Executive Summary

This report presents an integer programming model (extended model) to solve the problem of allocating children to kindergartens in Norway (Geitle et al., 2021). The extended model not only considers the children's prioritized choice of kindergartens, siblings being placed in kindergarten, and travel time, but also takes into account the gender and age balance in kindergarten and set stronger constraints for travelling time of each child. The main objectives of this models are to minimize the sum of penalties not allocated to each child's most preferred choice of kindergarten.

After discussing the strengths, limitations, and other aspects of the approach, this report simulates a reasonable size of city where there are some kindergartens and children and then test the basic model using different generated test datasets. The results show that if the capacity of kindergartens is sufficient, every child will be allocated. Among those children who have been allocated, over 90% of them are assigned to their top 3 priorities and almost all of them are with their siblings. Even though they are placed in priority kindergartens, the maximum distance of the allocated distance is no further than 2 km. Besides, almost every kindergarten has achieved their gender and age targets. This indicates that the extended model is very robust with the children's prioritized choice of kindergartens, siblings being placed in kindergarten, and the gender and age targets of each kindergarten.

I. Introduction

Before children are allocated to the specific kindergartens, their parents are asked to provide information about their kids' specific situations (i.e., age, gender, etc.), and then the related personnel will develop allocation plans based on it. During this process, how to balance all aspects of needs is what we focus on in this report. To be more specific, we need to meet children's priority choices and maintain gender/age balance of kindergartens. Besides, we also need consider children's priorities, which means not only do we need to consider the factors mentioned before but attempt to allocate these children to their preferred kindergartens.

This problem can be defined as an OR allocation problem, which can be solved by introducing weighting factors as penalty to minimize the objectives where our targets are violated.

The rest of this report will introduce an integer model in Section II and discuss the model in Section III. In Section IV, computational experiments of the models will be conducted, and the results will be shown and discussed. Section V concludes the whole report.

II. Two Integer Models

Parameters

C	All children	C^S	Children with special requirements
C^H	Children without special requirements	C^Y	Young children
C^O	Old children	K^D	A dummy kindergarten
K	All available kindergartens	k	The index of kindergarten
Q^Y	Kindergartens only accepting young children (0-2 years old)	K^O	Kindergartens only accepting old children (3-5 years old)
D	All weekdays	d	A weekday
$\overline{Q}_{kd}$	The capacity of children of kindergarten k for weekday d	O_{cd}	Equals 1 if child c wants kindergarten placement on weekday d, 0 otherwise.
P_{ck}^{m}	Equals 1 if child c ranks kindergarten k	K_c^m	The m^{th} kindergarten of child c (if $P_{ck}^{v} =$

	at the mth choice, 0 otherwise.		$1,\ K_c^v = k\ \ v = 1,2,3)$
S_{ck}	Equals 1 if child c has siblings in kindergarten k, 0 otherwise.	T_{ck}^W	Equals 1 if the traveling time between child c and kindergarten k is classified as walking distance, 0 otherwise.
T_{ck}^D	Short driving distance	T_{ck}^F	Driving distance farther than D
W_1	The penalty for child that does not get a place in any kindergarten (K^D)	W_2	The penalty for children not getting their second priority
W_3	The penalty for children not getting their third priority	W_4	The penalty for children not getting any of their priorities
W_5	The penalty for children not in the same kindergarten of their siblings	W_6	The penalty for children placed in kindergartens outside walking distance
W_7	The penalty for children placed in kindergartens outside driving distance	W_8	The penalty while gender target is violated
W_9	The penalty while age target is violated	W_{10}	The penalty while traveling time target is violated

Variables

X_{ck}	Equals 1 if child c is placed in kindergarten k, 0 otherwise	q_c	Equals 1 if child c is placed in a different kindergarten than its siblings, equals 0 if child c is placed in the same kindergarten as its siblings or child c does not have a sibling
δ_k^m	Equals 1 if all children have kindergarten k as the m^{th} choice are placed in k, 0 otherwise	δ_k	Equals 1 if $\delta_k^1 = \delta_k^2 = \delta_k^3 = 1$, 0 otherwise
G_c	Equals 1 if child is male, 0 otherwise	$\overline{G}_k$	Upper limit on the number of the same gender in kindergarten k
$\underline{G}_k$	lower limit on the number of the same gender in kindergarten k	g_k	Equals 1 if gender target in kindergarten k is violated, 0 otherwise
Y_{oa}	Equals 1 if child c is of age a, 0 otherwise	$\underline{L}_{ka}$	Lower limits of children of age a that can be placed in kindergarten k
$\overline{L}_{ka}$	Upper limits of children of age a that can be placed in kindergarten k	a_k	Equals 1 if age target of kindergarten k is violated, 0 otherwise
T_{ck}	The travel time for child c to go to kindergarten k	t^{max}	The upper limit on traveling time

