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Variational Inequality Modeling of a Strawberry Supply Chain

Supply Chain Engineering

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Contents

1. Introduction

In this report, we use a strawberry supply chain network equilibrium model as a case study to illustrate the application of variational inequality theory. This model captures the competitive dynamics among agricultural producers, competition among retailers, and consumer purchasing behavior in the demand market. Importantly, it accounts for the perishability of agricultural products and the significant time sensitivity associated with their quality degradation.

Fruit and vegetables are fundamental components of a healthy modern diet. Despite the fact that the global fruit output nearly tripled between 1968 and 2018, and vegetable production increased more than fivefold over the same period (FAOSTAT, 2020), consumption in many regions remains below the recommended levels. This gap results from both population growth and chronic supply chain inefficiencies, which push prices higher at the retail level. Major contributors to food loss include inadequate infrastructure, outdated technology, insufficient cold-chain logistics, and suboptimal packaging (Gustavsson et al., 2011). For instance, Kelly et al. (2019) examined how various stages in the supply chain affect the quality of strawberries, highlighting the cumulative impact of each step. In response, Arias Bustos and Moors (2018) investigated how innovative collaborations among stakeholders can mitigate post-harvest losses and enhance overall supply chain performance.

Regarding the modeling approach, Yu and Nagurney (2012) developed a network-based supply chain model employing variational inequality theory. Their formulation incorporates a perishability coefficient to reflect the quality deterioration of produce over time, within the context of oligopolistic competition. This mathematical framework allows for the analysis of equilibrium conditions across the supply chain while accounting for product perishability and competitive behavior.

2. Mathematical Modeling

2.1 Network and Assumptions

We consider a three-tier supply chain composed of i farms, j retailers, and k demand markets.

Let q_{ij}denote the quantity transported from farm i to retailer j. Since strawberries are perishable, the quality of products decays over time. We model this using the decay function: $\theta(t_{ij}) = e^{-\lambda t_{ij}}$, where $\lambda = 0.1$ is the decay rate and t_{ij} is the transportation time between farm i and retailer j. In the base case, $t_{ij} = 1$. The effective perishability cost is given by:$z(q_{ij}) = 2\big(1 - \theta(t_{ij})q_{ij}\big)$.

The cost of handling perished strawberries is modeled as: $z(q_{ij}) = 2\big(1 - \theta(t_{ij})q_{ij}\big)$, where q_{ij} is the quantity shipped from farm i to retailer j, and $1 - \theta(t_{ij})q_{ij}$ represents the effective quantity delivered after decay.

We assume the cost functions to be as follows, consistent with Example 2 in Nagurney (2006, p. 26). To align the model outputs with real-world market data, we define 1 unit of flow in the supply chain network as 10 kilograms (kg) of strawberries. This scaling ensures that the equilibrium prices and quantities match observable industry benchmarks. The production cost of farm i is $f_i(Q^1) = (\sum_{j=1}^{j} q_{ij})^2 + 5\big(\sum_{j=1}^{j} q_{ij}\big)$. The distribution cost from the farm i to retail j is $c_{ij}\big(q_{ij}\big) = q_{ij}^2 + 2q_{ij}$. Since the retailer must pick out the decayed strawberries when receiving them from the farm, the handling cost is still assumed to be a function of q_{ij} : $c_j(Q^1) = 2q_{ij}^2$. The collection cost borne by the demand market k is $c_{jk}(Q^2) = q_{jk} + 1$. And the demand function is $d_k(\rho_3) = -\rho_{3k} + 100$.

2.2 Variational Inequality Formulations

Farm-Level Equilibrium Condition

The equilibrium condition at the farm level is expressed as a variational inequality (VI):

$\sum_{i=1}^{m}\sum_{j=1}^{n}[[\frac{\partial f_i(Q^{1*})}{\partial q_{ij}} + \frac{\partial c_{ij}(q_{ij}^*)}{\partial q_{ij}} + \frac{\partial z(q_{ij})}{\partial q_{ij}} - \rho_{1ij}^*] \times [q_{ij} - q_{ij}^*] \geq 0, \ \forall Q^1 \in R_+^{mn}$.

