CS3230 Tutorial 9 (P, NP, and NP-Completeness) Sample Solutions

March 6, 2017

1 Question 1

Graph 2-Coloring: An algorithm based on BFS (or DFS) traversal can be implemented:

(a) Pick a start node: color it red (for example);

(b) Color all of its neighbors blue;

(c) Color all of their neighbors red, etc.;

(d) Check if the solution satisfies the requirements, i.e., if no two adjacent nodes have the same color.

The output of the algorithm is either that (a) it finds a 2-coloring, or (b) the graph is not 2-colorable.

Note: The general n-coloring graph problem is NP-complete. The intuition is that there are multiple choices to color each node and we might need to backtrack to change a previous color assignment and try again.

2 Question 2

Knapsack decision problem: Given k does there a subset of objects exist that fits in the knapsack and has a total value of at least k?

Bin-packing decision problem: Given k are there k bins each of capacity one and n objects with sizes $s_1, s_2, \ldots, s_n$, where $0 \leq s_j \leq 1$, such that all objects fit into the k bins?

3 Question 3

The chess decision problem is decidable (though not in polynomial time). A decision tree could be created to analyze all possible moves. In order for the chess position to be deemed a winning position, one needs to have at least one move that always leads to a win for every decision that one's opponent could possibly make. Given the rules regarding a draw (the exact same board position three times in a row, or fifty moves by each player since the last capture or pawn move), one can assume that there will always be terminal nodes in this decision tree.

4 Question 4

Polynomial time reduction between **vertex cover** and **independent set**.

VERTEX-COVER: Given a graph $G = (V, E)$ and an integer k, is there a subset of vertices $S \subseteq V$ such that $|S| \leq k$, and for each edge at least one of its endpoints is in S?

INDEPENDENT-SET: Given a graph $G = (V, E)$ and an integer k, is there a subset of vertices $S \subseteq V$ such that $|S| \geq k$, and for each edge at most one of its endpoints is in S?

$$\text{VERTEX-COVER} \equiv_p \text{INDEPENDENT-SET}$$

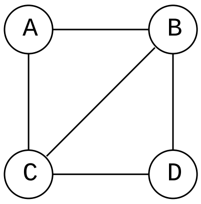
Proof: We show that the vertex set S is an independent set iff the set $V - S$ is a vertex cover.

$\Longleftarrow$ (a) Let S be an independent set;
(b) Consider an arbitrary edge (u, v);
(c) Since S is an independent set $\Rightarrow u \notin S$ or $v \notin S \Rightarrow u \in V - S$ or $v \in V - S$.
(d) Thus, $V - S$ covers (u, v).

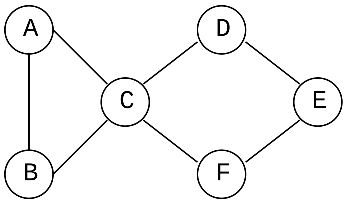
$\Longrightarrow$ (a) Let $V - S$ be any vertex cover;
(b) Consider two nodes $u \in S$ and $v \in S$.
(c) Observe that $(u, v) \notin E$ since $V - S$ is a vertex cover;
(d) Thus, no two nodes in S are joined by an edge $\Rightarrow S$ is an independent set.

5 Question 5

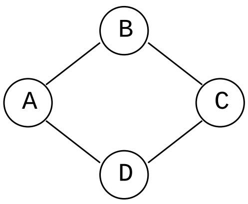
5.a) A graph with a Hamiltonian circuit but without an Eulerian circuit.



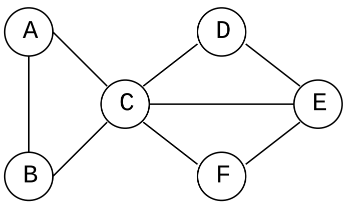
5.b) A graph with an Eulerian circuit but without a Hamiltonian circuit.



5.c) A graph with both a Hamiltonian circuit and an Eulerian circuit.



5.d) A graph with a cycle but neither an Eulerian circuit nor a Hamiltonian circuit.



6 Question 6

Arthur's Round Table: The problem can be modelled as a Hamiltonian circuit. Consider each of the 150 knights as a vertex in a graph. For each knight, put an edge between that knight and all of the other knights he does no quarrel with. Now, if there is a Hamiltonian circuit in this graph then that gives King Arthur the seating arrangement.

Any bugs and typos, please report to Roger Zimmermann (rogerz@comp.nus.edu.sg).