Article

Preliminaries of Petri Nets

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1. Preliminaries

1.1. Basics of Petri nets

Petri nets (PNs) are frequently and extensively utilized to model the product flow that occurs in an automated manufacturing system (AMS). As a mathematical tool, PNs have a number of interesting and useful properties. When understood in the context of a modelled manufacturing system, these attributes enable one to determine the presence or absence of the system's functional properties. The reader is referred to [1] for an overview of PN theory and its applications. This essay briefly reviews certain definitions that are used in our research.

A Petri net system is a five-tuple $PN = (P, T, F, W, M_0)$, where $P = \{p_1, p_2, \ldots, p_m\}$ is a finite set of places ($m \geq 0$) and $T = \{t_1, t_2, \ldots, t_n\}$ is a finite set of transitions ($n \geq 0$) with $P \cap T \neq \emptyset$ and $P \cup T \neq \emptyset$. Actually, a PN is a bipartite-directed graph. From a graph viewpoint, the elements in $P \cup T$ are called nodes. The set of all directed arcs among the nodes is $F \subseteq (T \times P) \cup (P \times T)$, where the output function $T \times P \to \mathbb{N}$ determines the set of directed arcs from T to P, and the input function is $P \times T \to \mathbb{N}$, which determines the set of directed arcs from P to T, where $\mathbb{N} = \{0, 1, 2, \ldots\}$ is a set of non-negative integers. By definition, there is no directed arc between any two elements of the same type, i.e., there is no arc between any two places or any two transitions. Mapping $W : F \to \mathbb{N}$ is the weight function and $M_0 : P \to \mathbb{N}$ is the initial marking. ${}^\bullet p$ (resp., $p^\bullet$) denotes the set of input (resp., output) transitions of a place p. Likewise, ${}^\bullet t$ (resp., $t^\bullet$) denotes the set of input (resp., output) places of a transition t. $G = (P, T, F, W)$ is called a Petri net structure without any specified initial marking. A net structure G with a specified initial marking M_0 is represented by (G, M_0) that is, as defined at the very beginning, a Petri net system, i.e., $PN = (G, M_0)$. A Petri net is deemed to be connected if its graph is connected.

A transition t is said to be firable or enabled if all input places $p \in {}^\bullet t$ are marked with at least $W(p, t)$ tokens, where $W(p, t)$ is the weight of the arc to t from p. If a transition is enabled, it may fire. When an enabled transition t fires, $W(p, t)$ tokens are removed from each input place $p \in {}^\bullet t$, and $W(t, p)$ tokens are added to each output place $p \in t^\bullet$, where $W(t, p)$ is the weight of the arc to p from t. This process is represented by $M[t\rangle M'$. A marking M indicates the number of tokens in each place, which represents the current status of the modeled system. When a marking M' is reached by firing a sequence of transitions $\sigma = t_0 t_1 t_2 \ldots t_k \in T^*$ from a marking M, the process is indicated by $M[\sigma\rangle M'$, where T^* is the Kleene closure of set T. $RM(G, M_0)$ denotes the set of all reachable markings of a net system with the initial marking M_0, i.e., $RM(G, M_0) = \{M \in \mathbb{N}^{|P|} \mid \exists \sigma \in T^* : M_0[\sigma\rangle M\}$.

If there exist a place p and a transition t such that both $p \in t^\bullet$ and $p \in {}^\bullet t$ hold, the pair of p and t is termed a self-loop. If a PN has no self-loops, it is said to be *pure*. If the weight of any arc is 1, the PN is said to be *ordinary*. A net (G, M_0) is said to be k-bounded or simply bounded if, at any reachable marking $M \in RM(G, M_0)$, the number of tokens at any place p is no more than a finite number k, i.e., $M(p) \leq k$, where $k \in \mathbb{N}$ is a non-negative integer. If the number of tokens in a place p at any marking is no greater than k, the place p is said to be k-bounded. If a net system is 1-bounded, it is said to be safe. A safe place is defined as a 1-bounded place. A transition t is said to be live at the initial marking M_0 if there exists a sequence of transitions σ firable from M_0 with $M_0[\sigma\rangle M$ such that t is enabled at M. If all of the transitions are live at M_0, the net system (G, M_0) is live. A deadlock exists in a net system (G, M_0) if there is a marking $M \in RM(G, M_0)$ at which no transition is enabled. This type of marking is known as a dead marking.

