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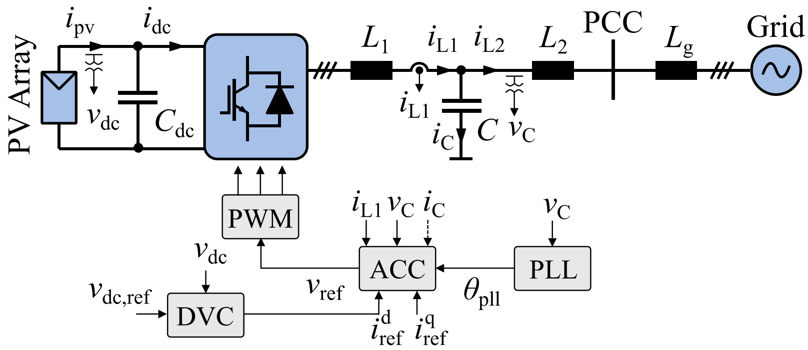
Small-Signal Impedance Modeling of Grid-Following Inverter



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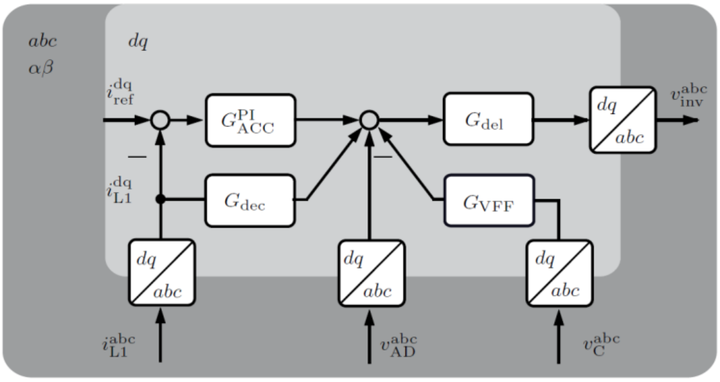
Alternating-Current Control (ACC)

Proportional-integral (PI) control



Control transfer function

$$F_{\mathrm{ACC}}^{\mathrm{PI,dq}}(s) = K_{\mathrm{p}}^{\mathrm{ACC}} + \frac{K_{\mathrm{i}}^{\mathrm{ACC}}}{s}$$

Control transfer matrix

$$G_{\mathrm{ACC}}^{\mathrm{PI}}(s) = \begin{bmatrix} F_{\mathrm{ACC}}^{\mathrm{PI,dq}}(s) & 0 \\ 0 & F_{\mathrm{ACC}}^{\mathrm{PI,dq}}(s) \end{bmatrix}$$

Decoupling transfer matrix

$$G_{\mathrm{dec}}(s) = \begin{bmatrix} 0 & \omega_{\mathrm{g}} L \\ -\omega_{\mathrm{g}} L & 0 \end{bmatrix}$$

Voltage feedforward filter (VFF) transfer matrix

$$G_{\mathrm{VFF}}(s) = \begin{bmatrix} \frac{\omega_{\mathrm{VFF}}}{s + \omega_{\mathrm{VFF}}} & 0 \\ 0 & \frac{\omega_{\mathrm{VFF}}}{s + \omega_{\mathrm{VFF}}} \end{bmatrix}$$

Active damping (AD) transfer matrix

$$G_{\mathrm{AD}}(s) = \begin{bmatrix} K_{\mathrm{AD}} & 0 \\ 0 & K_{\mathrm{AD}} \end{bmatrix}$$

Delay matrix in Padé approximation form

$$G_{\mathrm{del}}(s) = \begin{bmatrix} \frac{2 - sT_{\mathrm{del}}}{2 + sT_{\mathrm{del}}} & 0 \\ 0 & \frac{2 - sT_{\mathrm{del}}}{2 + sT_{\mathrm{del}}} \end{bmatrix}$$

Frequency-domain relation

$$\begin{bmatrix} m^{\mathrm{d,c}} \\ m^{\mathrm{q,c}} \end{bmatrix} \frac{v_{\mathrm{dc}}}{2} = G_{\mathrm{del}}(s) G_{\mathrm{ACC}}^{\mathrm{PI}}(s) \begin{bmatrix} i_{\mathrm{ref}}^{\mathrm{d}} \\ i_{\mathrm{ref}}^{\mathrm{q}} \end{bmatrix} - G_{\mathrm{del}}(s) \left(G_{\mathrm{ACC}}^{\mathrm{PI}}(s) + G_{\mathrm{dec}}(s) \right) \begin{bmatrix} i_{\mathrm{L1}}^{\mathrm{d,c}} \\ i_{\mathrm{L1}}^{\mathrm{q,c}} \end{bmatrix}$$

