

Model selection

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1 Load Required Libraries

```
library(boot)
library(pROC)

## Type 'citation("pROC")' for a citation.

##
## Attaching package: 'pROC'

## The following objects are masked from 'package:stats':
##
##      cov, smooth, var

library(ROCR)
```

2 Load the data

```
data.credit = read.csv("Credit.csv")
# Transform categorical variables
data.credit$credit_risk = as.factor(data.credit$credit_risk)
data.credit$status     = as.factor(data.credit$status)
data.credit$savings    = as.factor(data.credit$savings)
data.credit$property   = as.ordered(data.credit$property)
data.credit$other_installment_plans = as.factor(data.credit$other_installment_plans)
```

3 Split the data into training set and testing set

```
set.seed(1006742107)

n = nrow(data.credit)
index = sample(n, round(0.75 * n), replace = FALSE)
traindata = data.credit[index, ]
testdata = data.credit[-index, ]
```

4 Main effect model

4.1 Training model

4.1.1 Forward method

```
step(glm(credit_risk ~ 1, family = binomial, data = traindata), scope =
  ~status + duration + savings + property + age +
  other_installment_plans, direction = "forward", test = "Chisq")
```

```
## Start:  AIC=921.66
## credit_risk ~ 1
##
##              Df Deviance    AIC    LRT Pr(>Chi)
## + status      3   828.43 836.43 91.225 < 2.2e-16 ***
## + duration    1   887.29 891.29 32.368 1.276e-08 ***
## + savings     4   892.66 902.66 27.003 1.985e-05 ***
## + property    3   903.76 911.76 15.899 0.001189 **
## + age         1   909.12 913.12 10.537 0.001170 **
## + other_installment_plans 2   909.77 915.77 9.889 0.007123 **
## <none>              919.66 921.66
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Step:  AIC=836.43
## credit_risk ~ status
##
##              Df Deviance    AIC    LRT Pr(>Chi)
## + duration    1   801.74 811.74 26.6915 2.387e-07 ***
## + property    3   811.90 825.90 16.5377 0.0008796 ***
## + savings     4   814.60 830.60 13.8362 0.0078365 **
## + other_installment_plans 2   820.32 832.32 8.1125 0.0173134 *
## + age         1   823.40 833.40 5.0337 0.0248586 *
## <none>              828.43 836.43
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Step:  AIC=811.74
## credit_risk ~ status + duration
##
##              Df Deviance    AIC    LRT Pr(>Chi)
## + savings     4   787.09 805.09 14.6529 0.005478 **
## + other_installment_plans 2   794.74 808.74 7.0039 0.030138 *
## + age         1   797.00 809.00 4.7411 0.029450 *
## + property    3   794.81 810.81 6.9303 0.074154 .
## <none>              801.74 811.74
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Step:  AIC=805.09
## credit_risk ~ status + duration + savings
##
```

```

##              Df Deviance    AIC    LRT Pr(>Chi)
## + other_installment_plans  2   779.60 801.60 7.4878 0.02366 *
## + age                      1   783.28 803.28 3.8072 0.05103 .
## + property                 3   780.56 804.56 6.5277 0.08858 .
## <none>                     3   787.09 805.09
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Step: AIC=801.6
## credit_risk ~ status + duration + savings + other_installment_plans
##
##              Df Deviance    AIC    LRT Pr(>Chi)
## + age          1   775.22 799.22 4.3864 0.03623 *
## <none>          3   779.60 801.60
## + property     3   774.53 802.53 5.0741 0.16645
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Step: AIC=799.22
## credit_risk ~ status + duration + savings + other_installment_plans +
##   age
##
##              Df Deviance    AIC    LRT Pr(>Chi)
## + property     3   767.65 797.65 7.5621 0.05598 .
## <none>          3   775.22 799.22
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Step: AIC=797.65
## credit_risk ~ status + duration + savings + other_installment_plans +
##   age + property
##
##
## Call: glm(formula = credit_risk ~ status + duration + savings + other_installment_plans +
##   age + property, family = binomial, data = traindata)
##
## Coefficients:
##              (Intercept)                status2                status3
##                -0.72146                  0.45381                  0.85533
##                status4                duration                savings2
##                 1.75141                -0.03046                  0.25614
##                savings3                savings4                savings5
##                 0.13951                  1.49757                  0.72922
## other_installment_plans2 other_installment_plans3                age
##                 0.19640                  0.59292                  0.02236
##                property.L                property.Q                property.C
##                -0.58463                  -0.16008                  -0.06641
##
## Degrees of Freedom: 749 Total (i.e. Null);  735 Residual
## Null Deviance:          919.7
## Residual Deviance: 767.7    AIC: 797.7