Retailer-Level Equilibrium Condition

The equilibrium conditions in the retailer j's expressed as variational inequality is: $\sum_{i=1}^{m}\sum_{j=1}^{n}[\frac{\partial c_{ij}(Q^{1*})}{\partial q_{ij}} + \rho_{1ij}^* - \gamma_j^*] \times [q_{ij} - q_{ij}^*] + \sum_{j=1}^{n}\sum_{k=1}^{o}[-\rho_{2j}^* + \gamma_j^*] \times [q_{jk} - q_{jk}^*] + \sum_{j=1}^{n}[\sum_{i=1}^{m}\theta(t_{ij})q_{ij} - \sum_{k=1}^{o} q_{jk}^*] \times [\gamma_j - \gamma_j^*] \geq 0, \ \forall (Q^1, Q^2, \gamma) \in R_+^{mn+no+o}$.

Market-Level Equilibrium Condition

The equilibrium conditions at the demand market level is $\sum_{j=1}^{n}\sum_{k=1}^{o}[\rho_{2j}^* + c_{jk}(Q^{2*}) - \rho_{3k}^*] \times [q_{jk} - q_{jk}^*] + \sum_{k=1}^{o}[\sum_{j=1}^{n} q_{jk}^* - d_k(\rho_3^*)] \times [\rho_{3k} - \rho_{3k}^*] \geq 0, \ \forall (Q^{2*}, \rho_3^*) \in R_+^{no+o}$.

3. Case Study

3.1 Only Retailers' Channels

GreenHarvest is a strawberry farm outside the city. Strawberries are transported from the farm to two urban retailers, then immediately delivered to the central market. We assume the delivery from retailers to the market is instantaneous. The current network structure is as shown in Figure 1 in the appendix.

The solution to this scenario is Q^{1*}: $q_{11}^* = q_{12}^* = 7.23$, Q^{2*}: $q_{11}^* = q_{21}^* = 6.51$, with the farm producing $q_1^* = 14.46$ units of strawberries. The equilibrium demand market price is now $\rho_{31}^* = \$86.99$, the shadow prices are $\gamma_1^* = \gamma_2^* = \$$ 79.53, and the prices the farm charges the retailers for the strawberries are $\rho_{111}^* = \rho_{112}^* = \$50.61$. At the demand market, a total of \$1132.61 with \$97.78 allocated to collection costs is paid. The profit of each retailer is \$50.03 and the profit of the farm is \$314.07.

Compared to Example 2 in Nagurney (2006, p. 26), the incorporation of the decay factor and the cost of handling spoiled strawberries significantly reduces total profit, with the retailer experiencing an almost 50% decline although raising its unit price more. To compensate, the farm and retailers increase transaction volumes to sustain profitability. However, since consumer demand remains stable, an excessive price increase by retailers to maximize profits may drive away customers, ultimately leading to a net loss in profit.

3.1 Retailer and Online Channels

Sometimes customers can choose to purchase strawberries directly from GreenHarvest's website, skipping the retailer. In this case, the transportation cost and handling cost of strawberries will be borne by the farm only, and the customers' cost of collection is removed since it will be delivered to their doorstep. The delivery time from the farm to the customer is still assumed to be 1 day. This network structure is plotted as Figure 2 in the appendix.

For the retailer channel, the optimality conditions are the same as the previous scenario. For the online channel, it is now a two-tier network of the farm and the market. A loss node is introduced to simulate the decay in the flow, capturing the impact of perishability and loss within the supply chain dynamics. The optimality conditions in the demand market expressed as variational inequality is: $\sum_{j=1}^{n}\sum_{k=1}^{o}[\rho_{1k}^{*}-\rho_{3k}^{*}]\times[q_{jk}-q_{jk}^{*}] + \sum_{k=1}^{o}\left[\sum_{i=1}^{n}q_{jk}^{*}-d_k(\rho_3^{*})\right]\times[\rho_{3k}-\rho_{3k}^{*}]\geq 0, \ \forall(Q^{2*}, \ \rho_3^{*})\in R_{+}^{no+o}$. The farm's optimality conditions expressed as variational inequality is: $\sum_{i=1}^{m}\sum_{j=1}^{o}[[\frac{\partial f_i(Q^{1*})}{\partial q_{ij}}+\frac{\partial c_{ij}(q_{ij}^{*})}{\partial q_{ij}}+\frac{\partial c_{ij}(Q^{1*})}{\partial q_{ij}}+\frac{\partial z(q_{ij})}{\partial q_{ij}}-\rho_{ij}^{*}]\times[q_{ij}-q_{ij}^{*}]\geq 0, \ \forall Q^1\in R_{+}^{mo}$.

