CO CW4

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Problem 1

	P_D	P_U
x_2	$\frac{3}{5}$	$\frac{8}{5}$
x_3	$\frac{4}{15}$	$\frac{18}{5}$

$\max_i \{P_D(x_i), P_U(x_i)\} = x_3$, $\frac{4}{15} < \frac{18}{5}$.

So, $P_D(x_3)$ first.

LP1	x_1	x_5	x_4	
x_3	$\frac{7}{5}$	$-\frac{1}{5}$	$\frac{3}{5}$	$\frac{22}{5}$
x_2	$\frac{1}{5}$	$\frac{2}{5}$	$-\frac{1}{5}$	$\frac{21}{5}$
s_3	$\frac{-7}{5}$	$\frac{1}{5}$	$\frac{-3}{5}$	$\frac{-3}{5}$
Z	$\frac{13}{5}$	$\frac{6}{5}$	$\frac{2}{5}$	$\frac{128}{5}$

$\Rightarrow$

LP_1	x_1	x_5	s_3	
x_3	0	0	1	4
x_2	$\frac{2}{3}$	$\frac{1}{3}$	$-\frac{1}{3}$	$\frac{13}{3}$
x_4	$\frac{7}{3}$	$\frac{-1}{3}$	$\frac{-5}{3}$	$\frac{2}{3}$
Z	$\frac{5}{3}$	$\frac{4}{3}$	$\frac{2}{3}$	$\frac{76}{3}$

Now, $x_1 = 0$ $x_2 = \frac{13}{3}$, $x_3 = 4$.

Now, we will explore $x_2 \le 4$ first.

LP_2	x_1	x_5	s_3	
x_3	0	0	1	4
x_2	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{-1}{3}$	$\frac{13}{3}$
x_4	$\frac{7}{3}$	$\frac{-1}{3}$	$\frac{-5}{3}$	$\frac{2}{3}$
s_2	$\frac{-2}{3}$	$\frac{-1}{3}$	$\frac{1}{3}$	$-\frac{1}{3}$
Z	$\frac{5}{3}$	$\frac{4}{3}$	$\frac{2}{3}$	$\frac{76}{3}$

$\Rightarrow$

LP_2	s_2	x_5	s_3	
x_3	0	0	1	4
x_2	1	0	0	4
x_4	$\frac{7}{2}$	$\frac{-3}{2}$	$\frac{-1}{2}$	$\frac{-1}{2}$
x_1	$\frac{-3}{2}$	$\frac{1}{2}$	$\frac{-1}{2}$	$\frac{1}{2}$
Z	$\frac{5}{2}$	$\frac{1}{2}$	$\frac{3}{2}$	$\frac{49}{2}$

LP_2	S_2	x_4	S_3	
x_3	0	0	1	4
x_2	1	0	0	4
x_5	$\frac{-7}{3}$	$\frac{-2}{3}$	$\frac{2}{3}$	$\frac{1}{3}$
$\Rightarrow x_1$	$\frac{-1}{3}$	$\frac{1}{3}$	$\frac{-2}{3}$	$\frac{1}{3}$
Z	$\frac{11}{3}$	$\frac{1}{3}$	$\frac{4}{3}$	$\frac{73}{3}$

Now, $x_1 = \frac{1}{3}$

$x_2 = 4$

$x_3 = 4$

$Z = \frac{73}{3}$

We would explore $x_1 \leq 0$ first and then $x_1 \geq 1$.

LP_3	S_2	x_4	S_3	
x_3	0	0	1	4
x_2	1	0	4	4
x_5	$\frac{-7}{3}$	$\frac{-2}{3}$	$\frac{1}{3}$	$\frac{1}{3}$
x_1	$\frac{-1}{3}$	$\frac{1}{3}$	$\frac{-2}{3}$	$\frac{1}{3}$
S_1	$\frac{1}{3}$	$\frac{-1}{3}$	$\frac{2}{3}$	$\frac{-1}{3}$
Z	$\frac{11}{3}$	$\frac{1}{3}$	$\frac{4}{3}$	$\frac{73}{3}$

LP_3	S_2	S_1	S_3	
x_3	0	0	1	4
x_2	1	0	0	4
$\Rightarrow x_5$	-3	-2	-1	1
x_1	0	1	0	0
x_4	-1	-3	-2	1
Z	4	1	2	24

Now, $x_1 = 0$, $x_2 = 4$, $x_3 = 4$, $Z = 24$ which is the new LB. We would then explore $x_1 \geq 1$.

