EECS101 Exam #1 February 6, 2018

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This is an 80 minute, CLOSED BOOK exam. Calculators and phones are not allowed. Show all of your work. GOOD LUCK!

Question 1:

Question 2:

Question 3:

Question 4:

Question 5:

Question 6:

Question 7:

Question 8:

Question 9:

Question 10:

TOTAL:

Question 1 (10 points) For the binary images shown, let 1 denote object and 0 denote background.

4 a) For the binary image below, circle the 4-connected object (1) regions.

1	1	0	1	1	1	1	0
1	0	1	0	0	0	0	0
0	1	0	0	1	0	1	0
1	1	0	1	1	1	1	0
1	0	1	0	1	1	0	0
1	0	0	1	0	1	1	0
0	1	1	1	0	0	0	0
1	1	1	1	1	1	0	1

1 b) How many 4-connected object (1) regions are there in the image for part a? 8

4 c) For the binary image below, circle the 8-connected object (1) regions.

0	0	0	0	0	1	0	1
1	0	1	0	0	0	0	0
0	1	0	0	1	0	1	0
1	0	0	1	0	1	1	0
0	1	1	1	0	0	0	0
1	1	1	1	1	1	0	1
1	0	1	0	1	1	1	0
1	0	1	0	1	1	0	0

1 d) How many 8-connected object (1) regions are there in the image for part c? 3

Question 2 (10 points) Given the gray level image below, circle the largest connected region (assume 8-connectedness) so that the mean μ of the pixels in the region is $\mu = 5$ and the variance σ^2 of the pixels in the region satisfies $\sigma^2 \leq 2$. Show that μ and σ^2 for your region satisfy these conditions. Define the variance σ^2 of a set of N pixel values $I_1, I_2, \ldots, I_N$ by

$$\sigma^2 = \frac{1}{N}\sum_{j=1}^{N}(I_j - \mu)^2$$

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1	2	3	4	5	6	7	8
1	2	3	4	5	6	7	8
1	2	3	4	5	6	7	8
1	2	3	4	5	6	7	8

$$\mu = \frac{4*3+4*4+4*5+4*6+4*7}{20} = 5$$ 2

$$\sigma^2 = \frac{1}{20}\left[4(3-5)^2 + 4(4-5)^2 + 4(5-5)^2 + 4(6-5)^2 + 4(7-5)^2\right] = 2$$ 2

4 **Question 3 (4 points)** What does it mean for a point P in the world to be out of focus in an image?

A point P is out of focus if light leaving P images to more than one point in the image plane.

Sobel $\frac{\partial E}{\partial x} = \begin{matrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{matrix}$ $\frac{\partial E}{\partial y} = \begin{matrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{matrix}$

Question 4 (9 points) Consider the small image with 20 pixels shown below.

y				
5	10	20	30	40
4	10	20	30	40
3	10	20	30	40
2	10	20	30	40
1	10	20	30	40
	1	2	3	4

x

For each point in the image that is not a boundary point, use the Sobel mask to compute $\frac{\partial E}{\partial x}$, $\frac{\partial E}{\partial y}$, and the squared gradient magnitude. Enter the values that you computed in the table below.

$\frac{1}{2}$ point for each correct answer

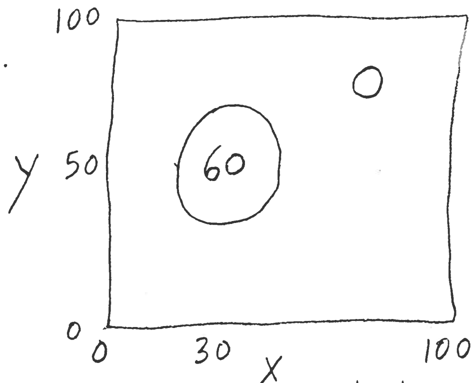
(x,y)	$\frac{\partial E}{\partial x}$	$\frac{\partial E}{\partial y}$	S.G.M.
(2,2)	80	0	6400
(2,3)	80	0	6400
(2,4)	80	0	6400
(3,2)	80	0	6400
(3,3)	80	0	6400
(3,4)	80	0	6400

Question 5 (14 points) A continuous image is defined over $0 \le x \le 100, 0 \le y \le 100$ by

$$E(x, y) = 60U\left(100 - (x - 30)^2 - (y - 50)^2\right)$$

where U is the unit step function. Note that the equation of a circle of radius r centered at $(0, 0)$ is $x^2 + y^2 = r^2$.

2 a) Draw the image $E(x, y)$.



