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Greeks

- 5.1 -

Delta

Delta

$$\Delta = \frac{\partial C}{\partial S}$$

- ▶ **Delta** is the rate of change of the option's price with respect to the price of the underlying asset.
- ▶ For a european call $\Delta = N(d_1)$.
- ▶ For a european put $\Delta = N(d_1) - 1$.
- ▶ Link with the Black-Scholes PDE. [▶ BS PDE](#)

Delta hedging

- ▶ Delta neutral portfolio.
- ▶ Dynamic delta hedging.
- ▶ Transaction costs.

- 5.2 -

Theta and Gamma

Theta

$$\Theta = \frac{\partial C}{\partial t}$$

- ▶ **Theta** is the rate of change of the option's price with respect to the passage of time.
- ▶ Unit: usually in years, divide by 365 to have theta per calendar day or by 252 to have theta per business day.
- ▶ **Intrinsic** and **optional** value of the option.
- ▶ Time decay is a certainty.

Gamma

$$\Gamma = \frac{\partial \Delta}{\partial S} = \frac{\partial^2 C}{\partial S^2}$$

- ▶ **Gamma** is the rate of change of the option's Delta with respect to the price of the underlying asset.
- ▶ For a european call or put

$$\Gamma = \frac{N'(d_1)}{S_0 \sigma \sqrt{T}} \quad \text{with} \quad N'(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$$

- ▶ Options are not “linear” products.

Relationship between Gamma and Theta

From the Black-Scholes PDE we have that the value Π of a portfolio of options must satisfy

$$\Theta + rS\Delta + \frac{1}{2}\sigma^2 S^2 \Gamma = r\Pi$$

When the portfolio is delta neutral

$$\Theta + \frac{1}{2}\sigma^2 S^2 \Gamma = r\Pi$$

Also, by a Taylor series expansion we obtain

$$d\Pi = \Theta dt + \frac{1}{2}\Gamma(dS)^2$$

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Vega and related greeks

Vega

$$\mathcal{V} = \frac{\partial C}{\partial \sigma}$$

- ▶ **Vega** is the rate of change of the option's price with respect to the volatility of the underlying asset.
- ▶ For a european call or put

$$\mathcal{V} = S_0 \sqrt{T} N'(d_1)$$

- ▶ The volatility surface.

Vanna and Volga

$$Vanna = \frac{\partial \mathcal{V}}{\partial S}$$

$$Volga = \frac{\partial \mathcal{V}}{\partial \sigma} = \frac{\partial^2 C}{\partial \sigma^2}$$

- ▶ **Vanna** is the rate of change of the option's vega with respect to the price of the underlying asset.
- ▶ **Volga** is the rate of change of the option's vega with respect to the volatility of the underlying asset.

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Rho

Rho

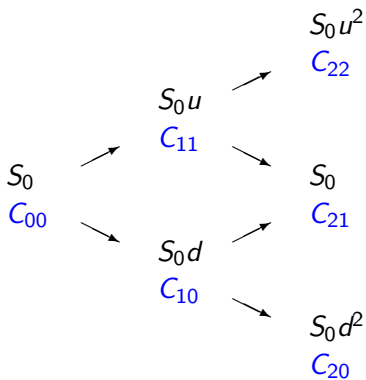
$$\rho = \frac{\partial C}{\partial r}$$

- **Rho** is the rate of change of the option's price with respect to the interest rate.

- 5.5 -

Numerical simulations

Using a tree



Using a tree

$$\Delta = \frac{C_{11} - C_{10}}{S_0 u - S_0 d}$$

$$\Gamma = \left(\frac{C_{22} - C_{21}}{S_0 u^2 - S_0} - \frac{C_{21} - C_{20}}{S_0 - S_0 d^2} \right) \cdot \frac{2}{(S_0 u^2 - S_0 d^2)}$$

$$\Theta = \frac{C_{21} - C_{00}}{2\Delta t}$$

$$\mathcal{V} = \frac{C' - C}{\Delta\sigma}$$

where C' is the price of the option obtained on a new tree built after changing the volatility to $\sigma + \Delta\sigma$.

Using Monte-Carlo

- ▶ The sensitivity is the difference between the price computed after changing the variable S , t , σ by a small amount δ and the initial price, difference divided by δ .
- ▶ To minimise the estimation error, the number of steps, number of trials and the values of the realisations of the standard normal law should be the same.