LP_4	S_2	x_4	S_3	
x_3	0	0	1	4
x_2	1	0	0	4
x_5	$\frac{-7}{3}$	$\frac{-2}{3}$	$\frac{1}{3}$	$\frac{1}{3}$
x_1	$\frac{-1}{3}$	$\frac{1}{3}$	$\frac{-2}{3}$	$\frac{1}{3}$
S_1	$\frac{-1}{3}$	$\frac{1}{3}$	$\frac{-2}{3}$	$\frac{-2}{3}$
Z	$\frac{11}{3}$	$\frac{1}{3}$	$\frac{4}{3}$	$\frac{73}{3}$

$\Rightarrow$

LP_4	S_2	x_4	S_1	
x_3	$\frac{-1}{2}$	$\frac{1}{2}$	$\frac{3}{2}$	3
x_2	1	0	0	4
x_5	$\frac{-5}{2}$	$\frac{-1}{2}$	$\frac{1}{2}$	0
x_1	0	0	-1	1
S_3	$\frac{1}{2}$	$\frac{-1}{2}$	$\frac{-3}{2}$	1
Z	3	1	2	23

Now, $x_1 = 1$, $x_2 = 4$, $x_3 = 3$. $Z = 23 < 24$.

We now explore $x_3 \le 4$ and $x_2 \geq 5$

LP5	x_1	x_5	s_3	
x_3	0	0	1	4
x_2	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{-1}{3}$	$\frac{13}{3}$
x_4	$\frac{7}{3}$	$\frac{-1}{3}$	$\frac{-5}{3}$	$\frac{2}{3}$
s_2	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{-1}{3}$	$\frac{-2}{3}$
Z	$\frac{5}{3}$	$\frac{4}{3}$	$\frac{2}{3}$	$\frac{76}{3}$

$\Rightarrow$

LP5	x_1	x_5	s_2	
x_3	2	1	3	2
x_2	0	0	-1	5
x_4	-1	-2	-5	4
s_3	-2	-1	-3	2
Z	3	2	2	24

Now, $x_1 = 0$ $x_2 = 5$, $x_3 = 2$, $Z = 24$

Now, we would explore $x_3 \geq 5$

LP6	x_1	x_5	x_4	
x_3	$\frac{7}{5}$	$\frac{-1}{5}$	$\frac{3}{5}$	$\frac{22}{5}$
x_2	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{-1}{5}$	$\frac{21}{5}$
s_3	$\frac{7}{5}$	$\frac{-1}{5}$	$\frac{3}{5}$	$\frac{-3}{5}$
Z	$\frac{13}{5}$	$\frac{6}{5}$	$\frac{2}{5}$	$\frac{128}{5}$

