dc}}}\end{bmatrix}$$

$$\boldsymbol{C}_{\mathrm{DC}}^{1\mathrm{x}1} = \boldsymbol{I}^{1\mathrm{x}1}$$

$$\boldsymbol{D}_{\mathrm{DC}}^{1\mathrm{x}5} = \boldsymbol{0}^{1\mathrm{x}5}$$

Grid



State equation

$$\frac{di_{\mathrm{g}}^{\mathrm{D}}}{dt} = \frac{1}{L_{\mathrm{g}}} v_{\mathrm{pcc}}^{\mathrm{D}} - \frac{1}{L_{\mathrm{g}}} v_{\mathrm{g}}^{\mathrm{D}} - \frac{R_{\mathrm{g}}}{L_{\mathrm{g}}} i_{\mathrm{g}}^{\mathrm{D}} + \omega_{\mathrm{g}} i_{\mathrm{g}}^{\mathrm{Q}}$$

$$\frac{di_{\mathrm{g}}^{\mathrm{Q}}}{dt} = \frac{1}{L_{\mathrm{g}}} v_{\mathrm{pcc}}^{\mathrm{Q}} - \frac{1}{L_{\mathrm{g}}} v_{\mathrm{g}}^{\mathrm{Q}} - \frac{R_{\mathrm{g}}}{L_{\mathrm{g}}} i_{\mathrm{g}}^{\mathrm{Q}} - \omega_{\mathrm{g}} i_{\mathrm{g}}^{\mathrm{D}}$$

Output equation

(virtual resistor to estimate the voltage at the PCC)

$$v_{\mathrm{pcc}}^{\mathrm{D}} = R_{\mathrm{v}} \left(i_{\mathrm{L2}}^{\mathrm{D}} - i_{\mathrm{g}}^{\mathrm{D}} \right)$$

$$v_{\mathrm{pcc}}^{\mathrm{Q}} = R_{\mathrm{v}} \left(i_{\mathrm{L2}}^{\mathrm{Q}} - i_{\mathrm{g}}^{\mathrm{Q}} \right)$$

Linearization

$$\frac{d\Delta i_{\mathrm{g}}^{\mathrm{D}}}{dt} = \frac{1}{L_{\mathrm{g}}} \Delta v_{\mathrm{pcc}}^{\mathrm{D}} - \frac{1}{L_{\mathrm{g}}} \Delta v_{\mathrm{g}}^{\mathrm{D}} - \frac{R_{\mathrm{g}}}{L_{\mathrm{g}}} \Delta i_{\mathrm{g}}^{\mathrm{D}} + \omega_{\mathrm{g}} \Delta i_{\mathrm{g}}^{\mathrm{Q}}$$

$$\frac{d\Delta i_{\mathrm{g}}^{\mathrm{Q}}}{dt} = \frac{1}{L_{\mathrm{g}}} \Delta v_{\mathrm{pcc}}^{\mathrm{Q}} - \frac{1}{L_{\mathrm{g}}} \Delta v_{\mathrm{g}}^{\mathrm{Q}} - \frac{R_{\mathrm{g}}}{L_{\mathrm{g}}} \Delta i_{\mathrm{g}}^{\mathrm{Q}} - \omega_{\mathrm{g}} \Delta i_{\mathrm{g}}^{\mathrm{D}}$$

$$\Delta v_{\mathrm{pcc}}^{\mathrm{D}} = R_{\mathrm{v}} \left(\Delta i_{\mathrm{L2}}^{\mathrm{D}} - \Delta i_{\mathrm{g}}^{\mathrm{D}} \right)$$

$$\Delta v_{\mathrm{pcc}}^{\mathrm{Q}} = R_{\mathrm{v}} \left(\Delta i_{\mathrm{L2}}^{\mathrm{Q}} - \Delta i_{\mathrm{g}}^{\mathrm{Q}} \right)$$

State-space model

$$\frac{d}{dt}\Delta\boldsymbol{x}_{\mathrm{g}} = \boldsymbol{A}_{\mathrm{g}}\Delta\boldsymbol{x}_{\mathrm{g}} + \boldsymbol{B}_{\mathrm{g}}\Delta\boldsymbol{u}_{\mathrm{g}}$$

$$\Delta\boldsymbol{y}_{\mathrm{g}} = \boldsymbol{C}_{\mathrm{g}}\Delta\boldsymbol{x}_{\mathrm{g}} + \boldsymbol{D}_{\mathrm{g}}\Delta\boldsymbol{u}_{\mathrm{g}}$$

$$\Delta\boldsymbol{x}_{\mathrm{g}}^{2\mathrm{x}1} = \begin{bmatrix}\Delta i_{\mathrm{g}}^{\mathrm{D}} & \Delta i_{\mathrm{g}}^{\mathrm{Q}}\end{bmatrix}^{T}$$

$$\Delta\boldsymbol{u}_{\mathrm{g}}^{4\mathrm{x}1} = \begin{bmatrix}\Delta i_{\mathrm{L2}}^{\mathrm{D}} & \Delta i_{\mathrm{L2}}^{\mathrm{Q}} & \Delta v_{\mathrm{g}}^{\mathrm{D}} & \Delta v_{\mathrm{g}}^{\mathrm{Q}}\end{bmatrix}^{T}$$

$$\Delta\boldsymbol{y}_{\mathrm{g}}^{4\mathrm{x}1} = \begin{bmatrix}\Delta v_{\mathrm{pcc}}^{\mathrm{D}} & \Delta v_{\mathrm{pcc}}^{\mathrm{Q}} & \Delta i_{\mathrm{g}}^{\mathrm{D}} & \Delta i_{\mathrm{g}}^{\mathrm{Q}}\end{bmatrix}^{T}$$

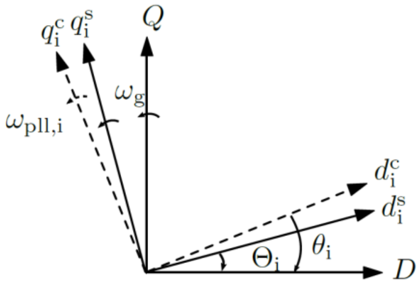
$$\boldsymbol{A}_{\mathrm{g}}^{2\mathrm{x}2} = \begin{bmatrix} -\frac{R_{\mathrm{g}}+R_{\mathrm{v}}}{L_{\mathrm{g}}} & \omega_{\mathrm{g}} \\ -\omega_{\mathrm{g}} & -\frac{R_{\mathrm{g}}+R_{\mathrm{v}}}{L_{\mathrm{g}}} \end{bmatrix}$$

$$\boldsymbol{B}_{\mathrm{g}}^{2\mathrm{x}4} = \begin{bmatrix} \frac{R_{\mathrm{v}}}{L_{\mathrm{g}}} & 0 & -\frac{1}{L_{\mathrm{g}}} & 0 \\ 0 & \frac{R_{\mathrm{v}}}{L_{\mathrm{g}}} & 0 & -\frac{1}{L_{\mathrm{g}}} \end{bmatrix}$$

$$\boldsymbol{C}_{\mathrm{g}}^{4\mathrm{x}2} = \begin{bmatrix} -R_{\mathrm{v}} & 0 \\ 0 & -R_{\mathrm{v}} \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\boldsymbol{D}_{\mathrm{g}}^{4\mathrm{x}4} = \begin{bmatrix} R_{\mathrm{v}} & 0 & 0 & 0 \\ 0 & R_{\mathrm{v}} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Reference Frame



Global frame → system frame

To facilitate the integration of multi-inverter system, a common global frame DQ is defined.

