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# Applications of options to corporate finance

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# Equity as a call option

# Introduction

- The seminal 1973 Black and Scholes paper is entitled "The Pricing of Options and Corporate Liabilities".
- The authors not only derive the price of european options,
- but also they propose a valuation method for the corporate liabilities (stocks and bonds) that uses options.

# Corporate Liabilities

- There are multiple instruments to finance a firm. A simplified classification is: debt or equity instruments.
- In case of liquidation, equity investors received whatever is left after all financial claims and debts are paid off, and they have a limited liability: they are not liable for the debts other than the value of their investment.

# Equity as a call option

- Assume the debt is in the form a single zero-coupon bond with face value $F$ and maturity $T$.
- If $V_T$ is the value of the assets of the firm when the debt matures, the payoff to equity investors is

$$\begin{cases} E_T = V_T - F & \text{if } V_T \geqslant F \\ E_T = 0 & \text{if } V_T < F \end{cases}$$

# Equity as a call option

- Hence equity investors are the owners of a call on the terminal value of the assets with strike equal to the face value of the debt.
- If $\sigma$ is the volatility of the assets of the firm and $r$ is the risk-free rate, then the value of equity as of today $(t = 0)$ is

$$E_0 = C(V_0, F, T, \sigma, r)$$

# Debt holders as the owners of the firm's assets

- If $D_0$ is the (market) value of the debt as of today, the balance sheet constraint gives

$$D_0 = V_0 - E_0 = V_0 - C$$

- Therefore it is possible to describe the debt holders as the owners of the firm's assets who have given to the stock holders the option to buy these assets back at the face value of the debt.

# Debt as a risk-free bond minus a Put option

- The put-call parity gives

$$C(V_0, F, T, \sigma, r) + Fe^{-rT} = P(V_0, F, T, \sigma, r) + V_0$$

- $B_0(F, T) = Fe^{-rT}$ is the value as of today of a risk-free zero-coupon bond that pays $F$ at time $T$.
- It follows that:

$$D_0 = B_0(F, T) - P(V_0, F, T, \sigma, r)$$

- Therefore it is possible to describe the debt holders as the owners of a risk-free zero-coupon bond and the sellers of a put option whereby they have to buy the firm's assets at the face value of the debt.

# Debt vs a risk-free bond

- If the firm's debt was risk-free, we would have

$$D_0 = B_0(F, T)$$

- Therefore the value of the put is the value of the option to default on the payment of the debt that is held by the equity holders of the firm.
- When there is a loan guarantee, the guarantor takes the value of the put on its balance sheet. Examples: "National Loan Guarantee Scheme" by the UK government, "Garantie développement des PME et TPE" by Bpifrance, ...

# Example

## Data

- The value of the total assets of the firm is $V_0 = 100$ M€.
- The volatility of the assets is $\sigma = 30\%$.
- The face value of the outstanding debt is $F = 80$ M€.
- The duration of the debt is $T = 10$ years.
- The 10 years risk-free rate is $r = 5\%$.

What are the values of the equity, the debt, the zero-coupon bond, the put option and the interest rate on the debt ?

# Example

## Answer

- The value of the equity is $E_0 = 59{,}4$ M€.
- The value of the outstanding debt is $D_0 = 40{,}6$ M€.
- This gives a interest rate on the debt of 7,0%.
- The value of the Zero-coupon bond is 48,5 M€.
- The value of the put option is 7,9 M€.

# Practical valuation

- Value of assets: use the market value of equity and debt if quoted or use security valuation techniques (discounted cash-flows, ...)
- Maturity of the debt: use the weighted average of the durations of the bonds outstanding, with the weights being proportional to the bond prices.[2].
- Variance of the assets: use the volatility of the equity, the volatility of the bonds and the correlation between the two. If they are not quoted, use the average variance of the values of firms in the same industry and the average variance of similarly rated bonds.

[2] See [Hull], chapter "Interest rates".

# Valuing a troubled firm

With the same data but $V_0 = 60$ M€.

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- Equity: $E_0 = 26{,}0$ M€.
- Debt: $D_0 = 34{,}0$ M€.
- Interest rate on the debt: 8,9%.
- Zero-coupon bond: 48,5 M€.
- Value of the put option: 14,5 M€.

# Valuing a merger

Consider the merger of two firms with identical characteristics and a correlation between firms cash-flows of 0,8.

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The correlation reduces the volatility of the merged companies to 28,5%. The values for half the merged companies are then

- Equity: $E_0 = 58{,}6$ M€.
- Debt: $D_0 = 41{,}4$ M€.
- Interest rate on the debt: 6,8%.
- Zero-coupon bond: 48,5 M€.
- Value of the put option: 7,1 M€.