Extended Model

The objective is to minimize the total penalty, i.e., try to satisfy children's priorities as well as assign kindergartens to them which are not too far from their homes and achieve balance between gender, age and traveling time. The objective function can be expressed as follow.

$$
\begin{aligned}
\min Z\colon \sum_{c\in C} \Bigg(& W_1 X_{ck^D} + W_2 X_{ck_c^2} + W_3 X_{ck_c^3} + W_4 \Bigg(\sum_{k\in K\setminus\{K_c^1,K_c^2,K_c^3,K_c^D\}} X_{ck} \Bigg) + W_5 q_c \\
& + W_6 \sum_{k\in K\setminus\{K_c^1,K_c^2,K_c^3,K_c^D\}} T_{ck}^D X_{ck} \quad + W_7 \sum_{k\in K\setminus\{K_c^1,K_c^2,K_c^3,K_c^D\}} T_{ck}^F X_{ck} \Bigg) \\
& + W_8 \sum_{k\in K\setminus\{K^D\}} g_k + W_9 \sum_{k\in K\setminus\{K^D\}} a_k + W_{10}\, t^{max}
\end{aligned}
$$

Constraint (1) means a child can only be placed in one kindergarten or not placed at all. (2) means the total number of children allocated on the given weekday should not exceed the kindergarten's capacity. (3) means if child c is placed in a different kindergarten than its siblings, $q_c = 1$. (4) means that δ_k^1 can only be 1 if all children with special requirements that have put kindergarten k as their first priorities are placed in kindergarten k. (5) means δ_k^2 can only be 1 if all children with special requirements that have put kindergarten k as their second priorities are placed in kindergarten k or their first priorities. (6) is like the previous one, and for this time, children are placed in kindergarten k or their first/second priorities. (7) means that only when $\delta_k^1 = \delta_k^2 = \delta_k^3 = 1$, can δ_k equal to 1. (8) means if not all the children with special requirements wanting a kindergarten are satisfied, children without special requirements should not be allocated to this kindergarten. Constraint (9) calculates the minimum number of the minority gender. (10) means that for old children, if gender target of kindergarten was violated, upper limit for the majority gender would increase to the amount of the kindergartens' capacity. (11) means that for old children, if gender target of kindergarten is not violated, the number of the minority gender should be higher than the lower limit, otherwise, it should at least be non-negative. (12), (13) and (14) are like gender balance, which discuss changes to the upper as well as lower limit when age balance is violated or not. Although we have already discussed distance in the basic model, it is still necessary to introduce an upper limit on the longest traveling time to avoid that some children are placed in the kindergartens which are too far from their home (15).

$$\sum_{k \in K} X_{ck} = 1 \quad c \in C \tag{1}$$

$$\sum_{c \in C} O_{cd} X_{ck} \leq \overline{Q}_{kd} \quad k \in K \setminus \{K^D\}, d \in D \tag{2}$$

$$\sum_{k \in K \setminus \{K^D\}} S_{ck}(1 - X_{ck}) \leq q_c \quad c \in C \tag{3}$$

$$\sum_{c \in C^S} P_{ck}^1 \delta_k^1 \leq \sum_{c \in C^S} X_{ck} \quad k \in K \setminus \{K^D\} \tag{4}$$

$$\sum_{c \in C^S} P_{ck}^2 \delta_k^2 \leq \sum_{c \in C^S} P_{ck}^2 \left(\sum_{k' \in K \setminus \{K^D\}} (P_{ck'}^1 X_{ck'}) + X_{ck} \right) \quad k \in K \setminus \{K^D\} \tag{5}$$

$$\sum_{c \in C^S} \left(P_{ck}^3 (\delta_k^3 - \sum_{k' \in K \setminus \{K^D\}} ((P_{ck'}^1 + P_{ck'}^2) X_{ck'} - X_{ck}) \right) \leq 0 \quad k \in K \setminus \{K^D\} \tag{6}$$

$$3\delta_k \leq \delta_k^1 + \delta_k^2 + \delta_k^3 \quad k \in K \tag{7}$$

$$X_{ck} \leq \delta_k \quad c \in C^H, k \in K \tag{8}$$

$$M = \overline{Q}_{kd} - \overline{G}_k \tag{9}$$

$$\sum_{c \in C}^{O_{cd}} G_c X_{ck} \leq \overline{G}_k + M g_k \quad k \in K, d \in D \tag{10}$$

$$\sum_{c \in C}^{O_{cd}} G_c X_{ck} \geq \underline{G}_k (1 - g_k) \quad k \in K, d \in D \tag{11}$$

$$M = \overline{Q}_{kd} - \overline{L}_{ka} \tag{12}$$

$$\sum_{c \in C} O_{cd} Y_{ca} X_{ck} \leq \overline{L}_{ka} + M a_k \quad k \in K, d \in D, a \in A \tag{13}$$

$$\sum_{c \in C} O_{cd} Y_{ca} X_{ck} \geq \underline{L}_{ka} (1 - a_k) \quad k \in K, d \in D, a \in A \tag{14}$$

$$\sum_{k \in K \setminus \{K_c^1, K_c^2, K_c^3, K_c^D\}} T_{ck} X_{ck} \leq t^{max} \quad c \in C \tag{15}$$

