Automatica 49 (2013) 2538–2545

Contents lists available at SciVerse ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Composite anti-disturbance control for Markovian jump nonlinear systems via disturbance observer[☆]

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ARTICLE INFO

Article history:
Received 11 April 2012
Received in revised form
27 February 2013
Accepted 1 May 2013
Available online 5 June 2013

Keywords:
Disturbance observer
Composite control
Markovian jump nonlinear systems
Multiple disturbances

ABSTRACT

This paper is concerned with the problems of composite disturbance-observer-based control (DOBC) and $\mathscr{H}_\infty$ control for Markovian jump systems with nonlinearity and multiple disturbances. Our aim is to design a disturbance observer to estimate the disturbance generated by an exogenous system, then construct the control scheme by integrating the output of the disturbance observer with state-feedback control law, such that, the closed-loop system can be guaranteed to be stochastically stable, and different types of disturbances can be attenuated and rejected. By constructing a proper stochastic Lyapunov–Krasovskii functional, sufficient conditions for the existence of the desired observer and the state-feedback controller are established in terms of linear matrix inequalities (LMIs), which can be readily solved by standard numerical software. Finally, a numerical example is provided to show the effectiveness of the proposed approaches.

1. Introduction

It is well known that disturbances exist in most practical controlled processes, due to the friction and load variation in mechanical and electrical systems, measurement noises, environment disturbance, the errors caused by sensors and actuators, and so on. Hence, to guarantee the stability and pursue performances, how to attenuate and reject the disturbances, especially for the nonlinear systems, becomes a crucial problem (Chen, 2003; Chen, Su, & Fukuda, 2004; Huang & Chen, 2004; Iwasaki, Shibata, & Matsui, 1999; Marino & Santosuosso, 2005; Nikiforov, 2001; She, Ohyama, & Nakano, 2005; Yang, Tsubakihara, Kanae, Wada, & Su, 2008). Here, disturbance attenuation means the influence of the disturbance can be decreased for the reference output with such as $\mathscr{H}_\infty$ controllers, while disturbance rejection denotes the compensation of the disturbance with the internal mode controllers or the disturbance observers (Guo, Wen, & Xin, 2010). Several elegant schemes have been proposed to combat the above problem, such as nonlinear DOBC theory, nonlinear regulation theory, nonlinear $\mathscr{H}_\infty$ theory and so on. Among them, the nonlinear DOBC approach is most favored (Chen, 2003, 2004; Guo & Chen, 2005; Iwasaki et al., 1999), and has found its applications in robotic systems (Chen, 2004), table drive systems (Iwasaki et al., 1999), missile systems (Guo & Wen, 2011) and so on. The basic idea of the DOBC scheme is to construct an observer to estimate the disturbance, and then based on the output of the observer, a feed-forward compensator plus conventional control laws are applied to reject the disturbance, and achieve the desired performance. The DOBC control for a class of MIMO nonlinear systems have been investigated in Guo and Chen (2005), and the disturbance considered in this note can be described by a linear exogenous system. However, it has been reported that when the disturbance has norm-bounded perturbations, the proposed approaches are unsatisfactory, which has been verified by the simulations in Guo and Chen (2005). Motivated by this, when the systems encounter multiple disturbances, hierarchical/composite control strategies consisting of DOBC and another control scheme, such as robust control (Wei & Guo, 2010), sliding mode control (Wei & Guo, 2009; Wei, Zhang, & Guo, 2009), adaptive control (Guo & Wen, 2011) and so on, are presented to achieve the anti-disturbance performance.

On another research front, Markovian jump linear systems (MJLs) have a strong practical background, since in practice many physical systems are subjected to abrupt variations in their structures due to random failures or repairs of components, sudden environmental disturbances, changing subsystem interconnections, and abrupt variations in the operating point of a nonlinear plant. MJLs include two parts: finite discrete jump modes and continuous states which are governed by Markov process and

[☆] The material in this paper was partially presented at the 2012 31st Chinese Control Conference (CCC'2012), July 25–27, 2012, Hefei, China. This paper was recommended for publication in revised form by Associate Editor James Lam under the direction of Editor Ian R. Petersen.

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http://dx.doi.org/10.1016/j.automatica.2013.05.002

differential equations, respectively. MJLs have found many applications in such systems as manufacturing systems, power systems, economics systems, communication systems and network-based control systems. Hence, MJLs have been drawing a continual interest from control theorists for decades. Recently, many papers have been published on MJLs (see, for example, Hu, Shi, & Frank, 2006; Shi, Xia, Liu, & Rees, 2006; Wang, Lam, & Liu, 2003; Xiong & Lam, 2006a; Yao, Wu, & Zheng, 2011, and the references therein). This literature covers a wide range of problems for MJLs. To just mention a few, the problem of stability analysis and stabilization is investigated in Sun, Lam, Xu, and Zou (2007) and Xiong and Lam (2006a), various control designs are presented in Hu et al. (2006), Lam, Shu, Xu, and Boukas (2007), Niu, Ho, and Wang (2007), Shi et al. (2006) and Xia, Zhang, and Boukas (2008), state estimation is reported in Wang et al. (2003); Wang, Yang, Ho, Liu, and Liu (2006), Xiong and Lam (2006b), Yao, Wu, Zheng, and Wang (2011), and the fault detection problem is considered in Yao, Wu, Zheng (2011) and Zhong, Ye, Shi, and Wang (2005). Notice that among the above mentioned literature of MJLs, the system has been assumed not to be subjected to any disturbance or to just one kind of disturbance. However, this is not the case in practice; the system is always accompanied by multiple disturbances, such as norm-bounded disturbance, disturbance with some known information and so on. Thus, how to design an unconventional control strategy to guarantee the system stability and achieve the desired performance in case of multiple disturbances becomes a thorny problem. To the best of the author's knowledge, the problem of controller design for MJLs with nonlinearity and multiple disturbances has not been investigated.