# The conflict between bond holders and equity holders

Assume that the equity holders can take a very risky project that will raise the volatility of the assets to 40%.

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- Interest rate on the debt: 8,8%.
- Zero-coupon bond: 48,5 M€.
- Value of the put option: 14,0 M€.

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# Real Options

# Introduction

- The usual approach to valuing a possible investment opportunity in real assets is the compute the NPV of the project. This implies to evaluates the possible future cash-flows and the discount rate for the project. This discount rate is usually obtained from the CAPM where the beta of the project has to be evaluated from similar companies.
- But many projects contain embedded options like the right to extend the project if it goes well or to abandon it if it goes badly. These optional projects usually have different betas than the base project.
- Another route is to use real options.

# Common Corporate Real Options

## Option to defer

- Management holds a lease on – or the option to buy – valuable land or natural resources. It can wait to see if output prices justify constructing a building or a plant, or developing a field.

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- Management holds a lease on – or the option to buy – valuable land or natural resources. It can wait to see if output prices justify constructing a building or a plant, or developing a field.
- This is an american call option on the value of the project with strike equal to the investment needed to enter the project.

# Common Corporate Real Options

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- If market conditions decline severely, management can abandon current operations permanently and realize on secondary markets the resale value of capital equipment and other assets.

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- This is an american put option on the value of the project with strike equal to the resale value.

# Common Corporate Real Options

## Options to expand or extend the life of a facility

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- If market demand turns out to be more favorable than expected, management may increase capacity or accelerate resource utilization. Management may also extend production if the life of the product is longer than expected.
- This is an american call option on the value of the additional capacity with strike equal to the discounted cost of creating this new capacity.

# Common Corporate Real Options

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## Options to contract

- If market demand turns out to be less favorable than expected, management may reduce the scale of operations.
- This is an american put option on the value of the reduced capacity with strike equal to the discounted saved cost that would have been incurred while running the closed capacity.

# Common Corporate Real Options

## Growth option

- An early investment (e.g., R & D, lease on undeveloped land or oil reserve or strategic acquisition) or a strategic investment is a prerequisite or a link in a chain of interrelated projects, opening up future growth opportunities (e.g., a new generation product or process, oil reserves, access to a new market).
- This is a compound option, an option on another option.

# Common Corporate Real Options

Also:

- Option to temporarily shut down the production process: if operations are less favorable than expected, management may temporarily halt and then start up again.
- Option to switch: if prices or demand changes, management may change the product mix of the facility ("product flexibility"). Alternatively, the same outputs can be produced by different production processes or inputs ("process flexibility").

# Typical industries

- Primary sector (natural resource extraction industries, farming) and real estate development.
- Industries that involve sequential investment processes (e.g. pharmaceuticals, electronics, oil, chemicals)
- Capital-intensive industries with tangible assets (e.g. airlines, railroads)
- New product introduction in uncertain markets.
- Facilities planning and construction in cyclical industries.

# Valuation of Real Options

- Binomial tree (CRR), Monte-Carlo, ...
- The risk-neutral probability is needed.
- If the underlying variable is not a traded asset, then an extension of the risk-neutral valuation framework is needed.

# The market price of risk

Assume that the prices of some derivatives $f_1$ and $f_2$ are a function of a variable $\theta$ (and of time $t$) whose price is driven by a Geometric Brownian Motion process:

$$\frac{d\theta}{\theta} = m\,dt + s\,dB_t$$

The only source of uncertainty in the prices of the derivatives comes from $B_t$, therefore we can assume that

$$\frac{df_1}{f_1} = \mu_1\,dt + \sigma_1\,dB_t$$
$$\frac{df_2}{f_2} = \mu_2\,dt + \sigma_2\,dB_t$$

# The market price of risk

It is possible to form a riskless portfolio $w_1 f_1 + w_2 f_2$ by setting the weights $w1 = \sigma_2/(\sigma_2 - \sigma_1)$ and $w_2 = -\sigma_1/(\sigma_2 - \sigma_1)$.
This portfolio being risk-free, its return has to be equal to the risk-free rate $r$, that is

$$\frac{\sigma_2}{\sigma_2 - \sigma_1}\mu_1 - \frac{\sigma_1}{\sigma_2 - \sigma_1}\mu_2 = r$$

which leads to

$$\frac{\mu_1 - r}{\sigma_1} = \frac{\mu_2 - r}{\sigma_2} = \lambda$$

where $\lambda$ is the market price of risk of $\theta$.

