one half the initial maximum may be hundreds of microseconds long. The duration of the wave is controlled by changing the effective value of shunt resistance in the discharge process, or by changing the value R_0 in the

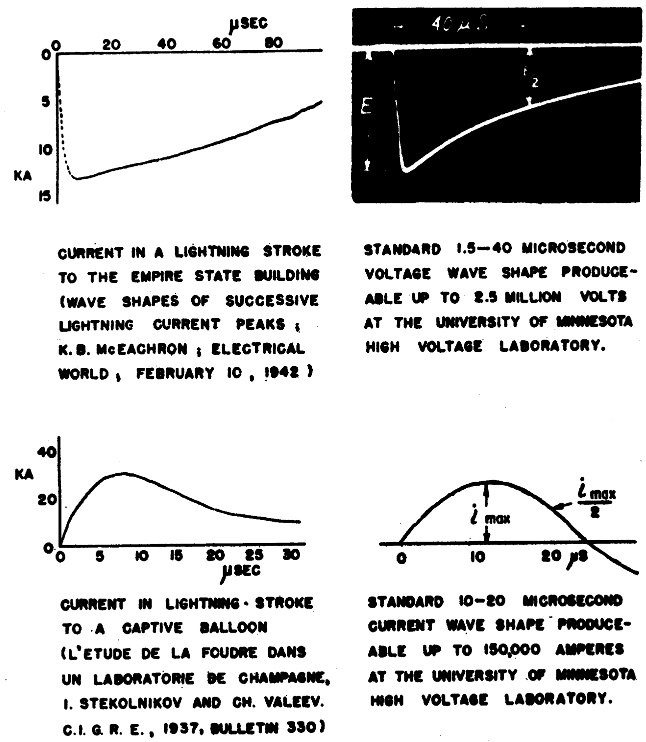

Fig. 12—Typical field records of lightning currents.

generally used impulse generator connection shown in Fig. 11(a). The equivalent circuit diagrams as given in the lower part of Fig. 11 have been proved a rather good approximation of the distributed network allowing fairly accurate calculation of the output wave shapes produced.

A comparison of typical laboratory wave shapes with similar field records of lightning surges is given in Fig. 12. By reconnecting the capacitors of the generator in parallel groupings, very high crest-current magnitudes are attainable which come quite close to duplicating field conditions. The natural-lightning record shown in Fig. 12 is quite easily reproducible, and even the greatest crest values recorded with natural lightning, of the order of 200,000 amperes, are readily possible in the laboratory. Where very large currents are encountered their duration is relatively short, and the laboratory generator with its Q of 5 coulombs in the parallel connection provides a very respectable imitation of the real thing. In cases of repeated lightning strokes or lower-current long-duration strokes where charge transfers of 100 coulombs have been observed in the field, the laboratory-lightning generator could be synchronized with a high-current power-line source at 60 cycles to duplicate pitting and burning effects of natural lightning. With such voltages and currents it is possible to duplicate fairly closely actual lightning-surge conditions.

The specially designed generator discussed in this paper has been used in the direct-current-connection series-compensating inductances incorporated directly in the lead connections between the capacitors as seen in Fig. 3. The similarity of the resultant set-up with that of the resistance-coupled impulse generator led in turn to a consideration of the possibility of a simple conversion-gap system capable of superposing surge potentials on the direct-current output voltage. The resultant combination circuit was worked out quite simply as shown in Fig. 3 and the schematic diagram of Fig. 4. The combination direct-current and impulse connection provides probably the nearest approach yet made to lightning-stress conditions where there is an electrostatic field preceding the discharge. This opens interesting possibilities for making a comprehensive study of lightning hazards in relation to aircraft[14] and studies on means of protection to minimize such hazards.

[14] J. M. Bryant and M. Newman, "Lightning discharge investigation—I," University of Minnesota Eng. Exp. Sta., Technical Paper No. 38; April, 1942.

A Note on a Simple Transmission Formula*

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***Summary*—A simple transmission formula for a radio circuit is derived. The utility of the formula is emphasized and its limitations are discussed.**

Introduction

THIS NOTE emphasizes the utility of the following simple transmission formula for a radio circuit made up of a transmitting antenna and a receiving antenna in free space:

$$P_r/P_t = A_r A_t / d^2 \lambda^2 \qquad (1)$$

where

P_t = power fed into the transmitting antenna at its input terminals.
P_r = power available at the output terminals of the receiving antenna.
(Same units of power)

A_r = effective area of the receiving antenna.
A_t = effective area of the transmitting antenna.
d = distance between antennas.
λ = wavelength.
(Same units of length)

The effective areas appearing in (1) are discussed in the next section and this is followed by a derivation of the formula and a discussion of its limitations.