Motivated by the above observations, in this paper, we propose a hierarchical/composite anti-disturbance control methodology, that is DOBC control plus $\mathscr{H}_\infty$ control, for Markovian jump systems with nonlinearity and multiple disturbances. The nonlinearity with known and unknown functions is considered in this paper, respectively. The multiple disturbances include two kinds: one is supposed to be a norm-bounded vector; the other is described by an exogenous system with perturbations. With the introduction of the notion of composite DOBC & $\mathscr{H}_\infty$ control and by choosing a proper stochastic Lyapunov–Krasovskii functional, disturbance observers and special controllers are solved, such that the composite system is stochastically stable, and meets certain performance requirements. Finally, a numerical example is used to illustrate the efficiency of the developed results.

The remainder of this paper is organized as follows. Section 2 formulates the composite DOBC & $\mathscr{H}_\infty$ control problem under consideration. Sections 3 and 4 present the theoretical results for the cases with known nonlinearity and unknown nonlinearity, respectively. A numerical example together with simulation results is given in Section 5. Finally, we conclude the paper in Section 6.

Notation. The notations used throughout the paper are standard. The superscript "T" stands for matrix transposition; $\mathbb{R}^n$ denotes the n-dimensional Euclidean space; the notation $P > 0$ means that P is real symmetric and positive definite; I and 0 represent the identity matrix and a zero matrix, respectively; diag{...} stands for a block-diagonal matrix; $\|\cdot\|$ denotes the Euclidean norm of a vector and its induced norm of a matrix; and the signals that are square integrable over $[0,\infty)$ are represented by $\mathscr{L}_2[0,\infty)$ with the norm $\|\cdot\|_2$. $\mathbf{E}\{\cdot\}$ denotes the expectation operator with respect to probability measure $\mathscr{P}$. In addition, in symmetric block matrices or long matrix expressions, a star ($\star$) is used to represent a term that is induced by symmetry. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations.

2. Problem formulation and preliminaries

Fix a probability space $(\Omega,\mathcal{F},\mathscr{P})$, where Ω is the sample space, $\mathcal{F}$ is the σ-algebra of subsets of the sample space and $\mathscr{P}$ is the probability measure on $\mathcal{F}$. Under this probability space, we consider the following MJLs with nonlinearity:

$$\begin{aligned}(\Sigma):\ \dot{x}(t) &= A(r_t)x(t)+F(r_t)f(x(t),r_t,t)\\ &\quad + G(r_t)[u(t)+d_1(t)]+H(r_t)d_2(t)\end{aligned} \tag{1}$$

where $x(t)\in\mathbb{R}^n$ is the state vector, $u(t)\in\mathbb{R}^m$ is the control input, $f(x(t),r_t,t)\in\mathbb{R}^q$ are nonlinear vector functions, $d_1(t)\in\mathbb{R}^m$ is supposed to satisfy bounded conditions described as Assumption 1, which can represent the constant and harmonic noises. $d_2(t)\in\mathbb{R}^q$ is another disturbance which is assumed to be an arbitrary signal in $\mathscr{L}_2[0,\infty)$. $\{r_t\}$ is a continuous-time Markov process with right continuous trajectories and taking values in a finite set $\mathscr{S}=\{1,2,\ldots,\mathscr{N}\}$ with transition probability matrix $\Pi\triangleq\{\pi_{ij}\}$ given by

$$\Pr\{r_{t+\Delta}=j|r_t=i\}=\begin{cases}\pi_{ij}\Delta+o(\Delta) & \text{if } j\neq i\\ 1+\pi_{ii}\Delta+o(\Delta) & \text{if } j=i\end{cases} \tag{2}$$

where $\Delta>0$, $\lim_{\Delta\to 0}(o(\Delta)/\Delta)=0$, $\pi_{ij}\geq 0$ is the transition rate from i at time t to j at time $t+\Delta$, and $\pi_{ii}=-\sum_{j=1,j\neq i}^{\mathscr{N}}\pi_{ij}$.

Assumption 1. The disturbance $d_1(t)$ in the control input path can be formulated by the following exogenous system

$$\begin{aligned}\dot{\omega}(t) &= W(r_t)\omega(t)+M(r_t)d_3(t)\\ d_1(t) &= V(r_t)\omega(t)\end{aligned} \tag{3}$$

where $W(r_t)\in\mathbb{R}^{r\times r}$, $M(r_t)\in\mathbb{R}^{r\times l}$, $V(r_t)\in\mathbb{R}^{m\times r}$ are known matrices. $d_3(t)\in\mathbb{R}^l$ is the additional disturbance which results from the perturbations and uncertainties in the exogenous system. It is also supposed that $d_3(t)$ belong to $\mathscr{L}_2[0,\infty)$. In the numerical simulation in Section 5, we choose $W(r_t)=\begin{bmatrix}0 & c\\ -c & 0\end{bmatrix}$ with $c>0$, then $d_1(t)$ represents the harmonic disturbance which is widespread in the practical engineering (Chen, 2004; Guo & Chen, 2005; Guo & Wen, 2011; Marino & Santosuosso, 2005; Serrani, 2006; Zarikian, 2007), and c denotes the frequency of the harmonic disturbance. Thus, Eq. (3) can represent a harmonic disturbance with known frequency (if c is known), but with unknown phase and amplitude. In this note, we demonstrate our approaches by the above mentioned case.