```

4.1.2 Backward method

```
step(glm(credit_risk ~status + duration + savings + property + age +
        other_installment_plans, family = binomial, data = traindata), test = "Chisq")

## Start:  AIC=797.65
## credit_risk ~ status + duration + savings + property + age +
##   other_installment_plans
##
##              Df Deviance    AIC    LRT Pr(>Chi)
## <none>              767.65 797.65
## - property          3   775.22 799.22  7.562  0.055984 .
## - other_installment_plans 2   774.23 800.23  6.577  0.037312 *
## - age               1   774.53 802.53  6.874  0.008744 **
## - savings           4   781.43 803.43 13.779  0.008036 **
## - duration          1   784.38 812.38 16.727 4.316e-05 ***
## - status            3   832.72 856.72 65.065 4.857e-14 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

##
## Call:  glm(formula = credit_risk ~ status + duration + savings + property +
##   age + other_installment_plans, family = binomial, data = traindata)
##
## Coefficients:
##              (Intercept)              status2              status3
##              -0.72146              0.45381              0.85533
##              status4              duration              savings2
##              1.75141              -0.03046              0.25614
##              savings3              savings4              savings5
##              0.13951              1.49757              0.72922
##              property.L              property.Q              property.C
##              -0.58463              -0.16008              -0.06641
##              age  other_installment_plans2  other_installment_plans3
##              0.02236              0.19640              0.59292
##
## Degrees of Freedom: 749 Total (i.e. Null);  735 Residual
## Null Deviance:          919.7
## Residual Deviance: 767.7    AIC: 797.7
```

From above coding, we could find that both forward selection and backward elimination choose the model:
`glm(credit_risk ~status + duration + savings + property + age + other_installment_plans, family = binomial, data = traindata)`

$$\text{logit}(\hat{\pi}) = -0.72 + 0.45 \cdot S_1 + 0.86 \cdot S_2 + 1.75 \cdot S_3 - 0.03 \cdot D + 0.26 \cdot SV_1 + 0.14 \cdot SV_2 + 1.50 \cdot SV_3 + 0.73 \cdot SV_4 - 0.58 \cdot P_L - 0.16 \cdot P_Q - 0.07 \cdot P_C + 0.$$

