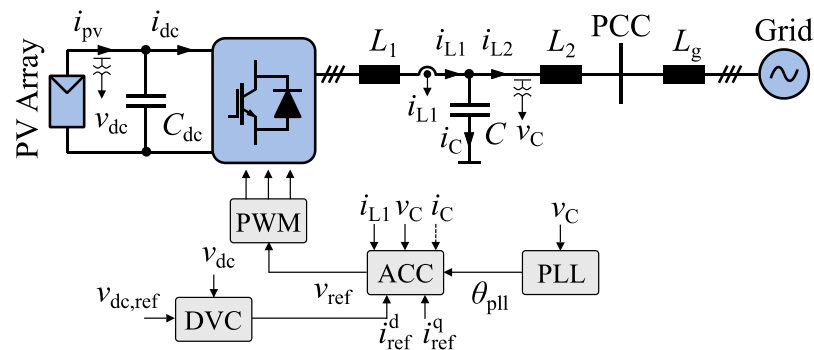


# Small-Signal Impedance Modeling of Grid-Following Inverter



Established by

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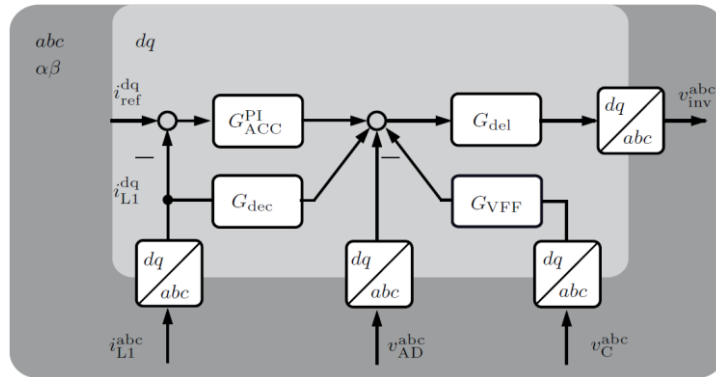
Aachen, on July 2021

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## Alternating-Current Control (ACC)

### Proportional-integral (PI) control



Control transfer function

$$F_{ACC}^{PI,dq}(s) = K_p^{ACC} + \frac{K_i^{ACC}}{s}$$

Control transfer matrix

$$G_{ACC}^{PI}(s) = \begin{bmatrix} F_{ACC}^{PI,dq}(s) & 0 \\ 0 & F_{ACC}^{PI,dq}(s) \end{bmatrix}$$

Decoupling transfer matrix

$$G_{dec}(s) = \begin{bmatrix} 0 & \omega_g L \\ -\omega_g L & 0 \end{bmatrix}$$

Voltage feedforward filter (VFF) transfer matrix

$$G_{VFF}(s) = \begin{bmatrix} \frac{\omega_{VFF}}{s + \omega_{VFF}} & 0 \\ 0 & \frac{\omega_{VFF}}{s + \omega_{VFF}} \end{bmatrix}$$

Active damping (AD) transfer matrix

$$G_{AD}(s) = \begin{bmatrix} K_{AD} & 0 \\ 0 & K_{AD} \end{bmatrix}$$

Delay matrix in Padé approximation form

$$G_{del}(s) = \begin{bmatrix} \frac{2 - sT_{del}}{2 + sT_{del}} & 0 \\ 0 & \frac{2 - sT_{del}}{2 + sT_{del}} \end{bmatrix}$$