Clockwise rotation

$$T_{\mathrm{C}} = \begin{bmatrix} \cos(\Theta) & -\sin(\Theta) \\ \sin(\Theta) & \cos(\Theta) \end{bmatrix}$$

Anti-clockwise rotation

$$T_{\mathrm{A}} = \begin{bmatrix} \cos(\Theta) & \sin(\Theta) \\ -\sin(\Theta) & \cos(\Theta) \end{bmatrix}$$

Grid-side inductor current of inverter $\Delta i_{\mathrm{L2}}^{\mathrm{dq,s}} \rightarrow \Delta i_{\mathrm{L2}}^{\mathrm{DQ}}$

$$\begin{bmatrix} \Delta i_{\mathrm{L2}}^{\mathrm{D}} \\ \Delta i_{\mathrm{L2}}^{\mathrm{Q}} \end{bmatrix} = T_{\mathrm{C}} \begin{bmatrix} \Delta i_{\mathrm{L2}}^{\mathrm{d,s}} \\ \Delta i_{\mathrm{L2}}^{\mathrm{q,s}} \end{bmatrix}$$

PCC voltage of the grid $\Delta v_{\mathrm{pcc}}^{\mathrm{DQ}} \rightarrow \Delta v_{\mathrm{pcc}}^{\mathrm{dq,s}}$

$$\begin{bmatrix} \Delta v_{\mathrm{pcc}}^{\mathrm{d,s}} \\ \Delta v_{\mathrm{pcc}}^{\mathrm{q,s}} \end{bmatrix} = T_{\mathrm{A}} \begin{bmatrix} \Delta v_{\mathrm{pcc}}^{\mathrm{D}} \\ \Delta v_{\mathrm{pcc}}^{\mathrm{Q}} \end{bmatrix}$$

System frame → control frame

Due to the PLL dynamics, variables with Park transformation are converted from the system to control frame.

$$\begin{bmatrix} \Delta x^{\mathrm{d,c}} \\ \Delta x^{\mathrm{q,c}} \end{bmatrix} = \begin{bmatrix} \Delta x^{\mathrm{d,s}} \\ \Delta x^{\mathrm{q,s}} \end{bmatrix} + \begin{bmatrix} X^{\mathrm{q}} \\ -X^{\mathrm{d}} \end{bmatrix} [\Delta\theta]$$

$$T_{\mathrm{X}}^{\mathrm{s2c}} = \begin{bmatrix} X^{\mathrm{q}} \\ -X^{\mathrm{d}} \end{bmatrix}$$

Inverter-side inductor current $\Delta i_{\mathrm{L1}}^{\mathrm{dq,s}} \rightarrow \Delta i_{\mathrm{L1}}^{\mathrm{dq,c}}$

$$\begin{bmatrix}\Delta i_{\mathrm{L1}}^{\mathrm{d,c}}\\ \Delta i_{\mathrm{L1}}^{\mathrm{q,c}}\end{bmatrix}=\begin{bmatrix}\Delta i_{\mathrm{L1}}^{\mathrm{d,s}}\\ \Delta i_{\mathrm{L1}}^{\mathrm{q,s}}\end{bmatrix}+\begin{bmatrix}I_{\mathrm{L1}}^{\mathrm{q}}\\ -I_{\mathrm{L1}}^{\mathrm{d}}\end{bmatrix}[\Delta\theta]$$

Grid-side capacitor current $\Delta v_{\mathrm{C}}^{\mathrm{dq,s}}\rightarrow\Delta v_{\mathrm{C}}^{\mathrm{dq,c}}$

$$\begin{bmatrix}\Delta v_{\mathrm{C}}^{\mathrm{d,c}}\\ \Delta v_{\mathrm{C}}^{\mathrm{q,c}}\end{bmatrix}=\begin{bmatrix}\Delta v_{\mathrm{C}}^{\mathrm{d,s}}\\ \Delta v_{\mathrm{C}}^{\mathrm{q,s}}\end{bmatrix}+\begin{bmatrix}V_{\mathrm{C}}^{\mathrm{q}}\\ -V_{\mathrm{C}}^{\mathrm{d}}\end{bmatrix}[\Delta\theta]$$

Grid-side inductor current $\Delta i_{\mathrm{L2}}^{\mathrm{dq,s}}\rightarrow\Delta i_{\mathrm{L2}}^{\mathrm{dq,c}}$

$$\begin{bmatrix}\Delta i_{\mathrm{L2}}^{\mathrm{d,c}}\\ \Delta i_{\mathrm{L2}}^{\mathrm{q,c}}\end{bmatrix}=\begin{bmatrix}\Delta i_{\mathrm{L2}}^{\mathrm{d,s}}\\ \Delta i_{\mathrm{L2}}^{\mathrm{q,s}}\end{bmatrix}+\begin{bmatrix}I_{\mathrm{L2}}^{\mathrm{q}}\\ -I_{\mathrm{L2}}^{\mathrm{d}}\end{bmatrix}[\Delta\theta]$$

Control frame → system frame

Due to the PLL dynamics, variables with inverse Park transformation are converted from the control to system frame.

$$\begin{bmatrix}\Delta x^{\mathrm{d,s}}\\ \Delta x^{\mathrm{q,s}}\end{bmatrix}=\begin{bmatrix}\Delta x^{\mathrm{d,c}}\\ \Delta x^{\mathrm{q,c}}\end{bmatrix}+\begin{bmatrix}-X^{\mathrm{q}}\\ X^{\mathrm{d}}\end{bmatrix}[\Delta\theta]$$

$$T_{\mathrm{X}}^{\mathrm{c2s}}=\begin{bmatrix}-X^{\mathrm{q}}\\ X^{\mathrm{d}}\end{bmatrix}$$

Modulation effect $\Delta v_{\mathrm{inv}}^{\mathrm{dq,c}}\rightarrow\Delta v_{\mathrm{inv}}^{\mathrm{dq,s}}$

$$\begin{bmatrix}\Delta v_{\mathrm{inv}}^{\mathrm{d,s}}\\ \Delta v_{\mathrm{inv}}^{\mathrm{q,s}}\end{bmatrix}=\begin{bmatrix}\Delta v_{\mathrm{inv}}^{\mathrm{d,c}}\\ \Delta v_{\mathrm{inv}}^{\mathrm{q,c}}\end{bmatrix}+\begin{bmatrix}-V_{\mathrm{inv}}^{\mathrm{q}}\\ V_{\mathrm{inv}}^{\mathrm{d}}\end{bmatrix}[\Delta\theta]$$

Derivation of small-signal relationship

This part provides the derivation of the small-signal relationship between the control and system frame. First consider a variable x that converts from the control to the global frame:

$$x^{\mathrm{DQ}}=T_{\mathrm{C}}(\theta)x^{\mathrm{dq,c}}=T_{\mathrm{C}}(\Theta+\Delta\theta)x^{\mathrm{dq,c}}$$