Remark 1. It should be noted that in some of the practical engineering, the frequency of harmonic disturbance may be not known. Thus, how to reject the harmonic disturbance when the frequency, phase and amplitude are all unknown is a more challenging and practical problem, which is one of our research directions in the future.

In this note, we make the following assumption on the nonlinear functions in the system (Σ).

Assumption 2. (i). $f(0,r_t,t)=0$.
(ii). $\|f(x_1(t),r_t,t)-f(x_2(t),r_t,t)\|\leq\|U(r_t)(x_1(r_t,t)-x_2(r_t,t))\|$ with $U(r_t)$ are given constant weighting matrices.

The following assumption is a necessary condition for the DOBC formulation.

Assumption 3. (i). $(A(r_t),G(r_t))$ is controllable.
(ii). $(W(r_t),G(r_t)V(r_t))$ is observable (Guo & Chen, 2005; Ji & Chizeck, 1988).

For notational simplification, $A(r_t)$ is denoted by A_i, and accordingly $G(r_t)$ and $H(r_t)$ are denoted by G_i and H_i, respectively, and so on.

In this note, we suppose that all of the system states are available. Then, only $d_1(t)$ is required to be estimated, and thus a reduced-order observer can be used. Here, we construct the reduced-order observers for $d_1(t)$ for the case with known nonlinearity and unknown nonlinearity, respectively. And then we design special controllers so that the disturbances can be rejected and attenuated, simultaneously, and the stochastically stability of the resulting composite system can also be guaranteed.

3. Composite DOBC & $\mathscr{H}_\infty$ control for the case with known nonlinearity

The disturbance observer is formulated as

$$\begin{aligned}\hat{d}_1(t) &= V_i\hat{\omega}(t)\\ \hat{\omega}(t) &= v(t) - L_i x(t)\\ \dot{v}(t) &= (W_i + L_iG_iV_i)(v(t) - L_ix(t))\\ &\quad + L_i(A_ix(t) + F_if_i(x(t), t) + G_iu(t)).\end{aligned} \tag{4}$$

In the DOBC scheme, the control can be constructed as

$$u(t) = -\hat{d}_1(t) + K_ix(t) \tag{5}$$

where $\hat{d}_1(t) \in \mathbb{R}^r$ is the estimation of $d_1(t)$, $K_i \in \mathbb{R}^{m\times n}$ and $L_i \in \mathbb{R}^{r\times n}$ are the controller gains, and observer gains, respectively.

Remark 2. A special form of observer-based controller is constructed in (5). Unlike the conventional observer-based control scheme, $u(t) = K\varrho(t)$ with K is the state-feedback gain and $\varrho(t)$ is the estimation of the original unknown state. Here the control law (5) includes two parts: one is the negative of the estimation of the disturbance $d_1(t)$ in (3), and the other is the classical mode-dependent state-feedback control laws. Obviously, with the unconventional scheme (5), the disturbance $d_1(t)$ generated by an exogenous system (3) can be compensated through the first part of the scheme, while the latter one plays a role in guaranteeing the dynamic system is stable and meets the required performances.

The estimation error is denoted as

$$e_\omega(t) \triangleq \omega(t) - \hat{\omega}(t). \tag{6}$$

Based on (1), (3), (4) and (6), it is shown that the error dynamics satisfies

$$\dot{e}_\omega(t) = (W_i + L_iG_iV_i)e_\omega(t) + M_id_3(t) + L_iH_id_2(t). \tag{7}$$

Combining (1), (5) and (7), the composite system yields

$$\dot{\xi}(t) = \bar{A}_i(t)\xi(t) + \bar{F}_if_i(\xi(t), t) + \bar{H}_id(t) \tag{8}$$

with $\xi(t) \triangleq \begin{bmatrix}x(t)^T & e_\omega(t)^T\end{bmatrix}^T$, $d(t) \triangleq \begin{bmatrix}d_2(t)^T & d_3(t)^T\end{bmatrix}^T$, $f_i(\xi(t), t) = f_i(x(t), t)$ and

$$\bar{A}_i \triangleq \begin{bmatrix}A_i + G_iK_i & G_iV_i\\ 0 & W_i + L_iG_iV_i\end{bmatrix}$$
$$\bar{F}_i \triangleq \begin{bmatrix}F_i\\ 0\end{bmatrix}, \qquad \bar{H}_i \triangleq \begin{bmatrix}H_i & 0\\ L_iH_i & M_i\end{bmatrix}.$$

The reference output is set to be

$$z(t) = C_{1i}x(t) + C_{2i}e_\omega(t) \triangleq \bar{C}_i\xi(t) \tag{9}$$

with $\bar{C}_i \triangleq \begin{bmatrix}C_{1i} & C_{2i}\end{bmatrix}$.

Remark 3. According to the composite system (8), $d(t) \in \mathscr{L}_2[0,\infty)$ can be deduced, due to $d_3(t) \in \mathscr{L}_2[0,\infty)$ and $d_2(t) \in \mathscr{L}_2[0,\infty)$. Hence, to attenuate the disturbance $d(t)$, $\mathscr{H}_\infty$ the control scheme is a good choice for the analysis and synthesis of the dynamical systems.

Now, with the Remarks 2 and 3, we give the notion of the composite DOBC & $\mathscr{H}_\infty$ control scheme.

