Leo-Wyndor

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Main Idea

01. Introduction of background

02. Workflow and preparation

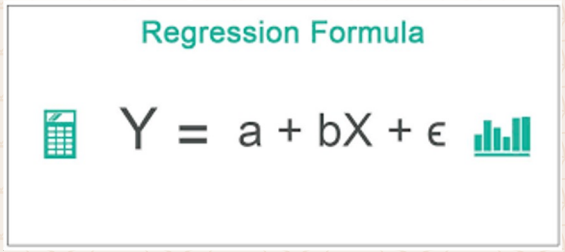
03. Model Interpretation

04. Comparison of each model

01.

Introduction

Background



In order to provide a strong mathematical foundation for comprehensive analysis, we need to use paradigms such as regression and stochastic programming based on very similar assumptions.

Next, we can use a company WYNDOR Glass as an example to help them maximize post-advertising profitability on the basis of existing funds.

Background: Goal

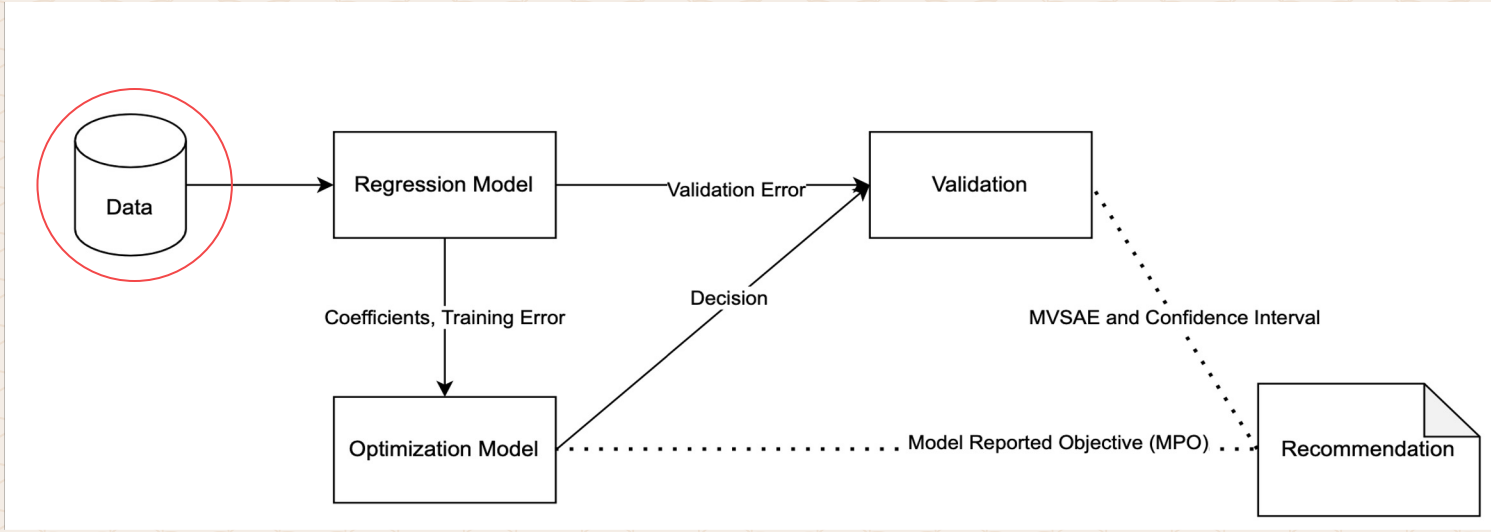
Use management's proposed advertising budget of $200,000 as a basis for determining how much advertising to place on radio and television to maximize expected profits. The point that needs to be considered here is to use three different methods to compare which method can maximize profits.