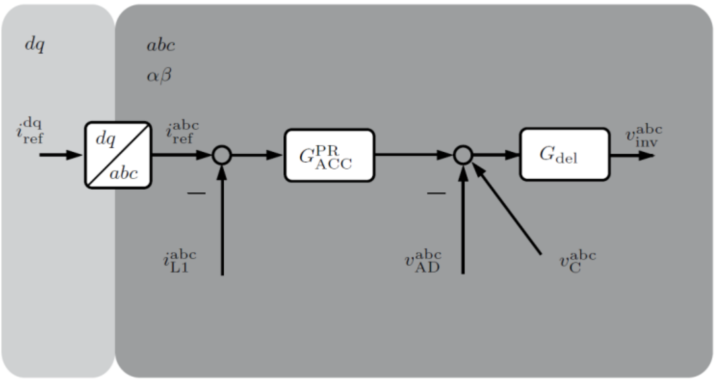
$$+ G_{\mathrm{del}}(s) G_{\mathrm{VFF}}(s) \begin{bmatrix} v_{\mathrm{C}}^{\mathrm{d,c}} \\ v_{\mathrm{C}}^{\mathrm{q,c}} \end{bmatrix} - G_{\mathrm{del}}(s) G_{\mathrm{AD}}(s) \begin{bmatrix} i_{\mathrm{C}}^{\mathrm{d,c}} \\ i_{\mathrm{C}}^{\mathrm{q,c}} \end{bmatrix}$$

Linearization

$$\begin{bmatrix} \Delta m^{\mathrm{d,c}} \\ \Delta m^{\mathrm{q,c}} \end{bmatrix} \frac{V_{\mathrm{dc}}}{2} + \begin{bmatrix} M^{\mathrm{d}} \\ M^{\mathrm{q}} \end{bmatrix} \frac{\Delta v_{\mathrm{dc}}}{2} = G_{\mathrm{del}}(s) G_{\mathrm{ACC}}^{\mathrm{PI}}(s) \begin{bmatrix} \Delta i_{\mathrm{ref}}^{\mathrm{d}} \\ \Delta i_{\mathrm{ref}}^{\mathrm{q}} \end{bmatrix} - G_{\mathrm{del}}(s) \left(G_{\mathrm{ACC}}^{\mathrm{PI}}(s) + G_{\mathrm{dec}}(s) \right) \begin{bmatrix} \Delta i_{\mathrm{L1}}^{\mathrm{d,c}} \\ \Delta i_{\mathrm{L1}}^{\mathrm{q,c}} \end{bmatrix}$$

$$+ G_{\mathrm{del}}(s) G_{\mathrm{VFF}}(s) \begin{bmatrix} \Delta v_{\mathrm{C}}^{\mathrm{d,c}} \\ \Delta v_{\mathrm{C}}^{\mathrm{q,c}} \end{bmatrix} - G_{\mathrm{del}}(s) G_{\mathrm{AD}}(s) \begin{bmatrix} \Delta i_{\mathrm{C}}^{\mathrm{d,c}} \\ \Delta i_{\mathrm{C}}^{\mathrm{q,c}} \end{bmatrix}$$

Proportional-resonant (PR) control



Control transfer function

$$F_{\mathrm{ACC}}^{\mathrm{PR},\alpha\beta}(s) = K_{\mathrm{p}}^{\mathrm{ACC}} + \frac{K_{\mathrm{r}}^{\mathrm{ACC}} s}{s^2 + \omega_{\mathrm{g}}^2} = K_{\mathrm{p}}^{\mathrm{ACC}} + \frac{K_{\mathrm{r}}^{\mathrm{ACC}}}{2}\left(\frac{1}{s - j\omega_{\mathrm{g}}} + \frac{1}{s + j\omega_{\mathrm{g}}}\right)$$

$$\Downarrow s^{\alpha\beta} \rightarrow s^{\mathrm{dq}} + j\omega_{\mathrm{g}}$$

$$\begin{aligned} F_{\mathrm{ACC}}^{\mathrm{PR,dq}}(s) = F_{\mathrm{ACC}}^{\mathrm{PR},\alpha\beta}(s + j\omega_{\mathrm{g}}) &= K_{\mathrm{p}}^{\mathrm{ACC}} + \frac{K_{\mathrm{r}}^{\mathrm{ACC}}}{2}\left(\frac{1}{s} + \frac{1}{s + j2\omega_{\mathrm{g}}}\right) \\ &= K_{\mathrm{p}}^{\mathrm{ACC}} + \frac{K_{\mathrm{r}}^{\mathrm{ACC}}}{2s} + \frac{K_{\mathrm{r}}^{\mathrm{ACC}} s}{2(s + 4\omega_{\mathrm{g}})^2} - j\frac{K_{\mathrm{r}}^{\mathrm{ACC}} \omega_{\mathrm{g}}}{2(s + 4\omega_{\mathrm{g}})^2} \end{aligned}$$

Control transfer matrix

$$G_{\mathrm{ACC}}^{\mathrm{PR}}(s) = \begin{bmatrix} \mathrm{Re}\left\{F_{\mathrm{ACC}}^{\mathrm{PR,dq}}(s)\right\} & -\mathrm{Im}\left\{F_{\mathrm{ACC}}^{\mathrm{PR,dq}}(s)\right\} \\ \mathrm{Im}\left\{F_{\mathrm{ACC}}^{\mathrm{PR,dq}}(s)\right\} & \mathrm{Re}\left\{F_{\mathrm{ACC}}^{\mathrm{PR,dq}}(s)\right\} \end{bmatrix}$$

