Spring 2017

CS3230 Tutorial (Transform and Conquer Algorithms, week of 6 March)

1. Given an array *A[0..n-1]* and a value *V*, design a better than quadratic time efficiency algorithm to determine if there exist $i \neq j$ such that $A[i] + A[j] = V$.

2. Consider a set of *n* open intervals

   $$(a_i, b_i) = \{X \in R: a_i < x < b_i\} \; ; \; i = 1,\ldots,n;$$

   where *R* is the set of real numbers. Design a better than quadratic efficiency algorithm to find the maximum number of overlapping intervals. Intervals overlap if their intersection is non-empty.

3. Draw a height 5 AVL tree with the minimal number of nodes.

4. Design and analyze (time efficiency) an algorithm for deleting from a heap respectively (1) the smallest element, (2) the largest element, and (3) the element with a given key.

5. An *n*×*n* matrix of lights is controlled by an *n*×*n* matrix of switches. Switch s_{ij} toggles lights L_{uv} where

   $$(u, v) \in \{(x, y): |x - i| + |y - j| \leq 1\}.$$

   Given an initial lighting pattern, describe an algorithm to transform it to another lighting pattern. For example, the following flipping of switches turns off an "identity matrix" of 2×2 lights:

   $$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + S_{11} \rightarrow \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} + S_{22} \rightarrow \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix};$$

   the following flipping of switches turns on an "identity matrix" of 2×2 lights:

   $$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + S_{12} \rightarrow \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} + S_{21} \rightarrow \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}.$$

6. Design an algorithm to find a^n based on the Horner evaluation of the ternary representation of *n*.

7. Design an algorithm to solve the farmer, cabbage, goat, wolf, river crossing problem:
   A farmer needs to bring a wolf, a goat, and a cabbage across the river. The boat is tiny and can only carry one passenger at a time. If he leaves the wolf and the goat alone together, the wolf will eat the goat. If he leaves the goat and the cabbage alone together, the goat will eat the cabbage. How can he bring all three safely across the river?