Question 1

Not answered

Marked out of 5.00

Assuming that the denominators are never zero, which of the following statements are true in general?

Select **all** the true statements - there may be more than one.

- [ ] $\dfrac{2a - 9ab}{a} = 2 - 9ab$
- [ ] $\dfrac{2a - 9ab}{a} = 2 - 9b$
- [ ] $\dfrac{1}{a} + \dfrac{1}{b} = \dfrac{1}{a+b}$
- [ ] $\dfrac{1}{x}\left(x + \dfrac{1}{x}\right) = 1 + \dfrac{1}{2x}$
- [ ] $\left(\dfrac{3}{x}\right)^2 = \dfrac{9}{x^2}$

A correct answer is:

- $\dfrac{2a - 9ab}{a} = 2 - 9b$
- $\left(\dfrac{3}{x}\right)^2 = \dfrac{9}{x^2}$

Question **2**

Not answered

Marked out of 5.00

Functions g and h are defined on suitable domains by $g(x) = 3^{3\,x}$ and $h(x) = \frac{x^2}{9} - 5$.

Given that $h(g(x)) = 3^{f(x)} - 5$, find an expression for $f(x)$.

Which one of the following is the correct expression for $f(x)$?

- $-12\,x$
- $6\,x - 2$
- $3\,x - 2$
- $\dfrac{x^2}{3} - 5$

We find the composite function $h(g(x)) = \frac{1}{9}\left(3^{3\,x}\right)^2 - 5$.

Putting this in the given form, $h(g(x)) = 3^{-2+2\,(3\,x)} - 5 = 3^{6\,x-2} - 5$.

So $f(x) = 6\,x - 2$.

A correct answer is:

- $6\,x - 2$

Question 3

Not answered

Marked out of 5.00

(a) Rewrite the quadratic function $f(x) = 2\,x^2 - 20\,x + 54$ in the form $f(x) = a(x-p)^2 + q$.

$f(x) =$?*(x-?)^2+?

(b) Which type of stationary point does this function have? (No answer given)

(c) What are the coordinates of the stationary point?

$(x, y) =($, $)$

To write $2\,x^2 - 20\,x + 54$ in completed square form we proceed as follows.

$$\begin{aligned} 2\,x^2 - 20\,x + 54 &= 2\,\left(x^2 - 10\,x\right) + 54 \\ &= 2\,(x-5)^2 - 2 \times 5^2 + 54 \\ &= 2\,(x-5)^2 + 4. \end{aligned}$$

This is a positive quadratic, and so has a minimum value. The coordinates of the turning point can be read off from (p, q) in the completed square form, giving $(5, 4)$.

A correct answer is $2\,(x-5)^2 + 4$, which can be typed in as follows: `2*(x-5)^2+4`

A correct answer is: `"Minimum turning point"`

A correct answer is 5, which can be typed in as follows: `5`

A correct answer is 4, which can be typed in as follows: `4`

Question 4

Not answered

Marked out of 5.00

Given that $\cos(x) = \frac{5}{9}$ for the acute angle x, find the value of $\cos(2x)$.

Give an exact answer as a fraction, for example $23/73$. Do not give the answer as a decimal number.

We can calculate the value of $\cos(2x)$ using the identity

$$\cos(2x) = 2\,\cos^2 x - 1.$$

Using the value of $\cos(x) = \frac{5}{9}$ this gives

$$\cos(2x) = -1 + 2\left(\frac{5}{9}\right)^2 = -\frac{31}{81}.$$

A correct answer is $-\frac{31}{81}$, which can be typed in as follows: `-(31/81)`

Question 5

Not answered

Marked out of 5.00

The expression $6\ \cos(x) - 8\ \sin(x)$ can be written in the form $A\sin(x+\varphi)$, where $A > 0$ and $-\pi < \varphi < \pi$.

Find the values of A and φ. Give the value of φ in radians, correct to at least three decimal places.

A =

φ =

Using one of the addition rules gives $A\sin(x+\varphi) = A\sin x\cos\varphi + A\cos x\sin\varphi$.

Comparing this to $6\ \cos(x) - 8\ \sin(x)$ gives that $A\sin\varphi = 6$ and $A\cos\varphi = -8$. Solving simultaneously gives $A = 10$ and $\varphi = 2.498$.