Active damping (AD) transfer matrix

$$G_{\mathrm{AD}}(s) = \begin{bmatrix} K_{\mathrm{AD}} & 0 \\ 0 & K_{\mathrm{AD}} \end{bmatrix}$$

Delay matrix in Padé approximation form

$$G_{\mathrm{del}}(s) = \begin{bmatrix} \dfrac{2 - sT_{\mathrm{del}}}{2 + sT_{\mathrm{del}}} & 0 \\ 0 & \dfrac{2 - sT_{\mathrm{del}}}{2 + sT_{\mathrm{del}}} \end{bmatrix}$$

Frequency-domain relation

$$\begin{bmatrix} m^{\mathrm{d,c}} \\ m^{\mathrm{q,c}} \end{bmatrix} \frac{v_{\mathrm{dc}}}{2} = G_{\mathrm{del}}(s) G_{\mathrm{ACC}}^{\mathrm{PR}}(s) \begin{bmatrix} i_{\mathrm{ref}}^{\mathrm{d}} \\ i_{\mathrm{ref}}^{\mathrm{q}} \end{bmatrix} - G_{\mathrm{del}}(s) G_{\mathrm{ACC}}^{\mathrm{PR}}(s) \begin{bmatrix} i_{\mathrm{L1}}^{\mathrm{d,c}} \\ i_{\mathrm{L1}}^{\mathrm{q,c}} \end{bmatrix} - G_{\mathrm{del}}(s) G_{\mathrm{AD}}(s) \begin{bmatrix} i_{\mathrm{C}}^{\mathrm{d,c}} \\ i_{\mathrm{C}}^{\mathrm{q,c}} \end{bmatrix}$$

Linearization

$$\begin{aligned} &\begin{bmatrix} \Delta m^{\mathrm{d,c}} \\ \Delta m^{\mathrm{q,c}} \end{bmatrix} \frac{V_{\mathrm{dc}}}{2} + \begin{bmatrix} M^{\mathrm{d}} \\ M^{\mathrm{q}} \end{bmatrix} \frac{\Delta v_{\mathrm{dc}}}{2} \\ &\qquad = G_{\mathrm{del}}(s) G_{\mathrm{ACC}}^{\mathrm{PR}}(s) \begin{bmatrix} \Delta i_{\mathrm{ref}}^{\mathrm{d}} \\ \Delta i_{\mathrm{ref}}^{\mathrm{q}} \end{bmatrix} - G_{\mathrm{del}}(s) G_{\mathrm{ACC}}^{\mathrm{PR}}(s) \begin{bmatrix} \Delta i_{\mathrm{L1}}^{\mathrm{d,c}} \\ \Delta i_{\mathrm{L1}}^{\mathrm{q,c}} \end{bmatrix} - G_{\mathrm{del}}(s) G_{\mathrm{AD}}(s) \begin{bmatrix} \Delta i_{\mathrm{C}}^{\mathrm{d,c}} \\ \Delta i_{\mathrm{C}}^{\mathrm{q,c}} \end{bmatrix} \end{aligned}$$

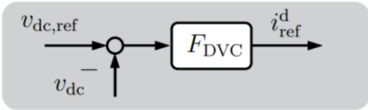
LCL Filter

$$Z_{\mathrm{L1}}(s) = \begin{bmatrix} sL_1 + R_1 & -\omega_{\mathrm{g}} L_1 \\ \omega_{\mathrm{g}} L_1 & sL_1 + R_1 \end{bmatrix}$$

$$Y_{\mathrm{C}}(s) = \begin{bmatrix} sC & -\omega_{\mathrm{g}} C \\ \omega_{\mathrm{g}} C & sC \end{bmatrix}$$

$$Z_{\mathrm{L2}}(s) = \begin{bmatrix} sL_2 + R_2 & -\omega_{\mathrm{g}} L_2 \\ \omega_{\mathrm{g}} L_2 & sL_2 + R_2 \end{bmatrix}$$

Direct-Voltage Control (DVC)



Control transfer function

$$F_{\mathrm{DVC}}(s) = K_{\mathrm{p}}^{\mathrm{DVC}} + \frac{K_{\mathrm{i}}^{\mathrm{DVC}}}{s}$$

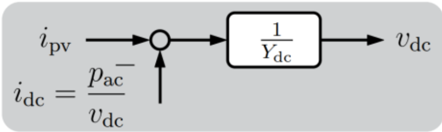
Frequency-domain relation

$$\begin{bmatrix} i_{\mathrm{ref}}^{\mathrm{d}} \\ i_{\mathrm{ref}}^{\mathrm{q}} \end{bmatrix} = \begin{bmatrix} F_{\mathrm{DVC}}(s) \\ 0 \end{bmatrix} \left(v_{\mathrm{dc,ref}} - v_{\mathrm{dc}} \right)$$