Circle is $(x-30)^2 + (y-50)^2 = 100$
center $(30, 50)$
radius 10

b) Find $\frac{\partial E}{\partial x}$. For what point(s) (x, y) in the image is $\left|\frac{\partial E}{\partial x}\right|$ the largest?

$$\frac{\partial E}{\partial x} = 60(-2)(x-30)\delta(100-(x-30)^2-(y-50)^2)$$
$$= -120(x-30)\delta(100-(x-30)^2-(y-50)^2)$$ 2

$\left|\frac{\partial E}{\partial x}\right|$ is largest for $x = 20$ and $x = 40$

Points are $(20, 50)$ and $(40, 50)$ 2

c) Find $\frac{\partial E}{\partial y}$. For what point(s) (x, y) in the image is $\left|\frac{\partial E}{\partial y}\right|$ the largest?

$$\frac{\partial E}{\partial y} = 60(-2)(y-50)\delta(100-(x-30)^2-(y-50)^2)$$
$$= -120(y-50)\delta(100-(x-30)^2-(y-50)^2)$$ 2

$\left|\frac{\partial E}{\partial y}\right|$ is largest for $y = 40$ and $y = 60$

Points are $(30, 40)$ and $(30, 60)$ 2

d) Find the squared gradient magnitude (SGM) for the image. For what point(s) (x, y) in the image is the SGM the largest?

2 $$SGM = \left(\frac{\partial E}{\partial x}\right)^2 + \left(\frac{\partial E}{\partial y}\right)^2 = 120^2\left[\delta(100-(x-30)^2-(y-50)^2)\right]^2\left[(x-30)^2+(y-50)\right]$$

The SGM has the same maximum value for all points on the circle $(x-30)^2 + (y-50)^2 = 100$ 2

Question 6 (13 points) Consider the imaging system used in class having a pinhole at the origin with the z axis horizontal (positive z to the left), the y axis vertical (positive y up), and the x axis forward (positive x out). The image plane is $z = f' > 0$ and points in the scene are located in the space $z < 0$.

2 a) Write the perspective projection equations for mapping a point (x, y, z) in the scene to a point (x', y') in the image plane.

$$x' = \frac{f'x}{z} \qquad y' = \frac{f'y}{z}$$

2 b) Write the orthographic projection equations for mapping a point (x, y, z) in the scene to a point (x', y') in the image plane.

$$x' = x \qquad y' = y$$

3 c) Suppose that a point P_1 in the scene has a perspective projection of $(x', y') = (6, 8)$. Find all possible locations (x, y, z) for the point P_1.

$$6 = \frac{f'x}{z} \qquad 8 = \frac{f'y}{z}$$

Possible Locations $(x, y, z) = \left(\frac{6z}{f'}, \frac{8z}{f'}, z\right)$ for all $z < 0$

3 d) Suppose that a point P_2 in the scene has a perspective projection (x', y') that is the same as the orthographic projection of P_2. Find all possible locations (x, y, z) for the point P_2.

for x' have $\frac{f'x}{z} = x$ for y' have $\frac{f'y}{z} = y$

Require $\frac{f'}{z} = 1$ or $(x, y) = (0, 0)$

f'/z can not be 1 since $f' > 0$ and $z < 0$

Possible Locations $(x, y, z) = (0, 0, z)$ for all $z < 0$

3 e) Ignoring the size and possible inversion of objects in an image, when will orthographic projection be a good approximation to perspective projection.

Orthographic projection is a good approximation to perspective projection when the z-distance to surfaces in the scene is much larger than the variation in z-distances among surfaces in the scene.

Question 7 (10 points) Suppose that in an imaging system using a lens the focal length of the lens is 5cm and the image plane is a distance 7.5cm behind the lens.

3 a) How far in front of the lens on the optical axis of the system must we place a point to get a perfectly focused image of the point?

$$\frac{1}{5} = \frac{1}{7.5} + \frac{1}{o} \qquad o = 15cm$$

b) If a blur diameter of 0.5cm is allowable and the lens diameter is 2cm, how far can we move the point in focus towards the lens before we exceed the allowable blur diameter?

$$b = \frac{d(\bar{z}' - z')}{\bar{z}'} \qquad 0.5 = \frac{2(\bar{z}' - 7.5)}{\bar{z}'} \rightarrow \bar{z}' = 10\,cm \quad 3$$

4

$$\frac{1}{5} = \frac{1}{\bar{z}'} + \frac{1}{\bar{z}} \qquad \frac{1}{5} = \frac{1}{10} + \frac{1}{-\bar{z}} \rightarrow \bar{z} = -10cm$$

We can move the point in focus 5cm (from $\bar{z} = -15$ to $\bar{z} = -10$)

Question 8 (11 points) a) What four conditions describe the regions $R_1, R_2, \ldots, R_N$ in an effectively segmented image?