(The method will be described in the next few chapters)

02

Workflow & Preparation

Workflow


Data
Regression Model
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Recommendation

WorkFlow - Data

1. Advertising allocation

Meaning	Value
Advertising cost on TV	100$
Advertising cost on Radio	500$
Total advertising budget	200$
Proportion of advertisement expenditures on Radio over TV	0.5

2. Each Product's production time and the profit

Plant	Prod. time A (Hr/Batch)	Prod. time A (Hr/Batch)	Total Hours
1	1	0	4
2	0	2	12
3	3	2	18
Profit	$3000	$5000	

WorkFlow - Data

3. Advertising dataset

x1 x2 Predict

Description: df [20' : 5]

X	TV	Radio	Newspaper	Sales
<int>	<dbl>	<dbl>	<dbl>	<dbl>
1	230.1	37.8	69.2	22.1
2	44.5	39.3	45.1	10.4
3	17.2	45.9	69.3	9.3
4	151.5	41.3	58.5	18.5
5	180.8	10.8	58.4	12.9
6	8.7	48.9	75.0	7.2
7	57.5	32.8	23.5	11.8
8	120.2	19.6	11.6	13.2

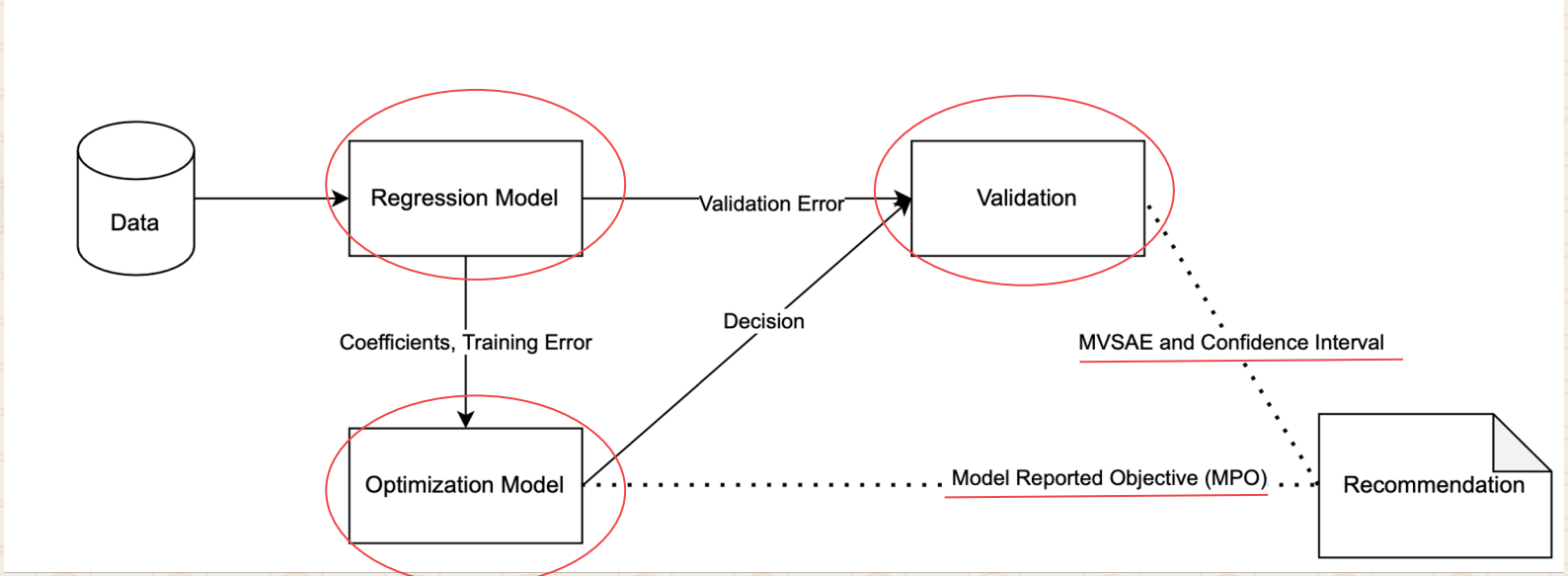
Preparation for Descriptive Analytics

Split data into training and validation set into 50-50 to have enough data for estimating the validation objective later