The small-signal relationship is derived as:

$$\Delta x^{\mathrm{D}}=\left[\frac{\partial x^{\mathrm{d,c}}\cos(\theta)-x^{\mathrm{q,c}}\sin(\theta)}{\partial x^{\mathrm{d,c}}}\right]_{X^{\mathrm{d}},X^{\mathrm{q}},\Theta}\Delta x^{\mathrm{d,c}}+\left[\frac{\partial x^{\mathrm{d,c}}\cos(\theta)-x^{\mathrm{q,c}}\sin(\theta)}{\partial x^{\mathrm{q,c}}}\right]_{X^{\mathrm{d}},X^{\mathrm{q}},\Theta}\Delta x^{\mathrm{q,c}}$$
$$+\left[\frac{\partial x^{\mathrm{d,c}}\cos(\theta)-x^{\mathrm{q,c}}\sin(\theta)}{\partial\theta}\right]_{X^{\mathrm{d}},X^{\mathrm{q}},\Theta}\Delta\theta$$

$$\Delta x^{\mathrm{Q}}=\left[\frac{\partial x^{\mathrm{d,c}}\sin(\theta)+x^{\mathrm{q,c}}\cos(\theta)}{\partial x^{\mathrm{d,c}}}\right]_{X^{\mathrm{d}},X^{\mathrm{q}},\Theta}\Delta x^{\mathrm{d,c}}+\left[\frac{\partial x^{\mathrm{d,c}}\sin(\theta)+x^{\mathrm{q,c}}\cos(\theta)}{\partial x^{\mathrm{q,c}}}\right]_{X^{\mathrm{d}},X^{\mathrm{q}},\Theta}\Delta x^{\mathrm{q,c}}$$
$$+\left[\frac{\partial x^{\mathrm{d,c}}\sin(\theta)-x^{\mathrm{q,c}}\cos(\theta)}{\partial\theta}\right]_{X^{\mathrm{d}},X^{\mathrm{q}},\Theta}\Delta\theta$$

Which leads to,

$$\Delta x^{\mathrm{D}} = \Delta x^{\mathrm{d,c}} \cos(\Theta) - \Delta x^{\mathrm{q,c}} \sin(\Theta) + \left(-X^{\mathrm{d}} \sin(\Theta) - X^{\mathrm{q}} \cos(\Theta)\right)\Delta\theta$$

$$\Delta x^{\mathrm{Q}} = \Delta x^{\mathrm{d,c}} \sin(\Theta) + \Delta x^{\mathrm{q,c}} \cos(\Theta) + \left(X^{\mathrm{d}} \cos(\Theta) - X^{\mathrm{q}} \sin(\Theta)\right)\Delta\theta$$

$$\Delta x^{\mathrm{DQ}} = T_{\mathrm{C}}(\Theta)\Delta x^{\mathrm{dq,c}} + \begin{bmatrix} -X^{\mathrm{d}} \sin(\Theta) - X^{\mathrm{q}} \cos(\Theta) \\ X^{\mathrm{d}} \cos(\Theta) - X^{\mathrm{q}} \sin(\Theta) \end{bmatrix}\Delta\theta$$

The relationship between $\Delta x^{\mathrm{dq,c}}$ and $\Delta x^{\mathrm{dq,s}}$

$$\Delta x^{\mathrm{dq,s}} = T_A(\Theta)\, \Delta x^{\mathrm{DQ}} = T_A(\Theta)\, T_{\mathrm{C}}(\Theta)\Delta x^{\mathrm{dq,c}} + T_A(\Theta) \begin{bmatrix} -X^{\mathrm{d}} \sin(\Theta) - X^{\mathrm{q}} \cos(\Theta) \\ X^{\mathrm{d}} \cos(\Theta) - X^{\mathrm{q}} \sin(\Theta) \end{bmatrix}\Delta\theta$$

$$T_A(\Theta) \begin{bmatrix} -X^{\mathrm{d}} \sin(\Theta) - X^{\mathrm{q}} \cos(\Theta) \\ X^{\mathrm{d}} \cos(\Theta) - X^{\mathrm{q}} \sin(\Theta) \end{bmatrix}$$

$$= \begin{bmatrix} -\cos(\Theta)\left(X^{\mathrm{d}} \sin(\Theta) + X^{\mathrm{q}} \cos(\Theta)\right) + \sin(\Theta)\left(X^{\mathrm{d}} \cos(\Theta) - X^{\mathrm{q}} \sin(\Theta)\right) \\ \sin(\Theta)\left(X^{\mathrm{d}} \sin(\Theta) + X^{\mathrm{q}} \cos(\Theta)\right) + \cos(\Theta)\left(X^{\mathrm{d}} \cos(\Theta) - X^{\mathrm{q}} \sin(\Theta)\right) \end{bmatrix} = \begin{bmatrix} -X^{\mathrm{q}} \\ X^{\mathrm{d}} \end{bmatrix}$$

Thus,

$$\Delta x^{\mathrm{dq,s}} = \Delta x^{\mathrm{dq,c}} + \begin{bmatrix} -X^{\mathrm{q}} \\ X^{\mathrm{d}} \end{bmatrix}\Delta\theta$$

$$\Delta x^{\mathrm{dq,c}} = \Delta x^{\mathrm{dq,s}} + \begin{bmatrix} X^{\mathrm{q}} \\ -X^{\mathrm{d}} \end{bmatrix}\Delta\theta$$

Component Connection Method (CCM)

State-space model: inverter

State equation

$$\frac{d}{dt}\Delta\boldsymbol{x}_{\mathrm{INV}} = \boldsymbol{A}_{\mathrm{INV}}\Delta\boldsymbol{x}_{\mathrm{INV}} + \boldsymbol{B}_{\mathrm{INV}}\Delta\boldsymbol{a}_{\mathrm{INV}}$$

$$\Delta\boldsymbol{b}_{\mathrm{INV}} = \boldsymbol{C}_{\mathrm{INV}}\Delta\boldsymbol{x}_{\mathrm{INV}} + \boldsymbol{D}_{\mathrm{INV}}\Delta\boldsymbol{a}_{\mathrm{INV}}$$

$\Delta\boldsymbol{x}$: State variable of all subsystems / the entire system

$$\Delta\boldsymbol{x}_{\mathrm{INV}} = [\Delta\boldsymbol{x}_{\mathrm{DVC}}^T \quad \Delta\boldsymbol{x}_{\mathrm{ACC}}^T \quad \Delta\boldsymbol{x}_{\mathrm{del}}^T \quad \Delta\boldsymbol{x}_{\mathrm{LCL}}^T \quad \Delta\boldsymbol{x}_{\mathrm{PLL}}^T \quad \Delta\boldsymbol{x}_{\mathrm{DC}}^T]^T$$

$$\Delta\boldsymbol{x}_{\mathrm{INV}}^{16\mathrm{x}1} = [1\mathrm{x}1 \quad 1\mathrm{x}4 \quad 1\mathrm{x}2 \quad 1\mathrm{x}6 \quad 1\mathrm{x}2 \quad 1\mathrm{x}1]^T$$

