- 6 -

Arbitrage relationships

Notations

Let C, $\mathcal{C}$, P and $\mathcal{P}$ be the prices of a european call, an american call, a european put and an american put respectively.

- 6.1 -

Put-Call Parity

Put-Call Parity

We consider options on non-dividend-paying stocks and we form two portfolios:

- (P_1): 1 Call Option (strike K, maturity T) and Ke^{-rT} in the risk-free asset.
- (P_2): 1 Put Option (strike K, maturity T) and 1 underlying asset.

The start values are

$$\begin{array}{ll} (P_1) & C + Ke^{-rT} \\ (P_2) & P + S_0 \end{array}$$

Put-Call Parity

In the absence of dividends, the end values at time T are:

$$
\begin{aligned}
(P_1) \quad & \max(S_T - K, 0) + K = \max(S_T, K) \\
(P_2) \quad & \max(K - S_T, 0) + S_T = \max(S_T, K)
\end{aligned}
$$

Therefore we must have

$$C + Ke^{-rT} = P + S_0$$

Example with the Black-Scholes formulas.

Put-Call Parity with dividends

Let $NPV(D)$ be the Net Present Value of the dividends paid during the life of the option, then

$$C + NPV(D) + Ke^{-rT} = P + S_0$$

- 6.2 -

American options on non-dividend-paying stocks

American call

From the put-call parity we get

$$C \geqslant S_0 - Ke^{-rT}$$

In fact, at any time t:

$$C_t \geqslant S_t - Ke^{-r(T-t)}$$

As it holds that $\mathcal{C} \geqslant C$, we have that, for any $t < T$,

$$\mathcal{C}_t > S_t - K$$

American options

- Therefore the american call is always worth strictly more than its intrinsic value hence early exercise is not optimal.
- This implies that there is no difference between a european call and an american call on a non-dividend-paying stock: $\mathcal{C} = C$.
- When there are some dividends, it may be optimal to exercise the call just before the dividend is paid.
- It may be optimal to exercise an american put that is deep in the money.

- 6.3 -

Bounds

Bounds for options price on non-dividend-paying stocks

By no arbitrage arguments, one can show that:

$$
\begin{aligned}
\max(S_0 - Ke^{-rT}; 0) &\leqslant C \leqslant S_0 \\
\max(S_0 - Ke^{-rT}; 0) &\leqslant \mathcal{C} \leqslant S_0 \\
\max(Ke^{-rT} - S_0; 0) &\leqslant P \leqslant Ke^{-rT} \\
\max(K - S_0; 0) &\leqslant \mathcal{P} \leqslant K
\end{aligned}
$$

Exercise: Put-call parity for american options on non-dividend-paying stocks

By no arbitrage arguments, prove that:

$$S_0 - K \leqslant \mathcal{C} - \mathcal{P} \leqslant S_0 - Ke^{-rT}$$

To prove that $S_0 - K \leqslant \mathcal{C} - \mathcal{P}$, that is $S_0 + \mathcal{P} \leqslant \mathcal{C} + K$, consider a portfolio that contains an american put and one share and a portfolio with one european call and an amount K invested in the risk-free asset.