- 1 -

Derivatives: Forwards, Futures, Swaps and Options

- 1.1 -

Definition

Derivatives

- A security whose value depends on the values of other assets.
- Underlying assets can be financial assets (stocks or debt instruments), real assets (commodities), interest rates, indices (inflation, FX), credit, ...
- Also known as contingent claims.
- Include Forwards, Futures, Swaps, Options and variations of these.

- 1.2 -

Forwards

Forwards characteristics

- Agreement to buy or sell an asset at a given date for a given price.
- Notations:

t	date of trade
T	maturity
F	delivery price
S_t	spot price of the asset at time t
S_T	spot price of the asset at time T

Forwards payoffs

- For a long position:

$$S_T - F$$

- For a short position:

$$F - S_T$$

Per one forward or one unit of currency.

Forwards delivery prices

Forward contract on a security with no income

$$F = S_t \exp^{r(T-t)}$$

Proof:
If $F > S_t \exp^{r(T-t)}$.

- At time t, sell the forward with delivery price F, and buy one unit of the asset at price S_t financed by borrowing.
- At time T, deliver the asset to the buyer of the forward contract and receive F. Pay back your loan: $S_t \exp^{r(T-t)}$. Your net receipt is $F - S_t \exp^{r(T-t)}$.

Conclude with a "no arbitrage" argument. □

Forwards delivery prices

Forward contract on a security with a known income

$$F = (S_t - NPV(income))\exp^{r(T-t)}$$

Forward contract on a security with a known dividend yield

$$F = S_t \exp^{(r-q)(T-t)}$$

Exercise: (i) What is the value of a forward contract at inception ? (ii) What is the value at time t_1 of a forward contract transacted at time t_0 and maturing at time T with $t_0 \leqslant t_1 \leqslant T$?

- 1.3 -

Futures

Futures characteristics

Same as forwards, but ...

- traded on exchanges vs. OTC,
- standardized characteristics vs. bespoke design,
- trade with a Clearinghouse vs. private contract,
- initial margin requirement, daily MtM with margin calls vs. gain or loss realised at maturity,
- no credit risk vs. risk of default,
- liquid secondary market vs. illiquid,
- often cash settlement vs. physical delivery.

Example

- Light Sweet Crude Oil (WTI) futures,
- NYMEX Rulebook (chapter 200).

Futures prices

Claim

When the risk-free rate is constant and the same for all maturities, the price of a future is the same as the price of a forward.

Proof:
See `http://www-2.rotman.utoronto.ca/~hull/TechnicalNotes/TechnicalNote24.pdf`. □

- 1.4 -

Swaps

Swaps

- An agreement to exchange cash flows at some future dates according to a prearranged formula.
- Can be seen as a portfolio of forward contracts or as a combined long position on one bond and a short position on another bond.
- Mainly interest rate swaps and cross currency swaps.
- But also equity swaps, volatility swaps, Credit Default Swaps, ...

- 1.5 -

Options

Options definition

- The buyer of the option pays a premium to the seller (writer).
- The buyer then has the right to buy, or to sell, an underlying asset at a future date for a predetermined price (strike price).
- The right to buy is a call option
- The right to sell is a put option.
- The seller has to fulfil her obligations under the terms of the option's contract.

Options payoffs

$$\begin{cases} T & \text{maturity} \\ K & \text{strike price} \\ S_T & \text{spot price of the asset at maturity} \end{cases}$$

- Long call:
$$(S_T - K)^+$$
- Long put:
$$(K - S_T)^+$$
- In the money (ITM), At the money (ATM), Out of the money (OTM).

Options exercice

- European: exercise on the expiration date only.
- American: exercise at any time up to the expiration date.
- Bermudan: exercise at predetermined dates during the life of the option.

also

- Binary options

Path Dependent Options

- Lookback options:

$$(S_T - \min_{t\in[t_0,T]} S_t)^+ \qquad (\max_{t\in[t_0,T]} S_t - S_T)^+$$

- Asian options:

$$(S_{average} - K)^+ \qquad (K - S_{average})^+$$

- Barrier and Parisian options. Call/Put, Up/Down, In/Out.

Options strategies

- Strategies involving the underlying and one option.
- Call-spread. Put-spread.
- Collar.
- Butterfly spread.
- Calendar spread.
- Straddle and Strangle.