03

Model Interpretation

Workflow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Model Interpretation

Predictive Analysis (Linear Regression)

Prescriptive

- Deterministic Linear Programming
  - Model
  - Validation
- Sample Average Approximation
  - Model
  - Validation
- Stochastic Decomposition
  - Model
  - Validation

Model Interpretation: Predictive Analysis

Multiple Linear Regression Model:

$$\tilde{\omega} = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \tilde{\epsilon}$$

Results:

$$\beta_0 = 3.14699, \beta_1 = 0.04585, \beta_2 = 0.18412$$

Training error:

$$\epsilon_{vi} \coloneqq \omega_i - \beta_0 - \beta_1 x_{1i} - \beta_2 x_{2i}$$

Validation error:

$$\epsilon_{ti} \coloneqq \omega_i - \beta_0 - \beta_1 x_{1i} - \beta_2 x_{2i}$$

Model Interpretation: Prescriptive Analysis

Linear Regression: Validation

1. F-test of ϵ_{ti} and ϵ_{vi} to check whether they are drawn from the same distribution.

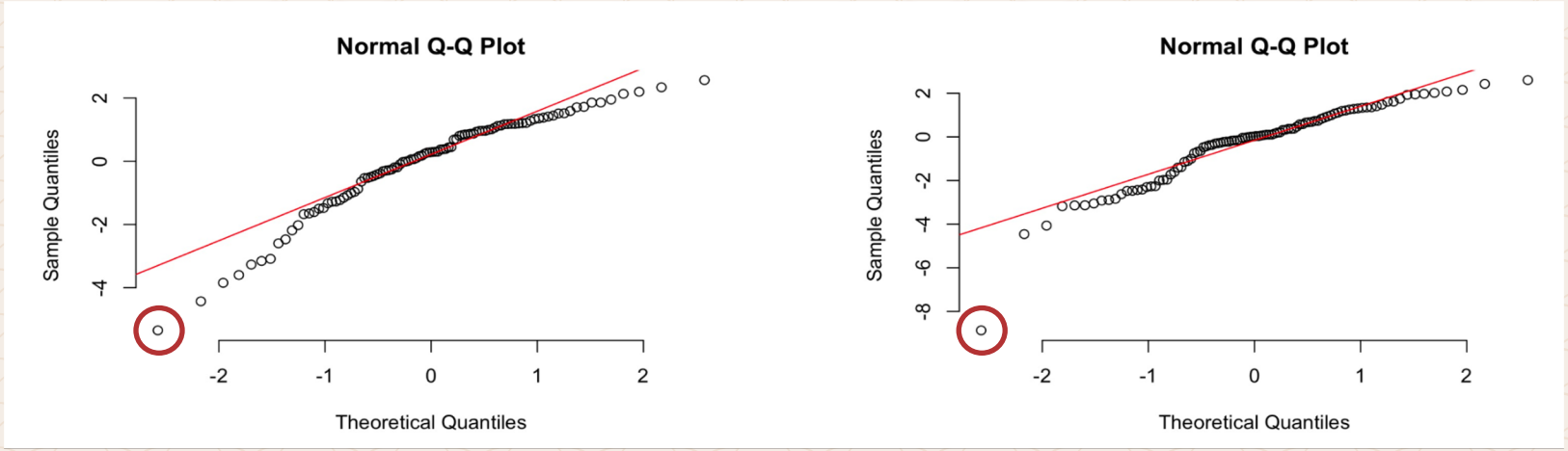
```
        F test to compare two variances

data:  ev and ei
F = 1.3391, num df = 99, denom df = 99, p-value = 0.148
alternative hypothesis: true ratio of variances is not equal to 1
95 percent confidence interval:
 0.9010256 1.9902672
sample estimates:
ratio of variances
          1.339135
```