A correct answer is 10, which can be typed in as follows: `10`

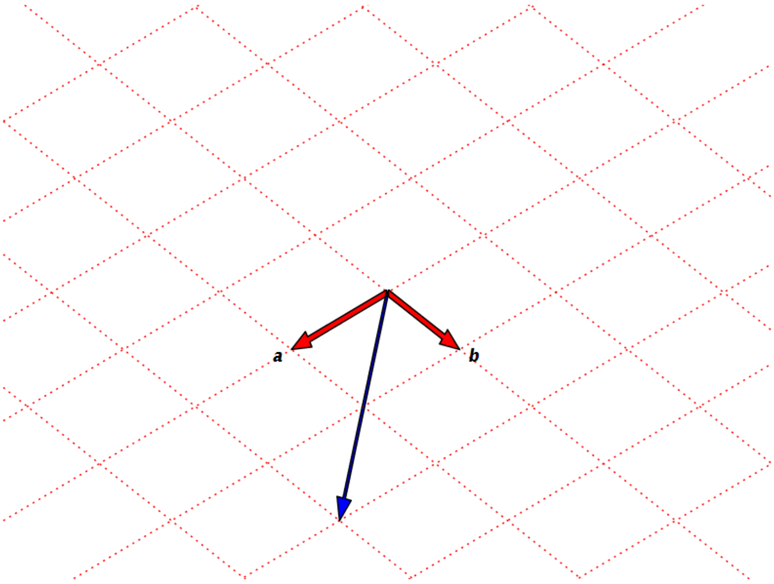
A correct answer is 2.498, which can be typed in as follows: `2.498`

Question 6

Not answered

Marked out of 5.00

The vectors $\mathbf{a}$ and $\mathbf{b}$ lie in the plane as indicated on the diagram. The other vector shown is $p\mathbf{a} + q\mathbf{b}$ where p and q are both integers.



Give the values of p and q:

$p =$

$q =$

A correct answer is 2, which can be typed in as follows: `2`

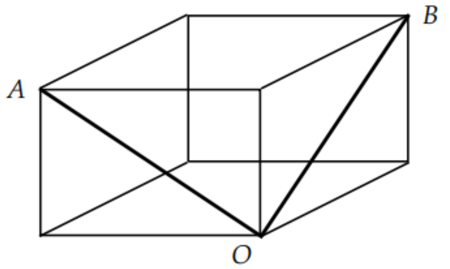
A correct answer is 2, which can be typed in as follows: `2`

Question 7

Not answered

Marked out of 5.00

A chemical factory has a rectangular room, with corners A, O and B as shown. The floor of the room is $4\,\mathrm{m} \times 5\,\mathrm{m}$ and the height of the room is $2\,\mathrm{m}$.



An engineer needs to bend a pipe at O so that it runs in a straight line from A to O, then bends at O, and then runs in a straight line from O to B.

What is the angle of the bend at O? Give your answer in degrees, correct to at least 1 decimal place.

We can form the vectors $\overrightarrow{OA} = \begin{bmatrix} -4 \\ 0 \\ 2 \end{bmatrix}$ and $\overrightarrow{OB} = \begin{bmatrix} 0 \\ 5 \\ 2 \end{bmatrix}$.

Then, $\cos(AOB) = \dfrac{\overrightarrow{OA}.\overrightarrow{OB}}{|\overrightarrow{OA}||\overrightarrow{OB}|} = \dfrac{4}{\sqrt{20}\sqrt{29}}$.

Thus $AOB = \cos^{-1}\left(\frac{2}{\sqrt{145}}\right) = 80.4^{\circ}$.

A correct answer is 80.4, which can be typed in as follows: `80.4`

Question **8**

Not answered

Marked out of 5.00

Express $2\ \ln(d\,x) - 3\ \ln\left(\frac{t}{x}\right)$ as a single logarithm.

Which one of the following is the correct result?

