Spring 2017

CS3230 Tutorial (Introduction, Euclid's Algorithm, week of 23 January)

1. Prove the soundness of Euclid's Algorithm. In other words, show that the following equality holds:
   gcd(*m, n*) = gcd(*n, m* mod *n*)
   for all positive integers *m* and *n*.

2. Start with two unequal positive integers, two players take turn to add a new positive number which is the difference of two existing numbers. The player who cannot add a new positive number loses. Given the two starting positive integers, will you choose to move first or second?

```
S <- { m, n }
    repeat
        if
            there are p, q in S such that 0 < p - q is not in S
        then
            S <- S U { p-q }
        else
            break
        fi
```

3. Extend Euclid's algorithm to find integers *p, q* such that *pm*+ *qn* = gcd(*m, n*).

4. Four burglars A, B, C, D want to sneak into the airport with a duplicate key. But they have to cross a drain. The time needed by A, B, C, D to cross the drain is respectively 1, 2, 5, 10 minutes. The plan is that two of them will cross together with the key. One of the two then brings the key back to those on the other side of the drain. For example, the plan:

   ```
   A and B cross and B returns (2 + 2 = 4 minutes), B and C cross and C
   returns (5 + 5 = 10 minutes), C and D cross (10 minutes)
   ```

   takes a total of 4+10+10 = 24 minutes. Devise a plan so that they can all be across in 17 minutes.

5. Given a bit string of *n* zeros, do

```
for i from 1 to n do
    flip the i-th 2*i-th, …, k*i-th bits
        where k = floor(n/i)
```

   After the above operations, which bits are one?