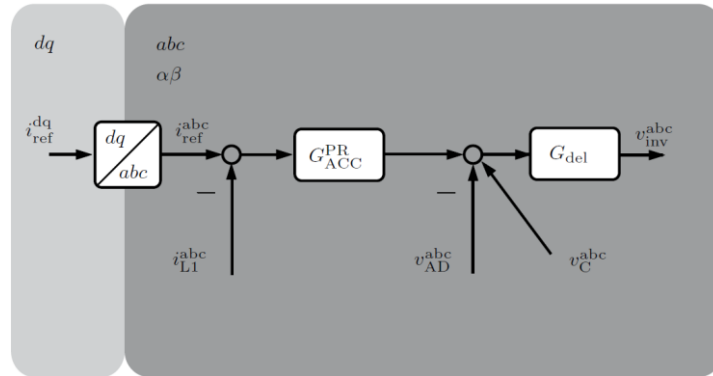
Frequency-domain relation

$$\begin{aligned} \begin{bmatrix} m^{d,c} \\ m^{q,c} \end{bmatrix} \frac{v_{dc}}{2} &= G_{del}(s) G_{ACC}^{PI}(s) \begin{bmatrix} i_{ref}^d \\ i_{ref}^q \end{bmatrix} - G_{del}(s) \left( G_{ACC}^{PI}(s) + G_{dec}(s) \right) \begin{bmatrix} i_{L1}^{d,c} \\ i_{L1}^{q,c} \end{bmatrix} \\ &+ G_{del}(s) G_{VFF}(s) \begin{bmatrix} v_C^{d,c} \\ v_C^{q,c} \end{bmatrix} - G_{del}(s) G_{AD}(s) \begin{bmatrix} i_C^{d,c} \\ i_C^{q,c} \end{bmatrix} \end{aligned}$$

Linearization

$$\begin{aligned} \begin{bmatrix} \Delta m^{d,c} \\ \Delta m^{q,c} \end{bmatrix} \frac{V_{dc}}{2} + \begin{bmatrix} M^d \\ M^q \end{bmatrix} \frac{\Delta v_{dc}}{2} &= G_{del}(s) G_{ACC}^{PI}(s) \begin{bmatrix} \Delta i_{ref}^d \\ \Delta i_{ref}^q \end{bmatrix} - G_{del}(s) \left( G_{ACC}^{PI}(s) + G_{dec}(s) \right) \begin{bmatrix} \Delta i_{L1}^{d,c} \\ \Delta i_{L1}^{q,c} \end{bmatrix} \\ &+ G_{del}(s) G_{VFF}(s) \begin{bmatrix} \Delta v_C^{d,c} \\ \Delta v_C^{q,c} \end{bmatrix} - G_{del}(s) G_{AD}(s) \begin{bmatrix} \Delta i_C^{d,c} \\ \Delta i_C^{q,c} \end{bmatrix} \end{aligned}$$

## Proportional-resonant (PR) control



Control transfer function

$$F_{ACC}^{PR,\alpha\beta}(s) = K_p^{ACC} + \frac{K_r^{ACC}s}{s^2 + \omega_g^2} = K_p^{ACC} + \frac{K_r^{ACC}}{2} \left( \frac{1}{s - j\omega_g} + \frac{1}{s + j\omega_g} \right)$$

$$\Downarrow s^{\alpha\beta} \rightarrow s^{dq} + j\omega_g$$

$$F_{ACC}^{PR,dq}(s) = F_{ACC}^{PR,\alpha\beta}(s + j\omega_g) = K_p^{ACC} + \frac{K_r^{ACC}}{2} \left( \frac{1}{s} + \frac{1}{s + j2\omega_g} \right)$$

$$= K_p^{ACC} + \frac{K_r^{ACC}}{2s} + \frac{K_r^{ACC}s}{2(s + j2\omega_g)^2} - j \frac{K_r^{ACC}\omega_g}{2(s + j2\omega_g)^2}$$

Control transfer matrix

$$G_{ACC}^{PR}(s) = \begin{bmatrix} \text{Re}\{F_{ACC}^{PR,dq}(s)\} & -\text{Im}\{F_{ACC}^{PR,dq}(s)\} \\ \text{Im}\{F_{ACC}^{PR,dq}(s)\} & \text{Re}\{F_{ACC}^{PR,dq}(s)\} \end{bmatrix}$$

Active damping (AD) transfer matrix

$$G_{AD}(s) = \begin{bmatrix} K_{AD} & 0 \\ 0 & K_{AD} \end{bmatrix}$$

Delay matrix in Padé approximation form

$$G_{del}(s) = \begin{bmatrix} \frac{2 - sT_{del}}{2 + sT_{del}} & 0 \\ 0 & \frac{2 - sT_{del}}{2 + sT_{del}} \end{bmatrix}$$