Linearization

$$\begin{bmatrix} \Delta i_{\mathrm{ref}}^{\mathrm{d}} \\ \Delta i_{\mathrm{ref}}^{\mathrm{q}} \end{bmatrix} = -\underbrace{\begin{bmatrix} F_{\mathrm{DVC}}(s) \\ 0 \end{bmatrix}}_{G_{\mathrm{DVC}}(s)} \Delta v_{\mathrm{dc}}$$

DC-Link Capacitor



Plant dynamics

$$Y_{\mathrm{dc}}(s) v_{\mathrm{dc}} = i_{\mathrm{pv}} - i_{\mathrm{dc}}$$

$$Y_{\mathrm{dc}}(s) = sC_{\mathrm{dc}}$$

Power balance

$$i_{\mathrm{dc}} \cancel{v_{\mathrm{dc}}} = p_{\mathrm{dc}} = p_{\mathrm{ac}} = \frac{3}{2}\left(i_{\mathrm{L1}}^{\mathrm{d,s}} m^{\mathrm{d,s}} + i_{\mathrm{L1}}^{\mathrm{q,s}} m^{\mathrm{q,s}}\right) \frac{\cancel{v_{\mathrm{dc}}}}{2}$$

$$\Downarrow$$

$$i_{\mathrm{dc}} = \frac{3}{4}\left(i_{\mathrm{L1}}^{\mathrm{d,s}} m^{\mathrm{d,s}} + i_{\mathrm{L1}}^{\mathrm{q,s}} m^{\mathrm{q,s}}\right)$$

Linearization

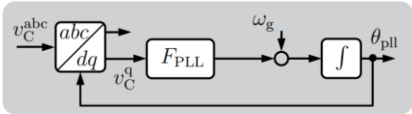
$$\Delta i_{\mathrm{dc}} = \frac{3}{4}\begin{bmatrix} M^{\mathrm{d}} & M^{\mathrm{q}} \end{bmatrix} \begin{bmatrix} \Delta i_{\mathrm{L1}}^{\mathrm{d,s}} \\ \Delta i_{\mathrm{L1}}^{\mathrm{q,s}} \end{bmatrix} + \frac{3}{4}\begin{bmatrix} I_{\mathrm{L1}}^{\mathrm{d}} & I_{\mathrm{L1}}^{\mathrm{q}} \end{bmatrix} \begin{bmatrix} \Delta m^{\mathrm{d,s}} \\ \Delta m^{\mathrm{q,s}} \end{bmatrix}$$

$$\Delta i_{\mathrm{dc}} = -Y_{\mathrm{dc}}(s) \Delta v_{\mathrm{dc}}$$

$$\Downarrow$$

$$\Delta v_{\mathrm{dc}} = -\left(\underbrace{\frac{3}{4Y_{\mathrm{dc}}(s)}\begin{bmatrix} M^{\mathrm{d}} & M^{\mathrm{q}} \end{bmatrix}}_{G_{\mathrm{DVC}}^{\mathrm{i}}(s)} \begin{bmatrix} \Delta i_{\mathrm{L1}}^{\mathrm{d,s}} \\ \Delta i_{\mathrm{L1}}^{\mathrm{q,s}} \end{bmatrix} + \underbrace{\frac{3}{4Y_{\mathrm{dc}}(s)}\begin{bmatrix} I_{\mathrm{L1}}^{\mathrm{d}} & I_{\mathrm{L1}}^{\mathrm{q}} \end{bmatrix}}_{G_{\mathrm{DVC}}^{\mathrm{m}}(s)} \begin{bmatrix} \Delta m^{\mathrm{d,s}} \\ \Delta m^{\mathrm{q,s}} \end{bmatrix} \right)$$

Phase-Locked Loop (PLL)



Control transfer function

$$F_{\mathrm{PLL}}(s) = K_{\mathrm{p}}^{\mathrm{PLL}} + \frac{K_{\mathrm{i}}^{\mathrm{PLL}}}{s}$$

Linearization according to [1, 2]

Derivation:

According to (E.11) in [2], a variable in the system frame observed in the control frame

$$\Delta v_{\mathrm{C}}^{\mathrm{q,c}} = \Delta v_{\mathrm{C}}^{\mathrm{q,s}} - V_{\mathrm{C}}^{\mathrm{d}} \Delta\theta$$

According to PLL control diagram

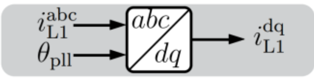
$$\frac{\Delta v_{\mathrm{C}}^{\mathrm{q,c}} F_{\mathrm{PLL}}(s)}{s} = \Delta\theta$$