The solution to this scenario is as follows. Q^{1*}: $q_{11}^{*}=7.18, q_{12}^{*}=7.92, Q^{2*}$: $q_{11}^{*}=$ 6.46, with farm producing 15.10 units of strawberries. The equilibrium demand market price is now ρ_3^{*}= $86.41. which is the same as the price the farm charges online. The price that the farm charges the retailer of is $50.28, and the shadow price for the retailer is $79.00. In terms of profits, the profit of the retailer is $45.10 while the farm's profit is now $425.30.

In comparison to the only-retailer equilibrium, the introduction of direct farm orders reshapes profit distribution between the farm and the retailer. The ability for customers to purchase strawberries directly from the farm enhances the farm's pricing autonomy, increasing their profit margins by 35.4% rather than relying on retail intermediaries. As a result, while the total transaction flow in the network increases, the retailer channel experiences a decline, suggesting that a portion of the market shifts towards direct transactions with the farm. This shift reflects a redistribution of sales volume, impacting profit margins and competition dynamics within the supply chain.

4. Conclusion

This report develops a strawberry supply chain equilibrium model to examine competition dynamics under product perishability and time sensitivity constraints. The analysis reveals that decay costs significantly erode profits in traditional retail channels, compelling upstream suppliers and retailers to expand transaction volumes to preserve margins. However, this volume-driven strategy risks losing customers due to consequent price inflation.

The introduction of direct farm-to-consumer sales channels demonstrates three key effects. Firstly, the farm gains enhanced pricing autonomy. Secondly, this creates channel conflict, compressing retailer profits as transactions migrate to the direct channel. Thirdly, the model suggests direct sales can reduce intermediary loss rates by bypassing traditional distribution nodes.

In conclusion, optimizing the supply chain to reduce agricultural product losses during transportation can significantly increase overall profitability. While the model suggests higher flow volumes in direct channels, real-world consumer behavior still favors traditional retail outlets. But it also suggests that farm-retailer collaboration such as shared inventory may mitigate efficiency losses from product decay.

5. Generative AI

Some parts of the text are refined using Generative AI to enhance the fluency and clarity of expression.

6. References

FAOSTAT. (2020). FAOSTAT database. Food and Agriculture Organization of the United Nations. Retrieved April 28, 2025, from http://www.fao.org/faostat/en/data/QC

Gustavsson, J., Cederberg, C., Sonesson, U., Van Otterdijk, R., Meybeck, A., 2011. Global Food Losses and Food Waste. Food and Agriculture Organization of the United Nations (FAO), Rome.

Kelly, K., Madden, R., Emond, J. P., & Nunes, M. C. N. (2019). A novel approach to determine the impact level of each step along the supply chain on strawberry quality. Postharvest Biology and Technology, 147, 78-88. https://doi.org/10.1016/j.postharvbio.2018.09.012

Arias Bustos, C., & Moors, E. H. M. (2018). Reducing post-harvest food losses through innovative collaboration: Insights from the Colombian and Mexican avocado supply chains. Journal of Cleaner Production, 199, 1020-1034. https://doi.org/10.1016/j.jclepro.2018.06.187

Yu, M., & Nagurney, A. (2012). Competitive food supply chain networks with application to fresh produce. European Journal of Operational Research, 224(2), 273-282. https://doi.org/10.2139/ssrn.2025471

Nagurney, A. (2006). Supply chain network economics: Dynamics of prices, flows, and profits. Edward Elgar Publishing.

7. Appendix

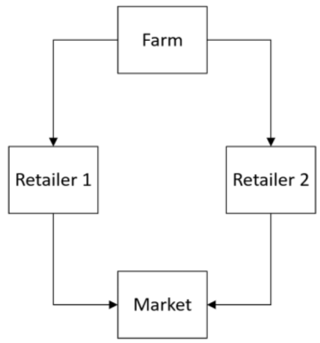

Figure 1. Network Structure for Retailers’ Channels

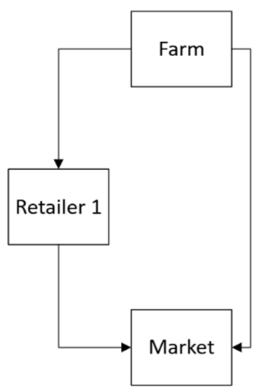

Figure 2. Network Structure for Hybrid Channels