Exercise 5.7

a)

Code:

```python
# YOUR CODE GOES HERE takes poly features of the input
def poly_features(x,D):
    F = np.ones((len(x),D+1))
    for i in range(len(x)):
        for j in range(D+1):
            F[i,j] = x[i]**j

    F = F.T

    return F
```

Figure:

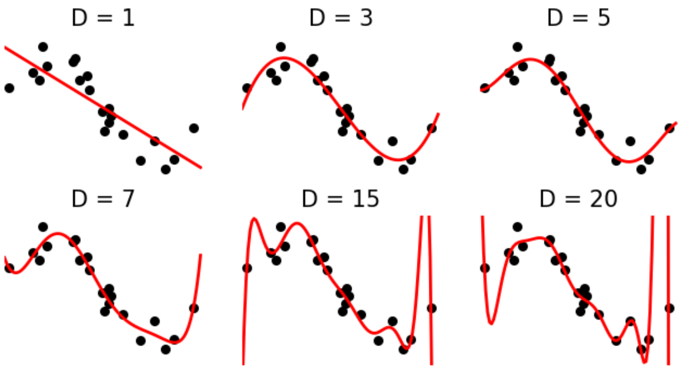


Figure1

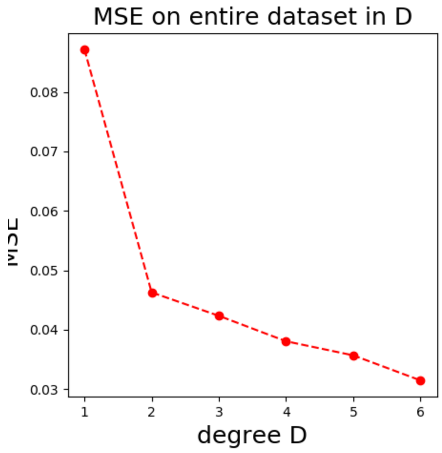


Figure2

b)

In the first figure, with the increasing of D, the indicator function fit the data better.

In the second figure, with the increasing of D, the mean squared error (MSE) decrease, which means the fitness becomes better and more stable.

Exercise 5.10

1) For Eric’s plot, I recommend 5 as the degree of polynomial, since at that degree, the testing error is at the least level and the training error is small.

2) For Stanley’s plot, I recommend choosing 8 as the degree of polynomial, since at that degree, the testing error is at the least level and the training error is small.

3) For Kyle’s plot, I recommend keeping increasing the number of D, since the errors keep decreasing, there may be some other degree which fits the datasets better.

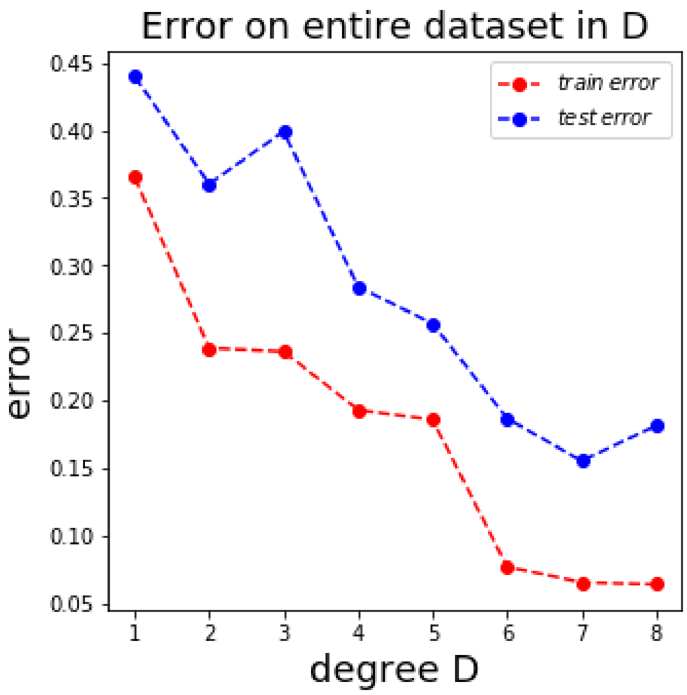
4) For Kenneth’s plot, I recommend choosing 6 as the degree of polynomial, since at that degree, the testing error is at the least level and the training error is small.