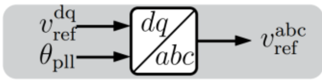
Substitute $\Delta v_{\mathrm{C}}^{\mathrm{q,c}}$, acquire

$$H_{\mathrm{PLL}}(s) = \frac{\Delta\theta}{\Delta v_{\mathrm{C}}^{\mathrm{q,s}}} = \frac{F_{\mathrm{PLL}}(s)}{s + V_{\mathrm{C}}^{\mathrm{d}} F_{\mathrm{PLL}}(s)}$$

$$\Rightarrow \Delta\theta = H_{\mathrm{PLL}}(s) \Delta v_{\mathrm{C}}^{\mathrm{q,s}}$$

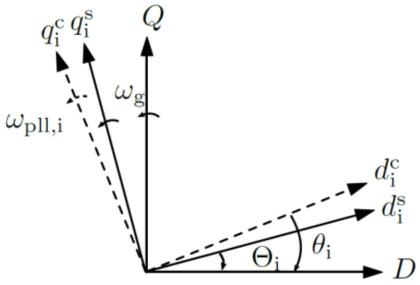
The small-signal transfer function $H_{\mathrm{PLL}}(s)$ influences the variables through Park and inverse Park transformation, e.g.





Detailed mathematical impacts are analyzed in the section reference frame

Reference Frame



Clockwise rotation

$$T_{\mathrm{C}}(\Theta) = \begin{bmatrix} \cos(\Theta) & -\sin(\Theta) \\ \sin(\Theta) & \cos(\Theta) \end{bmatrix}$$

Anti-clockwise rotation

$$T_{\mathrm{A}}(\Theta) = \begin{bmatrix} \cos(\Theta) & \sin(\Theta) \\ -\sin(\Theta) & \cos(\Theta) \end{bmatrix}$$

Note: Usually an anti-clockwise rotation is defined as $\begin{bmatrix} \cos(\Theta) & -\sin(\Theta) \\ \sin(\Theta) & \cos(\Theta) \end{bmatrix}$, however the rotation in the above depicted figure presents the relation of reference frames. A variable in a frame has a reverse rotation reflected in another frame. For convenience, the rotation angle is defined as positive for clockwise rotation, so that a variable observed in different frames can be evaluated according to the rotation direction of frames.

Example: a variable in the dq^{s} frame align with the direction of $d_{\mathrm{i}}^{\mathrm{s}}$ should be with a rotation of $\begin{bmatrix} \cos(\Theta) & \sin(\Theta) \\ -\sin(\Theta) & \cos(\Theta) \end{bmatrix}$ when being observed from the dq^{c}.

System frame → global frame

To facilitate the integration of multi-inverter system, a common global frame DQ is defined. Each inverter has its own rotation frame. They can be aggregated only after being converted into the same global frame.

According to (E.3) in [2], a variable in the system frame observed in the global frame

$$\begin{bmatrix} x^{\mathrm{D}} \\ x^{\mathrm{Q}} \end{bmatrix} = T_{\mathrm{C}}(\Theta) \begin{bmatrix} x^{\mathrm{d,s}} \\ x^{\mathrm{q,s}} \end{bmatrix}$$

Consider i^{th} inverter

$$I_{\mathrm{i}}^{\mathrm{DQ}} = T_{\mathrm{C}} \cdot I_{\mathrm{i}}^{\mathrm{dq,s}}$$

$$V_{\mathrm{i}}^{\mathrm{DQ}} = Z_{\mathrm{i}}^{\mathrm{DQ}} \cdot I_{\mathrm{i}}^{\mathrm{DQ}}$$

$$V_{\mathrm{i}}^{\mathrm{dq,s}} = Z_{\mathrm{i}}^{\mathrm{dq,s}} \cdot I_{\mathrm{i}}^{\mathrm{dq,s}}$$

Then

$$V_\mathrm{i}^\mathrm{DQ} = T_\mathrm{C} \cdot V_\mathrm{i}^\mathrm{dq,s}$$

$$Z_\mathrm{i}^\mathrm{DQ} \cdot I_\mathrm{i}^\mathrm{DQ} = T_\mathrm{C} \cdot Z_\mathrm{i}^\mathrm{dq,s} \cdot I_\mathrm{i}^\mathrm{dq,s}$$

$$Z_\mathrm{i}^\mathrm{DQ} \cdot I_\mathrm{i}^\mathrm{DQ} = T_\mathrm{C} \cdot Z_\mathrm{i}^\mathrm{dq,s} \cdot T_\mathrm{A} \cdot I_\mathrm{i}^\mathrm{DQ}$$

$$\Downarrow$$

$$Z_\mathrm{i}^\mathrm{DQ} = T_\mathrm{C} \cdot Z_\mathrm{i}^\mathrm{dq,s} \cdot T_\mathrm{A}$$

System frame → control frame

According to (E.11) in [2], a variable in the system frame observed in the control frame

$$\begin{bmatrix} \Delta x^\mathrm{d,c} \\ \Delta x^\mathrm{q,c} \end{bmatrix} = \begin{bmatrix} \Delta x^\mathrm{d,s} \\ \Delta x^\mathrm{q,s} \end{bmatrix} + \begin{bmatrix} X^\mathrm{q} \\ -X^\mathrm{d} \end{bmatrix} \Delta\theta$$