Definition 1. For dynamic system (1) with multiple disturbances $d_1(t)$ and $d_2(t)$, $d_2(t)$ is supposed to be a $\mathscr{H}_2$ norm-bounded vector; $d_1(t)$ is described by an exogenous system with $\mathscr{H}_2$ norm-bounded perturbations $d_3(t)$. Estimating the disturbance $d_1(t)$ with the disturbance observer (4), then based on the output of the observer, constructing the controller with the special form (5), we get the composite system (8) with $\mathscr{H}_2$ norm-bounded $d(t)$. Hence, the synthesis of the problem can be deduced to $\mathscr{H}_\infty$ control problem for the composite system (8) and (9). We name such a scheme *composite DOBC & $\mathscr{H}_\infty$ control* scheme.

Remark 4. In the composite control scheme, $\mathscr{H}_\infty$ control generally achieves the attenuation performance with respect to the disturbances belonging to $\mathscr{L}_2[0,\infty)$, while DOBC is used to reject the influence of the disturbance with some known information.

Before proposing the main problem, we give the following definition based on Cao and Lam (2000).

Definition 2. The Markovian jump system in (1) under (3) with $d_2(t) = d_3(t) = 0$ is said to be stochastically stable if there exists a matrix $M > 0$ such that for any initial states x_0, and initial mode $r_0 \in \mathscr{S}$,

$$\lim_{T\to\infty} \mathbf{E}\left\{\int_0^T x^T(t, x_0, r_0)x(t, x_0, r_0)dt \mid x_0, r_0\right\} \le x_0^TMx_0 \tag{10}$$

where $x(t, x_0, r_0)$ denotes the solution of the system (1) with (3) at the time t under the initial condition x_0 and r_0, and x_0 represents $x(t, x_0, r_0)$ at $t = 0$.

Therefore, the composite anti-disturbance control problem for system (1) with (3) can be formulated as follows.

Composite anti-disturbance control problem: Given the Markovian jump nonlinear system (1) with (3), design a reduced-order observer of the form (4) and controller of the form (5) such that the following requirements are satisfied:

(R1) The composite system in (8) and (9) with $d(t) = 0$ is stochastically stable.

(R2) Under the zero initial conditions, the following inequality holds:

$$\|z(t)\|_{E_2} < \gamma\|d(t)\|_2 \tag{11}$$

for all nonzero $d(t) \in \mathscr{L}_2[0,\infty)$, where $\gamma > 0$ is a prescribed scalar, and $\|z(t)\|_{E_2} = \mathbf{E}\left\{\int_0^\infty z^T(t)z(t)dt\right\}$.

In the following, we will present a sufficient condition in terms of linear matrix inequalities (LMIs), under which the augmented system in (8) and (9) is stochastically stable and satisfies the $\mathscr{H}_\infty$ performance inequality (11).

Theorem 1. *Consider system (1) with the disturbance (3) under Assumptions 2 and 3. Given parameters $\lambda_i > 0$ and $\gamma > 0$, there exists a disturbance observer in the form of (4), and a controller in the form of (5) such that the augmented system in (8) and (9) is stochastically stable and satisfies the $\mathscr{H}_\infty$ performance* *inequality (11) if there exist matrices $Q_i > 0$, $P_{2i} > 0$, X_i and Y_i such that for $i = 1, 2, \ldots, \mathcal{N}$,*

$$\begin{bmatrix} \Pi_{11i} & G_iV_i & F_i & \Pi_{14i} & Q_iC_{1i}^T & Q_iU_i^T & \Xi_i \\ \star & \Pi_{22i} & 0 & \Pi_{24i} & C_{2i}^T & 0 & 0 \\ \star & \star & -\frac{1}{\lambda_i^2}I & 0 & 0 & 0 & 0 \\ \star & \star & \star & -\gamma^2 I & 0 & 0 & 0 \\ \star & \star & \star & \star & -I & 0 & 0 \\ \star & \star & \star & \star & \star & -\lambda_i^2 I & 0 \\ \star & \star & \star & \star & \star & \star & \Lambda_i \end{bmatrix} < 0 \tag{12}$$

with

$$\Xi_i \triangleq \begin{bmatrix} Q_i & \ldots & Q_i & \ldots & Q_i \end{bmatrix}_{\mathcal{N}-1}$$
$$\Lambda_i \triangleq -\mathrm{diag}\left\{\pi_{i1}^{-1}Q_1, \ldots, \pi_{ij}^{-1}Q_j, \ldots, \pi_{i\mathcal{N}}^{-1}Q_{\mathcal{N}}\right\}_{j\neq i}$$
$$\Pi_{11i} \triangleq A_iQ_i + Q_iA_i^T + G_iX_i + X_i^TG_i^T + \pi_{ii}Q_i$$
$$\Pi_{14i} \triangleq \begin{bmatrix} H_i & 0 \end{bmatrix}$$
$$\Pi_{22i} \triangleq P_{2i}W_i + Y_iG_iV_i + (P_{2i}W_i + Y_iG_iV_i)^T + \bar{P}_{2i}$$
$$\Pi_{24i} \triangleq \begin{bmatrix} Y_iH_i & P_{2i}M_i \end{bmatrix}$$
$$\bar{P}_{2i} \triangleq \sum_{j=1}^{\mathcal{N}} \pi_{ij}P_{2j}.$$

Moreover, if the above conditions are feasible, the gains of the desired observer in the form of (4) and the desired controller in the form of (5) are given by

$$K_i = X_iQ_i^{-1}, \qquad L_i = P_{2i}^{-1}Y_i \tag{13}$$

Proof. Define a Lyapunov functional candidate as follows:

$$V(\xi(t), r_t, t) \triangleq V_1(\xi(t), r_t, t) + V_2(\xi(t), r_t, t) \tag{14}$$

with

$$\begin{cases} V_1(\xi(t), r_t, t) \triangleq \xi^T(t)P_i\xi(t) \\ V_2(\xi(t), r_t, t) \triangleq \dfrac{1}{\lambda_i^2}\displaystyle\int_0^t \|U_ix(\tau)\|^2 - \|f_i(x(\tau), \tau)\|^2 \, d\tau \end{cases}$$

with $P_i > 0$, $i \in \mathcal{S}$.