$\Delta\boldsymbol{u}$: Input vector of all subsystems

$$\Delta\boldsymbol{u}_{\mathrm{INV}} = [\Delta\boldsymbol{u}_{\mathrm{DVC}}^T \quad \Delta\boldsymbol{u}_{\mathrm{ACC}}^T \quad \Delta\boldsymbol{u}_{\mathrm{del}}^T \quad \Delta\boldsymbol{x}_{\mathrm{LCL}}^T \quad \Delta\boldsymbol{u}_{\mathrm{PLL}}^T \quad \Delta\boldsymbol{u}_{\mathrm{DC}}^T]^T$$

$$\Delta\boldsymbol{u}_{\mathrm{INV}}^{22\mathrm{x}1} = [1\mathrm{x}2 \quad 1\mathrm{x}8 \quad 1\mathrm{x}2 \quad 1\mathrm{x}4 \quad 1\mathrm{x}1 \quad 1\mathrm{x}5]^T$$

$\Delta\boldsymbol{y}$: Output vector of all subsystems

$$\Delta\boldsymbol{y}_{\mathrm{INV}} = [\Delta\boldsymbol{y}_{\mathrm{DVC}}^T \quad \Delta\boldsymbol{y}_{\mathrm{ACC}}^T \quad \Delta\boldsymbol{y}_{\mathrm{del}}^T \quad \Delta\boldsymbol{y}_{\mathrm{LCL}}^T \quad \Delta\boldsymbol{y}_{\mathrm{PLL}}^T \quad \Delta\boldsymbol{y}_{\mathrm{DC}}^T]^T$$

$$\Delta\boldsymbol{y}_{\mathrm{INV}}^{13\mathrm{x}1} = [1\mathrm{x}1 \quad 1\mathrm{x}2 \quad 1\mathrm{x}2 \quad 1\mathrm{x}6 \quad 1\mathrm{x}1 \quad 1\mathrm{x}1]^T$$

$\Delta\boldsymbol{a}$: Input vector of the entire system

$$\Delta\boldsymbol{a}_{\mathrm{INV}}^{5\mathrm{x}1} = \left[\Delta v_{\mathrm{dc,ref}} \quad \Delta i_{\mathrm{ref}}^{\mathrm{q}} \quad \Delta i_{\mathrm{pv}} \quad \Delta v_{\mathrm{pcc}}^{\mathrm{D}} \quad \Delta v_{\mathrm{pcc}}^{\mathrm{Q}}\right]^T$$

$\Delta\boldsymbol{b}$: Output vector of the entire system

$$\Delta\boldsymbol{b}_{\mathrm{INV}}^{2\mathrm{x}1} = \left[\Delta i_{\mathrm{L2}}^{\mathrm{D}} \quad \Delta i_{\mathrm{L2}}^{\mathrm{Q}}\right]^T$$

Interconnection equation

$$\Delta\boldsymbol{u}_{\mathrm{INV}}^{22\mathrm{x}1}=\boldsymbol{L}_{\mathrm{INV},1}^{22\mathrm{x}13}\Delta\boldsymbol{y}_{\mathrm{INV}}^{13\mathrm{x}1}+\boldsymbol{L}_{\mathrm{INV},2}^{22\mathrm{x}5}\Delta\boldsymbol{a}_{\mathrm{INV}}^{5\mathrm{x}1}$$

$$\Delta\boldsymbol{b}_{\mathrm{INV}}^{2\mathrm{x}1}=\boldsymbol{L}_{\mathrm{INV},3}^{2\mathrm{x}13}\Delta\boldsymbol{y}_{\mathrm{INV}}^{13\mathrm{x}1}+\boldsymbol{L}_{\mathrm{INV},4}^{2\mathrm{x}5}\Delta\boldsymbol{a}_{\mathrm{INV}}^{5\mathrm{x}1}$$

$$\begin{bmatrix}\Delta v_{\mathrm{dc,ref}}\\ \Delta v_{\mathrm{dc}}\\ \Delta i_{\mathrm{ref}}^{\mathrm{d}}\\ \Delta i_{\mathrm{ref}}^{\mathrm{q}}\\ \Delta i_{\mathrm{L1}}^{\mathrm{d,c}}\\ \Delta i_{\mathrm{L1}}^{\mathrm{q,c}}\\ \Delta v_{\mathrm{C}}^{\mathrm{d,c}}\\ \Delta v_{\mathrm{C}}^{\mathrm{q,c}}\\ \Delta i_{\mathrm{L2}}^{\mathrm{d,c}}\\ \Delta i_{\mathrm{L2}}^{\mathrm{q,c}}\\ \Delta v_{\mathrm{ref}}^{\mathrm{d}}\\ \Delta v_{\mathrm{ref}}^{\mathrm{q}}\\ \Delta v_{\mathrm{inv}}^{\mathrm{d,s}}\\ \Delta v_{\mathrm{inv}}^{\mathrm{q,s}}\\ \Delta v_{\mathrm{pcc}}^{\mathrm{d,s}}\\ \Delta v_{\mathrm{pcc}}^{\mathrm{q,s}}\\ \Delta v_{\mathrm{C}}^{\mathrm{q,c}}\\ \Delta v_{\mathrm{inv}}^{\mathrm{d,s}}\\ \Delta v_{\mathrm{inv}}^{\mathrm{q,s}}\\ \Delta i_{\mathrm{L1}}^{\mathrm{d,s}}\\ \Delta i_{\mathrm{L1}}^{\mathrm{q,s}}\\ \Delta i_{\mathrm{pv}}\end{bmatrix}=\begin{bmatrix}0&0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0&1\\ 1&0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&1&0&0&0&0&0&I_{\mathrm{L1}}^{\mathrm{q}}&0\\ 0&0&0&0&0&0&1&0&0&0&0&-I_{\mathrm{L1}}^{\mathrm{d}}&0\\ 0&0&0&0&0&0&0&1&0&0&0&V_{\mathrm{C}}^{\mathrm{q}}&0\\ 0&0&0&0&0&0&0&0&1&0&0&-V_{\mathrm{C}}^{\mathrm{d}}&0\\ 0&0&0&0&0&0&0&0&0&1&0&I_{\mathrm{L2}}^{\mathrm{q}}&0\\ 0&0&0&0&0&0&0&0&0&0&1&-I_{\mathrm{L2}}^{\mathrm{d}}&0\\ 0&1&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&1&0&0&0&0&0&0&0&-V_{\mathrm{inv}}^{\mathrm{q}}&0\\ 0&0&0&0&1&0&0&0&0&0&0&V_{\mathrm{inv}}^{\mathrm{d}}&0\\ 0&0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&1&0&0&-V_{\mathrm{C}}^{\mathrm{d}}&0\\ 0&0&0&1&0&0&0&0&0&0&0&-V_{\mathrm{inv}}^{\mathrm{q}}&0\\ 0&0&0&0&1&0&0&0&0&0&0&V_{\mathrm{inv}}^{\mathrm{d}}&0\\ 0&0&0&0&0&1&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&1&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0&0\end{bmatrix}\begin{bmatrix}\Delta i_{\mathrm{ref}}^{\mathrm{d}}\\ \Delta v_{\mathrm{ref}}^{\mathrm{d}}\\ \Delta v_{\mathrm{ref}}^{\mathrm{q}}\\ \Delta v_{\mathrm{inv}}^{\mathrm{d,c}}\\ \Delta v_{\mathrm{inv}}^{\mathrm{q,c}}\\ \Delta i_{\mathrm{L1}}^{\mathrm{d,s}}\\ \Delta i_{\mathrm{L1}}^{\mathrm{q,s}}\\ \Delta v_{\mathrm{C}}^{\mathrm{d,s}}\\ \Delta v_{\mathrm{C}}^{\mathrm{q,s}}\\ \Delta i_{\mathrm{L2}}^{\mathrm{d,s}}\\ \Delta i_{\mathrm{L2}}^{\mathrm{q,s}}\\ \Delta\theta\\ \Delta v_{\mathrm{dc}}\end{bmatrix}+\begin{bmatrix}1&0&0&0&0\\ 0&0&0&0&0\\ 0&0&0&0&0\\ 0&1&0&0&0\\ 0&0&0&0&0\\ 0&0&0&0&0\\ 0&0&0&0&0\\ 0&0&0&0&0\\ 0&0&0&0&0\\ 0&0&0&0&0\\ 0&0&0&0&0\\ 0&0&0&0&0\\ 0&0&0&0&0\\ 0&0&0&0&0\\ 0&0&0&T_{\mathrm{A}}^{1,1}&T_{\mathrm{A}}^{1,2}\\ 0&0&0&T_{\mathrm{A}}^{2,1}&T_{\mathrm{A}}^{2,2}\\ 0&0&0&0&0\\ 0&0&0&0&0\\ 0&0&0&0&0\\ 0&0&0&0&0\\ 0&0&0&0&0\\ 0&0&1&0&0\end{bmatrix}\begin{bmatrix}\Delta v_{\mathrm{dc,ref}}\\ \Delta i_{\mathrm{ref}}^{\mathrm{q}}\\ \Delta i_{\mathrm{pv}}\\ \Delta v_{\mathrm{pcc}}^{\mathrm{D}}\\ \Delta v_{\mathrm{pcc}}^{\mathrm{Q}}\end{bmatrix}$$