According to the derivation in the PLL section

$$\Delta\theta = H_\mathrm{PLL}(s)\Delta v_\mathrm{C}^\mathrm{q,s}$$

Due to the PLL dynamics, variables with Park transformation are converted from the system to control frame.

$$\begin{bmatrix} \Delta x^\mathrm{d,c} \\ \Delta x^\mathrm{q,c} \end{bmatrix} = \begin{bmatrix} \Delta x^\mathrm{d,s} \\ \Delta x^\mathrm{q,s} \end{bmatrix} + \begin{bmatrix} 0 & X^\mathrm{q} H_\mathrm{PLL}(s) \\ 0 & -X^\mathrm{d} H_\mathrm{PLL}(s) \end{bmatrix} \begin{bmatrix} \Delta v_\mathrm{C}^\mathrm{d,s} \\ \Delta v_\mathrm{C}^\mathrm{q,s} \end{bmatrix}$$

Inverter-side inductor current $\Delta i_\mathrm{L1}^\mathrm{dq,s} \rightarrow \Delta i_\mathrm{L1}^\mathrm{dq,c}$

$$\begin{bmatrix} \Delta i_\mathrm{L1}^\mathrm{d,c} \\ \Delta i_\mathrm{L1}^\mathrm{q,c} \end{bmatrix} = \begin{bmatrix} \Delta i_\mathrm{L1}^\mathrm{d,s} \\ \Delta i_\mathrm{L1}^\mathrm{q,s} \end{bmatrix} + \underbrace{\begin{bmatrix} 0 & I_\mathrm{L1}^\mathrm{q} H_\mathrm{PLL}(s) \\ 0 & -I_\mathrm{L1}^\mathrm{d} H_\mathrm{PLL}(s) \end{bmatrix}}_{G_\mathrm{PLL}^\mathrm{i}(s)} \begin{bmatrix} \Delta v_\mathrm{C}^\mathrm{d,s} \\ \Delta v_\mathrm{C}^\mathrm{q,s} \end{bmatrix}$$

Grid-side capacitor current $\Delta i_\mathrm{C}^\mathrm{dq,s} \rightarrow \Delta i_\mathrm{C}^\mathrm{dq,c}$

$$\begin{bmatrix} \Delta i_\mathrm{C}^\mathrm{d,c} \\ \Delta i_\mathrm{C}^\mathrm{q,c} \end{bmatrix} = \begin{bmatrix} \Delta i_\mathrm{C}^\mathrm{d,s} \\ \Delta i_\mathrm{C}^\mathrm{q,s} \end{bmatrix} + \underbrace{\begin{bmatrix} 0 & I_\mathrm{C}^\mathrm{q} H_\mathrm{PLL}(s) \\ 0 & -I_\mathrm{C}^\mathrm{d} H_\mathrm{PLL}(s) \end{bmatrix}}_{G_\mathrm{PLL}^\mathrm{ic}(s)} \begin{bmatrix} \Delta v_\mathrm{C}^\mathrm{d,s} \\ \Delta v_\mathrm{C}^\mathrm{q,s} \end{bmatrix}$$

Grid-side capacitor voltage $\Delta v_\mathrm{C}^\mathrm{dq,s} \rightarrow \Delta v_\mathrm{C}^\mathrm{dq,c}$

$$\begin{bmatrix} \Delta v_\mathrm{C}^\mathrm{d,c} \\ \Delta v_\mathrm{C}^\mathrm{q,c} \end{bmatrix} = \begin{bmatrix} \Delta v_\mathrm{C}^\mathrm{d,s} \\ \Delta v_\mathrm{C}^\mathrm{q,s} \end{bmatrix} + \underbrace{\begin{bmatrix} 0 & V_\mathrm{C}^\mathrm{q} H_\mathrm{PLL}(s) \\ 0 & -V_\mathrm{C}^\mathrm{d} H_\mathrm{PLL}(s) \end{bmatrix}}_{G_\mathrm{PLL}^\mathrm{v}(s)} \begin{bmatrix} \Delta v_\mathrm{C}^\mathrm{d,s} \\ \Delta v_\mathrm{C}^\mathrm{q,s} \end{bmatrix}$$

Control frame → system frame

Due to the PLL dynamics, variables with inverse Park transformation are converted from the control to system frame.

$$\begin{bmatrix} \Delta x^{\mathrm{d,s}} \\ \Delta x^{\mathrm{q,s}} \end{bmatrix} = \begin{bmatrix} \Delta x^{\mathrm{d,c}} \\ \Delta x^{\mathrm{q,c}} \end{bmatrix} + \begin{bmatrix} 0 & -X^{\mathrm{q}} H_{\mathrm{PLL}}(s) \\ 0 & X^{\mathrm{d}} H_{\mathrm{PLL}}(s) \end{bmatrix} \begin{bmatrix} \Delta v_{\mathrm{C}}^{\mathrm{d,s}} \\ \Delta v_{\mathrm{C}}^{\mathrm{q,s}} \end{bmatrix}$$