- P-value > 0.05
- Cannot reject the null hypothesis
- Illustrate the fit using a Q-Q plot and check the normality of error term

Model Interpretation: Prescriptive Analysis

Linear Regression: Validation

2. Q-Q Plot of ϵ_{ti} and ϵ_{vi}



Model Interpretation: Prescriptive Analysis

Linear Regression: Validation

3. Box Plot of ϵ_{ti} and ϵ_{vi}

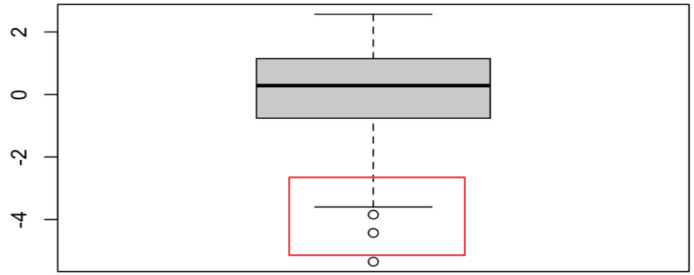


Figure 3: Box Plot for Training Data

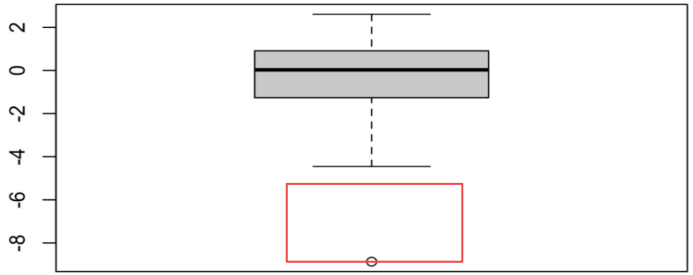


Figure 4: Box Plot for Validation Data

Model Interpretation: Prescriptive Analysis

Re-do Regression Model

Dataset	β_0	β_1	β_2
Full data set	3.14699	0.04585	0.18412
Outlier Deleted	3.157054	0.044285	0.194257

* The following analysis are all based on the dataset without outliers.

Model Interpretation: Prescriptive Analysis

Deterministic Linear Programming: Model

$$
\begin{aligned}
\text{Max } & -0.1x_1 - 0.5x_2 + 3y_A + 5y_B \\
s.t. \quad & x_1 + x_2 \leq 200 \\
& x_1 - 0.5x_2 \geq 0 \\
& y_A \leq 8 \\
& 2y_B \leq 24 \\
& 3y_A + 2y_B \leq 36 \\
& -\beta_1 x_1 - \beta_2 x_2 + y_A + y_B \leq \beta_0 \\
& y_A, y_B \geq 0 \\
& L_1 \leq x_1 \leq U_1, L_2 \leq x_2 \leq U_2
\end{aligned}
$$

Methodology	x_1	x_2	MPO (in $)
Deterministic LP	173.3636	26.6363	41,3454

Model Interpretation: Prescriptive Analysis

Deterministic Linear Programming: Validation

Steps:

1. Input validation data are the sales generated by plugging in the first stage decision (x1,x2) obtained from the LP
2. Use all the validation sales value to calculate the profit in each scenarios by solving the second-stage problem with validation sales data
3. Calculate Model Validation Sample Average Estimate (MVSAE)

Second - Stage Problem:

$$
\begin{aligned}
Profit(\omega_i) = \ & \text{Max } 3y_A + 5y_B \\
s.t. \quad & y_A \leq 8 \\
& 2y_B \leq 24 \\
& 3y_A + 2y_B \leq 36 \\
& y_A + y_B \leq \omega \\
& y_A, y_B \geq 0
\end{aligned}
$$

Methodology	x_1	x_2	MPO (in $)	MVSAE	STD
Deterministic LP	173.422	26.578	$41,369	$[38,634, 39,966]	3.364

