

$$\text{perplexity} = \prod_{n=1}^N \left(\frac{1}{P_{LM}(x^{(n+1)} | (x^{(1)} \dots x^{(n)}))} \right)^{\frac{1}{N}}$$

$$= \prod_{n=1}^N \left(\frac{1}{y_{x_{n+1}}^{(n+1)}} \right)^{\frac{1}{N}}$$

$$= \exp \left(\frac{1}{N} \sum_{n=1}^N -\log y_{x_{n+1}}^{(n+1)} \right)$$

$$= \exp \left(\frac{\text{total loss}}{\text{number of predictions}} \right)$$