

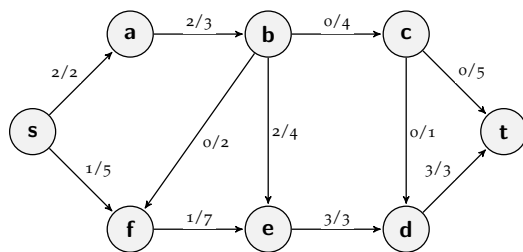
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Collaborators: list your collaborators

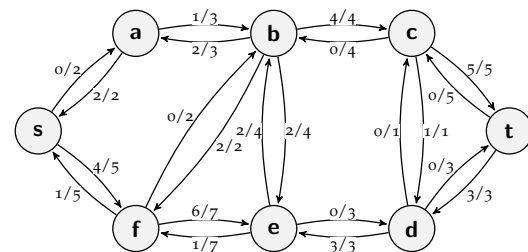
Sources: list your sources

PROBLEM 1 Max Flow

Given the following Flow Network G and Residual Graph G' :



Flow Network G



Residual Graph G'

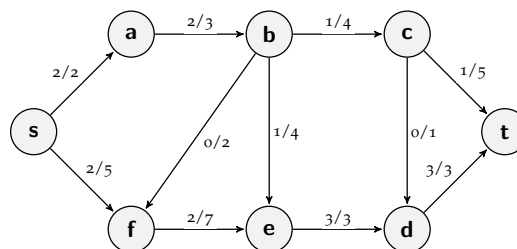
- Find an augmenting path in the graph G' . List the nodes in the path you found in order (e.g., $s \rightarrow a \rightarrow b \rightarrow c \rightarrow d \rightarrow t$).

Solution:

$s \rightarrow f \rightarrow e \rightarrow b \rightarrow c \rightarrow t$

- Update the Flow Network G above. You **must** edit the graph below (do not upload a picture).

Solution:



- Find the min cut of the graph. List the nodes on each side of the cut.

Solution:

$S = \{s, a, b, e, f\}$

$T = \{c, d, t\}$

PROBLEM 2 *Element Uniqueness Reduction*

Reduce Element Uniqueness to Mode in $O(n)$ time. Element Uniqueness is defined as: given a list of numbers, return true if no number appears more than once (i.e., every number is distinct). Mode is defined as: given a list of numbers, return one of the numbers which appears most frequently in the list; i.e. if everything was unique it will return an arbitrary element.

Solution:

From Element Uniqueness to Mode, we map the input of the Element Uniqueness Problem to the input of the Mode Problem, so input the exact same list of numbers from Element Uniqueness to Mode. This map takes $O(1)$.

From Mode to Element Uniqueness, we map the result of Mode back to Element Uniqueness by counting how many times the result element returned by Mode shows up in the list of numbers input in Element Uniqueness. If the number takes place only once, return True, otherwise return False. This can be done by traversing the whole list of numbers and counting the total number of the resulting number that shows up, which is $O(n)$.

Thus, we reduce Element Uniqueness to Mode in $O(n)$ time.

PROBLEM 3 *Reading and Evaluating Proofs*

Generative AI systems are exciting – *and scary*. They can answer many questions, but how much can we trust the results?

For this problem you will choose an generative AI system (e.g., ChatGPT, Bing (inside of Edge)) and ask it to do a proof for an algorithm we've studied in this unit—specifically, the proof that the reduction of Bi-Partite Matching to Max-Flow is correct. You'll then carefully read the proof it gives you and compare it to the version of that proof in our textbook, noting any issues or significant differences.

Here's a suggested prompt to give the system. You may use this unchanged, or alter it to try to get a better result.

Answer this question as if you were a computer scientist. Formally prove that the Bi-Partite Matching algorithm using a max-flow algorithm is correct (i.e, it always find the optimal matching between nodes in the bi-partite graph).

In your solution, provide the following;

1. Give the name and version number of the generative AI system you've used.

Solution:

ChatGPT 3.5

2. In a sentence, describe the proof strategy used by the AI.

Solution:

The AI approaches with direct proof and proves the result with three sub-claims: 1. each edge in a matching of the bipartite graph is represented in the network flow; 2. Max-flow equals Max matching because each edge has capacities 1. 3. The construction of the network flow separates two sides of the bipartite graph and results in a Min Cut.

3. Study the textbook's proof of this algorithm, Lemma 24.9 in Section 24.3 of the 4th edition of the textbook. In no more than 5 or 6 sentences, describe any issues or problems you see in the AI's result or how it differs from the textbook's proof. *Your answer might address the following questions: Do you think it successfully proves correctness? Are there gaps or odd logical jumps in the proof it provides? How different is it from the proof in the textbook? (If there are no issues to report, just say that.)*

Solution:

I don't think AI successfully proves the correctness. It oversimplifies the proof and loses rigorousness and completeness compared to the proof in the textbook. For example, when proving that a match can be represented on the network flow, the AI failed to clearly show that for any edge in a matching, we can find a corresponding flow in the network flow. Without giving an instance for illustrating the proof, the AI lost the mathematical rigorousness and completeness in proving. More critically, the AI failed to prove the converse way, proving that a flow in the network flow represents a matching in the bipartite graph. Therefore, the AI's proof is incomplete.