Model Interpretation: Prescriptive Analysis

Sample Average Approximation: Model

$$\text{Max } -0.1x_1 - 0.5x_2 + \frac{1}{N}\sum_{i=1}^{N}(13y_{Ai} + 5y_{Bi})$$

$$\begin{aligned} s.t. \quad x_1 + x_2 &\le 200 \\ x_1 - 0.5x_2 &\ge 0 \\ y_{Ai} &\le 8, i = 1, \dots, N \\ 2y_{Bi} &\le 24, i = 1, \dots, N \\ 3y_{Ai} + 2y_{Bi} &\le 36, i = 1, \dots, N \\ -\beta_1 x_1 - \beta_2 x_2 + y_{Ai} + y_{Bi} &\le \beta_0 + \epsilon_{ti}, i = 1, \dots, N \\ y_A, y_B &\ge 0 \\ L_1 \le x_1 \le U_1, L_2 \le x_2 \le U_2, y_{Ai}, y_{Bi} &\ge 0 \end{aligned}$$

Methodology	x_1	x_2	MPO (in $)
SLP with SAA	182.333	17.667	40,593

Model Interpretation: Prescriptive Analysis

Sample Average Approximation: Validation

The validation steps are the same as DLP

Methodology	x_1	x_2	MPO (in $)	MVSAE	STD
SLP with SAA	182.333	17.677	$40,593	$[39,083, 40,920]	4.639

Model Interpretation: Prescriptive Analysis

Stochastic Decomposition: Model

$$
\begin{aligned}
\text{Max } & -0.1x_1 - 0.5x_2 + 3y_A + 5y_B \\
s.t. \quad & x_1 + x_2 \le 200 \\
& x_1 - 0.5x_2 \ge 0 \\
& y_A \le 8 \\
& 2y_B \le 24 \\
& 3y_A + 2y_B \le 36 \\
& -\beta_1 x_1 - \beta_2 x_2 + y_A + y_B \le \beta_0 \\
& y_A, y_B \ge 0 \\
& L_1 \le x_1 \le U_1, L_2 \le x_2 \le U_2
\end{aligned}
$$

Import all the constraints and objective specified in Deterministic Linear Programming problem as a template.

Model Interpretation: Prescriptive Analysis

Stochastic Decomposition: Model

$$-\beta_1 x_1 - \beta_2 x_2 + y_{Ai} + y_{Bi} \leq \beta_0 + \epsilon_{ti}$$

```
function mystoc()
    d = rand(rng, ei)
    binding = [Position(c4g, "RHS") => d+b0]
    return OneRealization(binding)
end;
```

Split the decision variables and constraints from first and second stage

```
user_mean = copy([b0]);
split_position = Position(c4d, ya);
solution = solve_sd(model, split_position, user_mean, mystoc)
```