# Link with the risk neutral valuation

It is possible to rewrite the diffusion equation of the derivative using the market price of risk:

$$\frac{\mathrm{d}f_1}{f_1} = \mu_1\,\mathrm{d}t + \sigma_1\,\mathrm{d}B_t = r\,\mathrm{d}t + \sigma_1(\lambda\,\mathrm{d}t + \mathrm{d}B_t)$$

Using Girsanov's Theorem, it is possible to show that the process defined by

$$\mathrm{d}B_t^* = \lambda\,\mathrm{d}t + \mathrm{d}B_t$$

is a Brownian motion under some probability $\mathbb{P}^*$ which is the risk-neutral probability.

# Extension of the risk-neutral valuation framework

- If the variable $\theta$ is a traded asset, then we also have that

$$\frac{m-r}{s}=\lambda$$

and the drift of $\theta$ in the risk-neutral world is $r$.

- If the variable $\theta$ is not traded then it can be shown that its drift in the risk-neutral world is

$$m-\lambda s$$

hence estimating its market price of risk $\lambda$ allows to obtain its expected growth rate in the risk-neutral world.

# Estimating the market price of risk

Its possible to use a continuous-time version of the Capital Asset Pricing Model to estimate the market price of risk of the variable $\theta$ and of any derivative that is a sole function of this variable:

$$\mu - r = \beta(\mu_m - r)$$

where $\mu_m$ is the expected return of a market index and

$$\beta = \frac{\rho\sigma}{\sigma_m}$$

with $\sigma_m$ the volatility of the market index and $\rho$ the correlation between the variable and the market index.

# Estimating the market price of risk

Using the relationship

$$\mu - r = \lambda\sigma$$

we obtain

$$\lambda = \frac{\rho}{\sigma_m}(\mu_m - r)$$

# A modification of the Black-Scholes formula

The call value for a traded underlying asset is given by:

$$C = S_0 N(d_1) - Ke^{-rT} N(d_2) = e^{-rT}(S_0 e^{rT} N(d_1) - KN(d_2))$$

where

$$\begin{aligned} d_1 &= \frac{1}{\sigma\sqrt{T}}\left(\ln\left(S_0/K\right) + \left(r + \tfrac{1}{2}\sigma^2\right)T\right) \\ &= \frac{1}{\sigma\sqrt{T}}\left(\ln\left(S_0 e^{rT}/K\right) + \tfrac{1}{2}\sigma^2 T\right) \\ d_2 &= d_1 - \sigma\sqrt{T} \end{aligned}$$

# A modification of the Black-Scholes formula

- If the asset is traded, $\widehat{S}_T = S_0 e^{rT}$ is the expected value under the risk-neutral probability of $S_T$.
- If the asset is not traded, this expected value is given by

$$\widehat{S}_T = S_0 e^{(m-\lambda s)T}$$

- and the BS formula becomes:

$$C = e^{-rT}\left(\widehat{S}_T N(d_1) - KN(d_2)\right)$$
$$d_1 = \frac{1}{\sigma\sqrt{T}}\left(\ln\left(\widehat{S}_T/K\right) + \frac{1}{2}\sigma^2 T\right)$$

# Example: Real Estate project

(See [Hull] chapter "Real Options")

- A company has the opportunity to pay 10 M€ now for the option to rent 10,000 $m^2$ at a price of 350 € per $m^2$ for a 5 years period starting in 2 years.
- The current cost for a 5 years rental agreement is 300 € per $m^2$ and it is expected to grow at 12% per year with a volatility of 20%. Rent is paid annually on advance.
- Given a risk-free rate of 5% and a market price of risk of 0.3, should the company buy the option ?

# Example: Real Estate project

## Answer

- Let $S_t$ be the cost of renting at time $t$ with $S_0 = 300$, the current cost of renting.
- The payoff of the option per square meter is

$$\text{Annuity}.C(S_0, K = 350, T = 2, \sigma = 20\%, r = 5\%)$$

where

$$\begin{aligned}\text{Annuity} &= 1 + e^{-0,05} + e^{-0,05.2} + e^{-0,05.3} + e^{-0,05.4} \\ &= 4.5355\end{aligned}$$

# Example: Real Estate project

## Numerical results

Finally:

- $m - \lambda s = 6\%$,
- $S_0 e^{(m-\lambda s)T} = 338,25$ €,
- $d_1 = 0,02068$, $d_2 = -0,26216$,
- $C = 29,96$
- The total value of the option for 10,000 m$^2$ is 1,358,620 €.