4. Copy the prompt you gave the AI below.

Solution:

Answer this question as if you were a computer scientist. Formally prove that the Bi-Partite Matching algorithm using a max-flow algorithm is correct (i.e, it always find the optimal matching between nodes in the bi-partite graph).

5. Copy the AI's response (the proof) below.

Solution:

Proof: Bipartite Matching Algorithm using Max-Flow

Consider a bipartite graph $G = (V, E)$ with bipartition $V = V_1 \cup V_2$. A matching M in G is a subset of edges $M \subseteq E$ such that no two edges in M share a common vertex.

Let G' be the flow network constructed from G as follows:

- Create a source node s connected to all vertices in V_1 with edge capacities of 1.
- Connect all vertices in V_2 to a sink node t with edge capacities of 1.
- Add edges between vertices in V_1 and V_2 based on the original bipartite graph G .

The algorithm uses a max-flow algorithm (e.g., Ford-Fulkerson) to find the maximum flow in G' .

Claim: The maximum flow in G' is equal to the maximum matching in G .

Proof of Claim:

- (a) The flow in G' from s to t corresponds to a valid matching in G . Each unit of flow from s to t represents a matched edge in G .
- (b) The max-flow in G' is equal to the maximum matching in G because the capacity constraints in G' ensure that the maximum flow is achieved while maintaining the property that no vertex in V_1 or V_2 is saturated beyond capacity 1.
- (c) By the Max-Flow Min-Cut theorem, the maximum flow in G' equals the minimum cut capacity separating s and t . This minimum cut represents a partition between V_1 and V_2 such that the flow across the cut is maximized.

Therefore, since the maximum flow in G' equals the maximum matching in the original bipartite graph G , and the algorithm constructs the flow network such that it adheres to the properties of bipartite graphs, the Bipartite Matching algorithm using a max-flow approach is correct and indeed finds the optimal matching between nodes in the bipartite graph.

PROBLEM 4 *Algorithms and Society: Ethical and Social Issues*

In society the term *algorithms* is frequently used for what some call *algorithmic decision systems*. These are systems that rely on large amounts of data and algorithms that use AI or machine learning to make decisions in a wide range of important issues in society.

https://en.wikipedia.org/wiki/Automated_decision-making

While these systems are different in nature that almost all the algorithms topics we've studied in this course, some kind of algorithmic process is at the heart of such systems. It is appropriate for a computing student studying algorithms to be aware of this use of the term and to understand examples of such systems and the social and ethical challenges they pose.

Below is a list of articles, etc., that touch on algorithm-based systems and social or ethical issues in society. Choose one article that interests you, and answer the following questions.

1. List the title of the of the reading that you chose.

Solution:

Designing Ethical Self-Driving Cars

2. In no more than five or six sentences, summarize how one algorithmic decision system discussed in the reading may lead to negative or undesirable consequences for individuals or societies. (The reading may discuss more than one, but you only need to write about one.)

Solution:

The article discusses the algorithmic decision system of programming Automated Vehicles (AVs) to deal with the "trolley dilemma", which is to deal with a potential collision that could be avoided by breaking traffic laws and creating damage to others. The current design is to use RSS responsibility-sensitive safety to avoid collision but does not handle unavoidable collisions caused by other people violating the laws and could result in injuries and damages to a third party who has not violated the laws when the AVs try to avoid the collisions. The author proposed the design that we should not choose actions that violate the duty of care we owe to other people. Thus, the design of AVs should only deal with the party that violates the traffic laws and caused the collision.

3. In a few sentences, what actions do you think the computing personnel or organizations that create such systems could do to reduce possible negative or undesirable consequences? (Keep your answers brief!)

Solution:

I think it is important to test the system with complicated cases where multiple stakeholders are involved to make sure the system still makes the decision we want it to make. Additionally, I think working along with the legal system to make a consensus on new regulations or laws on automated algorithms would make things clearer and less controversial in application.

Readings (choose one):

1. *Bias in AI-based models for medical applications: challenges and mitigation strategies*
<https://www.nature.com/articles/s41746-023-00858-z>

2. *Machine Bias* (This is a longer article on risk assessments in criminal sentencing that got a lot of national attention.)
<https://www.propublica.org/article/machine-bias-risk-assessments-in-criminal-sentencing>
3. *Why colleges are using algorithms to determine financial aid levels*
<https://www.highereddive.com/news/colleges-enrollment-algorithms-aid-students/692601>
4. *Designing Ethical Self-Driving Cars*
<https://hai.stanford.edu/news/designing-ethical-self-driving-cars>
5. *Algorithms that Run the World, an interview with Cathy O'Neil* (author of the book "Weapons of Math Destruction")
<https://thedecisionlab.com/podcasts/algorithms-that-run-the-world-with-cathy-oneil>