III. Discussions

Strengths and limitations of the model

Strengths: This model has the advantages of robustness, strong interpretation, and fast to get results. The model considers not only children's priorities, but also more soft conditions, i.e., parents suffer from the problem of children in the different kindergartens as their siblings. Besides, the model limits the computing time to three hours and proposes a heuristic variable reduction scheme to solve the largest instances in Norway, which reduces the computation time considerably and achieve a solution very near to the optimum, i.e., most children can meet their top 3 priorities.

Limitations: The model takes into account the presence of allocated siblings but does not consider how to allocate the children if the family has more than one child who needs to attend kindergarten at the same time. Moreover, how to allocate children if there is more than one allocated sibling and they are in different kindergartens is also a problem. It did not consider the relationship between the two children, who may come from the same family.

Reasonableness of the model

Assumptions: the model has the following main assumptions to capture the complex situation：

- Children need be placed in a priority kindergarten, where the higher priority is the better.
- Children need be placed in the same kindergarten as their sibling.

- Travel times are divided into three zones: walking, short driving and far driving distance.
- The priority of a child with special requirements is determined by government data, while a child without special requirements is randomly assigned a kindergarten as his/her priority.

The assumptions of this paper are almost all reasonable, but a child without special requirement is randomly assigned to a kindergarten sounds less convincing. It is better to give all children a list of priorities for the sake of fairness. For these unreasonable assumptions, we have improved the model by giving each child a score for each kindergarten, which considers the distance and the hardware of each kindergarten and will choose three kindergartens with the highest scores as their priorities.

The model does a good job for the criteria for the choice of approach, i.e., the degree to which the model meets the priority of each child and the age and gender target of each kindergarten.

Implications and savings

Based on the result we obtained, it is easy to find out that our model can make sure all the children are allocated when the number of children is less than all kindergartens' total capacity. However, our model might not be able to output a final solution with some actual data and if this situation happened, we should combine other methods to make sure that all the children are allocated or make some relaxations to our original constraints.

Since the solutions we obtain generally satisfy our constraints, allocators (i.e., the government) can avoid extreme situations (i.e., lots of children are allocated to the kindergartens which are too far from their home) and therefore meet the needs of most children and their parents which can increase this crowd's satisfaction degree towards the government and achieve fair allocation to a greater extent. At the same time, it is obvious that the government would pay extra for people testing the solutions by manpower only without our model, which could cause additional expenses.

Extensions

Except the factor we have already discussed above, family's economic condition can also be a problem that we should take into consideration.

IV. Computational Experiments

Hypothetical Data

Assumptions

1. All kindergartens are open and admit the same number of children every weekday.
2. All children are required to attend kindergartens from Monday to Friday.
3. All children have special requirements (3 priority kindergartens) for the sake of fairness.
4. The allocation of every child is independent of each other.

Information about Kindergartens

Indicators	Explanation	Realization
Q_k	Capacity of each kindergarten	Normal distribution in [20,50]
G_k	Quality of each kindergarten	Normal distribution in {3,4,5}
$\overline{L}_{ka}$	Age upper limit of each kindergarten	$\frac{3}{10}Q_k$
$\underline{L}_{ka}$	Age lower limit of each kindergarten	$min(\frac{1}{10}Q_k, \frac{\#Children}{10 * \#Kindergartens})$
$\overline{G}_k$	Gender upper limit of each kindergarten	$\frac{3}{4}Q_k$

$\underline{G}_k$	Gender lower limit of each kindergarten	$min(\frac{1}{4}Q_k, \frac{\#Children}{4 * \#Kindergartens})$

Information about Children

Indicators	**Explanation**	**Realization**
Ac	Age of each child	Uniform distribution in {1,2,3,4,5}
Xc	Gender of each child	Binomial distribution with $p = 0.5$
Sc	Binary: 1 if c has a sibling, 0 otherwise.	Random 10% of children have a sibling who attends a specific kindergarten.
KSc	Integer: kindergarten k of c's sibling	If $Sc = 1$, then $KSc = k.$ Random 60% k in walking distance (W), 35% driving (D), 5% far (F).
K_c^v	Integer: $v = 1,2,3$ The v^{th} priority kindergarten of c.	K_c^v is chosen by a ranking system, where each child ranks all kindergartens by the following formula: $F = -\text{distance(c, k)} + G_k + 1000 \cdot S_c$

Travel Time and Distance between Children and Kindergartens

We generate a city of a 10×10 km^2 square, where both kindergartens and children are normally distributed with the same mean value but different variances in the square. The variance of the kindergarten distribution is smaller than that of children because kindergartens are more likely to be built in the city center. The travel time is proportional to travel distance. The distance between each child and each kindergarten is calculated by the generated data above using Euclidean distance. The distance below 1.5 km is walking distance (W), between 1.5 km and 6 km as driving distance (D), over 6 km as far distance (F).

