Spring 2017

CS3230 Tutorial (P, NP, and NP-Completeness; week of 27 March)

1. Graph 2-coloring (textbook page 402).
   Design a polynomial-time algorithm for the graph 2-coloring problem: determine whether vertices of a given graph can be colored in no more than two colors so that no two adjacent vertices are colored the same color. Can we find a polynomial-time algorithm for the n-coloring problem?

2. State the decision version for each of the following problems and outline a polynomial-time algorithm that verifies whether or not a proposed solution solves the problem. (You may assume that a proposed solution represents a legitimate input to your verification algorithm.)
   a. knapsack problem b. bin-packing problem

3. A game of chess can be posed as the following decision problem: given a legal position of chess pieces and information about which side is to move, determine whether that side can win. Is this decision problem decidable?

4. Polynomial reduction (textbook page 403).
   Show that the following two problems are polynomially reducible to each other.
   Determine, for a given graph $G=<V,E>$ and a positive integer $m \leq |V|$, whether there is a **vertex cover** of size m or less for G. (A vertex cover of size k for a graph $G=<V,E>$ is a subset $V' \subseteq V$ such that $|V'|=k$ and, for each edge $(u,v) \in E$, at least one of u and v belongs to V'.)
   Determine, for a given graph $G=<V,E>$ and a positive integer $m \leq |V|$, whether G contains an **independent set** of size m or more. (An independent set of size k for a graph $G=<V,E>$ is a subset $V' \subseteq V$ such that $|V'|=k$ and, for all $u,v \in V'$, vertices u and v are *not* adjacent in G.)

5. Give examples of the following graphs or explain why such examples cannot exist (textbook page 402).
   a. A graph with a Hamiltonian circuit but without an Eulerian circuit.
   b. A graph with an Eulerian circuit but without a Hamiltonian circuit.
   c. A graph with both a Hamiltonian circuit and an Eulerian circuit.
   d. A graph with a cycle that includes all the vertices but with neither a Hamiltonian circuit not an Eulerian circuit.

6. King Arthur expects 150 knights for an annual dinner at Camelot. Unfortunately, some of the knights quarrel with each other, and Arthur knows who quarrels with whom. Arthur wants to seat his guests around a table such that no two quarreling knights sit next to each other. Which standard problem can be used to model King Arthur's task? Graph Coloring