Exercise 5.11

Code:

```
from __future__ import division
import numpy as np
import numpy.matlib
import matplotlib.pyplot as plt
import pylab
from sklearn.model_selection import train_test_split

# load data
def load_data():
    data = np.array(np.genfromtxt('C:/Users/10448/Desktop/wavy_data.csv', delimiter=','))
    x = np.reshape(data[:,0],(np.size(data[:,0]),1))
    y = np.reshape(data[:,1],(np.size(data[:,1]),1))
    return x,y

# Fourier features
def four_features(x,D):
    F = np.zeros((len(x),D+1))
    for i in range(len(x)):
        for j in range(D+1):
            if j%2 ==0:
                F[i,j] = np.cos(2*np.pi*((j+2)/2)*x[i])
            else:
                F[i,j] = np.sin(2*np.pi*((j+1)/2)*x[i])
    F = F.T

    return F

# plot train error and test error over all D tested
def plot_error(train,test,deg):
    plt.plot(np.arange(1,np.size(train)+1),train,'ro--')
    plt.plot(np.arange(1,np.size(test)+1),test,'bo--')
    plt.title('Error on entire dataset in D', fontsize=18)
    plt.xlabel('degree D', fontsize=18)
    plt.ylabel('error       ', fontsize=18)
    plt.legend([r'$train\:error$',r'$test\:error$'])

# run over all the degrees and calculate errors
def try_all_degs(x,y,deg_range):

    # split data
    x_train, x_test, y_train, y_test = train_test_split(x, y, test_size=1/3, random_state=19)

    # generate train and test error features
    train_error = []
    test_error = []

    # calculate errors
    for D in np.arange(0,np.size(deg_range)):
        # generate fourier feature transformation
        F_train = four_features(x_train,deg_range[D])
        F_test = four_features(x_test,deg_range[D])

        # concatenate ones for F
        tr_o = np.ones((np.shape(F_train)[1],1))
        F_train_new = np.concatenate((tr_o,F_train.T),axis = 1)
        F_train_new = F_train_new.T
        te_o = np.ones((np.shape(F_test)[1],1))
        F_test_new = np.concatenate((te_o,F_test.T),axis = 1)
        F_test_new = F_test_new.T

        # get error
        temp_train = np.linalg.pinv(np.dot(F_train_new,F_train_new.T))
        w_train = np.dot(np.dot(temp_train,F_train_new),y_train)
        tr_error = np.linalg.norm(np.dot(F_train_new.T,w_train)-y_train)/np.size(y_train)
        temp_test = np.linalg.pinv(np.dot(F_test_new,F_test_new.T))
        w_test = w_train
        te_error = np.linalg.norm(np.dot(F_test_new.T,w_test)-y_test)/np.size(y_test)
        train_error.append(tr_error)
        test_error.append(te_error)

    # make plot of train and test errors
    fig = plt.figure(figsize = (5,5))
    plot_error(train_error,test_error,deg_range)
    plt.show()

# load data and defined degree range
x, y = load_data()
deg_range = [1,2,3,4,5,6,7,8]           # degree fourier to try

# run all over degree range
try_all_degs(x,y,deg_range)
```

Figure:

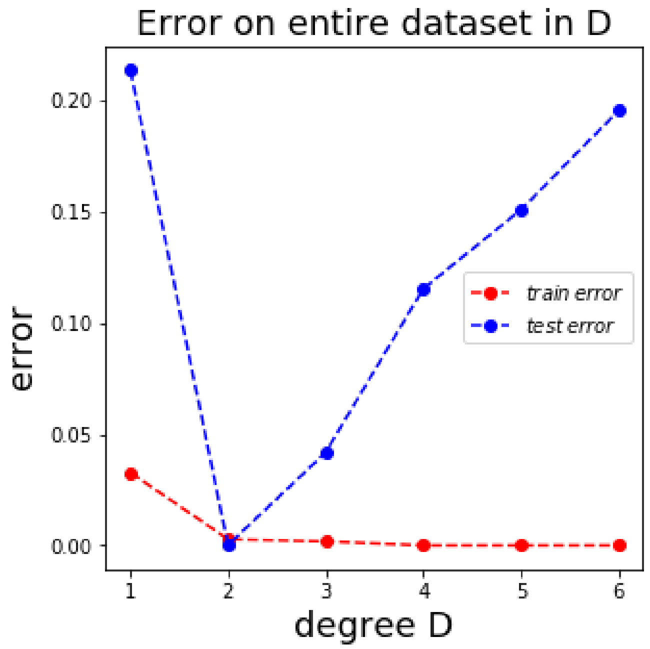


Exercise 5.12

Code:

```python
from __future__ import division
import numpy as np
import numpy.matlib
import matplotlib.pyplot as plt
import pylab
from sklearn.model_selection import train_test_split
from sklearn.model_selection import KFold

# load data
def load_data():
    data = np.array(np.genfromtxt('C:/Users/10448/Desktop/galileo_ramp_data.csv', delimiter=','))
    x = np.reshape(data[:,0],(np.size(data[:,0]),1))
    y = np.reshape(data[:,1],(np.size(data[:,1]),1))
    return x,y

# Fourier features
def poly_features(x,D):
    F = np.ones((len(x),D+1))
    for i in range(len(x)):
        for j in range(D+1):
            F[i,j] = x[i]**j
    F = F.T

    return F

# plot train error and test error over all D tested
def plot_error(train,test,deg):
    plt.plot(np.arange(1,np.size(train)+1),train,'ro--')
    plt.plot(np.arange(1,np.size(test)+1),test,'bo--')
    plt.title('Error on entire dataset in D', fontsize=18)
    plt.xlabel('degree D', fontsize=18)
    plt.ylabel('error       ', fontsize=18)
    plt.legend([r'$train\:error$',r'$test\:error$'])

# run over all the degrees and calculate errors
def try_all_degs(x,y,deg_range):

    # split data
    kf = KFold(n_splits = 6,shuffle = False)
    x_train = []
    x_test = []
    y_train = []
    y_test = []
    for train_index, test_index in kf.split(x):
        x_train.append(x[train_index])
        x_test.append(x[test_index])
        y_train.append(y[train_index])
        y_test.append(y[test_index])

    # generate train and test error features
    train_error = []
    test_error = []
    train_error_ave = 0
    test_error_ave = 0

    # calculate errors
    for D in np.arange(0,np.size(deg_range)):
        # generate poly feature transformation
        for i in range(np.size(x_train,0)):
            F_train = poly_features(x_train[i],deg_range[D])
            F_test = poly_features(x_test[i],deg_range[D])
            # get error
            temp_train = np.linalg.pinv(np.dot(F_train,F_train.T))
            w_train = np.dot(np.dot(temp_train,F_train),y_train[i])
            tr_error = np.linalg.norm(np.dot(F_train.T,w_train)-y_train[i])/np.size(y_train[i])
            temp_test = np.linalg.pinv(np.dot(F_test,F_test.T))
            w_test = w_train
            te_error = np.linalg.norm(np.dot(F_test.T,w_test)-y_test[i])/np.size(y_test[i])
            train_error_ave = train_error_ave + tr_error/(np.size(x_train,0))
            test_error_ave = test_error_ave + te_error/(np.size(x_train,0))

        train_error.append(tr_error)
        test_error.append(te_error)
        train_error_ave = 0
        test_error_ave = 0

    # make plot of train and test errors
    fig = plt.figure(figsize = (5,5))
    plot_error(train_error,test_error,deg_range)
    plt.show()

# load data and defined degree range
x, y = load_data()
deg_range = [1,2,3,4,5,6]           # degree polynomial to try

# run all over degree range
try_all_degs(x,y,deg_range)
```

Figure:


Error on entire dataset in D
error
degree D
train error
test error