Modulation index $\Delta m^{\mathrm{dq,c}} \rightarrow \Delta m^{\mathrm{dq,s}}$

$$\begin{bmatrix} \Delta m^{\mathrm{d,s}} \\ \Delta m^{\mathrm{q,s}} \end{bmatrix} = \begin{bmatrix} \Delta m^{\mathrm{d,c}} \\ \Delta m^{\mathrm{q,c}} \end{bmatrix} + \underbrace{\begin{bmatrix} 0 & -M^{\mathrm{q}} H_{\mathrm{PLL}}(s) \\ 0 & M^{\mathrm{d}} H_{\mathrm{PLL}}(s) \end{bmatrix}}_{G_{\mathrm{PLL}}^{\mathrm{m}}(s)} \begin{bmatrix} \Delta v_{\mathrm{C}}^{\mathrm{d,s}} \\ \Delta v_{\mathrm{C}}^{\mathrm{q,s}} \end{bmatrix}$$

If PR control is implemented, reference current obtained in the dq frame should be converted to the $\alpha\beta$ frame, which also requires inverse Park transformation, i.e. $\Delta i_{\mathrm{ref}}^{\mathrm{dq}} \rightarrow \Delta i_{\mathrm{ref}}^{\alpha\beta}$

$$\begin{bmatrix} \Delta i_{\mathrm{ref}}^{\alpha} \\ \Delta i_{\mathrm{ref}}^{\alpha} \end{bmatrix} = \begin{bmatrix} \Delta i_{\mathrm{ref}}^{\mathrm{d}} \\ \Delta i_{\mathrm{ref}}^{\mathrm{q}} \end{bmatrix} + \underbrace{\begin{bmatrix} 0 & -I_{\mathrm{ref}}^{\mathrm{q}} H_{\mathrm{PLL}}(s) \\ 0 & I_{\mathrm{ref}}^{\mathrm{d}} H_{\mathrm{PLL}}(s) \end{bmatrix}}_{G_{\mathrm{PLL}}^{\mathrm{iref}}(s)} \begin{bmatrix} \Delta v_{\mathrm{C}}^{\mathrm{d,s}} \\ \Delta v_{\mathrm{C}}^{\mathrm{q,s}} \end{bmatrix}$$

Modulation Delay

The modulation delay in the complex frequency domain can be precisely modeled as:

$$\underline{v}_{\mathrm{inv}}^{\mathrm{dq}} = v_{\mathrm{inv}}^{\mathrm{d}} + jv_{\mathrm{inv}}^{\mathrm{q}} = \underline{v}_{\mathrm{ref}}^{\mathrm{dq}} \cdot e^{-sT_{\mathrm{del}}}$$

If the delay is expanded in Euler's form,

$$e^{-sT_{\mathrm{del}}} = \cos \omega_{\mathrm{g}} T_{\mathrm{del}} - j \sin \omega_{\mathrm{g}} T_{\mathrm{del}}$$

The delay transfer matrix can be described as below.

$$G_{\mathrm{del}}(s) = \begin{bmatrix} \cos \omega_{\mathrm{g}} T_{\mathrm{del}} & \sin \omega_{\mathrm{g}} T_{\mathrm{del}} \\ -\sin \omega_{\mathrm{g}} T_{\mathrm{del}} & \cos \omega_{\mathrm{g}} T_{\mathrm{del}} \end{bmatrix}$$

If the delay is expanded in Padé approximation,

$$e^{-sT_{\mathrm{del}}} \approx \frac{2 - sT_{\mathrm{del}}}{2 + sT_{\mathrm{del}}}$$

The delay transfer matrix can be described as below.

$$G_{\mathrm{del}}(s) = \begin{bmatrix} \frac{2 - sT_{\mathrm{del}}}{2 + sT_{\mathrm{del}}} & 0 \\ 0 & \frac{2 - sT_{\mathrm{del}}}{2 + sT_{\mathrm{del}}} \end{bmatrix}$$

It should be noticed that, the following format is not very strict, since all the elements in a transfer matrix should be real transfer functions according to [3].

$$G_{\mathrm{del}}(s) = \begin{bmatrix} e^{-sT_{\mathrm{del}}} & 0 \\ 0 & e^{-sT_{\mathrm{del}}} \end{bmatrix}$$