Define

$$P_i \triangleq \begin{bmatrix} P_{1i} & 0 \\ 0 & P_{2i} \end{bmatrix} \tag{15}$$

with $P_{1i} \geq 0$ and $P_{2i} \geq 0$. Let $\mathcal{A}$ be the weak infinitesimal generator (Dynkin, 1965) of the random process $\{\xi(t), r_t\}$. Then, for each $r_t = i$, $i \in \mathcal{S}$, it can be shown that (Yao, Wu, Zheng, & Wang, 2010)

$$\begin{aligned} \mathcal{A}V_1(\xi(t), i, t) &= \xi^T(t)(P_i\bar{A}_i + \bar{A}_i^TP_i)\xi(t) + \xi^T(t)\bar{P}_i\xi(t) \\ &\quad + 2\xi^T(t)P_i\bar{F}_if_i(x(t), t) + 2\xi^T(t)P_i\bar{H}_id(t) \end{aligned} \tag{16}$$

$$\mathcal{A}V_2(\xi(t), i, t) = \frac{1}{\lambda_i^2}x^T(t)U_i^TU_ix(t) - \frac{1}{\lambda_i^2}f_i^T(x(t), t)f_i(x(t), t). \tag{17}$$

Combining (15)–(17), we can derive

$$\begin{aligned} \mathcal{A}V(\xi(t), i, t) &= \mathcal{A}V_1(\xi(t), i, t) + \mathcal{A}V_2(\xi(t), i, t) \\ &= \eta^T(t)\begin{bmatrix} \Phi_{1i} & P_{1i}G_iV_i & P_{1i}F_i & \Phi_{2i} \\ \star & \Phi_{3i} & 0 & \Phi_{4i} \\ \star & \star & -\frac{1}{\lambda_i^2}I & 0 \\ \star & \star & \star & 0 \end{bmatrix}\eta(t) \end{aligned} \tag{18}$$

with $\eta(t) \triangleq \begin{bmatrix} x^T(t) & e_\omega^T(t) & f_i^T(t, x(t)) & d^T(t) \end{bmatrix}^T$ and

$$\Phi_{1i} \triangleq P_{1i}(A_i + G_iK_i) + (A_i + G_iK_i)^TP_{1i} + \bar{P}_{1i} + \frac{1}{\lambda_i^2}U_i^TU_i$$
$$\Phi_{2i} \triangleq \begin{bmatrix} P_{1i}H_i & 0 \end{bmatrix}$$
$$\Phi_{3i} \triangleq P_{2i}(W_i + L_iG_iV_i) + (W_i + L_iG_iV_i)^TP_{2i} + \bar{P}_{2i}$$
$$\Phi_{4i} \triangleq \begin{bmatrix} P_{2i}L_iH_i & P_{2i}M_i \end{bmatrix}$$
$$\bar{P}_{1i} \triangleq \sum_{j=1}^{\mathcal{N}} \pi_{ij}P_{1j}$$
$$\bar{P}_{2i} \triangleq \sum_{j=1}^{\mathcal{N}} \pi_{ij}P_{2j}.$$

Consider the following index

$$J(T) \triangleq \mathbf{E}\left\{\int_0^T \left[z^T(t)z(t) - \gamma^2d^T(t)d(t)\right]dt\right\}.$$

Then, under the zero initial conditions, it follows from (9) and (18) that

$$\begin{aligned} J(T) &= \mathbf{E}\left\{\int_0^T \left[z^T(t)z(t) - \gamma^2d^T(t)d(t)\right]dt\right\} + \mathbf{E}V(\xi(T), i, T) \\ &= \mathbf{E}\left\{\int_0^T \eta^T(t)\Theta_i\eta(t)dt\right\} \end{aligned} \tag{19}$$

with

$$\Theta_i \triangleq \begin{bmatrix} \Phi_{1i} + C_{1i}^TC_{1i} & P_{1i}G_iV_i + C_{1i}^TC_{2i} & P_{1i}F_i & \Phi_{2i} \\ \star & \Phi_{3i} + C_{2i}^TC_{2i} & 0 & \Phi_{4i} \\ \star & \star & -\frac{1}{\lambda_i^2}I & 0 \\ \star & \star & \star & -\gamma^2I \end{bmatrix}$$

with $\eta(t)$, Φ_{1i}, Φ_{2i}, Φ_{3i} and Φ_{4i} is defined in (18).

Now, we begin to verify that if (12) holds, then $\Theta_i < 0$.