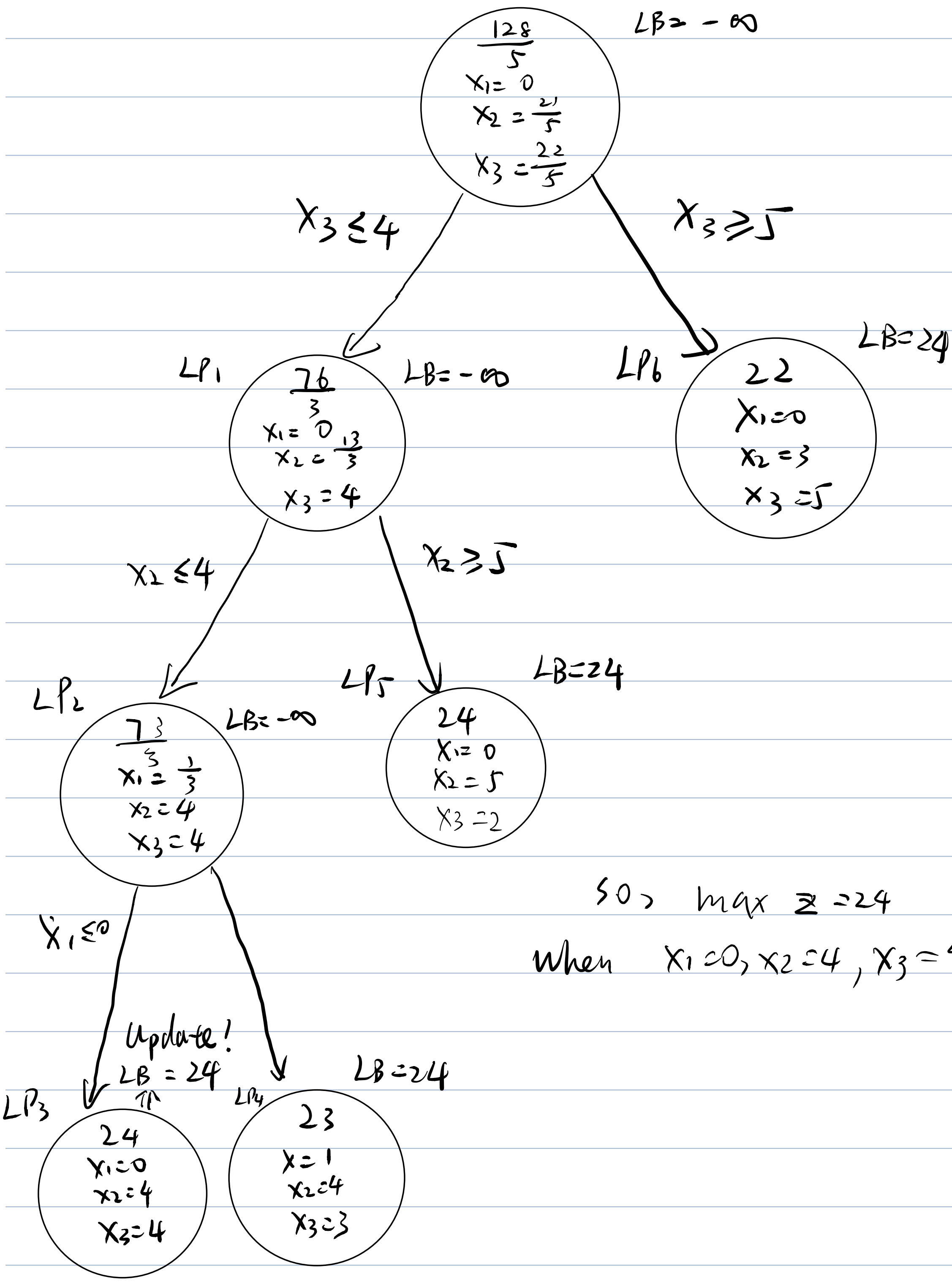
$\Rightarrow$

LP6	x_1	s_3	x_4	
x_3	0	-1	0	5
x_2	3	2	1	3
x_5	-7	-5	-3	3
Z	11	6	4	22

Now, $x_1 = 0$, $x_2 = 3$, $x_3 = 5$. $Z = 22$.

So, max $Z = 24$ when $x_1 = 0$, $x_2 = 4$, $x_3 = 4$.

The whole search tree will be illustrated in the next page :



so, max $z = 24$

when $x_1 = 0, x_2 = 4, x_3 = 4$

Problem 2

(a) According to property P, we cannot find a proper R_1 and R_2. So, no.

(b) $R_1 = \{row1, row2, row3, row4\}$
$R_2 = \emptyset$ Yes, it is unimodular

(c) $R_1 = \{row1, row2, row3, row4\}$
$R_2 = \{row5, row6\}$, it is unimodular.

Problem 3

(a) According to the constraints, at most 2 products can be selected.

So, as for (1), due to $21 > 20 > 18$.

So, $3x_1 + 4x_2 + 2x_3 \leq 21 + 20 \leq 30 + M'_1 \delta$

$\Rightarrow M_1' \geq 11$

As for (2), $30 > 28 > 18$

so $4x_1 + 6x_2 + 2x_3 \leq 30 + 28 \leq 40 + M'\delta$

$\Rightarrow M' \geq 18$.

So, a smaller value of M' can be 18.

(b) $M' < 36$ which is $M' = 18$. It can reduce the search space by offering a smaller feasible region.