Frequency-domain relation

$$\begin{bmatrix} m^{d,c} \\ m^{q,c} \end{bmatrix} \frac{v_{dc}}{2} = G_{del}(s) G_{ACC}^{PR}(s) \begin{bmatrix} i_{ref}^d \\ i_{ref}^q \end{bmatrix} - G_{del}(s) G_{ACC}^{PR}(s) \begin{bmatrix} i_{L1}^{d,c} \\ i_{L1}^{q,c} \end{bmatrix} - G_{del}(s) G_{AD}(s) \begin{bmatrix} i_C^{d,c} \\ i_C^{q,c} \end{bmatrix}$$

Linearization

$$\begin{aligned} \begin{bmatrix} \Delta m^{d,c} \\ \Delta m^{q,c} \end{bmatrix} \frac{V_{dc}}{2} + \begin{bmatrix} M^d \\ M^q \end{bmatrix} \frac{\Delta v_{dc}}{2} \\ = G_{del}(s) G_{ACC}^{PR}(s) \begin{bmatrix} \Delta i_{ref}^d \\ \Delta i_{ref}^q \end{bmatrix} - G_{del}(s) G_{ACC}^{PR}(s) \begin{bmatrix} \Delta i_{L1}^{d,c} \\ \Delta i_{L1}^{q,c} \end{bmatrix} - G_{del}(s) G_{AD}(s) \begin{bmatrix} \Delta i_C^{d,c} \\ \Delta i_C^{q,c} \end{bmatrix} \end{aligned}$$

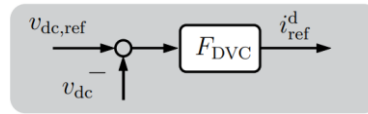
## LCL Filter

$$Z_{L1}(s) = \begin{bmatrix} sL_1 + R_1 & -\omega_g L_1 \\ \omega_g L_1 & sL_1 + R_1 \end{bmatrix}$$

$$Y_C(s) = \begin{bmatrix} sC & -\omega_g C \\ \omega_g C & sC \end{bmatrix}$$

$$Z_{L2}(s) = \begin{bmatrix} sL_2 + R_2 & -\omega_g L_2 \\ \omega_g L_2 & sL_2 + R_2 \end{bmatrix}$$

## Direct-Voltage Control (DVC)



Control transfer function

$$F_{\text{DVC}}(s) = K_p^{\text{DVC}} + \frac{K_i^{\text{DVC}}}{s}$$

Frequency-domain relation

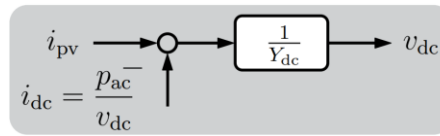
$$\begin{bmatrix} i_{\text{ref}}^d \\ i_{\text{ref}}^q \end{bmatrix} = \begin{bmatrix} F_{\text{DVC}}(s) \\ 0 \end{bmatrix} (v_{\text{dc,ref}} - v_{\text{dc}})$$

Linearization

$$\begin{bmatrix} \Delta i_{\text{ref}}^d \\ \Delta i_{\text{ref}}^q \end{bmatrix} = - \underbrace{\begin{bmatrix} F_{\text{DVC}}(s) \\ 0 \end{bmatrix}}_{G_{\text{DVC}}(s)} \Delta v_{\text{dc}}$$



## DC-Link Capacitor



Plant dynamics

$$Y_{dc}(s)v_{dc} = i_{pv} - i_{dc}$$

$$Y_{dc}(s) = sC_{dc}$$

Power balance

$$i_{dc}v_{dc} = p_{dc} = p_{ac} = \frac{3}{2}(i_{L1}^{d,s}m^{d,s} + i_{L1}^{q,s}m^{q,s})\frac{v_{dc}}{2}$$