- $\ln\left(\frac{d^2\,x^5}{t^3}\right)$
- $\ln\left(2\,d\,x - \frac{3\,t}{x}\right)$
- $\ln\left(\frac{d^2\,t^3}{x}\right)$
- $\ln\left(\frac{x^3}{t^3} + d^2\,x^2\right)$

For any base $c > 0$ with $c \neq 1$:

$$\log_c(a) = b, \text{ means } a = c^b$$

From this definition we have

$$\log_c(a) + \log_c(b) = \log_c(ab)$$

$$\log_c(a) - \log_c(b) = \log_c\left(\frac{a}{c}\right)$$

Applying these to $2\ \ln(d\,x) - 3\ \ln\left(\frac{t}{x}\right)$ to re-write it as a single logarithm we have

$$2\ \ln(d\,x) - 3\ \ln\left(\frac{t}{x}\right) = \ln\left(\frac{d^2\,x^5}{t^3}\right).$$

A correct answer is:

- $\ln\left(\frac{d^2\,x^5}{t^3}\right)$

Question 9

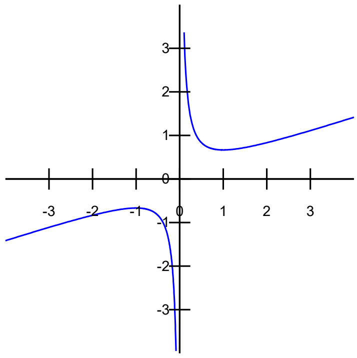
Not answered

Marked out of 5.00

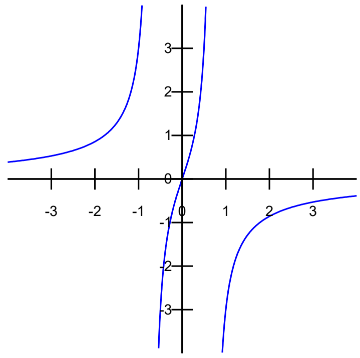
Complete the statements below:

- The curve with equation $y = -\dfrac{3\,x}{2\,x^2+1}$ is plotted in graph (No answer given)
- The curve with equation $y = -\dfrac{-3\,x^2-3}{9\,x}$ is plotted in graph (No answer given)

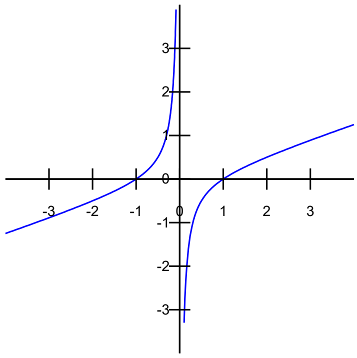
Graph A



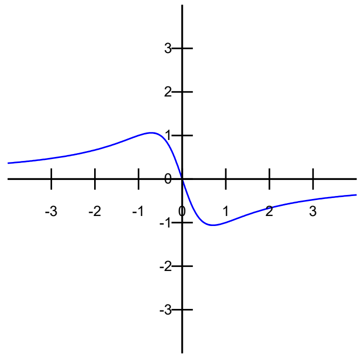
Graph B



Graph C



Graph D



A correct answer is: `D`

A correct answer is: `A`

Question **10**

Not answered

Marked out of 5.00

You are given the two equations

$$\left.\begin{array}{r} x+a\,y+b=0 \\ 2\,x-3\,y-4=0 \end{array}\right\} \qquad (*)$$

where a and b are constants.

For each of the following statements, decide if it is always, sometimes or never true.

For those which you decide are "sometimes" true, give examples of values for a and b which make the statement true.

1. The system (*) has no solutions. (No answer given)

   If you think "sometimes" then give an example: $a =$ $b =$
2. The system (*) has precisely one solution. (No answer given)

   If you think "sometimes" then give an example: $a =$ $b =$
3. The system (*) has precisely two solutions. (No answer given)

   If you think "sometimes" then give an example: $a =$ $b =$
4. The system (*) has infinitely many solutions. (No answer given)

   If you think "sometimes" then give an example: $a =$ $b =$

The system (*) can have no solutions sometimes. This occurs when the two lines are parallel, but do not coincide. In this case we have to rearrange $2\,x-3\,y-4=0$ so that the coefficient of x is one. Dividing through we have any line of the form

$$x-\frac{3\,y}{2}+b=0$$

where $b \neq -2$.

The system (*) can have a unique solution. This occurs when the two lines are not parallel, and so the lines intersect at the point (x, y) which is the unique solution. For example if the equation is

$$x-y-2=0$$

then the unique solution is

$$[x=2, y=0].$$

The system (*) can have never have precisely two solutions.