Using the Schur complement for inequalities (12), we obtain the following inequalities

$$\begin{bmatrix} \Delta_i & G_iV_i & F_i & \Pi_{14i} & Q_iC_{1i}^T & Q_iU_i^T \\ \star & \Pi_{22i} & 0 & \Pi_{24i} & C_{2i}^T & 0 \\ \star & \star & -\frac{1}{\lambda_i^2}I & 0 & 0 & 0 \\ \star & \star & \star & -\gamma^2I & 0 & 0 \\ \star & \star & \star & \star & -I & 0 \\ \star & \star & \star & \star & \star & -\lambda_i^2I \end{bmatrix} < 0 \tag{20}$$

with $\Delta_i \triangleq \Pi_{11i} + \sum_{j=1, j\neq i}^{\mathcal{N}} \pi_{ij}Q_iQ_j^{-1}Q_i$, Π_{11i}, Π_{14i}, Π_{22i}, and Π_{24i} defined in (12). Define

$$P_{1i} \triangleq Q_i^{-1}, \qquad X_i \triangleq K_iP_{1i}^{-1}, \qquad Y_i \triangleq P_{2i}L_i \tag{21}$$

then performing a congruence transformation to (20) by diag $\{P_{1i}, I, I, I, I\}$, we readily obtain the following inequalities

$$\begin{bmatrix} \bar{\Phi}_{1i} & P_{1i}G_iV_i & P_{1i}F_i & \Phi_{2i} & C_{1i}^T & U_i^T \\ \star & \Phi_{3i} & 0 & \Phi_{4i} & C_{2i}^T & 0 \\ \star & \star & -\frac{1}{\lambda_i^2}I & 0 & 0 & 0 \\ \star & \star & \star & -\gamma^2I & 0 & 0 \\ \star & \star & \star & 0 & -I & 0 \\ \star & \star & \star & 0 & 0 & -\lambda_i^2I \end{bmatrix} < 0 \tag{22}$$

with $\bar{\Phi}_{1i} \triangleq P_{1i}(A_i + G_iK_i) + (A_i + G_iK_i)^TP_{1i} + \bar{P}_{1i}$.

Using the Schur complement with (22), we can readily derive $\Theta_i < 0$.

Thus, $J(T) \leq 0$ by taking (19) into account. Under the zero initial conditions and for any nonzero $d(t) \in \mathscr{L}_2(0, \infty)$, letting $T \to \infty$, we obtain $\|z(t)\|_2 \leq \gamma \|d(t)\|_2$.

Moreover, based on (21), the gains of the observer (4) and the gains of the controller (5) are given by (13). The proof is completed. □

Remark 5. It is worth noting that if the composite MJLs in (8) and (9) guarantees $\mathscr{H}_\infty$ disturbance attenuation level γ according to Theorem 1, then the stochastic stability of the composite system with $d(t) = 0$ is also guaranteed. This is briefly shown as follows. First, we define the Lyapunov–Krasovskii function as in (14). Then, by following along lines similar to the proof of Theorem 1, one can see that the weak infinitesimal to $V(\xi(t), i, t)$ along the solution of (8) with $d(t) = 0$ is given by

$$\mathcal{A}V(x_t, i, t) \leq \hat{\eta}^T(t) \begin{bmatrix} \Phi_{1i} & P_{1i}G_iV_i & P_{1i}F_i \\ \star & \Phi_{3i} & 0 \\ \star & \star & -\dfrac{1}{\lambda_i^2}I \end{bmatrix} \hat{\eta}(t) \tag{23}$$

with $\hat{\eta}(t) = \begin{bmatrix} x^T(t) & e_\omega^T(t) & f_i^T(x(t), t) \end{bmatrix}^T$.

Again, using the similar arguments to the proof of Theorem 1, one can see that (12) guarantees

$$\begin{bmatrix} \Phi_{1i} & P_{1i}G_iV_i & P_{1i}F_i \\ \star & \Phi_{3i} & 0 \\ \star & \star & -\dfrac{1}{\lambda_i^2}I \end{bmatrix} < 0.$$

Finally, following along the lines similar to Yao, Wu, Zheng, Wang (2011) and Definition 2, we have that the composite MJLs in (8) with $d(t) = 0$ are stochastically stable.

4. Composite DOBC & $\mathscr{H}_\infty$ control for the case with unknown nonlinearity

In this section, we suppose Assumptions 1–3 hold, but nonlinear functions $f_i(x(t), t)$ are unknown. Unlike Section 3, $f_i(x(t), t)$ are unavailable in observer design.

In this section, we choose the following disturbance observer

$$\begin{aligned} \hat{d}_1(t) &= V_i\hat{\omega}(t) \\ \hat{\omega}(t) &= v(t) - L_ix(t) \\ \dot{v}(t) &= (W_i + L_iG_iV_i)(v(t) - L_ix(t)) + L_i(A_ix(t) + G_iu(t)) \end{aligned} \tag{24}$$

the controller can be constructed as

$$u(t) = -\hat{d}_1(t) + K_ix(t). \tag{25}$$

The estimation error is denoted as

$$e_\omega(t) \triangleq \omega(t) - \hat{\omega}(t). \tag{26}$$

Based on (1), (3), (24) and (26), it is shown that the error dynamics satisfies

$$\dot{e}_\omega(t) = (W_i + L_iG_iV_i)e_\omega(t) + M_id_3(t) + L_iH_id_2(t). \tag{27}$$

Combining (1), (5) and (7), the composite system yields

$$\dot{\xi}(t) = \bar{A}_i(t)\xi(t) + \bar{F}_if_i(\xi(t), t) + \bar{H}_id(t) \tag{28}$$

with $\xi(t) \triangleq \begin{bmatrix} x(t)^T & e_\omega(t)^T \end{bmatrix}^T$, $d(t) \triangleq \begin{bmatrix} d_2(t)^T & d_3(t)^T \end{bmatrix}^T$, $f_i(\xi(t), t) = f_i(x(t), t)$ and

$$\bar{A}_i \triangleq \begin{bmatrix} A_i + G_iK_i & G_iV_i \\ 0 & W_i + L_iG_iV_i \end{bmatrix}$$

$$\bar{F}_i \triangleq \begin{bmatrix} F_i \\ L_iF_i \end{bmatrix}, \qquad \bar{H}_i \triangleq \begin{bmatrix} H_i & 0 \\ L_iH_i & M_i \end{bmatrix}.$$

The reference output is set to be

$$z(t) = C_{1i}x(t) + C_{2i}e_\omega(t) \triangleq \bar{C}_i\xi(t) \tag{29}$$

with $\bar{C}_i \triangleq \begin{bmatrix} C_{1i} & C_{2i} \end{bmatrix}$.