1.2. S^3PR Models

This section mostly discusses the fundamental principles and ideas of a class of Petri net, S^3PR, which is a system of simple sequential processes with resources [2]. An S^3PR is a regular net that models an automated manufacturing system that produces various products sequentially utilizing diverse resource types, where each processing stage requires only one unit of a resource type, and one resource cannot participate in two or more consecutive processing stages [3].

Definition 1. A simple sequential process (S^2P) is defined as a Petri net $N = (\{p^0\} \cup P_A, T, F)$ that fulfills the following statements: 1) The idle place $p^0 \notin P_A$; 2) the set of activity places $P_A \neq \emptyset$; 3) N is a strongly connected state machine; 4) place p^0 is present in every circuit of N;

Definition 2. An (S^2P) with resources (S^2PR) is a Petri net $N = (P_A \cup \{p^0\} \cup P_R, T, F)$ that satisfies the following conditions: 1) $\forall p \in P_A, \forall t \in {}^\bullet p, \forall t' \in p^\bullet, \exists r_p \in P_R, {}^\bullet t \cap P_R = t'^\bullet \cap P_R = \{r_p\}$; 2) $P_R \neq \emptyset$; $(\{p^0\} \cup P_A) \cap P_R = \emptyset$; 3) the subnet generated from $X = \{p^0\} \cup P_A \cup T$ is an S^2P; 4) ${}^{\bullet\bullet}(p^0) \cap P_R = (p^0)^{\bullet\bullet} \cap P_R = \emptyset$; 5) $\forall r \in P_R, {}^{\bullet\bullet}r \cap P_A = r^{\bullet\bullet} \cap P_A \neq \emptyset; \forall r \in P_R, {}^\bullet r \cap r^\bullet = \emptyset$.

Definition 3. Given an S^2PR N=$(\{p^0\} \cup P_A \cup P_R, T, F)$, an initial marking M_0 is said to be an acceptable one for N if the following conditions are satisfied:

1) $M_0(p^0) \geq 1$; 2) $M_0(r) \geq 1, \forall r \in P_R$; 3) $M_0(p) = 0, \forall p \in P_A$;

Definition 4. An S^3PR, i.e., a system of S^3PR, can be formulated as follows in a recursive way: 1) An S^2PR is an S^3PR; 2) Let $N_1 = (\{p_1^0\} \cup P_{A_1} \cup P_{R_1}, T_1, F_1)$ and $N_2 = (\{p_2^0\} \cup P_{A_2} \cup P_{R_2}, T_2, F_2)$ be two S^3PR, satisfying $(\{p_1^0\} \cup P_{A_1}) \cap (\{p_2^0\} \cup P_{A_2}) = \emptyset, P_{R_1} \cap P_{R_2} = P_C \neq \emptyset$, and $T_1 \cap T_2 \neq \emptyset$. The Petri net $N = (P^0 \cup P_A \cup P_R, T, F)$ composed by N_1 and N_2 through P_C, denoted by $N = N_1 \circ N_2$, is still an S^3PR, defined as $P^0 = \{p_1^0\} \cup \{p_2^0\}$, $P_A = P_{A_1} \cup P_{A_2}$, $P_R = P_{R_1} \cup P_{R_2}$, $T = T_1 \cup T_2$, and $F = F_1 \cup F_2$.

Definition 5. Let N be an S^3PR. (N, M_0) is called an acceptably marked S^3PR if one of the following conditions is satisfied: 1) (N, M_0) is an acceptably marked S^3PR. 2) $N = N_1 \circ N_2$, where $(N_i, M_{0_i})(i = 1, 2)$ is an acceptably marked S^3PR. Moreover, for all $i \in \{1, 2\}$ and $p \in P_{A_i} \cup \{p_i^0\}, M_0(p) = M_{0_i}(p)$; for all $i \in \{1, 2\}$ and $r \in P_{R_i} \backslash P_C, M_0(r) = M_{0_i}(r)$; for all $r \in P_C, M_0(r) = \max\{M_{0_1}(r), M_{0_2}(r)\}$.

When a manufacturing system is modeled with an S^3PR, transitions in $(P^0)^\bullet({}^\bullet(P^0))$ are referred to as source (sink) transitions, representing the entry (exit) of raw materials (completed products), where $P^0 = \{p^0\}$ contains the unique idle place in the S^3PR.

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