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Effective Areas

The effective area of any antenna, whether transmitting or receiving, is defined for the condition in which the antenna is used to *receive* a linearly polarized, plane electromagnetic wave. The author suggests the adoption of the following definition:

$$A_{\text{eff.}} = P_r/P_0 \tag{2}$$

or

$$P_r = P_0 A_{\text{eff.}} \tag{3}$$

where P_r is the received power as defined above and P_0 is the power flow per unit area of the incident field at the antenna. In words, (3) states that the received power is equal to the power flow through an area that is equal to the effective area of the antenna. Note that the definition does not impose the condition of no heat loss in the antenna. Equation (3) shows that the effective area of an antenna is proportional to its power gain.

The effective areas of antennas of special interest are given in the following:

A. Small Dipole with No Heat Loss

For a small uniform current element the available output power is equal to the induced voltage squared, divided by four times the radiation resistance. Thus

$$P_r = E^2 a^2 / 4R_{\text{rad.}}$$

where

E = effective value of the electric field of the wave.
a = length of the current element.
$R_{\text{rad.}}$ = radiation resistance of the current element ($R_{\text{rad.}} = 80\pi^2 a^2/\lambda^2$).[1]

Since the power flow per unit area is equal to the electric field squared divided by the impedance of free space, i.e., $P_0 = E^2/120\pi$, we have

$$A_{\text{dip.}} = P_r/P_0 = 3\lambda^2/8\pi = 0.1193\lambda^2. \tag{4}$$

The effective area of a half-wavelength dipole with no heat loss is only 9.4 per cent, 0.39 decibels,[2] larger than the effective area of the small dipole. Therefore

$$A_{0.5\lambda} = 0.1305\lambda^2. \tag{5}$$

The area of a rectangle with one-half wavelength and one-quarter wavelength sides is $0.125\lambda^2$ and it is, therefore, a good approximation for the effective areas of small dipoles and half-wavelength dipoles.

B. Isotropic Antenna with No Heat Loss

The hypothetical isotropic antenna has the same radiation intensity in all directions. It has two thirds of the gain[3] or effective area of the small dipole. Therefore

$$A_{\text{isotr.}} = \lambda^2/4\pi. \tag{6}$$

C. Broadside Arrays (Pine-Tree Antennas)

The effective area of an antenna array made up of a curtain of rows of half-wave dipoles spaced half a wavelength was calculated several years ago by the method of Pistolkors.[4] Equal amplitude and phase of the currents in all the dipoles and no heat loss were assumed. The effective area of such an array with a reflector that doubled the gain was found to be approximately equal to the actual area occupied by the array; thus

$$A_{\text{pine-tree}} \approx n \times 0.5\lambda \times 0.5\lambda \tag{7}$$

where n is the total number of half-wave dipoles in the front curtain. Formula (7) is a good approximation for large antennas. For example, an antenna of 6 rows of 17 dipoles each gave a calculated effective area only 3 per cent below the value obtained by (7). It should be pointed out that the heat loss in the connecting transmission lines will reduce the effective areas in actual antennas.

D. Parabolic Reflectors

The effective area of the parabolic type of antenna with a proper feed has been found experimentally to be approximately two thirds of the projected area of the reflector.

E. Electric Horns—Aperture Sides$\gg\lambda$

The effective area of a very long horn with small aperture dimensions is 81 per cent of the area of the aperture. For an optimum horn, where the aperture is dimensioned to give maximum gain for a given length of the horn, the effective area is approximately 50 per cent of the area of the aperture.[5]

Derivation of Transmission Formula (1)

Having defined the effective area of an antenna, it is a simple matter to derive (1). As shown in Fig. 1, consider a radio circuit made up of an isotropic transmitting

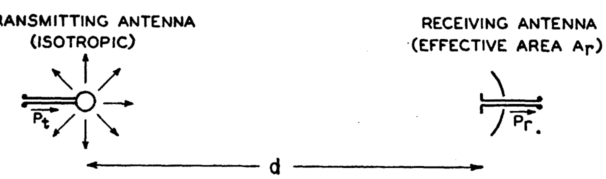

Fig. 1—Free-space radio circuit.

antenna and a receiving antenna with effective area A_r. The power flow per unit area at the distance d from the transmitter is

$$P_0 = P_t/4\pi d^2. \tag{8}$$

Assuming a plane wave front at the distance d, definition (2) for the effective area and formula (8) give

$$P_r/P_t = A_r/4\pi d^2. \tag{9}$$

Replacing the isotropic transmitting antenna in the

[1] S. A. Schelkunoff, "Electromagnetic Waves," D. Van Nostrand Company, Inc., New York, N. Y., 1943, p. 134, equation (3-3).
[2] See p. 341 of footnote reference 1.
[3] See p. 337, equation (5-2), of footnote reference 1.
[4] A. A. Pistolkors, "The radiation resistance of beam antennas," Proc. I.R.E., vol. 17, pp. 562–579; March, 1929. See also Table I in "Report of Radio Research in Japan," vol. 3, no. 1, June, 1933.
[5] See pp. 364 and 365 of footnote reference 1.

illustration with a transmitting antenna with effective area A_t will increase the received power by the ratio $A_t/A_{\text{isotr.}}$, and we obtain

$$P_r/P_t = A_r A_t / 4\pi d^2 A_{\text{isotr.}} \tag{10}$$

Introducing the effective area (6) for the isotropic antenna, we have (1).