The system (*) can have infinitely many solutions sometimes. This occurs when the two lines are parallel, and do coincide. In this case we have to rearrange $2\,x-3\,y-4=0$ so that the coefficient of x is one. Dividing through we have the line

$$x-\frac{3\,y}{2}-2=0.$$

A correct answer is: `"Sometimes true"`

A correct answer is $-\frac{3}{2}$, which can be typed in as follows: `-(3/2)`

A correct answer is -1, which can be typed in as follows: `-1`

A correct answer is: `"Sometimes true"`

A correct answer is -1, which can be typed in as follows: `-1`

A correct answer is -2, which can be typed in as follows: `-2`

A correct answer is: `"Never true"`

A correct answer is a, which can be typed in as follows: `a`

A correct answer is b, which can be typed in as follows: `b`

A correct answer is: `"Sometimes true"`

A correct answer is $-\frac{3}{2}$, which can be typed in as follows: `-(3/2)`

A correct answer is -2, which can be typed in as follows: `-2`

Question 11

Not answered

Marked out of 5.00

Given two integers a and b,

- $\max(a, b)$ denotes the maximum of a and b, e.g. $\max(10, 20) = 20$,
- $\min(a, b)$ denotes the minimum of a and b, e.g. $\min(10, 20) = 10$.

(a) Evaluate the following expressions:

$\max(\min(12, 5), 17) =$

$\min(\max(12, 5), 17) =$

(b) Give values of a, b, c for which the following inequality is **false**:

$$\max(\min(a, b), c) > \min(\max(a, b), c).$$

$a =$

$b =$

$c =$

A correct answer is 17, which can be typed in as follows: `17`

A correct answer is 12, which can be typed in as follows: `12`

A correct answer is 0, which can be typed in as follows: `0`

A correct answer is 0, which can be typed in as follows: `0`

A correct answer is 0, which can be typed in as follows: `0`

Question 12

Not answered

Marked out of 5.00

At what point on the graph of $y = 2\,x^2 + x + 5$ is the slope equal to -7?

$(x, y) = ($ ______ , ______ $)$.

The derivative of $y = 2\,x^2 + x + 5$ is $\frac{dy}{dx} = 4\,x + 1$. To find the point at which this equals -7 we set up an equation

$$4\,x + 1 = -7$$

and solve this to get $x = -2$.

To find the y-coordinate we evaluate $y = 2\,x^2 + x + 5$ at $x = -2$ to get $y = 11$.

A correct answer is -2, which can be typed in as follows: `-2`

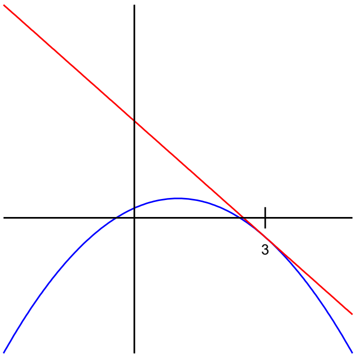
A correct answer is 11, which can be typed in as follows: `11`

Question 13

Not answered

Marked out of 5.00

The curve with equation $y = -x^2 + 2\,x + 1$ has a tangent at $x = 3$, as shown in the diagram.



The tangent has equation $y = mx + c$. What are the values of m and c?

$m =$

$c =$

Evaluating $\frac{dy}{dx} = 2 - 2\,x$ when $x = 3$ gives the gradient of the tangent, $m = -4$.

When $x = 3$, $y = -2$, so the line passes through the point $(3, -2)$.

From this we find the equation of the line is $y + 2 = -4\,(x - 3)$ which can be rearranged to give $y = 10 - 4\,x$, hence $c = 10$.

A correct answer is -4, which can be typed in as follows: `-4`

A correct answer is 10, which can be typed in as follows: `10`

Question 14

Not answered

Marked out of 5.00

A curve has equation $y = -x^3 - 6\,x^2 - 22\,x - 2$.

The line $y = mx + c$ is a tangent to the curve at the point (a, b).

(a) Find the values of m to complete the following statements:

- When $a = -2$, $m =$
- When $a = 1$, $m =$

(b) What is the maximum value of m, over all possible values of a?.

(a) We have $\frac{dy}{dx} = -3\,x^2 - 12\,x - 22$, and evaluating this at $x = -2, 1$ gives the gradient of the tangent to the curve at these points, i.e. $-10, -37$ respectively.

(b) We need to find the maximum value of the quadratic $\frac{dy}{dx} = -3\,x^2 - 12\,x - 22$.

Completing the square, $\frac{dy}{dx} = -3\,(x+2)^2 - 10$.

So the maximum value is -10.