8

1) $\text{Image} = \bigcup_{K=1}^{N} R_K$ 2) $R_i \cap R_j = \phi \quad i \neq j$

3) $H(R_i) = \text{TRUE}$ for all i

4) $H(R_i \cup R_j) = \text{FALSE}$ for all adjacent regions R_i and R_j

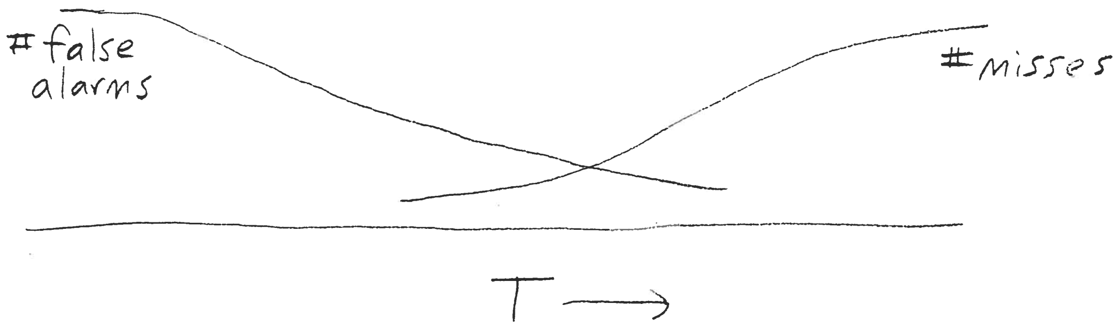
where $H(\cdot)$ is a homogeneity measure

b) If we assign each pixel to a separate region R_i which condition will this proposed segmentation usually violate?

3 This will usually violate condition 4.

Question 9 (8 points) a) Suppose that we threshold the squared gradient magnitude for edge detection. Draw a typical plot of the number of false alarms and the number of misses as a function of the threshold T for the case where any threshold chosen will lead to errors where an error is defined to be either a false alarm or a miss.

5



b) Explain how to best choose the threshold T for a given application.

3

In an application there will be some cost associated with false alarms and misses. T should be chosen to minimize this cost.

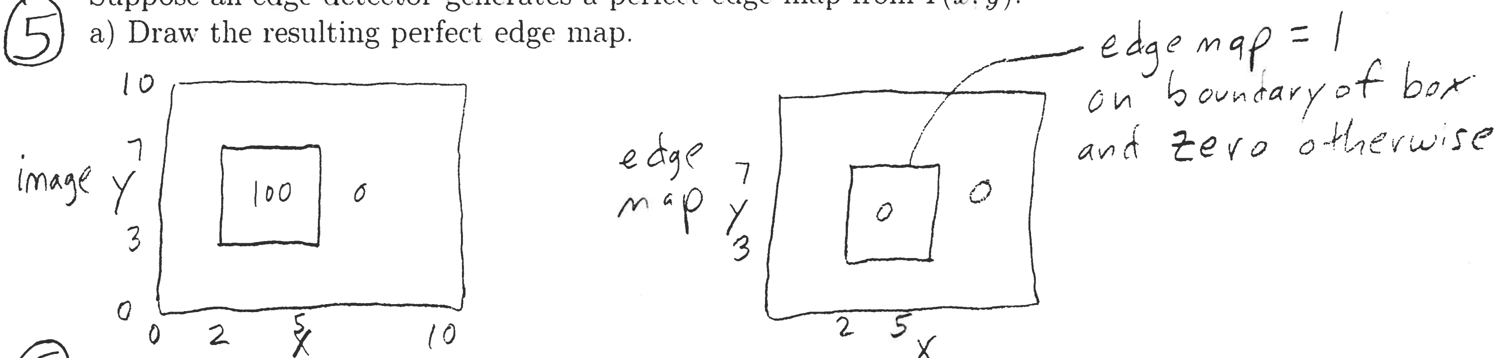
Question 10 (11 points) Consider an image $I(x, y)$ of a bright box on a dark background defined over the coordinates $0 \leq x \leq 10, 0 \leq y \leq 10$ so that

If $(2 \leq x \leq 5$ AND $3 \leq y \leq 7)$ then $I(x, y) = 100$, otherwise $I(x, y) = 0$

Suppose an edge detector generates a perfect edge map from $I(x, y)$.

5

a) Draw the resulting perfect edge map.



6

b) Assume that lines in this image are represented according to $x sin\theta - y cos\theta + \rho = 0$. Show where maxima will occur in the (ρ, θ) space when using the Hough transform for finding lines.

line	θ	ρ
$x = 2$	$\pi/2$	-2
$x = 5$	$\pi/2$	-5
$y = 3$	0	3
$y = 7$	0	7

4 peaks