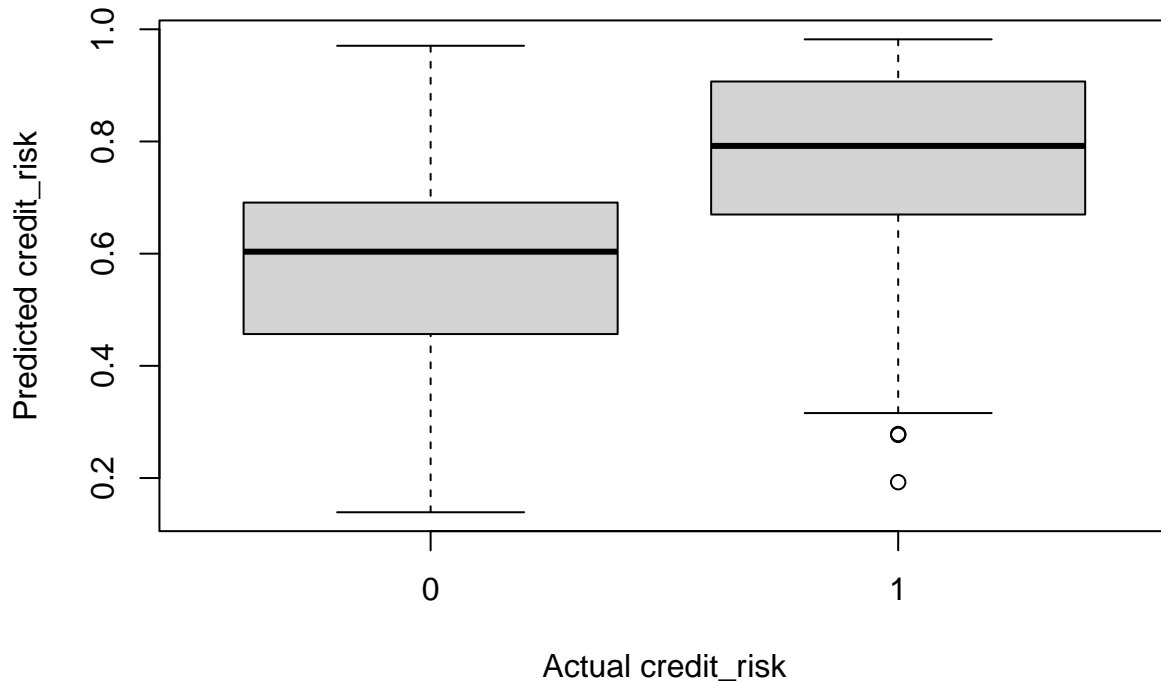
where * S_i 's are dummy variables for status * D is duration * SV 's are dummy variables for savings * P_i 's are dummy variables for property * A is age * O_i 's are dummy variables for other_installment_plans

```
bestmodel.1 = glm(credit_risk ~ status + duration + savings + property + age + other_installment_plans,
summary(bestmodel.1)
```

```
##
## Call:
## glm(formula = credit_risk ~ status + duration + savings + property +
##      age + other_installment_plans, family = binomial, data = traindata)
##
## Deviance Residuals:
##      Min       1Q   Median       3Q      Max
## -2.7517  -0.9595   0.4902   0.8265   1.8215
##
## Coefficients:
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept)    -0.721459   0.436812  -1.652  0.09861 .
## status2         0.453812   0.222048   2.044  0.04098 *
## status3         0.855325   0.366951   2.331  0.01976 *
## status4         1.751411   0.235658   7.432 1.07e-13 ***
## duration       -0.030462   0.007502  -4.061 4.89e-05 ***
## savings2        0.256136   0.302805   0.846  0.39762
## savings3        0.139505   0.392170   0.356  0.72205
## savings4        1.497575   0.657083   2.279  0.02266 *
## savings5        0.729215   0.264897   2.753  0.00591 **
## property.L      -0.584630   0.213990  -2.732  0.00629 **
## property.Q      -0.160077   0.189526  -0.845  0.39832
## property.C      -0.066411   0.173897  -0.382  0.70254
## age             0.022359   0.008708   2.568  0.01024 *
## other_installment_plans2 0.196405   0.440276   0.446  0.65553
## other_installment_plans3 0.592919   0.240369   2.467  0.01364 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##      Null deviance: 919.66  on 749  degrees of freedom
## Residual deviance: 767.65  on 735  degrees of freedom
## AIC: 797.65
##
## Number of Fisher Scoring iterations: 5
```

4.2 Testing model

```
pred.1 = predict(bestmodel.1, newdata = testdata)
plot(testdata$credit_risk, inv.logit(pred.1), xlab = "Actual credit_risk", ylab = "Predicted credit_risk")
```



From the plot, we find that the main effect model can describe the actual data fairly well.