$$\begin{bmatrix}\Delta i_{\mathrm{L2}}^{\mathrm{D}}\\ \Delta i_{\mathrm{L2}}^{\mathrm{Q}}\end{bmatrix}=\begin{bmatrix}0&0&0&0&0&0&0&0&0&T_{\mathrm{C}}^{1,1}&T_{\mathrm{C}}^{1,2}&0&0\\ 0&0&0&0&0&0&0&0&0&T_{\mathrm{C}}^{2,1}&T_{\mathrm{C}}^{2,2}&0&0\end{bmatrix}\begin{bmatrix}\Delta i_{\mathrm{ref}}^{\mathrm{d}}\\ \Delta v_{\mathrm{ref}}^{\mathrm{d}}\\ \Delta v_{\mathrm{ref}}^{\mathrm{q}}\\ \Delta v_{\mathrm{inv}}^{\mathrm{d,c}}\\ \Delta v_{\mathrm{inv}}^{\mathrm{q,c}}\\ \Delta i_{\mathrm{L1}}^{\mathrm{d,s}}\\ \Delta i_{\mathrm{L1}}^{\mathrm{q,s}}\\ \Delta v_{\mathrm{C}}^{\mathrm{d,s}}\\ \Delta v_{\mathrm{C}}^{\mathrm{q,s}}\\ \Delta i_{\mathrm{L2}}^{\mathrm{d,s}}\\ \Delta i_{\mathrm{L2}}^{\mathrm{q,s}}\\ \Delta\theta\\ \Delta v_{\mathrm{dc}}\end{bmatrix}+\begin{bmatrix}0&0&0&0&0\\ 0&0&0&0&0\end{bmatrix}\begin{bmatrix}\Delta v_{\mathrm{dc,ref}}\\ \Delta i_{\mathrm{ref}}^{\mathrm{q}}\\ \Delta i_{\mathrm{pv}}\\ \Delta v_{\mathrm{pcc}}^{\mathrm{D}}\\ \Delta v_{\mathrm{pcc}}^{\mathrm{Q}}\end{bmatrix}$$

State-space equation

$$\frac{d}{dt}\Delta\boldsymbol{x}_{\mathrm{INV}}^{16\mathrm{x}1} = \boldsymbol{A}_{\mathrm{INV}}^{16\mathrm{x}16}\Delta\boldsymbol{x}_{\mathrm{INV}}^{16\mathrm{x}1} + \boldsymbol{B}_{\mathrm{INV}}^{16\mathrm{x}5}\Delta\boldsymbol{a}_{\mathrm{INV}}^{5\mathrm{x}1}$$

$$\Delta\boldsymbol{b}_{\mathrm{INV}}^{2\mathrm{x}1} = \boldsymbol{C}_{\mathrm{INV}}^{2\mathrm{x}16}\Delta\boldsymbol{x}_{\mathrm{INV}} + \boldsymbol{D}_{\mathrm{INV}}^{2\mathrm{x}5}\Delta\boldsymbol{a}_{\mathrm{INV}}^{5\mathrm{x}1}$$

Where,

$$\boldsymbol{A}_{\mathrm{INV,diag}}^{16\mathrm{x}16} = \mathrm{diag}\{\boldsymbol{A}_{\mathrm{DVC}}^{1\mathrm{x}1} \quad \boldsymbol{A}_{\mathrm{ACC}}^{4\mathrm{x}4} \quad \boldsymbol{A}_{\mathrm{del}}^{2\mathrm{x}2} \quad \boldsymbol{A}_{\mathrm{LCL}}^{6\mathrm{x}6} \quad \boldsymbol{A}_{\mathrm{PLL}}^{2\mathrm{x}2} \quad \boldsymbol{A}_{\mathrm{DC}}^{1\mathrm{x}1}\}$$

$$\boldsymbol{B}_{\mathrm{INV,diag}}^{16\mathrm{x}22} = \mathrm{diag}\{\boldsymbol{B}_{\mathrm{DVC}}^{1\mathrm{x}2} \quad \boldsymbol{B}_{\mathrm{ACC}}^{4\mathrm{x}8} \quad \boldsymbol{B}_{\mathrm{del}}^{2\mathrm{x}2} \quad \boldsymbol{A}_{\mathrm{LCL}}^{6\mathrm{x}4} \quad \boldsymbol{A}_{\mathrm{PLL}}^{2\mathrm{x}1} \quad \boldsymbol{A}_{\mathrm{DC}}^{1\mathrm{x}5}\}$$

$$\boldsymbol{C}_{\mathrm{INV,diag}}^{13\mathrm{x}16} = \mathrm{diag}\{\boldsymbol{C}_{\mathrm{DVC}}^{1\mathrm{x}1} \quad \boldsymbol{C}_{\mathrm{ACC}}^{2\mathrm{x}4} \quad \boldsymbol{C}_{\mathrm{del}}^{2\mathrm{x}2} \quad \boldsymbol{A}_{\mathrm{LCL}}^{6\mathrm{x}6} \quad \boldsymbol{A}_{\mathrm{PLL}}^{1\mathrm{x}2} \quad \boldsymbol{A}_{\mathrm{DC}}^{1\mathrm{x}1}\}$$