Impedance Model

$$Z_{\text{inv}} = -\begin{bmatrix}\Delta v_{\text{C}}^{\text{d,s}} \\ \Delta v_{\text{C}}^{\text{q,s}}\end{bmatrix} \Big/ \begin{bmatrix}\Delta i_{\text{L1}}^{\text{d,s}} \\ \Delta i_{\text{L1}}^{\text{q,s}}\end{bmatrix} = \begin{bmatrix}Z_{\text{inv}}^{\text{dd}} & Z_{\text{inv}}^{\text{dq}} \\ Z_{\text{inv}}^{\text{qd}} & Z_{\text{inv}}^{\text{qq}}\end{bmatrix}$$

$$Y_{\text{inv}} = Z_{\text{inv}}^{-1}$$

$$Z_{\text{pcc}} = -\begin{bmatrix}\Delta v_{\text{pcc}}^{\text{d,s}} \\ \Delta v_{\text{pcc}}^{\text{q,s}}\end{bmatrix} \Big/ \begin{bmatrix}\Delta i_{\text{L2}}^{\text{d,s}} \\ \Delta i_{\text{L2}}^{\text{q,s}}\end{bmatrix} = \left(Y_{\text{inv}} + \left(Z_{\text{Rd}} + Y_{\text{C}}{}^{-1}\right)^{-1}\right)^{-1} + Z_{\text{L2}}$$

Control	Without DVC	With DVC
PI	$Y_{\text{inv}} = \dfrac{I - G_{\text{C}}^{\text{PI}}}{G_{\text{del}}(G_{\text{ACC}}^{\text{PI}} + G_{\text{dec}}) + Z_{\text{L1}}}$	$Y_{\text{inv}} = \dfrac{(V_{\text{dc}}I - G_{\text{M}}G_{\text{DVC}}^{\text{m}})G_{\text{C}}^{\text{PI}} - 2G_{\text{A}}^{\text{PI}}}{(V_{\text{dc}}I - G_{\text{M}}G_{\text{DVC}}^{\text{m}})G_{\text{B}}^{\text{PI}} - G_{\text{M}}G_{\text{DVC}}^{\text{i}}G_{\text{A}}^{\text{PI}} - 2G_{\text{A}}^{\text{PI}}Z_{\text{L1}}}$
PR	$Y_{\text{inv}} = \dfrac{I - G_{\text{C}}^{\text{PR}}}{G_{\text{del}}G_{\text{ACC}}^{\text{PR}} + Z_{\text{L1}}}$	$Y_{\text{inv}} = \dfrac{(V_{\text{dc}}I - G_{\text{M}}G_{\text{DVC}}^{\text{m}})G_{\text{C}}^{\text{PR}} - 2G_{\text{D}}^{\text{PR}}}{(2G_{\text{B}}^{\text{PR}} + G_{\text{M}}G_{\text{DVC}}^{\text{i}})G_{\text{A}}^{\text{PR}} + 2G_{\text{B}}^{\text{PR}}G_{\text{D}}^{\text{PR}} - 2G_{\text{D}}^{\text{PR}}Z_{\text{L1}}}$

Where,

$$G_{\text{M}} = \begin{bmatrix}M^{\text{d}} \\ M^{\text{q}}\end{bmatrix}$$

$$I = \begin{bmatrix}1 & 0 \\ 0 & 1\end{bmatrix}$$

$$G_{\text{A}}^{\text{PI}} = \frac{V_{\text{dc}}}{2}I - \frac{1}{2}G_{\text{M}}G_{\text{DVC}}^{\text{m}} - G_{\text{del}}G_{\text{ACC}}^{\text{PI}}G_{\text{DVC}}G_{\text{DVC}}^{\text{m}}$$

$$G_{\text{B}}^{\text{PI}} = G_{\text{del}}G_{\text{ACC}}^{\text{PI}}G_{\text{DVC}}G_{\text{DVC}}^{\text{i}} - G_{\text{del}}\left(G_{\text{ACC}}^{\text{PI}} + G_{\text{dec}}\right) + \frac{1}{2}G_{\text{M}}G_{\text{DVC}}^{\text{i}}$$

$$G_{\text{C}}^{\text{PI}} = \frac{V_{\text{dc}}}{2}G_{\text{PLL}}^{\text{m}} + G_{\text{del}}G_{\text{VFF}}G_{\text{PLL}}^{\text{v}} - G_{\text{del}}\left(G_{\text{ACC}}^{\text{PI}} + G_{\text{dec}}\right)G_{\text{PLL}}^{\text{i}} - G_{\text{del}}G_{\text{AD}}(G_{\text{PLL}}^{\text{ic}} + Y_{\text{C}})$$

$$G_{\text{A}}^{\text{PR}} = G_{\text{del}}G_{\text{ACC}}^{\text{PR}}G_{\text{DVC}}G_{\text{DVC}}^{\text{m}}$$

$$G_{\text{B}}^{\text{PR}} = G_{\text{del}}G_{\text{ACC}}^{\text{PR}}\left(G_{\text{DVC}}G_{\text{DVC}}^{\text{i}} - I\right)$$

$$G_{\text{C}}^{\text{PR}} = G_{\text{del}}\left(G_{\text{ACC}}^{\text{PR}}G_{\text{PLL}}^{\text{iref}} + I - G_{\text{AD}}Y_{\text{C}}\right)$$

$$G_{\text{D}}^{\text{PR}} = \frac{V_{\text{dc}}}{2}I - \frac{1}{2}G_{\text{M}}G_{\text{DVC}}^{\text{m}} - G_{\text{A}}^{\text{PR}}$$

Reference

The model development can refer [4, 5, 6, 7, 2]

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