5 Interaction model

5.1 Training model

5.1.1 Forward method

```
bestmodel.3 <- step(glm(credit_risk ~ 1, family = binomial, data = traindata), scope = ~status * duration)
```

```
## Start: AIC=921.66
## credit_risk ~ 1
##
##               Df Deviance    AIC    LRT Pr(>Chi)
## + status       3   828.43 836.43 91.225 < 2.2e-16 ***
## + duration     1   887.29 891.29 32.368 1.276e-08 ***
## + savings      4   892.66 902.66 27.003 1.985e-05 ***
## + property     3   903.76 911.76 15.899 0.001189 **
## + age          1   909.12 913.12 10.537 0.001170 **
## + other_installment_plans 2   909.77 915.77 9.889 0.007123 **
## <none>                919.66 921.66
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Step: AIC=836.43
## credit_risk ~ status
##
##               Df Deviance    AIC    LRT Pr(>Chi)
## + duration     1   801.74 811.74 26.6915 2.387e-07 ***
```

```

## + property          3   811.90 825.90 16.5377 0.0008796 ***
## + savings           4   814.60 830.60 13.8362 0.0078365 **
## + other_installment_plans 2   820.32 832.32  8.1125 0.0173134 *
## + age               1   823.40 833.40  5.0337 0.0248586 *
## <none>              828.43 836.43
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Step: AIC=811.74
## credit_risk ~ status + duration
##
##              Df Deviance    AIC      LRT Pr(>Chi)
## + savings      4   787.09 805.09 14.6529 0.005478 **
## + other_installment_plans 2   794.74 808.74  7.0039 0.030138 *
## + age          1   797.00 809.00  4.7411 0.029450 *
## + property     3   794.81 810.81  6.9303 0.074154 .
## <none>         801.74 811.74
## + status:duration 3   797.97 813.97  3.7701 0.287383
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Step: AIC=805.09
## credit_risk ~ status + duration + savings
##
##              Df Deviance    AIC      LRT Pr(>Chi)
## + duration:savings 4   773.61 799.61 13.4777 0.009163 **
## + other_installment_plans 2   779.60 801.60  7.4878 0.023662 *
## + age            1   783.28 803.28  3.8072 0.051032 .
## + property       3   780.56 804.56  6.5277 0.088578 .
## <none>           787.09 805.09
## + status:duration 3   783.47 807.47  3.6232 0.305134
## + status:savings 12   769.40 811.40 17.6913 0.125392
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Step: AIC=799.61
## credit_risk ~ status + duration + savings + duration:savings
##
##              Df Deviance    AIC      LRT Pr(>Chi)
## + other_installment_plans 2   765.10 795.10  8.5114 0.01418 *
## + age                  1   769.17 797.17  4.4427 0.03505 *
## + property             3   766.99 798.99  6.6259 0.08483 .
## <none>                 773.61 799.61
## + status:duration      3   770.18 802.18  3.4347 0.32934
## + status:savings       12   760.49 810.49 13.1277 0.35983
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Step: AIC=795.1
## credit_risk ~ status + duration + savings + other_installment_plans +
##      duration:savings
##
##              Df Deviance    AIC      LRT Pr(>Chi)
## + age          1   760.07 792.07  5.0305 0.0249 *