$$\boldsymbol{D}_{\mathrm{INV,diag}}^{13\mathrm{x}22} = \mathrm{diag}\{\boldsymbol{D}_{\mathrm{DVC}}^{1\mathrm{x}2} \quad \boldsymbol{D}_{\mathrm{ACC}}^{2\mathrm{x}8} \quad \boldsymbol{D}_{\mathrm{del}}^{2\mathrm{x}2} \quad \boldsymbol{A}_{\mathrm{LCL}}^{6\mathrm{x}4} \quad \boldsymbol{A}_{\mathrm{PLL}}^{1\mathrm{x}1} \quad \boldsymbol{A}_{\mathrm{DC}}^{1\mathrm{x}5}\}$$

$$\boldsymbol{A}_{\mathrm{INV}}^{16\mathrm{x}16} = \boldsymbol{A}_{\mathrm{INV,diag}}^{16\mathrm{x}16} + \boldsymbol{B}_{\mathrm{INV,diag}}^{16\mathrm{x}22}\boldsymbol{L}_{\mathrm{INV,1}}^{22\mathrm{x}13}\left(\boldsymbol{I}^{13\mathrm{x}13} - \boldsymbol{D}_{\mathrm{INV,diag}}^{13\mathrm{x}22}\boldsymbol{L}_{\mathrm{INV,1}}^{22\mathrm{x}13}\right)^{-1}\boldsymbol{C}_{\mathrm{INV,diag}}^{13\mathrm{x}16}$$

$$\boldsymbol{B}_{\mathrm{INV}}^{16\mathrm{x}5} = \boldsymbol{B}_{\mathrm{INV,diag}}^{16\mathrm{x}22}\boldsymbol{L}_{\mathrm{INV,1}}^{22\mathrm{x}13}\left(\boldsymbol{I}^{13\mathrm{x}13} - \boldsymbol{D}_{\mathrm{INV,diag}}^{13\mathrm{x}22}\boldsymbol{L}_{\mathrm{INV,1}}^{22\mathrm{x}13}\right)^{-1}\boldsymbol{D}_{\mathrm{INV,diag}}^{13\mathrm{x}22}\boldsymbol{L}_{\mathrm{INV,2}}^{22\mathrm{x}5} + \boldsymbol{B}_{\mathrm{INV,diag}}^{16\mathrm{x}22}\boldsymbol{L}_{\mathrm{INV,2}}^{22\mathrm{x}5}$$

$$\boldsymbol{C}_{\mathrm{INV}}^{2\mathrm{x}16} = \boldsymbol{L}_{\mathrm{INV,3}}^{2\mathrm{x}13}\left(\boldsymbol{I}^{13\mathrm{x}13} - \boldsymbol{D}_{\mathrm{INV,diag}}^{13\mathrm{x}22}\boldsymbol{L}_{\mathrm{INV,1}}^{22\mathrm{x}13}\right)^{-1}\boldsymbol{C}_{\mathrm{INV,diag}}^{13\mathrm{x}16}$$

$$\boldsymbol{D}_{\mathrm{INV}}^{2\mathrm{x}5} = \boldsymbol{L}_{\mathrm{INV,3}}^{2\mathrm{x}13}\left(\boldsymbol{I}^{13\mathrm{x}13} - \boldsymbol{D}_{\mathrm{INV,diag}}^{13\mathrm{x}22}\boldsymbol{L}_{\mathrm{INV,1}}^{22\mathrm{x}13}\right)^{-1}\boldsymbol{D}_{\mathrm{INV,diag}}^{13\mathrm{x}22}\boldsymbol{L}_{\mathrm{INV,2}}^{22\mathrm{x}5} + \boldsymbol{L}_{\mathrm{INV,4}}^{2\mathrm{x}5}$$

State-space model: inverter + grid

State equation

$$\frac{d}{dt}\Delta\boldsymbol{x}_{\mathrm{SYS}} = \boldsymbol{A}_{\mathrm{SYS}}\Delta\boldsymbol{x}_{\mathrm{SYS}} + \boldsymbol{B}_{\mathrm{SYS}}\Delta\boldsymbol{a}_{\mathrm{SYS}}$$

$$\Delta\boldsymbol{b}_{\mathrm{SYS}} = \boldsymbol{C}_{\mathrm{SYS}}\Delta\boldsymbol{x}_{\mathrm{SYS}} + \boldsymbol{D}_{\mathrm{SYS}}\Delta\boldsymbol{a}_{\mathrm{SYS}}$$

$\Delta\boldsymbol{x}$: State variable of all subsystems / the entire system

$$\Delta\boldsymbol{x}_{\mathrm{SYS}} = \left[\Delta\boldsymbol{x}_{\mathrm{INV}}^{T} \quad \Delta\boldsymbol{x}_{\mathrm{g}}^{T}\right]^{T}$$

$$\Delta\boldsymbol{x}_{\mathrm{INV}}^{\mathrm{18x1}} = [\mathrm{1x16} \quad \mathrm{1x2}]^{T}$$

$$\Delta\boldsymbol{x}_{\mathrm{SYS}} = \left[\Delta\psi_{\mathrm{dc}} \quad \Delta\gamma^{\mathrm{d}} \quad \Delta\gamma^{\mathrm{q}} \quad \Delta v_{\mathrm{Cf}}^{\mathrm{d,c}} \quad \Delta v_{\mathrm{Cf}}^{\mathrm{q,c}} \quad \Delta x_{\mathrm{del}}^{\mathrm{d}} \quad \Delta x_{\mathrm{del}}^{\mathrm{q}} \quad \Delta i_{\mathrm{L1}}^{\mathrm{d,s}} \quad \Delta i_{\mathrm{L1}}^{\mathrm{q,s}} \quad \Delta v_{\mathrm{C}}^{\mathrm{d,s}} \quad \Delta v_{\mathrm{C}}^{\mathrm{q,s}} \quad \Delta i_{\mathrm{L2}}^{\mathrm{d,s}} \quad \Delta i_{\mathrm{L2}}^{\mathrm{q,s}} \quad \Delta\varepsilon \quad \Delta\theta \quad \Delta v_{\mathrm{dc}} \quad \Delta i_{\mathrm{g}}^{\mathrm{D}} \quad \Delta i_{\mathrm{g}}^{\mathrm{Q}}\right]^{T}$$

$\Delta\boldsymbol{u}$: Input vector of all subsystems

$$\Delta\boldsymbol{u}_{\mathrm{SYS}} = \left[\Delta\boldsymbol{a}_{\mathrm{INV}}^{T} \quad \Delta\boldsymbol{u}_{\mathrm{g}}^{T}\right]^{T}$$

$$\Delta\boldsymbol{u}_{\mathrm{SYS}}^{\mathrm{9x1}} = [\mathrm{1x5} \quad \mathrm{1x4}]^{T}$$

$\Delta\boldsymbol{y}$: Output vector of all subsystems

$$\Delta\boldsymbol{y}_{\mathrm{SYS}} = \left[\Delta\boldsymbol{b}_{\mathrm{INV}}^{T} \quad \Delta\boldsymbol{y}_{\mathrm{g}}^{T}\right]^{T}$$

$$\Delta\boldsymbol{y}_{\mathrm{SYS}}^{\mathrm{6x1}} = [\mathrm{1x2} \quad \mathrm{1x4}]^{T}$$