In the following, we will present sufficient conditions in terms of LMIs, under which the augmented system in (28) and (29) is stochastically stable and satisfies the $\mathscr{H}_\infty$ performance inequality (11).

Theorem 2. *Consider system (1) with the disturbance (3) under Assumptions 2 and 3. Given parameters $\lambda_i > 0$ and $\gamma > 0$, there exists a disturbance observer in the form of (24), and a controller in the form of (25) such that the augmented system in (28) and (29) is stochastically stable and satisfies the $\mathscr{H}_\infty$ performance inequality (11) if there exist matrices $Q_i > 0$, $P_{2i} > 0$, X_i and Y_i such that for $i = 1, 2, \ldots, \mathcal{N}$,*

$$\begin{bmatrix} \Pi_{11i} & G_iV_i & F_i & \Pi_{14i} & Q_iC_{1i}^T & Q_iU_i^T & \Xi_i \\ \star & \Pi_{22i} & Y_iF_i & \Pi_{24i} & C_{2i}^T & 0 & 0 \\ \star & \star & -\dfrac{1}{\lambda_i^2}I & 0 & 0 & 0 & 0 \\ \star & \star & \star & -\gamma^2 I & 0 & 0 & 0 \\ \star & \star & \star & \star & -I & 0 & 0 \\ \star & \star & \star & \star & \star & -\lambda_i^2 I & 0 \\ \star & \star & \star & \star & \star & \star & \Lambda_i \end{bmatrix} < 0 \tag{30}$$

with Ξ_i, Λ_i, Π_{11i}, Π_{14i}, Π_{22i} and Π_{24i} defined in the (12).

Moreover, if the above conditions are feasible, the gains of the desired observer in the form of (24) and the desired controller in the form of (25) are given by

$$K_i = X_iQ_i^{-1}, \qquad L_i = P_{2i}^{-1}Y_i \tag{31}$$

Proof. Comparing the system matrices in (28), (29) with the system matrices in (8), (9), and following along the similar arguments in Theorem 1, we can readily obtain Theorem 2. Hence, we omit the process of the proof. □

5. Numerical example

In this section, we will present a illustrative example to demonstrate the effectiveness of the proposed approaches. Consider the system in (1) with (3) involving two modes. The parameters of the system are given as follows:

Mode 1:

$$A_1 = \begin{bmatrix} -2.2 & 1.5 \\ 0 & 1.2 \end{bmatrix}, \qquad F_1 = \begin{bmatrix} 1.1 \\ 0.1 \end{bmatrix}$$

$$G_1 = \begin{bmatrix} -1.5 \\ 2.0 \end{bmatrix}, \qquad H_1 = \begin{bmatrix} 1.2 \\ 1.0 \end{bmatrix}$$

$$C_{11} = \begin{bmatrix} 0.5 & 0.1 \end{bmatrix}, \qquad C_{21} = \begin{bmatrix} 0.1 & 0 \end{bmatrix}$$

$$W_1 = \begin{bmatrix} 0 & 0.5 \\ -0.5 & 0 \end{bmatrix}, \qquad V_1 = \begin{bmatrix} 2.0 & 0 \end{bmatrix}, \qquad M_1 = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}$$

Mode 2:

$$A_2 = \begin{bmatrix} 1.9 & 0.5 \\ 0.2 & -1.2 \end{bmatrix}, \qquad F_2 = \begin{bmatrix} 0.2 \\ 0.1 \end{bmatrix}$$

$$G_2 = \begin{bmatrix} -1.0 \\ 1.0 \end{bmatrix}, \qquad H_2 = \begin{bmatrix} 0.6 \\ 0.4 \end{bmatrix}$$

$$C_{12} = \begin{bmatrix} 1.2 & 0.1 \end{bmatrix}, \qquad C_{22} = \begin{bmatrix} 0 & 0.1 \end{bmatrix}$$

$$W_2 = \begin{bmatrix} 0 & 0.5 \\ -0.5 & 0 \end{bmatrix}, \qquad V_2 = \begin{bmatrix} 1.0 & 0 \end{bmatrix}, \qquad M_2 = \begin{bmatrix} 0.2 \\ 0.2 \end{bmatrix}.$$

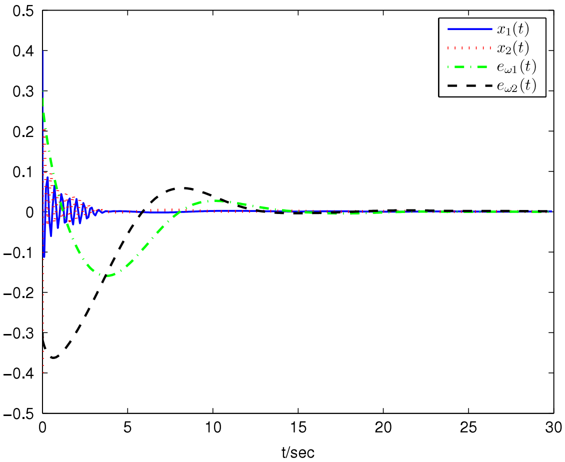

Fig. 1. The average of the states of the composite system (8) by Monte Carlo simulations.