```

```

## <none> 765.10 795.10
## + property 3 759.95 795.95 5.1531 0.1609
## + status:duration 3 761.92 797.92 3.1767 0.3652
## + duration:other_installment_plans 2 765.05 799.05 0.0514 0.9746
## + savings:other_installment_plans 8 756.37 802.37 8.7308 0.3655
## + status:other_installment_plans 6 761.30 803.30 3.8027 0.7034
## + status:savings 12 753.00 807.00 12.1014 0.4376
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Step: AIC=792.07
## credit_risk ~ status + duration + savings + other_installment_plans +
## age + duration:savings
##
## Df Deviance AIC LRT Pr(>Chi)
## + age:other_installment_plans 2 750.02 786.02 10.0493 0.006574 **
## + property 3 752.24 790.24 7.8305 0.049647 *
## <none> 760.07 792.07
## + duration:age 1 759.61 793.61 0.4620 0.496669
## + status:duration 3 756.45 794.45 3.6226 0.305204
## + duration:other_installment_plans 2 760.04 796.04 0.0335 0.983389
## + status:age 3 758.23 796.23 1.8404 0.606175
## + savings:age 4 757.98 797.98 2.0917 0.718903
## + savings:other_installment_plans 8 750.31 798.31 9.7585 0.282388
## + status:other_installment_plans 6 756.14 800.14 3.9290 0.686284
## + status:savings 12 747.44 803.44 12.6261 0.396794
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Step: AIC=786.02
## credit_risk ~ status + duration + savings + other_installment_plans +
## age + duration:savings + other_installment_plans:age
##
## Df Deviance AIC LRT Pr(>Chi)
## + property 3 742.30 784.30 7.7188 0.0522 .
## <none> 750.02 786.02
## + duration:age 1 749.76 787.76 0.2605 0.6098
## + status:duration 3 745.89 787.89 4.1336 0.2474
## + status:age 3 747.65 789.65 2.3749 0.4983
## + duration:other_installment_plans 2 749.97 789.97 0.0476 0.9765
## + savings:age 4 747.70 791.70 2.3220 0.6768
## + status:other_installment_plans 6 745.83 793.83 4.1928 0.6506
## + savings:other_installment_plans 8 742.20 794.20 7.8179 0.4515
## + status:savings 12 739.02 799.02 10.9992 0.5290
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Step: AIC=784.3
## credit_risk ~ status + duration + savings + other_installment_plans +
## age + property + duration:savings + other_installment_plans:age
##
## Df Deviance AIC LRT Pr(>Chi)
## + status:property 9 722.44 782.44 19.8671 0.01875 *
## <none> 742.30 784.30

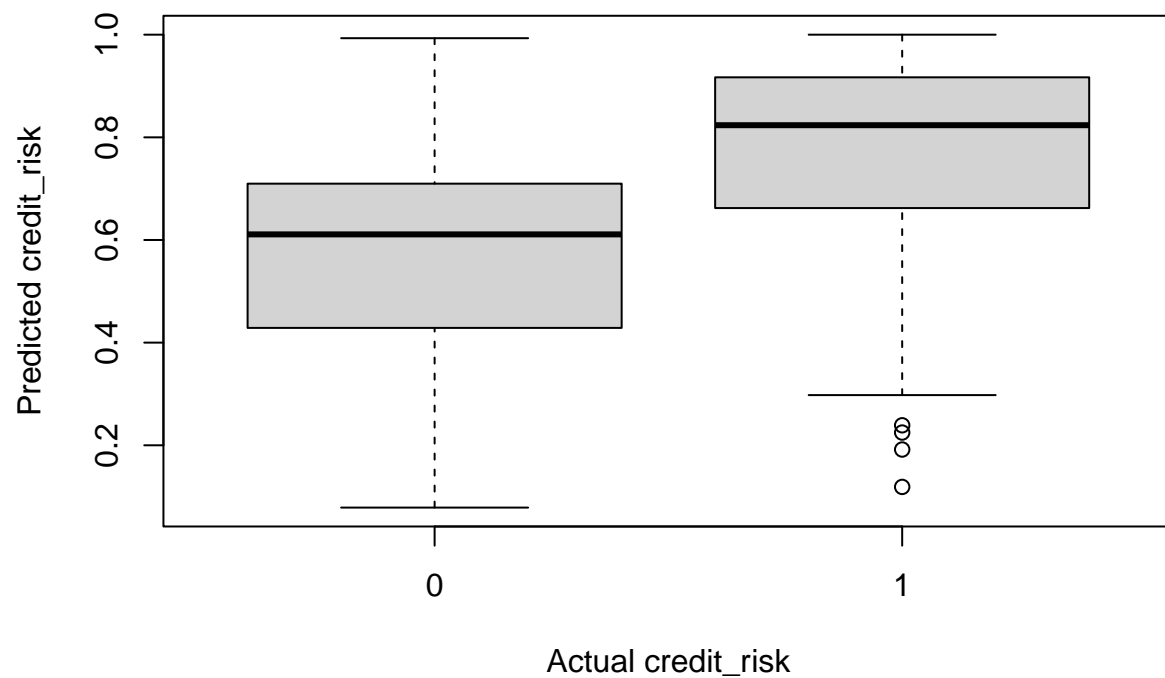
```