$\Delta\boldsymbol{a}$: Input vector of the entire system

$$\Delta\boldsymbol{a}_{\mathrm{SYS}}^{\mathrm{5x1}} = \left[\Delta v_{\mathrm{dc,ref}} \quad \Delta i_{\mathrm{ref}}^{\mathrm{q}} \quad \Delta i_{\mathrm{pv}} \quad \Delta v_{\mathrm{g}}^{\mathrm{D}} \quad \Delta v_{\mathrm{g}}^{\mathrm{Q}}\right]^{T}$$

$\Delta\boldsymbol{b}$: Output vector of the entire system

$$\Delta\boldsymbol{b}_{\mathrm{SYS}}^{\mathrm{18x1}} = \Delta\boldsymbol{x}_{\mathrm{SYS}}^{\mathrm{18x1}}$$

Interconnection equation

$$\Delta \boldsymbol{u}_{\mathrm{SYS}}^{9\mathrm{x}1} = \boldsymbol{L}_{\mathrm{INV},1}^{9\mathrm{x}6} \Delta \boldsymbol{y}_{\mathrm{SYS}}^{6\mathrm{x}1} + \boldsymbol{L}_{\mathrm{SYS},2}^{9\mathrm{x}5} \Delta \boldsymbol{a}_{\mathrm{SYS}}^{5\mathrm{x}1}$$

$$\begin{bmatrix} \Delta v_{\mathrm{dc,ref}} \\ \Delta i_{\mathrm{ref}}^{\mathrm{q}} \\ \Delta i_{\mathrm{pv}} \\ \Delta v_{\mathrm{pcc}}^{\mathrm{D}} \\ \Delta v_{\mathrm{pcc}}^{\mathrm{Q}} \\ \Delta i_{\mathrm{L2}}^{\mathrm{D}} \\ \Delta i_{\mathrm{L2}}^{\mathrm{Q}} \\ \Delta v_{\mathrm{g}}^{\mathrm{D}} \\ \Delta v_{\mathrm{g}}^{\mathrm{Q}} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta i_{\mathrm{L2}}^{\mathrm{D}} \\ \Delta i_{\mathrm{L2}}^{\mathrm{Q}} \\ \Delta v_{\mathrm{pcc}}^{\mathrm{D}} \\ \Delta v_{\mathrm{pcc}}^{\mathrm{Q}} \\ \Delta i_{\mathrm{g}}^{\mathrm{D}} \\ \Delta i_{\mathrm{g}}^{\mathrm{Q}} \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \Delta v_{\mathrm{dc,ref}} \\ \Delta i_{\mathrm{ref}}^{\mathrm{q}} \\ \Delta i_{\mathrm{pv}} \\ \Delta v_{\mathrm{g}}^{\mathrm{D}} \\ \Delta v_{\mathrm{g}}^{\mathrm{Q}} \end{bmatrix}$$

State-space equation

$$\frac{d}{dt}\Delta\boldsymbol{x}_{\text{SYS}}^{16\text{x}1} = \boldsymbol{A}_{\text{SYS}}^{16\text{x}16}\Delta\boldsymbol{x}_{\text{SYS}}^{16\text{x}1} + \boldsymbol{B}_{\text{SYS}}^{16\text{x}5}\Delta\boldsymbol{a}_{\text{SYS}}^{5\text{x}1}$$

$$\Delta\boldsymbol{b}_{\text{SYS}}^{18\text{x}1} = \boldsymbol{A}_{\text{SYS}}^{18\text{x}18}\Delta\boldsymbol{x}_{\text{SYS}}^{18\text{x}1} + \boldsymbol{D}_{\text{SYS}}^{18\text{x}5}\Delta\boldsymbol{a}_{\text{SYS}}^{5\text{x}1}$$

Where,

$$\boldsymbol{A}_{\text{SYS,diag}}^{18\text{x}18} = \text{diag}\{\boldsymbol{A}_{\text{INV}}^{16\text{x}16} \quad \boldsymbol{A}_{\text{g}}^{2\text{x}2}\}$$

$$\boldsymbol{B}_{\text{SYS,diag}}^{18\text{x}9} = \text{diag}\{\boldsymbol{B}_{\text{INV}}^{16\text{x}5} \quad \boldsymbol{B}_{\text{g}}^{2\text{x}4}\}$$

$$\boldsymbol{C}_{\text{SYS,diag}}^{6\text{x}18} = \text{diag}\{\boldsymbol{C}_{\text{INV}}^{2\text{x}16} \quad \boldsymbol{C}_{\text{g}}^{4\text{x}2}\}$$

$$\boldsymbol{D}_{\text{SYS,diag}}^{6\text{x}9} = \text{diag}\{\boldsymbol{D}_{\text{INV}}^{2\text{x}5} \quad \boldsymbol{D}_{\text{g}}^{4\text{x}4}\}$$

$$\boldsymbol{A}_{\text{SYS}}^{16\text{x}16} = \boldsymbol{A}_{\text{SYS,diag}}^{18\text{x}18} + \boldsymbol{B}_{\text{SYS,diag}}^{18\text{x}9}\boldsymbol{L}_{\text{INV,1}}^{9\text{x}6}\left(\boldsymbol{I}^{13\text{x}13} - \boldsymbol{D}_{\text{SYS,diag}}^{6\text{x}9}\boldsymbol{L}_{\text{INV,1}}^{9\text{x}6}\right)^{-1}\boldsymbol{C}_{\text{SYS,diag}}^{6\text{x}18}$$

$$\boldsymbol{B}_{\text{SYS}}^{16\text{x}5} = \boldsymbol{B}_{\text{SYS,diag}}^{18\text{x}9}\boldsymbol{L}_{\text{INV,1}}^{9\text{x}6}\left(\boldsymbol{I}^{13\text{x}13} - \boldsymbol{D}_{\text{SYS,diag}}^{6\text{x}9}\boldsymbol{L}_{\text{INV,1}}^{9\text{x}6}\right)^{-1}\boldsymbol{D}_{\text{SYS,diag}}^{6\text{x}9}\boldsymbol{L}_{\text{SYS,2}}^{9\text{x}5} + \boldsymbol{B}_{\text{SYS,diag}}^{18\text{x}9}\boldsymbol{L}_{\text{SYS,2}}^{9\text{x}5}$$

$$\boldsymbol{C}_{\text{INV}}^{2\text{x}16} = \boldsymbol{I}^{18\text{x}18}$$

$$\boldsymbol{D}_{\text{INV}}^{2\text{x}5} = \boldsymbol{0}^{18\text{x}5}$$

Padé Approximation

The time delay of the digital control system can be modeled as:

$$v_{\text{inv}}^{\text{dq}} = e^{-sT_{\text{del}}} \cdot v_{\text{ref}}^{\text{dq}}$$

