Transshipment Problem

Yuan Luo, Yidan Sun, Jinhu Qi, Zhiwei Li

TABLE OF CONTENTS

01

Background

Misson, Goal, and Data

Background

Mission

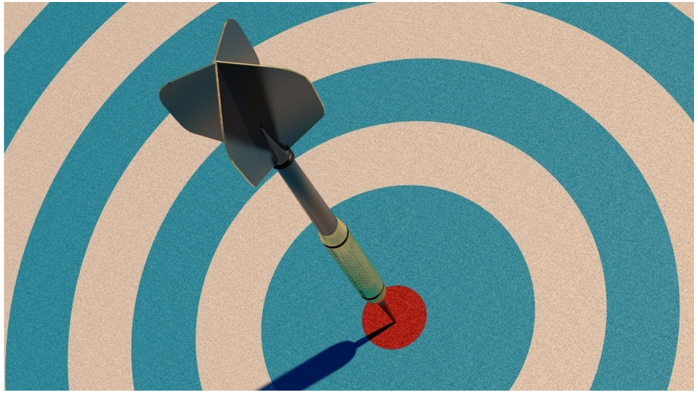
To meet customer demand and reduce expected long-run average cost by replenishment (order-up-to S) policy and transshipment policy.

Given an order-up-to S policy, we show how to determine an optimal transshipment policy, using a all-in-one linear programming and a benders decomposition model.

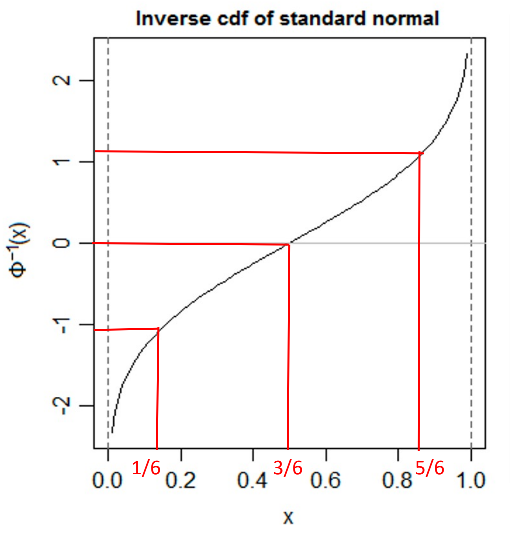
Background

Goal

Solve and report **objective** and **first stage decisions: order-up-to quantities**



Background: Data



Demand (Independent & Normally Distributed)

Location	1	2	3	4	5	6	7
Mean	100	200	150	170	180	170	170
Std Dev	20	50	30	50	40	30	50

Discretize the Normal distribution

- Low - Medium - High
- Normal density → 3 intervals with equal probability
- [1/6, 3/6, 5/6] quantile

Equal probability for each of the 3^7 scenarios.

Background: Data

Cost (Uniform)

Symbol	Unit	Meaning	Value
h	dollar/unit	Per unit holding cost when there is excess inventory at one location	1.0
c	dollar/unit	Per unit transshipment cost to move inventory from one location to satisfy demand at another location	0.5
p	dollar/unit	Per unit shortage cost when demand cannot be met at one location	4.0

Assumption

- Transshipment cost, holding cost and shortage cost are uniform across all locations

02

All-In-One LP

Two-stage stochastic, All-in-one model

All-In-One LP

1. Parameters
2. Decision Variables
3. Objective Function
4. Constraints

Two-stage Stochastic Problem

Stage 1: Find optimal order-up-to quantity

$$\begin{aligned} \min \quad & c^\top x + E[h(x, \tilde{\omega})] \\ \text{s.t.} \quad & Ax \le b \end{aligned}$$

Decision Variable: x

Stage 2: Find transshipment quantity to minimize COST

$$\begin{aligned} h(x, \omega) = \min g^\top y \\ \text{s.t.} \quad Wy = r(\omega) - Tx \\ y \ge 0 \end{aligned}$$

Decision Variable: y
Random Variable: $\boldsymbol{\omega}$

All - In - One Model

1. Parameters

1. $\mathbf{d}_{3^7 \times 7}$: Demand for 7 locations with low, medium, high scenarios
2. $\mathbf{h}$: Per unit holding cost when there is excess inventory for all locations
3. $\mathbf{p}$: Per unit transshipment cost from one location to satisfy demand at another location
4. $\mathbf{c}$: Per unit shortage cost when demand cannot be met for all locations

All - In - One Model

2. Decision Variables

1. $\mathbf{s}_{[1\times 7]}$: Order-up-to quantity for each of the 7 locations
2. $\mathbf{e}_{[3^7\times 7]}$: Ending inventory held at retailer i for scenario s
3. $\mathbf{f}_{[3^7\times 7]}$: Stock at retailer i used to satisfy demand at retailer i for scenario s
4. $\mathbf{q}_{[3^7\times 7]}$: Inventory at retailer i increased through replenishment for scenario s
5. $\mathbf{r}_{[3^7\times 7]}$: Amount of shortage met after replenishment at retailer i for scenario s
6. $\mathbf{t}_{[3^7\times 7\times 7]}$: Stock at retailer i used to meet demand at retailer j for scenario s

