Supplementary Material for "Identifying Integer-Variable-Induced Disjointness Within Security Region Considering Unit Commitment Adjustments"

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APPENDEX

A. Pseudocode of the IISI Algorithm

IISI Algorithm: Identifying Disjoint Regions Within $\Omega_{\mathrm{SR}}^{\mathcal{Z}}$

1. *initialization*: Input parameter $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$ and $\boldsymbol{D}$, outer convex approximation $\mathcal{E}_{\mathrm{SR}}^{\mathcal{Z}}$, set $\mathcal{I}_{\mathrm{IR}}=\emptyset$, and accuracy threshold $\varepsilon>0$;
2. **if** $\Pi^{+}(\mathcal{E}_{\mathrm{SR}}^{\mathcal{Z}})<\varepsilon$ **then return** $\mathcal{I}_{\mathrm{IR}}$ and **terminate**
3. **while** 1 (processes A) and B))
   - *identify disjointness*: solve the **RDD**:
   - $\Pi^{+}(\mathcal{E}_{\mathrm{SR}}^{\mathcal{Z}}\backslash\mathcal{I}_{\mathrm{IR}})=\max\limits_{\boldsymbol{X}\in\mathcal{E}_{\mathrm{SR}}^{\mathcal{Z}}}\min\limits_{\boldsymbol{Y}\in\mathcal{Y},\boldsymbol{Z}\in\mathcal{Z},\boldsymbol{S}^{+}}\{\mathbf{1}^{\mathrm{T}}\boldsymbol{S}^{+}:\mathrm{s.t.}\,(6),(11)\}$;
   - **if** $\Pi^{+}(\mathcal{E}_{\mathrm{SR}}^{\mathcal{Z}}\backslash\mathcal{I}_{\mathrm{IR}})<\varepsilon$ **then break**
   - **else** obtain optimal $(\boldsymbol{X}^{*},\boldsymbol{Z}^{*}),i\leftarrow1,\mathcal{I}_{\mathrm{IR},k}^{(i)}\leftarrow\mathcal{E}_{\mathrm{SR}}^{\mathcal{Z}}$;
   - **while** 1 (processes a) and b))
     - $\boldsymbol{Z}_{(i)}^{*}=\arg\min\limits_{\boldsymbol{X}\in\mathcal{I}_{\mathrm{IR}}^{(i)},\boldsymbol{Y}\in\mathcal{Y},\boldsymbol{S}^{+},\boldsymbol{Z}\in\mathcal{Z}}\{\mathbf{1}^{\mathrm{T}}\boldsymbol{S}^{+}:\mathrm{s.t.}\,(6),(11)\}$
     - **if** $\mathbf{1}^{\mathrm{T}}\boldsymbol{S}^{+}>\varepsilon$ **then break**
     - **else** compute $\mathrm{LEC}_{(i)}$ by $\left(\boldsymbol{X}^{*},\boldsymbol{Z}_{(i)}^{*}\right)$;
     - $i\leftarrow i+1,\mathcal{I}_{\mathrm{IR},k}^{(i)}\leftarrow\mathcal{I}_{\mathrm{IR},k}^{(i)}\bigcap\mathrm{LEC}_{(i)}$;
   - *update logic-based exclusion* constraints by (11);
5. **return** $\mathcal{I}_{\mathrm{IR}}$ and **terminate**

B. Proof of Convergence of LEC-Based Iterative Refinement

Let the continuous relaxation of the SR under UC adjustments be denoted as $\mathcal{E}_{\mathrm{SR}}^{\mathcal{Z}}$, and let $\mathcal{Z}$ denote the finite set of all possible quick-start unit combinations (UC configurations). Define the iterative infeasible subregion sequence as

$$\mathcal{I}_{\mathrm{IR}}^{(0)}=\mathcal{E}_{\mathrm{SR}}^{\mathcal{Z}}\bigcap\mathrm{LEC}(\boldsymbol{Z}^{*}),\tag{A1}$$

$$\mathcal{I}_{\mathrm{IR}}^{(i+1)}=\mathcal{I}_{\mathrm{IR}}^{(i)}\bigcap\mathrm{LEC}_{(i+1)},\tag{A2}$$

where $\mathrm{LEC}_{(i+1)}=\left\{\boldsymbol{X}:\boldsymbol{\lambda}_{(i+1)}^{\mathrm{T}}\boldsymbol{A}\boldsymbol{X}\leq\boldsymbol{\lambda}_{(i+1)}^{\mathrm{T}}\left(\boldsymbol{D}-\boldsymbol{C}\boldsymbol{Z}_{(i+1)}^{*}\right)\right\}$ is the LEC constructed from the infeasible pair $\left(\boldsymbol{X}^{*},\boldsymbol{Z}_{(i+1)}^{*}\right)$ according to **Lemma 1**. To prove the convergence of LEC-based iterative refinement, the following properties are demonstrated.