↓

$$i_{dc} = \frac{3}{4}(i_{L1}^{d,s}m^{d,s} + i_{L1}^{q,s}m^{q,s})$$

Linearization

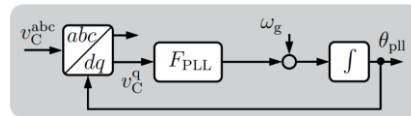
$$\Delta i_{dc} = \frac{3}{4}[M^d \quad M^q] \begin{bmatrix} \Delta i_{L1}^{d,s} \\ \Delta i_{L1}^{q,s} \end{bmatrix} + \frac{3}{4}[I_{L1}^d \quad I_{L1}^q] \begin{bmatrix} \Delta m^{d,s} \\ \Delta m^{q,s} \end{bmatrix}$$

$$\Delta i_{dc} = -Y_{dc}(s)\Delta v_{dc}$$

↓

$$\Delta v_{dc} = - \left( \underbrace{\frac{3}{4Y_{dc}(s)}[M^d \quad M^q]}_{G_{DVC}^i(s)} \begin{bmatrix} \Delta i_{L1}^{d,s} \\ \Delta i_{L1}^{q,s} \end{bmatrix} + \underbrace{\frac{3}{4Y_{dc}(s)}[I_{L1}^d \quad I_{L1}^q]}_{G_{DVC}^m(s)} \begin{bmatrix} \Delta m^{d,s} \\ \Delta m^{q,s} \end{bmatrix} \right)$$

## Phase-Locked Loop (PLL)



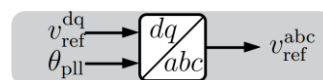
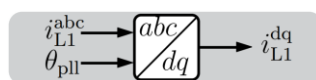
Control transfer function

$$F_{\text{PLL}}(s) = K_{\text{p}}^{\text{PLL}} + \frac{K_{\text{i}}^{\text{PLL}}}{s}$$

Linearization according to [1]

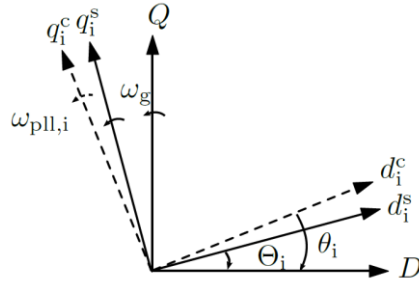
$$H_{\text{PLL}}(s) = \frac{\Delta\theta}{\Delta v_{\text{C}}^{\text{q}}} = \frac{F_{\text{PLL}}(s)}{s + V_{\text{C}}^{\text{d}} F_{\text{PLL}}(s)}$$

The small-signal transfer function  $H_{\text{PLL}}(s)$  influences the variables through Park and inverse Park transformation, e.g.



Detailed mathematical impacts are analyzed in the section reference frame

## Reference Frame



### System frame → control frame

Due to the PLL dynamics, variables with Park transformation are converted from the system to control frame.

$$\begin{bmatrix} \Delta x^{d,c} \\ \Delta x^{q,c} \end{bmatrix} = \begin{bmatrix} \Delta x^{d,s} \\ \Delta x^{q,s} \end{bmatrix} + \begin{bmatrix} 0 & X^q H_{PLL}(s) \\ 0 & -X^d H_{PLL}(s) \end{bmatrix} \begin{bmatrix} \Delta v_C^{d,s} \\ \Delta v_C^{q,s} \end{bmatrix}$$

Inverter-side inductor current  $\Delta i_{L1}^{dq,s} \rightarrow \Delta i_{L1}^{dq,c}$

$$\begin{bmatrix} \Delta i_{L1}^{d,c} \\ \Delta i_{L1}^{q,c} \end{bmatrix} = \begin{bmatrix} \Delta i_{L1}^{d,s} \\ \Delta i_{L1}^{q,s} \end{bmatrix} + \underbrace{\begin{bmatrix} 0 & I_{L1}^q H_{PLL}(s) \\ 0 & -I_{L1}^d H_{PLL}(s) \end{bmatrix}}_{G_{PLL}^i(s)} \begin{bmatrix} \Delta v_C^{d,s} \\ \Delta v_C^{q,s} \end{bmatrix}$$

