

CS3230 Tutorial (Dynamic Programming; week of 13 March)

1. Binomial Coefficients (textbook page 282).
 - a. Compute $C(6,3)$ by applying the dynamic programming algorithm.
 - b. Is it also possible to compute $C(n,k)$ by filling the algorithm's dynamic programming table column by column rather than row by row?

2. Prove that

$$C(n,k) = C(n,n-k) \quad \text{for } n \geq k \geq 0$$

and explain how this formula can be utilized in computing $C(n,k)$.

3. Dynamic Programming (textbook page 292).
 - a. Apply Warshall's algorithm to find the transitive closure of the digraph defined by the following adjacency matrix

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

- b. Explain how Warshall's algorithm can be used to determine whether a given digraph is a dag (directed acyclic graph). Is it a good algorithm for this problem?

4. Dynamic Programming (textbook page 292).
 - a. Solve the all-pairs shortest-path problem for the digraph with the weight matrix

$$\begin{bmatrix} 0 & 2 & \infty & 1 & 8 \\ 6 & 0 & 3 & 2 & \infty \\ \infty & \infty & 0 & 4 & \infty \\ \infty & \infty & 2 & 0 & 3 \\ 3 & \infty & \infty & \infty & 0 \end{bmatrix}$$

- b. Give an example of a graph or digraph with negative weights for which Floyd's algorithm does not yield the correct result.