Property 1 (*Inclusiveness*): Each LEC **strictly encloses the detected infeasible point $\boldsymbol{X}^{*}$.**

Proof: Since the infeasible point $\boldsymbol{X}^{*}$ has been identified by the **RDD**, there exists no $(\boldsymbol{Y},\boldsymbol{Z})\in\mathcal{Y}\times\mathcal{Z}$ such that $\boldsymbol{A}\boldsymbol{X}+\boldsymbol{B}\boldsymbol{Y}+\boldsymbol{C}\boldsymbol{Z}\leq\boldsymbol{D}$. Therefore, each cut $\mathrm{LEC}_{(i+1)}$ necessarily satisfies $\boldsymbol{X}^{*}\in\mathrm{LEC}_{(i+1)}$, guaranteeing that all LECs enclose and surround the identified infeasible point $\boldsymbol{X}^{*}$. ∎

Property 2 (*Monotonicity*): Sequence $\left\{\mathcal{I}_{\mathrm{IR}}^{(i)}\right\}$ is monotonically non-expanding and converges in a finite number of iterations.

Proof: Since each iteration introduces one additional LEC, it follows directly that

$$\mathcal{I}_{\mathrm{IR}}^{(i+1)}=\left(\mathcal{I}_{\mathrm{IR}}^{(i)}\bigcap\mathrm{LEC}_{(i+1)}\right)\subseteq\mathcal{I}_{\mathrm{IR}}^{(i)}.\tag{A3}$$

Let z denote the total number of UC configurations in $\mathcal{Z}$, corresponding to all possible on/off combinations of quick-start units. At each iteration, the *detect-and-refine* step identifies at least one new configuration $\boldsymbol{Z}_{(i+1)}^{*}\in\mathcal{Z}$, introducing a unique exclusion cut $\mathrm{LEC}_{(i+1)}$. Since $\mathcal{Z}$ is finite and each iteration identifies one distinct component associated with a specific UC configuration $\boldsymbol{Z}_{(i+1)}^{*}$, the iterative process must terminate after at most z iterations, i.e.,

$$\lim_{i\to i^{*}}\mathcal{I}_{\mathrm{IR}}^{(i)}\subseteq\mathcal{I}_{\mathrm{IR}}^{(*)},i^{*}\leq z.\tag{A4}$$

Hence, the LEC-based iterative refinement converges finitely to the complete local infeasible subregion $\mathcal{I}_{\mathrm{IR}}^{(*)}$. ∎

Property 3 (Accuracy): no feasible points are erroneously included in $\mathcal{I}_{\mathrm{IR}}^{(*)}$.

Proof: Recall Step a) **Infeasibility effectiveness detection**. Each iteration first verifies the feasibility of the current subregion under all UC configurations. A new $\mathrm{LEC}_{(i+1)}$ is constructed only when a feasible pair $\left(\boldsymbol{X}_{(i)}^{*},\boldsymbol{Z}_{(i)}^{*}\right)$ is detected from (6). When $\mathcal{I}_{\mathrm{IR}}^{(*)}$ converges, no further valid LEC can be constructed. According to **Lemma 1**, this implies that there no longer exists any nonnegative vector $\boldsymbol{\lambda}\geq\mathbf{0}$ satisfying $\boldsymbol{\lambda}^{\mathrm{T}}\boldsymbol{B}=\mathbf{0}$ and $\boldsymbol{\lambda}^{\mathrm{T}}(\boldsymbol{A}\boldsymbol{X}^{*}+\boldsymbol{C}\boldsymbol{Z}^{*}-\boldsymbol{D})>\mathbf{0}$ for any $\boldsymbol{X}\in\mathcal{I}_{\mathrm{IR}}^{(*)}$. ∎

Therefore, the LEC-based iterative refinement process is both monotonically non-expanding and finitely convergent, producing the complete infeasible subregion that satisfies **Properties 1-3**.