Limitations of Transmission Formula (1)

In deriving (1), a plane wave front was assumed at the distance d. Formula (1), therefore, should not be used when d is small. W. D. Lewis, of these Laboratories, has made a theoretical study of transmission between large antennas of equal areas with plane phase fronts at their apertures and he finds that (1) is correct to within a few per cent when

$$d \geqq 2a^2/\lambda \tag{11}$$

where a is the largest linear dimension of either of the antennas.

Formula (1) applies to free space only, a condition which designers of microwave circuits seek to approximate. Application of the formula to other conditions may require corrections for the effect of the "ground," and for absorption in the transmission medium, which are beyond the scope of this note.

The advantage of (1) over other formulations is that, fortunately, it has no numerical coefficients. It is so simple that it may be memorized easily. Almost 7 years of intensive use has proved its utility in transmission calculations involving wavelengths up to several meters, and it may become useful also at longer wavelengths. It is suggested that radio engineers hereafter give the radiation from a transmitting antenna in terms of the power flow per unit area which is equal to $P_t A_t/\lambda^2 d^2$, instead of giving the field strength in volts per meter. It is also suggested that an antenna be characterized by its effective area, instead of by its power gain or radiation resistance.[6] The ratio of the effective area to the actual area of the aperture of an antenna is also of importance in antenna design, since it gives an indication of how efficiently the antenna is utilizing the physical space it occupies.

[6] The directional pattern, which has not been discussed in this note, is, of course, always an important characteristic of an antenna

Nonlinearity in Frequency-Modulation Radio Systems Due to Multipath Propagation*

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Summary—**A theoretical study is made to determine the effects of multipath propagation on over-all transmission characteristics in frequency-modulation radio circuits. The analysis covers a simplified case where the transmitted carrier is frequency-modulated by a single modulating frequency and is propagated over two paths having relative delay and amplitude differences. Equations are derived for the receiver output in terms of the transmitter input for fundamental and harmonics of the modulating frequency. Curves are plotted and discussed for various values of relative carrier- and signal-frequency phase shift and relative amplitude difference of the received waves.**

The results show that a special kind of amplitude nonlinearity is produced in the input-output characteristics of an over-all frequency-modulation radio system. Under certain conditions, sudden changes in output-signal amplitude accompany the passage of the input-signal amplitude through certain critical values. Transmission irregularities of this type are proposed as a possible explanation of so-called "volume bursts" sometimes encountered in frequency-modulation radio circuits. In general, it appears that amplitude and frequency distortion are most severe where the relative delay between paths is large and the amplitude difference is small.

THE INFLUENCE of multipath propagation on the transmission properties of frequency-modulation radio circuits is of considerable interest. The subject has been treated at some length in previous papers from both experimental and theoretical standpoints.[1,2,3] It is the purpose here to extend the theoretical side in an effort to obtain a clearer understanding of the true nature of the over-all circuit transmission changes induced by multipath propagation. Experimental support of the conclusions has not been obtained, due to the lack of time and facilities brought on by the pressure of war work.

Many of the causes of multiple paths over which radio waves sometimes travel from transmitter to receiver are well known and need not be recounted here. It is sufficient to state that when these paths exist simultaneously and are of different lengths and time of travel, interference at the receiver takes place between arriving waves. This interference is manifest by alterations in the amplitude and phase-versus-frequency characteristics of the resultant received wave as compared to the wave which is transmitted. In all types of radio systems such alterations in the received wave usually result in a

* Decimal classification: R630.11. Original manuscript received by the Institute, November 13, 1945.

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[1] Murray G. Crosby, "Frequency-modulation-propagation characteristics," Proc. I.R.E., vol. 24, pp. 898–913; June, 1936.

[2] Murray G. Crosby, "Observations of frequency-modulation propagation on 26 megacycles," Proc. I.R.E., vol. 29, pp. 398–403; July, 1941.

[3] Murlan S. Corrington, "Frequency-modulation distortion caused by multipath transmission," Proc. I.R.E., vol. 33, pp. 878–891; December, 1945.