All - In - One Model

3. Objective

$$\min \quad \sum_s Prob_s \times (\sum_i \underbrace{h \times e_{si}}_{\text{Holding Cost}} + \sum_{i \neq j} \underbrace{c \times t_{sij}}_{\text{Shipment Cost}} + \sum_i \underbrace{p \times r_{si}}_{\text{Shortage Cost}})$$

Based on the equal probability of each scenarios

$$E[h(S, \tilde{D})] = \sum_s (\sum_i h \times e_{si} + \sum_{i \neq j} c \times t_{sij} + \sum_i p \times r_{si})/M$$

$$M = 3^7$$

All - In - One Model

4. Constraint

$$
\begin{aligned}
\text{s.t.} \quad & f_i + \sum_{j \neq i} t_{sij} + e_{si} = s_i, \forall i, s \\
& f_i + \sum_{j \neq i} t_{sji} + r_{si} = d_i, \forall i, s \\
& \sum_i r_{si} + \sum_i q_{si} = \sum_i d_{si}, \forall s \\
& e_{si} + q_{si} = s_i, \forall i, s \\
& e_{si}, f_{si}, q_{si}, r_{si}, s_i, t_{sij} \geq 0, \forall i, j, s
\end{aligned}
$$

03

Benders Decomposition

Master problem, Sub-problem, Build Cut

Benders Decomposition

1. **Solve Master Problem**

   Check for stopping criteria

2. **Solve Subproblem**

   Obtain the dual extreme points

3. **Build Benders Cut**

   Add cut to master; repeat

Benders Decomposition

1. Objective

Master problem:

$$\min_{S} \eta$$

Sub-problem:

$$\min h(S, \tilde{D})$$

$$E[h(S, \tilde{D})] = \sum_{i} h \times e_i + \sum_{i \neq j} c \times t_{ij} + \sum_{i} p \times r_i$$

The parameters and variable used in this model are the same as the all-in-one model.

Benders Decomposition

2. Constraints

Master problem

Add one more constraint (cut) in each iteration k

$$\eta \geq \alpha_k + \beta_k \times S^k$$

$$\alpha = \sum_i d_i * (\underline{\pi_{Mi}} + \underline{\pi_{Ri}})$$

$$\beta = \underline{\pi_B} + \underline{\pi_E}$$

π : The dual variables of the constraints
In the sub-problems

Sub-problem

Constraint B $$f_i + \sum_{j \neq i} t_{ij} + e_i = s_i, \forall i$$

Constraint M $$f_i + \sum_{j \neq i} t_{ji} + r_i = d_i, \forall i$$

Constraint R $$\sum_i r_i + \sum_i q_i = \sum_i d_i$$

Constraint E $$e_i + q_i = s_i, \forall i$$

$$e_i, f_i, q_i, r_i, t_{ij} \geq 0, \forall i, j$$

Benders Decomposition

3. Algorithm

In the k-th iteration:

- Solve the master problem to get a **lower bound LB_k** and a **first stage decision S_k**.
- Use the new first stage decision to solve the sub-problem and get **the coefficients (α_k, β_k)** of the new cut, and **upper bound UB_k.**
- Add **a new cut (constraint) η >= α_k + β_k * S_k** to the master problem.
- If **UB_k - LB_k <=** Stopping Criteria, then stop; else, go on to the next iteration.

Stopping Criteria: Upper Bound-Lower Bound<=1e-6

04

Results Comparison

All-in-one v.s. Bender's, All-in-one LP v.s. IP

Results Comparison
All-In-One v.s. Benders

Model	Time (second)	Objective Value	Value of s
All-In-One	6.598277	144.5956842831869	**Location 1:** 107.2557 **Location 2:** 213.3020 **Location 3:** 158.4650 **Location 4:** 183.3020 **Location 5:** 190.8835 **Location 6:** 180.8835 **Location 7:** 183.3020
Benders	99.032718（52 iterations)	144.59568428320085	**Location 1:** 99.9999 **Location 2:** 219.3484 **Location 3:** 179.0226 **Location 4:** 169.9999 **Location 5:** 180.0000 **Location 6:** 199.0226 **Location 7:** 169.9999

Results Comparison w/wo integer restriction

Model	Time (second)	Objective Value	Value of s
All-In-One	6.598277	144.5956842831869	**Location 1:** 107.2557 **Location 2:** 213.3020 **Location 3:** 158.4650 **Location 4:** 183.3020 **Location 5:** 190.8835 **Location 6:** 180.8835 **Location 7:** 183.3020
All-In-One (Int) * Decision variables are integers; Demands are rounded to integers	8.608392	143.93209876543145	**Location 1:** 103 **Location 2:** 210 **Location 3:** 159 **Location 4:** 188 **Location 5:** 190 **Location 6:** 179 **Location 7:** 188

Plan

1. 4-level scenarios: Very Low - Low - High - Very High

   [1/8, 3/8, 5/8, 7/8] quantile

Model	Time (second)	Objective Value	Value of s
All-In-One (4-level of scenarios)	\	**150.56341192779723**	**Location 1:** 106.3728 **Location 2:** 215.9320 **Location 3:** 159.5592 **Location 4:** 185.9320 **Location 5:** 192.7456 **Location 6:** 179.5592 **Location 7:** 185.9320

2. Compare the second stage decisions of Bender's method and all-in-one model.

3. Holding cost, transshipment cost, shortage cost could vary at different locations.

Code

Link:
https://github.com/YuanLuo/Transshipment.git

THANKS!

Team 2