C. Test System Models and Numerical Settings

The numerical experiments are conducted on the modified IEEE 9-bus, 30-bus, and 118-bus test systems. For all cases, the original network topology (bus, branch, and admittance data) is preserved exactly as in the standard IEEE test systems, which can be obtained from MATPOWER [A1]. Modifications are made only to the unit-commitment-related parameters, including quick-start capability limits, start-up/shut-down constraints, and generator locations and capacities.

Modification in 9-bus system: The observed SR variables are the branch power flows $P_{B,4-5}$, and $P_{B,8-2}$. The corresponding UC settings for the 9-bus (a) and 9-bus (b) system are listed in Table A-I and Table A-II, respectively, and the active loads are uniformly increased to 1.25 times their original values.

TABLE A-I
MODIFIED UC SETTINGS IN 9-BUS (A) SYSTEM

Generator	Location	P_G^{min} (MW)	P_G^{max} (MW)	UC adjustable (Y/N)
1	Bus 1#	10.000	150.000	Y
2	Bus 1#	50.000	150.000	Y
3	Bus 2#	10.000	300.000	N
4	Bus 3#	10.000	270.000	N

TABLE A-II
MODIFIED UC SETTINGS IN 9-BUS (B) SYSTEM

Generator	Location	P_G^{min} (MW)	P_G^{max} (MW)	UC adjustable (Y/N)
1	Bus 1#	10.000	80.000	Y
2	Bus 1#	100.000	220.000	Y
3	Bus 2#	10.000	300.000	N
4	Bus 3#	10.000	90.000	Y
5	Bus 3#	100.000	180.000	Y

Modification in 30-bus system: The observed SR variables are the branch power flows $P_{B,6-10}$, $P_{B,4-12}$ and $P_{B,28-27}$. The corresponding UC settings are listed in Table A-III, while all other system parameters remain unchanged.

TABLE A-III
MODIFIED UC SETTINGS IN 30-BUS SYSTEM

Generator	Location	P_G^{min} (MW)	P_G^{max} (MW)	UC adjustable (Y/N)
1	Bus 1#	0.000	80.000	N
2	Bus 2#	5.000	80.000	N
3	Bus 13#	5.000	40.000	N
4	Bus 22#	0.000	10.000	Y
5	Bus 22#	15.000	40.000	Y
6	Bus 23#	5.000	30.000	N
7	Bus 27#	0.000	15.000	Y
8	Bus 27#	20.000	40.000	Y

Modification in 118-bus system: The observed SR variables are the branch power flows $P_{B,4-11}$, $P_{B,12-16}$ and $P_{B,23-24}$. The corresponding UC settings are listed in Table A-IV, while all other system parameters remain unchanged.

TABLE A-IV
MODIFIED UC SETTINGS IN 118-BUS SYSTEM

Generator	Location	P_G^{min} (MW)	P_G^{max} (MW)	UC adjustable (Y/N)
1	Bus 10#	0.000	100.000	Y
2	Bus 10#	200.000	550.000	Y
3	Bus 12#	0.000	185.000	N
4	Bus 25#	0.000	320.000	N
5	Bus 26#	0.000	414.000	N
6	Bus 31#	0.000	107.000	N
7	Bus 46#	0.000	119.000	N
8	Bus 49#	0.000	304.000	N
9	Bus 54#	0.000	148.000	N
10	Bus 59#	0.000	255.000	N
11	Bus 61#	0.000	260.000	N
12	Bus 65#	0.000	200.000	Y
13	Bus 65#	50.000	291.000	Y
14	Bus 66#	0.000	200.000	Y
15	Bus 66#	50.000	292.000	Y
16	Bus 69#	0.000	100.000	Y
17	Bus 69#	400.000	805.200	Y
18	Bus 80#	0.000	577.000	N
19	Bus 87#	0.000	104.000	N
20	Bus 89#	0.000	707.000	N
21	Bus 100#	0.000	352.000	N
22	Bus 103#	0.000	140.000	N
24	Bus 111#	0.000	136.000	Y

REFERENCES

[A1] R. D. Zimmerman, C. E. Murillo-Sánchez, and R. J. Thomas, "MAT POWER: Steady-state operations, planning, and analysis tools for power systems research and education," *IEEE Trans. Power Syst.*, vol. 26, no. 1, pp. 12-19, Feb. 2011.