Small-Signal State-Space Modeling of Grid-Following Inverter

\\Oled\lea\Forschung\ogP\8316_BMWi_PV_Kraftwerk2025\Intern\Daten\zya\01_Data\01_Figure\01_Circuit\01_Three-Phase Grid-Tied Inverter\System_Single_Inv_LCL_Lg_ACC_PLL_DVC.emf

Established by

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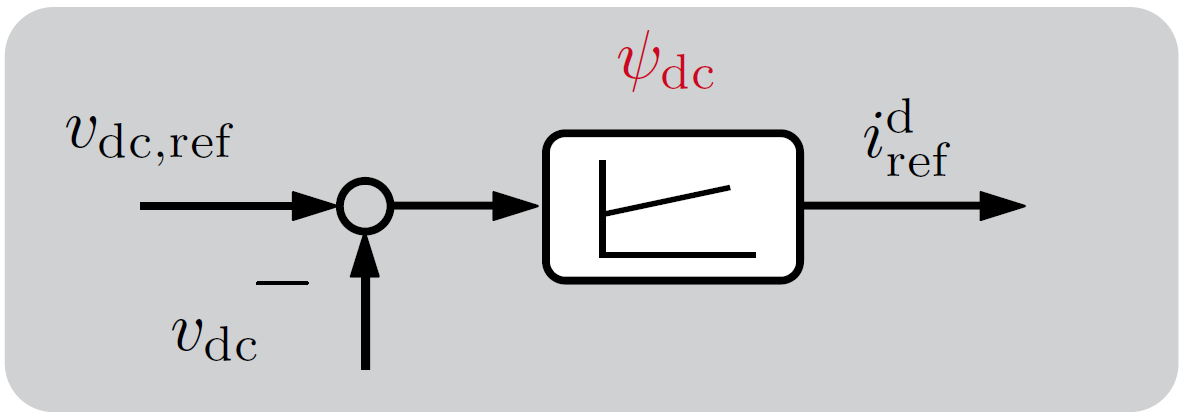
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# Direct-Voltage Control (DVC)