Results and Sensitivity Analysis

Test Datasets

1. Keep the number of children to be 500 and change the number of kindergartens as follows, 8, 12, 16, 20 and 25.

2. Keep the number of kindergartens to be 15 and change the number of children as follows, 100, 200, 300, 400 and 500.

Extended model

W1-W10: 0.27, 0.03, 0.06, 0.09, 0.26, 0.02, 0.04, 0.11, 0.10, 0.01

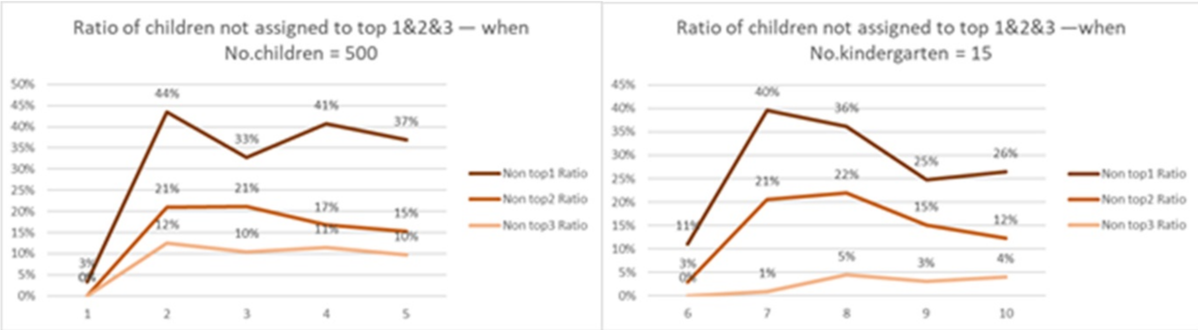
No.	#Children	#Kindergartens	#Available places	#Unallocated children
1	500	8	118	382
2	500	12	177	323
3	500	16	232	268
4	500	20	287	213
5	500	25	360	140
6	100	15	219	0
7	200	15	219	0
8	300	15	219	81
9	400	15	219	81
10	500	15	219	281

Here we can find that

$$No.of\ Unallocated\ Children = \begin{cases} 0, & if\ No.of\ Available\ places\ \geq\ No.of\ children \\ No.of\ children\ -\ No.of\ Available\ places, & otherwise. \end{cases}$$

This illustrates the reasonableness of the allocation, which tries to ensure that all children have a kindergarten to allocate.

When the number of children is fixed at 500, the number of children assigned to kindergartens increases as the number of kindergartens increases, i.e., there are more places to choose from, and this leads to a higher proportion of children not being assigned to the first choice. Nevertheless, the model ensures that approximately 90% are allocated to at least the first three choices, indicating that the model's allocation remains stable as the number of children increases.



Similarly, if the number of kindergartens is fixed to 15, as the number of children increases, the number of children allocated increases until the maximum capacity of the kindergarten is reached. The graph shows that the proportion of children not allocated to the first three choices eventually remains at around 4%. This implies that as the model increases with the number of children, it maintains a good allocation effect while still meeting the various allocation needs and gender-age ratios.

No.	1	2	3	4	5	6	7	8	9	10
Violated Gender Ratio	0.0%	0.0%	0.0%	0.0%	0.0%	6.7%	0.0%	0.0%	0.0%	0.0%
Violated Age Ratio	0.0%	8.3%	6.3%	10.0%	24.0%	0.0%	13.3%	6.7%	6.7%	6.7%
Max Distance	0	1	1	1	2	0	2	1	1	1

V. Conclusions

This report presented a new integer programming (IP) model to the Kindergarten Allocation Problem (KAP) and applied it for simulated kindergarten and children’s datasets. The results show that the model is very robust with the children's prioritized choice of kindergartens, siblings being placed in kindergarten and the gender and age targets of each kindergarten.

References

A. H. Geitle, Ø. K. Johnsen, H. F. E. Ruud, K. Fagerholt & C. A. Julsvoll (2021) Kindergarten allocation in Norway: An integer programming approach, *Journal of the Operational Research Society*, 72:7, 1664-1673, DOI: 10.1080/01605682.2020.1727786