A correct answer is -10, which can be typed in as follows: `-10`

A correct answer is -37, which can be typed in as follows: `-37`

A correct answer is -10, which can be typed in as follows: `-10`

Question **15**

Not answered

Marked out of 5.00

The curve $y = -\frac{x^3}{3} + 5\,x^2 - 21\,x - 3$ has two stationary points. Complete the table below to show the x-coordinates of the stationary points and their nature.

Note : Enter the x-coordinates in ascending order, i.e. with the smaller first.

x	Nature
	(No answer given)
	(No answer given)

To find stationary points, solve $\frac{dy}{dx} = 0$.

In this case $\frac{dy}{dx} = -x^2 + 10\,x - 21$, so there are stationary points at $x = 3$ and $x = 7$.

It remains to determine their nature, which can be done by thinking about the sign of the coefficient of x^3 and what this means for the shape of the cubic curve.

A correct answer is 3, which can be typed in as follows: `3`

A correct answer is: `"Local minimum"`

A correct answer is 7, which can be typed in as follows: `7`

A correct answer is: `"Local maximum"`

Question **16**

Not answered

Marked out of 5.00

Which one of the following is the derivative of $\sin\left(a\,x^2+b\right)$ with respect to x?

- [ ] $-2\,a\,x\,\cos\left(a\,x^2+b\right)$
- [ ] $2\,a\,x\,\cos\left(a\,x^2+b\right)$
- [ ] $2\,a\,x\,\cos(2\,a\,x)$
- [ ] $-2\,a\,x\,\sin\left(a\,x^2+b\right)$

By the chain rule, the derivative is $2\,a\,x\,\cos\left(a\,x^2+b\right)$.

A correct answer is:

- $2\,a\,x\,\cos\left(a\,x^2+b\right)$

Question 17

Not answered

Marked out of 5.00

Which one of the following is the derivative of $\left(3\,x^2+7\right)^3$ with respect to x?

- $18\,x\,\left(3\,x^2+7\right)^2$
- $3\,\left(3\,x^2+7\right)^2$
- $162\,x^5$
- $\frac{\left(3\,x^2+7\right)^4}{24\,x}$

By the chain rule, the derivative is $18\,x\,\left(3\,x^2+7\right)^2$.

A correct answer is:

- $18\,x\,\left(3\,x^2+7\right)^2$

Question **18**

Not answered

Marked out of 5.00

Find the exact value of $\displaystyle\int_1^3 \frac{5}{x^3}\,dx$.

Give your answer as a fraction, for example 17/33.

Remember that for $n \neq -1$, $\int (ax^n)dx = \frac{a}{n+1}x^{n+1} + C$.

Using this result gives $5\int_1^3 \frac{1}{x^3}\,\mathrm{d}x = \left[-\frac{5}{2\,x^2}\right]_1^3 = \frac{20}{9}$.

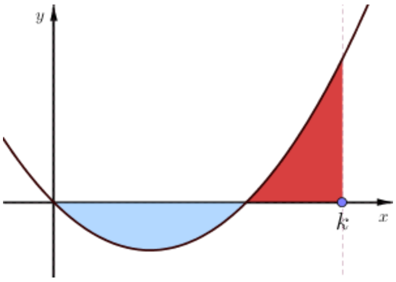
A correct answer is $\frac{20}{9}$, which can be typed in as follows: `20/9`

Question 19

Not answered

Marked out of 5.00

The curve with equation $y = 2\,x^2 - 4\,x$ is shown in the diagram.



Find the value of k for which the two shaded areas are equal.

$k =$

The value of k is such that $\int_0^k 2\,x^2 - 4\,x\,dx = 0$ since the area above and below the axis should "cancel out".

So k satisfies $\frac{2\,k^3}{3} - 2\,k^2 = 0$, hence $k = 3$.

A correct answer is 3, which can be typed in as follows: `3`

Question 20

Not answered

Marked out of 5.00

The function $f(x)$ is such that $f(4) = 11$ and its derivative $f'(4) = -7$.

Given that $g(x) = xf(x)$, what is the value of $g'(4)$?

$g'(4) =$

We can make use of the product rule from calculus

$$\frac{\mathrm{d}}{\mathrm{d}x} u(x)v(x) = u'(x)v(x) + u(x)v'(x).$$

This gives us

$$g'(x) = 1 \times f(x) + xf'(x).$$

Using the values supplied we have

$$g'(4) = -17.$$

A correct answer is -17, which can be typed in as follows: `-17`

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