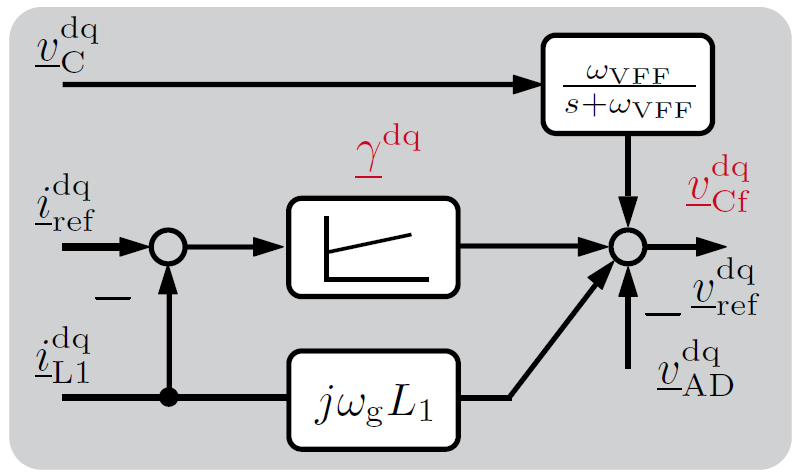
State equation

Output equation

Linearization

## State-space model

# Alternating-Current Control (ACC)



State equation

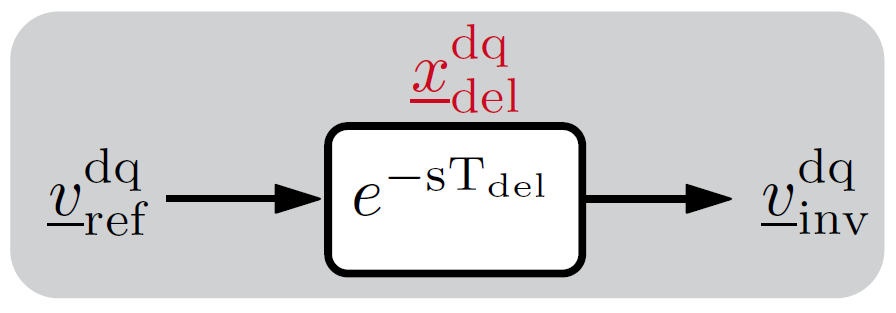
Output equation

Linearization

## State-space model

# Modulation Delay

Refer to Padé approximation



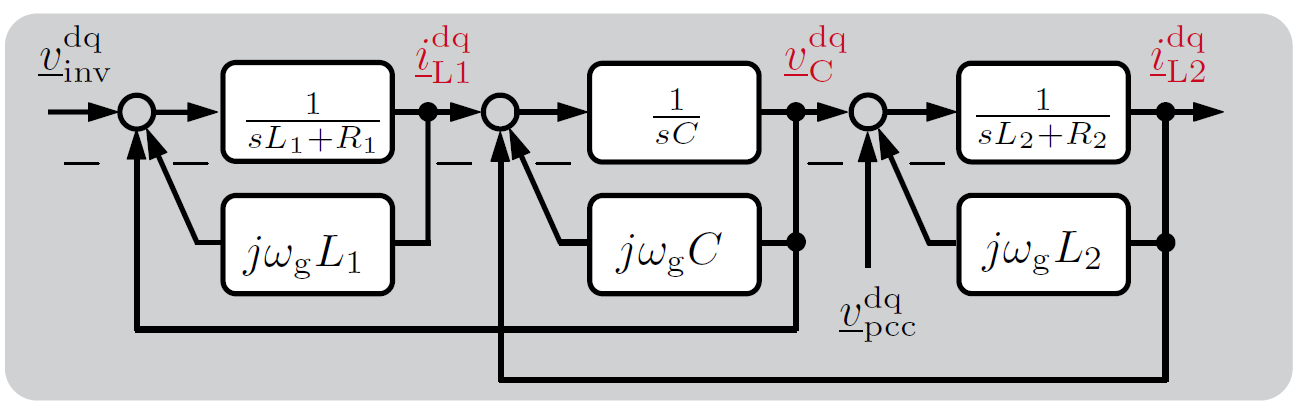
State equation

Output equation

Linearization

## State-space model

# LCL Filter

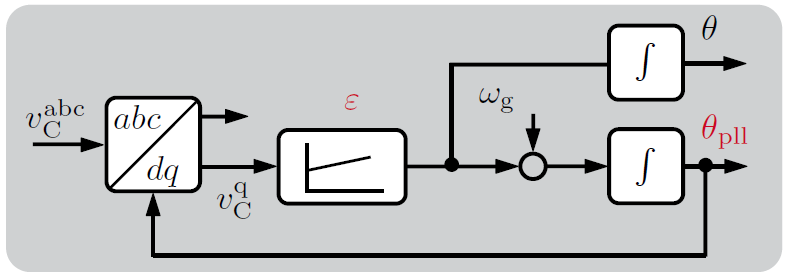


State equation

Linearization

## State-space model

# Phase-Locked Loop (PLL)



State equation

Linearization

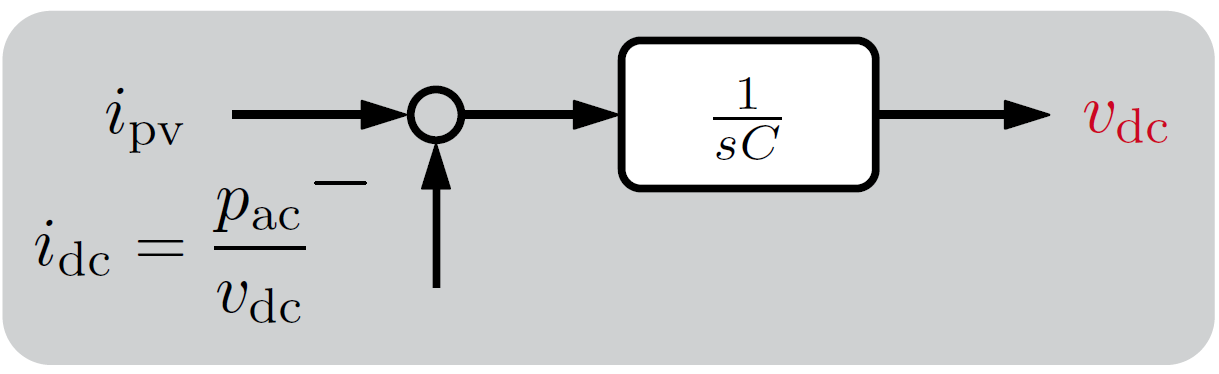
If directly convert from the control to system frame

Refer to power angle relationship

## State-space model

If consider direct conversion from the control to system frame:

# DC-Link Capacitor

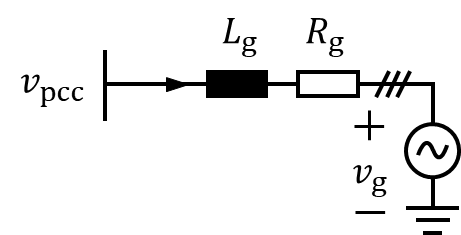


State equation

Linearization

## State-space model

# Grid



State equation

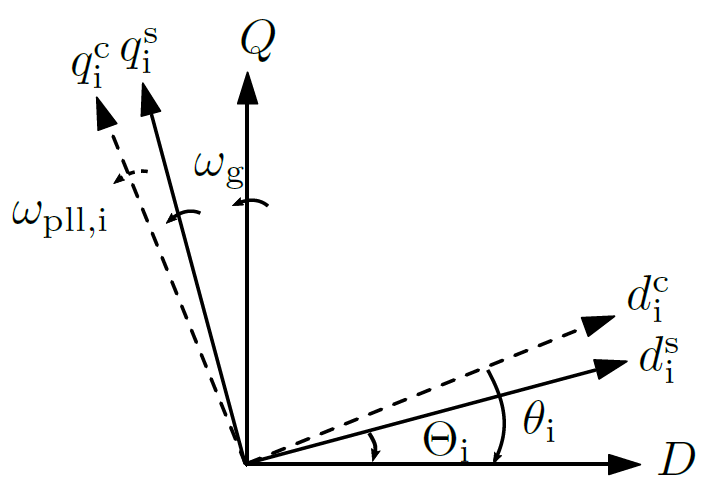
Output equation

(virtual resistor to estimate the voltage at the PCC)

Linearization

## State-space model

# Reference Frame



## Global frame 🡪 system frame

To facilitate the integration of multi-inverter system, a common global frame is defined.

Clockwise rotation

Anti-clockwise rotation

Grid-side inductor current of inverter

PCC voltage of the grid

## System frame 🡪 control frame

Due to the PLL dynamics, variables with Park transformation are converted from the system to control frame.

Inverter-side inductor current

Grid-side capacitor current

Grid-side inductor current

## Control frame 🡪 system frame

Due to the PLL dynamics, variables with inverse Park transformation are converted from the control to system frame.

Modulation effect

## Derivation of small-signal relationship

This part provides the derivation of the small-signal relationship between the control and system frame. First consider a variable that converts from the control to the global frame:

The small-signal relationship is derived as:

Which leads to,

The relationship between and

Thus,

# Component Connection Method (CCM)

## State-space model: inverter

State equation

: State variable of all subsystems / the entire system

: Input vector of all subsystems

: Output vector of all subsystems

: Input vector of the entire system

: Output vector of the entire system

Interconnection equation

State-space equation

Where,

## State-space model: inverter + grid

State equation

: State variable of all subsystems / the entire system

: Input vector of all subsystems

: Output vector of all subsystems

: Input vector of the entire system

: Output vector of the entire system

Interconnection equation

State-space equation

Where,

# Padé Approximation

The time delay of the digital control system can be modeled as:

Padé approximation is a mathematical tool for analyzing nonlinear plant by using the Taylor series:

Where,

Transfer function for time-delay:

Create an intermediate variable :

In time domain,

Assume state variables

Then,

The plant is converted as:

## State-space model

First-order Padé approximation

# Reference

The model development can refer [1, 2, 3]. The component connection method is introduced in [4].

|  |  |
| --- | --- |
| [1] | Z. Yang, Q. Wang, J. Warmuz and R. W. De Doncker, "Stability Assessment of a Three-Phase Grid-Tied PV Inverter with Eigenvalue-Based Method," in *2019 IEEE 10th International Symposium on Power Electronics for Distributed Generation Systems (PEDG)*, Xi'an, 2019. |
| [2] | Z. Yang, C. Bendfeld, J. Qiang, M. Benedict und R. W. De Doncker, „Stability Investigation of Large-Scale PV Parks with Eigenvalue-Based Method,“ in *2020 22nd European Conference on Power Electronics and Applications (EPE'20 ECCE Europe)*, Lyon, 2020. |
| [3] | Z. Yang, On the Stability of Three-Phase Grid-Tied Photovoltaic Inverter Systems, Bd. 90, Aachen: E.ON Energy Research Center, RWTH Aachen University, 2021. |
| [4] | G. Gaba, S. Lefebvre und D. Mukhedkar, „Comparative analysis and study of the dynamic stability of AC/DC systems,“ *IEEE Transactions on Power Systems,* Bd. 3, Nr. 3, pp. 978-985, Aug 1988. |