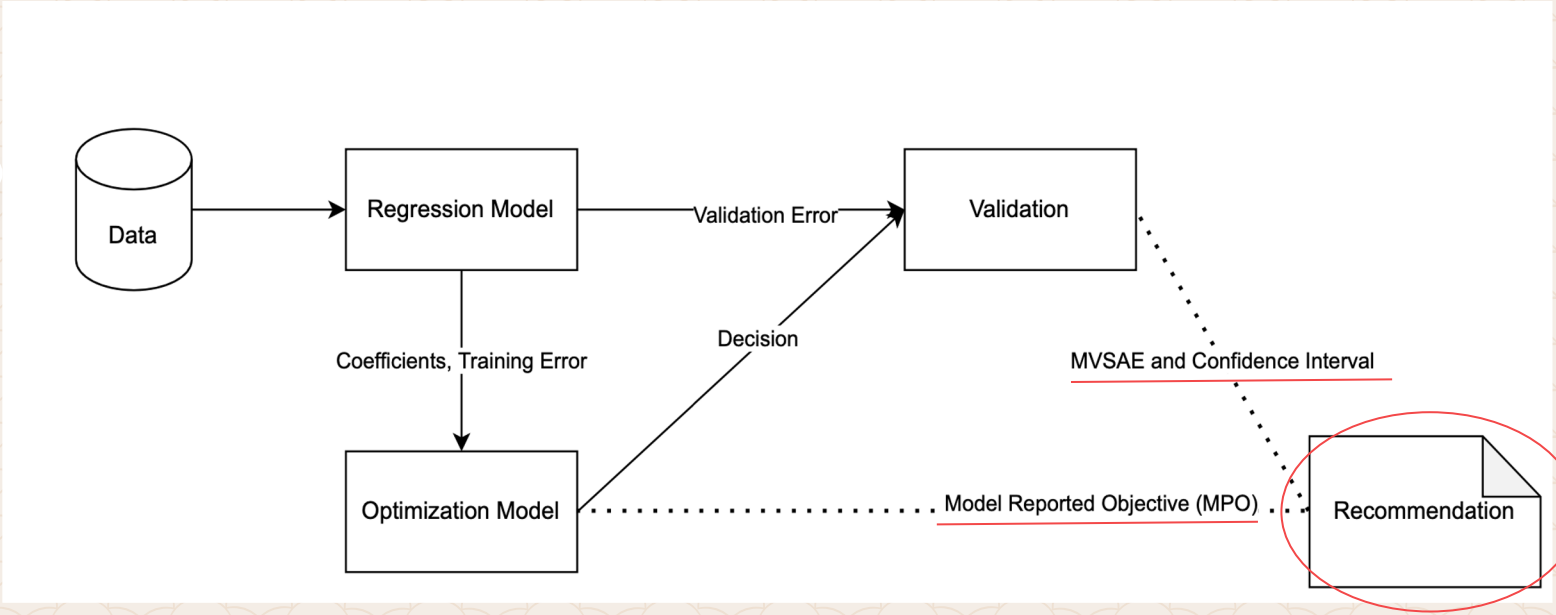
Model Interpretation: Prescriptive Analysis

Stochastic Decomposition: Validation

The validation steps are the same as DLP

Methodology	x_1	x_2	MPO (in $)	MVSAE	STD
SLP with SD	182.293	17.707	$40,860 $[40,575, 41,145]	$[39,084, 40,919]	4.633

Workflow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04

Model Comparison

Model Comparison

Methodology	x_1	x_2	MPO (in $)	MVSAE	STD
Deterministic LP	173.422	26.578	$41,369	$[38,634, 39,966]	3.364
SLP with SAA	182.333	17.677	$40,593	$[39,083, 40,920]	4.639
SLP with SD	182.293	17.707	$40,860 $[40,575, 41,145]	$[39,084, 40,919]	4.633

Observation:

- **Deterministic LP**: MPO is larger than MVSAE (out of 95% C.I.)
- **SAA & SD**: MPO fall in side of the MVSAE 95% C.I.

- **First stage decision (x_1, x_2)** of deterministic LP is quite different from SAA and SD
- **MPO** of deterministic LP is slightly larger than SAA and SD
- **Validated expected profit** of Deterministic LP is slightly lower than SAA and SD

Model Comparison

Methodology	x_1	x_2	MPO (in $)	MVSAE	STD
Deterministic LP	173.422	26.578	$41,369	$[38,634, 39,966]	3.364
SLP with SAA	182.333	17.677	$40,593	$[39,083, 40,920]	4.639
SLP with SD	182.293	17.707	$40,860 $[40,575, 41,145]	$[39,084, 40,919]	4.633

Recommendation:

- We recommend **(182, 18)** (decision of SAA and SD) over **(173, 27)** (decision of Deterministic LP).
- Prediction from SAA and SD are much closer to the validated Objective function estimates.
- Deterministic LP is over optimistic on objective.
- Compare to SAA, SD also has a slightly better variance reduction properties.

Code

https://github.com/TonyQJH/ISE562_Project3_Group2.git

Thank YOU!