Design and Analysis of Algorithms	**Exam #1**
CS3230	**29 September 2008**

Open book, no sharing of materials, no laptops, NOTP = "None of the Preceeding."

1. What does Euclid's $gcd(n, m)$ algorithm do for a pair of numbers n and m in which n is smaller than m? (A) Nothing. (B) Fail. (C) Swap n and m. (D) NOTP.

2. The height h of a binary tree with n nodes is: (A) $h < \lfloor \log_2 n \rfloor$. (B) $h = \lfloor \log_2 n \rfloor$. (C) $h \le n - 1$. (D) $\lfloor \log_2 n \rfloor \le h \le n - 1$. (E) NOTP.

3. For any $f(n), g(n) \in \Theta(h(n))$, what is the efficiency class of $f(n) + g(n)$ if $f(n) \ge g(n)$? (A) $\Omega(h(n))$. (B) $\Theta(h(n))$. (C) $O(h(n))$. (D) NOTP.

4. The result of the sum $\sum_{i=3}^{n+1} 1$ is: (A) $n + 1$. (B) n. (C) $n - 1$. (D) $n - 2$. (E) NOTP.

5. Consider two functions $f(n) \in \Theta(\log_2 n)$ and $g(n) \in \Theta(\log_{10} n)$. The order of growth for the two functions is: (A) Not the same. (B) The same.

6. Consider two functions $f(n) = n!$ and $g(n) = 2^n$. Which of the following is true: (A) $f(n) \in \Theta(g(n))$. (B) $f(n) \in O(g(n))$. (C) $f(n) \in \Omega(g(n))$. (D) NOTP.

7. For any eventually positive functions $f(n), g(n)$, if $f(n) \in \Omega(g(n))$, then $L = \lim_{n\to\infty} \frac{f(n)}{g(n)}$ exists and $0 < L \le \infty$. (A) True. (B) False.

8. Computing the Fibonacci sequence recursively through $F(n) = F(n-1) + F(n-2)$ is equally efficient compared with computing it iteratively. (A) True. (B) False.

9. What is the maximum number of substrings that begin with 'A' and end with 'Z' in a text of length n? (A) $\frac{n(n+1)}{2}$. (B) $\frac{n(n-1)}{2}$. (C) $\lfloor \frac{n}{2} \rfloor \lceil \frac{n}{2} \rceil$. (D) $\lfloor \frac{n+1}{2} \rfloor$. (E) NOTP.

10. Consider the *basic* Gaussian elimination algorithm discussed in class. We have two computers A and B, where B is 1,000 times faster than A. If A can solve a problem of input size n in time T, what problem size can we solve on B in approximately the same time T? (A) $10 \times n$. (B) $33.3 \times n$. (C) $333 \times n$. (D) $1,000 \times n$. (E) NOTP.

11. The closest-pair problem can be posed on the k-dimensional space in which the Euclidean distance between two points $P' = (x'_1, \ldots, x'_k)$ and $P'' = (x''_1, \ldots, x''_k)$ is defined as $d(P', P'') = \sqrt{\sum_{s=1}^{k} (x'_s - x''_s)^2}$. What is the efficiency class of the brute-force k-dimensional algorithm? (A) $\Theta(n^k)$. (B) $\Theta(kn^3)$. (C) $\Theta(kn^2)$. (D) NOTP.

12. What is the largest number of key comparisons made by binary search in searching for a key in the following array: 3, 14, 27, 31, 39, 42, 55, 70, 74, 81, 85, 93, 98. (A) 2. (B) 3. (C) 4. (D) 5. (E) NOTP.

13. For any nonempty, *full binary tree* the number of internal (non-leaf) nodes is: (A) $\lfloor \frac{n}{2} \rfloor$. (B) $\frac{n}{2}$. (C) $n - 1$. (D) NOTP.

14. The *preorder* node traversal of a binary tree visits the nodes in reverse order compared with a *postorder* traveral. (A) True. (B) False.

15. The best case input sequence for *insertion sort* is when the array of input values is already sorted. (A) True. (B) False.

16. The best case input sequence for *quicksort* is when the array of input values is already sorted. (A) True. (B) False.

17. In the divide-and-conquer version of the closest-pair algorithm which part of the algorithm dominates in its average case runtime complexity? (A) sorting the points. (B) combining the subproblems. (C) both sorting the points and combining the subproblems equally. (D) NOTP.

18. Let $G = (V, E)$ be a graph with n vertices and m edges. All its DFS forests (for traversals starting at different vertices) will have the same number of trees. (A) True. (B) False.

19. Let $G = (V, E)$ be a graph with n vertices and m edges. All its DFS forests will have the same number of tree edges and the same number of back edges. (A) True. (B) False.

20. For a digraph with n distinct vertices, what is the largest number of distinct solutions the topological sorting problem can have? (A) n^2. (B) $\frac{n(n-1)}{2}$. (C) 2^n. (D) $n!$. (E) NOTP.

21. The Johnson-Trotter algorithm has a runtime efficiency of $\Theta(n!)$. There exist more efficient algorithms to generate all n-element permutations. (A) True. (B) False.

22. What is the Jonson-Trotter permutation immediately after .4 .2 3. .1 5. 6. ? (A) 4. .2 .1 3. .6 .5 (B) .4 .2 3. .1 6. 5. (C) 4. .2 .1 6. 3. .5 (D) 4. .2 .1 3. .5 .6 (E) NOTP.

23. The solution to the Josephus problem is 1 for every n that is a power of 2. (A) True. (B) False.

24. Gaussian elimination solves an $n \times n$ linear system $Ax = b$. The algorithm cannot be used to compute the determinant $|A|$ when A is singular. (A) True. (B) False.

25. If a topological sort has the property that all pairs of consecutive vertices in the sorted order are connected by edges, then the topological sort order is unique. (A) True. (B) False.

26. There exist no sorting algorithms that have an average case runtime complexity that is better than $O(n \log n)$. (A) True. (B) False.

27. If an $n \times n$ matrix is non-singular then it can be decomposed into two matrices L and U (*LU decomposition*) such that $A = LU$. In both matrices L and U the diagonal elements have value 1. (A) True. (B) False.

28. An undirected graph $G = (V, E)$ is a tree if and only if $|V| - |E| = 1$. (A) True. (B) False.

29. In the average case DFS is more space efficient than BFS. (A) True. (B) False.

30. Which permutation *immediately* follows the permutation 46728953 in lexographic order? (A) 46782953. (B) 46798532. (C) 47628953. (D) 46729358. (E) NOTP.

31. A BFS traversal of a graph $G = (V, E)$ can be used to determine the following properties of G: (A) Acyclicity only. (B) Connectivity only. (C) Both acyclicity and connectivity. (D) NOTP.

32. Strassen's matrix multiplication algorithm reduces the number of multiplication operations, but increases the number of additions and subtractions. (A) True. (B) False.

33. The *interpolation search* algorithms has an improved complexity class for the average case as compared with *binary search*. (A) True. (B) False.