Grid-side capacitor current  $\Delta i_C^{dq,s} \rightarrow \Delta i_C^{dq,c}$

$$\begin{bmatrix} \Delta i_C^{d,c} \\ \Delta i_C^{q,c} \end{bmatrix} = \begin{bmatrix} \Delta i_C^{d,s} \\ \Delta i_C^{q,s} \end{bmatrix} + \underbrace{\begin{bmatrix} 0 & I_C^q H_{PLL}(s) \\ 0 & -I_C^d H_{PLL}(s) \end{bmatrix}}_{G_{PLL}^{ic}(s)} \begin{bmatrix} \Delta v_C^{d,s} \\ \Delta v_C^{q,s} \end{bmatrix}$$

Grid-side capacitor voltage  $\Delta v_C^{dq,s} \rightarrow \Delta v_C^{dq,c}$

$$\begin{bmatrix} \Delta v_C^{d,c} \\ \Delta v_C^{q,c} \end{bmatrix} = \begin{bmatrix} \Delta v_C^{d,s} \\ \Delta v_C^{q,s} \end{bmatrix} + \underbrace{\begin{bmatrix} 0 & V_C^q H_{PLL}(s) \\ 0 & -V_C^d H_{PLL}(s) \end{bmatrix}}_{G_{PLL}^v(s)} \begin{bmatrix} \Delta v_C^{d,s} \\ \Delta v_C^{q,s} \end{bmatrix}$$

### Control frame → system frame

Due to the PLL dynamics, variables with inverse Park transformation are converted from the control to system frame.

$$\begin{bmatrix} \Delta x^{d,s} \\ \Delta x^{q,s} \end{bmatrix} = \begin{bmatrix} \Delta x^{d,c} \\ \Delta x^{q,c} \end{bmatrix} + \begin{bmatrix} 0 & -X^q H_{PLL}(s) \\ 0 & X^d H_{PLL}(s) \end{bmatrix} \begin{bmatrix} \Delta v_C^{d,s} \\ \Delta v_C^{q,s} \end{bmatrix}$$

Modulation index  $\Delta m^{\text{dq,c}} \rightarrow \Delta m^{\text{dq,s}}$

$$\begin{bmatrix} \Delta m^{\text{d,s}} \\ \Delta m^{\text{q,s}} \end{bmatrix} = \begin{bmatrix} \Delta m^{\text{d,c}} \\ \Delta m^{\text{q,c}} \end{bmatrix} + \underbrace{\begin{bmatrix} 0 & -M^{\text{q}}H_{\text{PLL}}(s) \\ 0 & M^{\text{d}}H_{\text{PLL}}(s) \end{bmatrix}}_{G_{\text{PLL}}^{\text{m}}(s)} \begin{bmatrix} \Delta v_{\text{C}}^{\text{d,s}} \\ \Delta v_{\text{C}}^{\text{q,s}} \end{bmatrix}$$

If PR control is implemented, reference current obtained in the  $dq$  frame should be converted to the  $\alpha\beta$  frame, which also requires inverse Park transformation, i.e.  $\Delta i_{\text{ref}}^{\text{dq}} \rightarrow \Delta i_{\text{ref}}^{\alpha\beta}$

$$\begin{bmatrix} \Delta i_{\text{ref}}^{\alpha} \\ \Delta i_{\text{ref}}^{\beta} \end{bmatrix} = \begin{bmatrix} \Delta i_{\text{ref}}^{\text{d}} \\ \Delta i_{\text{ref}}^{\text{q}} \end{bmatrix} + \underbrace{\begin{bmatrix} 0 & -I_{\text{ref}}^{\text{q}}H_{\text{PLL}}(s) \\ 0 & I_{\text{ref}}^{\text{d}}H_{\text{PLL}}(s) \end{bmatrix}}_{G_{\text{PLL}}^{\text{iref}}(s)} \begin{bmatrix} \Delta v_{\text{C}}^{\text{d,s}} \\ \Delta v_{\text{C}}^{\text{q,s}} \end{bmatrix}$$

## System → global frame

To facilitate the integration of multi-inverter system, a common global frame  $DQ$  is defined.