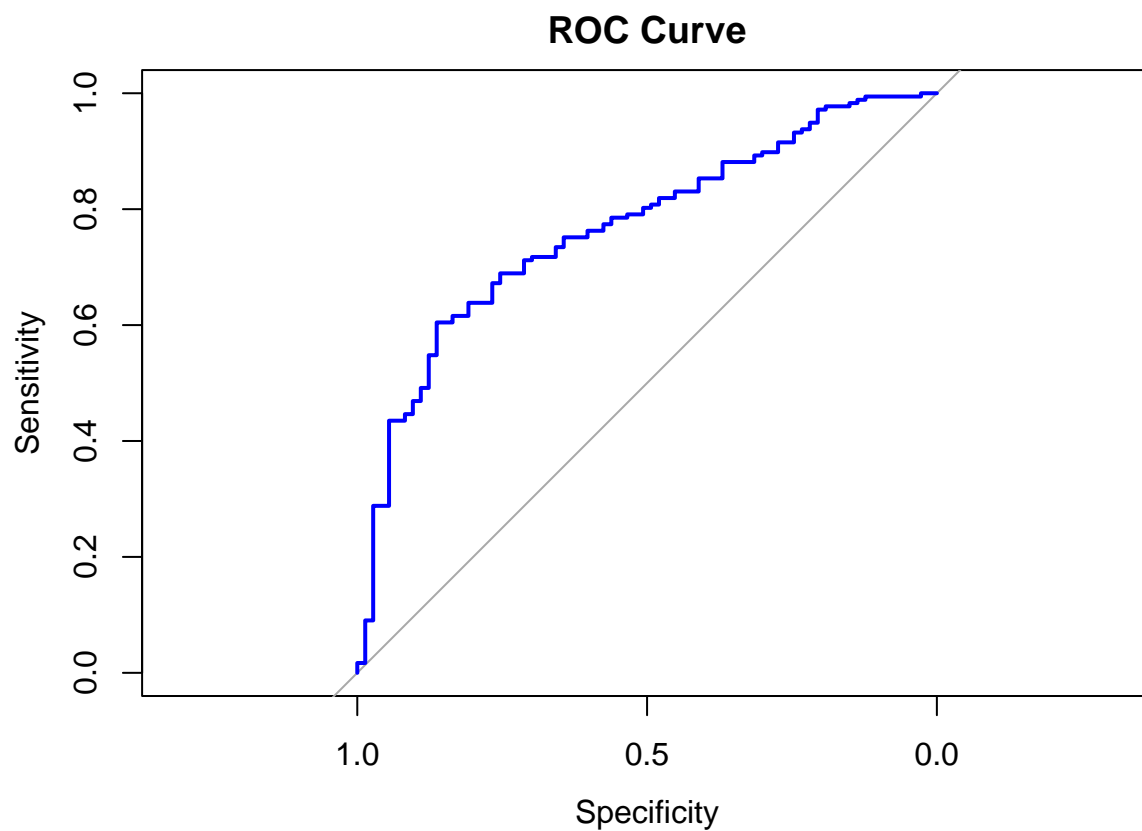
```
## + duration:age 1 742.22 786.22 0.0787 0.77913
## + status:duration 3 738.37 786.37 3.9359 0.26847
## + duration:property 3 738.94 786.94 3.3634 0.33891
## + status:age 3 740.05 788.05 2.2542 0.52135
## + duration:other_installment_plans 2 742.29 788.29 0.0141 0.99300
## + property:age 3 741.47 789.47 0.8287 0.84258
## + savings:age 4 739.79 789.79 2.5115 0.64258
## + property:other_installment_plans 6 737.35 791.35 4.9562 0.54944
## + status:other_installment_plans 6 738.23 792.23 4.0688 0.66737
## + savings:other_installment_plans 8 734.97 792.97 7.3372 0.50073
## + savings:property 12 729.92 795.92 12.3800 0.41566
## + status:savings 12 731.61 797.61 10.6915 0.55552
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Step: AIC=782.44
## credit_risk ~ status + duration + savings + other_installment_plans +
## age + property + duration:savings + other_installment_plans:age +
## status:property
##
## Df Deviance AIC LRT Pr(>Chi)
## <none> 722.44 782.44
## + duration:age 1 722.35 784.35 0.0832 0.7730
## + status:age 3 718.69 784.69 3.7428 0.2906
## + duration:property 3 718.92 784.92 3.5143 0.3189
## + status:duration 3 720.15 786.15 2.2823 0.5159
## + duration:other_installment_plans 2 722.22 786.22 0.2146 0.8983
## + savings:age 4 719.83 787.83 2.6071 0.6256
## + property:age 3 721.88 787.88 0.5560 0.9064
## + property:other_installment_plans 6 717.17 789.17 5.2635 0.5105
## + savings:other_installment_plans 8 715.61 791.61 6.8292 0.5552
## + status:other_installment_plans 6 720.08 792.08 2.3606 0.8837
## + savings:property 12 709.66 793.66 12.7724 0.3858
## + status:savings 12 710.09 794.09 12.3503 0.4180
```

5.2 Testing model

```
pred.3 <- predict(bestmodel.3, newdata = testdata)
plot(testdata$credit_risk, inv.logit(pred.3), xlab = "Actual credit_risk", ylab = "Predicted credit_risk")
```



```
roc(testdata$credit_risk~inv.logit(pred.3), plot=TRUE, main="ROC Curve", col="blue")
```



```
##
## Call:
## roc.formula(formula = testdata$credit_risk ~ inv.logit(pred.3),      plot = TRUE, main = "ROC Curve",
```

```
##  
## Data: inv.logit(pred.3) in 73 controls (testdata$credit_risk 0) < 177 cases (testdata$credit_risk 1)  
## Area under the curve: 0.7659
```

```
auc(testdata$credit_risk~inv.logit(pred.3))
```

```
## Area under the curve: 0.7659
```