Padé approximation is a mathematical tool for analyzing nonlinear plant by using the Taylor series:

$$e^{-sT_{\text{del}}} = \frac{b_0 + \cdots + b_i(sT_{\text{del}})^i + \cdots + b_l(sT_{\text{del}})^l}{a_0 + \cdots a_j(sT_{\text{del}})^j + \cdots + a_k(sT_{\text{del}})^k}$$

Where,

$$a_j = \frac{(l+k-j)!\,k!}{j!\,(k-j)!}, j = 0, \cdots, k$$

$$b_i = (-1)^i \frac{(l+k-i)!\,l!}{i!\,(l-i)!}, i = 0, \cdots, l\ (l = k)$$

Transfer function for time-delay:

$$G(s) = \frac{Y}{U} = e^{-sT_{\text{del}}} = \frac{b_0 + \cdots + b_i(sT_{\text{del}})^i + \cdots + b_l(sT_{\text{del}})^l}{a_0 + \cdots a_j(sT_{\text{del}})^j + \cdots + a_k(sT_{\text{del}})^k}$$

Create an intermediate variable X:

$$G(s) = \frac{Y}{X} \cdot \frac{X}{U} = \frac{\frac{b_0}{a_k}{T_{\text{del}}}^{-k} + \cdots + \frac{b_i}{a_k}{T_{\text{del}}}^{-k+i}s^i + \cdots + \frac{b_l}{a_k}{T_{\text{del}}}^{-k+l}s^l}{\frac{a_0}{a_k}{T_{\text{del}}}^{-k} + \cdots \frac{a_j}{a_k}{T_{\text{del}}}^{-k+j}s^j + \cdots + \frac{a_{k-1}}{a_k}{T_{\text{del}}}^{-1}s^{k-1} + s^k}$$

$$\frac{X}{U} = \frac{1}{\frac{a_0}{a_k}{T_{\text{del}}}^{-k} + \cdots \frac{a_j}{a_k}{T_{\text{del}}}^{-k+j}s^j + \cdots + \frac{a_{k-1}}{a_k}{T_{\text{del}}}^{-1}s^{k-1} + s^k}$$

$$\frac{Y}{X} = \frac{b_0}{a_k}{T_{\text{del}}}^{-k} + \cdots + \frac{b_i}{a_k}{T_{\text{del}}}^{-k+i}s^i + \cdots + \frac{b_l}{a_k}{T_{\text{del}}}^{-k+l}s^l$$

In time domain,

$$u = (\dot{x})^k + \frac{a_{k-1}}{a_k}{T_{\text{del}}}^{-1}(\dot{x})^{k-1} + \cdots + \frac{a_j}{a_k}{T_{\text{del}}}^{-k+j}(\dot{x})^j + \cdots + \frac{a_1}{a_k}{T_{\text{del}}}^{-k+1}(\dot{x}) + \frac{a_0}{a_k}{T_{\text{del}}}^{-k}(x)$$

$$y = \frac{b_l}{a_k}{T_{\text{del}}}^{-k+l}(\dot{x})^l + \cdots + \frac{b_i}{a_k}{T_{\text{del}}}^{-k+i}(\dot{x})^i + \cdots + \frac{b_1}{a_k}{T_{\text{del}}}^{-k+1}(\dot{x}) + \frac{b_0}{a_k}{T_{\text{del}}}^{-k}(x)$$

Assume state variables

$$x_1 = (x), x_2 = (\dot{x}), \cdots, x_k = (\dot{x})^{k-1}, x_{k+1} = (\dot{x})^k$$

Then,

$$\dot{x}_1 = x_2,$$

$$\dot{x}_2 = x_3,$$

$$\cdots$$

$$\dot{x}_k = x_{k+1} = (\dot{x})^k = u - \frac{a_0}{a_k}{T_{\mathrm{del}}}^{-k}x_1 - \frac{a_1}{a_k}{T_{\mathrm{del}}}^{-k+1}x_2 - \cdots - \frac{a_j}{a_k}{T_{\mathrm{del}}}^{-k+j}x_{j+1} - \frac{a_{k-1}}{a_k}{T_{\mathrm{del}}}^{-1}x_k$$

The plant is converted as:

$$X = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \cdots \\ \dot{x}_{k-1} \\ \dot{x}_k \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & 1 \\ -\frac{a_0}{a_k}{T_{\mathrm{del}}}^{-k} & -\frac{a_1}{a_k}{T_{\mathrm{del}}}^{-k+1} & -\frac{a_2}{a_k}{T_{\mathrm{del}}}^{-k+2} & \cdots & -\frac{a_{k-1}}{a_k}{T_{\mathrm{del}}}^{-1} \end{bmatrix} X + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix} u$$

State-space model

$$\boldsymbol{A}_d = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & 1 \\ -\frac{a_0}{a_k}{T_{\mathrm{del}}}^{-k} & -\frac{a_1}{a_k}{T_{\mathrm{del}}}^{-k+1} & -\frac{a_2}{a_k}{T_{\mathrm{del}}}^{-k+2} & \cdots & -\frac{a_{k-1}}{a_k}{T_{\mathrm{del}}}^{-1} \end{bmatrix}$$

$$\boldsymbol{B}_d = [0 \quad 0 \quad 0 \quad \cdots \quad 1]^T$$

$$\boldsymbol{C}_d = \frac{1}{a_k^2}[(a_k b_0 - a_0 b_k){T_{\mathrm{del}}}^{-k} \quad (a_k b_1 - a_1 b_k){T_{\mathrm{del}}}^{-k+1} \quad \cdots \quad (a_k b_{k-1} - a_{k-1} b_k){T_{\mathrm{del}}}^{-1}]$$

$$\boldsymbol{D}_d = \frac{b_k}{a_k}$$

First-order Padé approximation

$k = 1$

$$\boldsymbol{A}_d = -\frac{a_0}{a_k}{T_{\mathrm{del}}}^{-k}$$

$$\boldsymbol{B}_d = 1$$

$$\boldsymbol{C}_d = \frac{1}{a_k^2}(a_k b_0 - a_0 b_k){T_{\mathrm{del}}}^{-k}$$

$$\boldsymbol{D}_d = \frac{b_k}{a_k}$$

Reference

The model development can refer [1, 2, 3]. The component connection method is introduced in [4].

[1] Z. Yang, Q. Wang, J. Warmuz and R. W. De Doncker, "Stability Assessment of a Three-Phase Grid-Tied PV Inverter with Eigenvalue-Based Method," in *2019 IEEE 10th International Symposium on Power Electronics for Distributed Generation Systems (PEDG)*, Xi'an, 2019.

[2] Z. Yang, C. Bendfeld, J. Qiang, M. Benedict und R. W. De Doncker, „Stability Investigation of Large-Scale PV Parks with Eigenvalue-Based Method," in *2020 22nd European Conference on Power Electronics and Applications (EPE'20 ECCE Europe)*, Lyon, 2020.

[3] Z. Yang, On the Stability of Three-Phase Grid-Tied Photovoltaic Inverter Systems, Bd. 90, Aachen: E.ON Energy Research Center, RWTH Aachen University, 2021.

[4] G. Gaba, S. Lefebvre und D. Mukhedkar, „Comparative analysis and study of the dynamic stability of AC/DC systems," *IEEE Transactions on Power Systems,* Bd. 3, Nr. 3, pp. 978-985, Aug 1988.