Clockwise rotation

$$T_{\text{C}} = \begin{bmatrix} \cos(\Theta) & -\sin(\Theta) \\ \sin(\Theta) & \cos(\Theta) \end{bmatrix}$$

Anti-clockwise rotation

$$T_{\text{A}} = \begin{bmatrix} \cos(\Theta) & \sin(\Theta) \\ -\sin(\Theta) & \cos(\Theta) \end{bmatrix}$$

Each inverter has its own rotation frame. They can be aggregated only after being converted into the same global frame.

Consider  $i^{\text{th}}$  inverter

$$I_{\text{i}}^{\text{DQ}} = T_{\text{C}} \cdot I_{\text{i}}^{\text{dq,s}}$$

$$V_{\text{i}}^{\text{DQ}} = Z_{\text{i}}^{\text{DQ}} \cdot I_{\text{i}}^{\text{DQ}}$$

$$V_{\text{i}}^{\text{dq,s}} = Z_{\text{i}}^{\text{dq,s}} \cdot I_{\text{i}}^{\text{dq,s}}$$

Then

$$V_{\text{i}}^{\text{DQ}} = T_{\text{C}} \cdot V_{\text{i}}^{\text{dq,s}}$$

$$Z_{\text{i}}^{\text{DQ}} \cdot I_{\text{i}}^{\text{DQ}} = T_{\text{C}} \cdot Z_{\text{i}}^{\text{dq,s}} \cdot I_{\text{i}}^{\text{dq,s}}$$

$$Z_{\text{i}}^{\text{DQ}} \cdot I_{\text{i}}^{\text{DQ}} = T_{\text{C}} \cdot Z_{\text{i}}^{\text{dq,s}} \cdot T_{\text{A}} \cdot I_{\text{i}}^{\text{DQ}}$$

⇓

$$Z_{\text{i}}^{\text{DQ}} = T_{\text{C}} \cdot Z_{\text{i}}^{\text{dq,s}} \cdot T_{\text{A}}$$

## Modulation Delay

The modulation delay in the complex frequency domain can be precisely modeled as:

$$\underline{v}_{\text{inv}}^{\text{dq}} = v_{\text{inv}}^{\text{d}} + jv_{\text{inv}}^{\text{q}} = \underline{v}_{\text{ref}}^{\text{dq}} \cdot e^{-sT_{\text{del}}}$$

If the delay is expanded in Euler's form,

$$e^{-sT_{\text{del}}} = \cos \omega_g T_{\text{del}} - j \sin \omega_g T_{\text{del}}$$

The delay transfer matrix can be described as below.

$$G_{\text{del}}(s) = \begin{bmatrix} \cos \omega_g T_{\text{del}} & \sin \omega_g T_{\text{del}} \\ -\sin \omega_g T_{\text{del}} & \cos \omega_g T_{\text{del}} \end{bmatrix}$$

If the delay is expanded in Padé approximation,

$$e^{-sT_{\text{del}}} \approx \frac{2 - sT_{\text{del}}}{2 + sT_{\text{del}}}$$

The delay transfer matrix can be described as below.

$$G_{\text{del}}(s) = \begin{bmatrix} \frac{2 - sT_{\text{del}}}{2 + sT_{\text{del}}} & 0 \\ 0 & \frac{2 - sT_{\text{del}}}{2 + sT_{\text{del}}} \end{bmatrix}$$

It should be noticed that, the following format is not very strict, since all the elements in a transfer matrix should be real transfer functions according to [2].

$$G_{\text{del}}(s) = \begin{bmatrix} e^{-sT_{\text{del}}} & 0 \\ 0 & e^{-sT_{\text{del}}} \end{bmatrix}$$

## Impedance Model

$$Z_{\text{inv}} = - \begin{bmatrix} \Delta v_C^{d,s} \\ \Delta v_C^{q,s} \end{bmatrix} / \begin{bmatrix} \Delta i_{L1}^{d,s} \\ \Delta i_{L1}^{q,s} \end{bmatrix} = \begin{bmatrix} Z_{\text{inv}}^{dd} & Z_{\text{inv}}^{dq} \\ Z_{\text{inv}}^{qd} & Z_{\text{inv}}^{qq} \end{bmatrix}$$

$$Y_{\text{inv}} = Z_{\text{inv}}^{-1}$$

$$Z_{\text{pcc}} = - \begin{bmatrix} \Delta v_{\text{pcc}}^{d,s} \\ \Delta v_{\text{pcc}}^{q,s} \end{bmatrix} / \begin{bmatrix} \Delta i_{L2}^{d,s} \\ \Delta i_{L2}^{q,s} \end{bmatrix} = \left( Y_{\text{inv}} + (Z_{\text{Rd}} + Y_C^{-1})^{-1} \right)^{-1} + Z_{L2}$$

Control	Without DVC	With DVC
PI	$Y_{\text{inv}} = \frac{I - G_C^{\text{PI}}}{G_{\text{del}}(G_{\text{ACC}}^{\text{PI}} + G_{\text{dec}}) + Z_{L1}}$	$Y_{\text{inv}} = \frac{(V_{\text{dc}}I - G_M G_{\text{DVC}}^{\text{m}})G_C^{\text{PI}} - 2G_A^{\text{PI}}}{(V_{\text{dc}}I - G_M G_{\text{DVC}}^{\text{m}})G_B^{\text{PI}} - G_M G_{\text{DVC}}^{\text{i}} G_A^{\text{PI}} - 2G_A^{\text{PI}} Z_{L1}}$
PR	$Y_{\text{inv}} = \frac{I - G_C^{\text{PR}}}{G_{\text{del}}G_{\text{ACC}}^{\text{PR}} + Z_{L1}}$	$Y_{\text{inv}} = \frac{(V_{\text{dc}}I - G_M G_{\text{DVC}}^{\text{m}})G_C^{\text{PR}} - 2G_D^{\text{PR}}}{(2G_B^{\text{PR}} + G_M G_{\text{DVC}}^{\text{i}})G_A^{\text{PR}} + 2G_B^{\text{PR}} G_D^{\text{PR}} - 2G_D^{\text{PR}} Z_{L1}}$

Where,

$$G_M = \begin{bmatrix} M^d \\ M^q \end{bmatrix}$$

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$G_A^{\text{PI}} = \frac{V_{\text{dc}}}{2}I - \frac{1}{2}G_M G_{\text{DVC}}^{\text{m}} - G_{\text{del}}G_{\text{ACC}}^{\text{PI}}G_{\text{DVC}}^{\text{m}}$$

$$G_B^{\text{PI}} = G_{\text{del}}G_{\text{ACC}}^{\text{PI}}G_{\text{DVC}}^{\text{i}} - G_{\text{del}}(G_{\text{ACC}}^{\text{PI}} + G_{\text{dec}}) + \frac{1}{2}G_M G_{\text{DVC}}^{\text{i}}$$

$$G_C^{\text{PI}} = \frac{V_{\text{dc}}}{2}G_{\text{PLL}}^{\text{m}} + G_{\text{del}}G_{\text{VFF}}G_{\text{PLL}}^{\text{v}} - G_{\text{del}}(G_{\text{ACC}}^{\text{PI}} + G_{\text{dec}})G_{\text{PLL}}^{\text{i}} - G_{\text{del}}G_{\text{AD}}(G_{\text{PLL}}^{\text{ic}} + Y_C)$$

$$G_A^{\text{PR}} = G_{\text{del}}G_{\text{ACC}}^{\text{PR}}G_{\text{DVC}}^{\text{m}}$$

$$G_B^{\text{PR}} = G_{\text{del}}G_{\text{ACC}}^{\text{PR}}(G_{\text{DVC}}^{\text{i}} - I)$$

$$G_C^{\text{PR}} = G_{\text{del}}(G_{\text{ACC}}^{\text{PR}}G_{\text{PLL}}^{\text{iref}} + I - G_{\text{AD}}Y_C)$$

$$G_D^{\text{PR}} = \frac{V_{\text{dc}}}{2}I - \frac{1}{2}G_M G_{\text{DVC}}^{\text{m}} - G_A^{\text{PR}}$$

## Reference

The model development can refer [3, 4, 5, 6, 7]

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