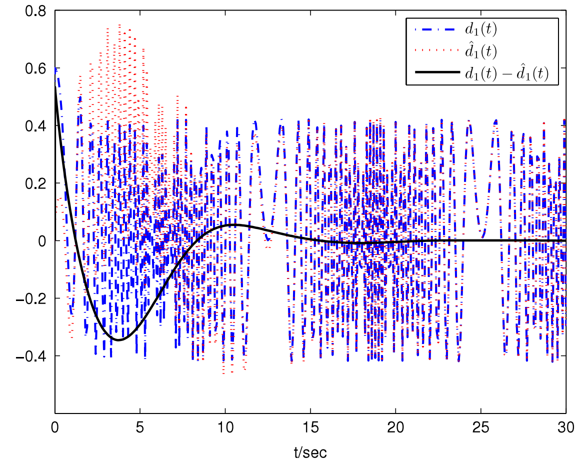

Fig. 2. Disturbance estimation error along an individual switching sequence for known nonlinearity.

The transition probability matrix is assumed to be $\Pi = \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$. Assume $\lambda_1 = \lambda_2 = \gamma = 1$ and $U_1 = U_2 = \begin{bmatrix} 1.3 & 0 \\ 0 & 1.0 \end{bmatrix}$.

Our intention here is to design disturbance-observer-based controllers in the form of (4), (5) and (24), (25), for the case with known nonlinearity and unknown nonlinearity, respectively, such that the composite system is stochastically stable and satisfies the prescribed performance.

Case 1: *with known nonlinearity:*

We resort to the LMI Toolbox in Matlab to solve the LMIs in (12), and the gains of the desired observer and controller are given by

$$K_1 = \begin{bmatrix} -5.3829 & -9.1748 \end{bmatrix}, \qquad K_2 = \begin{bmatrix} 17.8473 & 6.5338 \end{bmatrix}$$
$$L_1 = \begin{bmatrix} -0.0715 & -0.3264 \\ -0.1480 & -0.6544 \end{bmatrix}, \qquad L_2 = \begin{bmatrix} 0.8905 & -2.1273 \\ -0.1023 & 0.2897 \end{bmatrix}.$$

Suppose $f_1(x(t), t) = f_2(x(t), t) = x_2(t)\sin(t)$, we can find $\|f_i(x(t), t)\| \le \|U_i x(t)\|, i = 1, 2$. Assume $d_2(t) = \frac{1}{5+10t}$. Given the initial condition as $\xi(0) = \begin{bmatrix} 0.4 & -0.4 & 0.3 & -0.3 \end{bmatrix}^T$.

To show the composite system is stochastically stable, we run Monte Carlo simulations (Costa & Aya, 2002): (1) generate a large number of the switching sequences, here we choose 200 sequences; (2) compute the average value of the states over 200 cases. The simulation results are given in Fig. 1, where the average of the states in composite system (8) are plotted.

To demonstrate the effectiveness of the proposed disturbance observer, the disturbance $d_1(t)$, its estimation $\hat{d}_1(t)$ and the estimation error $d_1(t)-\hat{d}_1(t)$ along a single switching sequence are presented in Fig. 2. From that, we can see the proposed disturbance observer is fine and effective.

To verify the advantage of our proposed composite control scheme, we compare our composite control method with single $\mathscr{H}_\infty$ control method. Fig. 3 represents the reference output $z(t)$ in (9) along an individual switching sequence via the above two approaches, which indicates the proposed scheme can reject and attenuate the multiple disturbances, and the single $\mathscr{H}_\infty$ scheme does not work.

Case 2: *with unknown nonlinearity:*

By solving the LMIs in (30), and the gains of the desired observer and controller are given by

$$K_1 = \begin{bmatrix} -5.0204 & -8.8665 \end{bmatrix}, \qquad K_2 = \begin{bmatrix} 17.3188 & 6.0287 \end{bmatrix}$$
$$L_1 = \begin{bmatrix} -0.1925 & -0.5198 \\ -0.0861 & -0.2264 \end{bmatrix}, \qquad L_2 = \begin{bmatrix} 0.5550 & -2.5943 \\ 0.0402 & -0.0388 \end{bmatrix}.$$

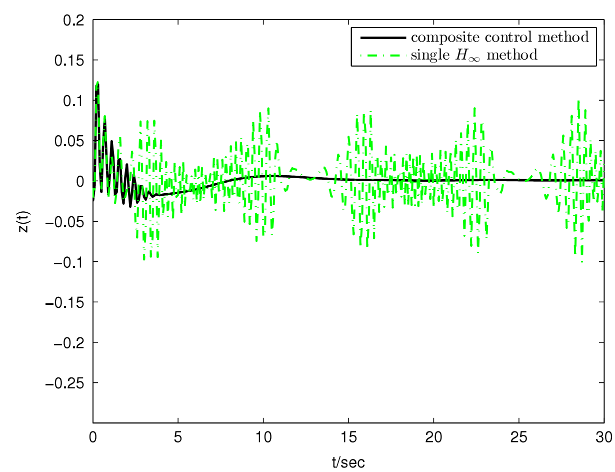

Fig. 3. Reference output along an individual switching sequence for known nonlinearity.

Different from Case 1, we suppose that $f_i(t, x(t))$ is unknown. Similar to Guo and Chen (2005), we assume $f_1(t, x(t)) = f_2(t, x(t)) = \alpha(t)x_2(t)$, where $\alpha(t)$ is assumed to be a random input with an upper bound 2.8. We can find $\|f_i(x(t), t)\| \le \|U_i(t, x_i(t))\|$, $i = 1, 2$. As in Case 1, we also do Monte Carlo simulations. The simulation results are given as follows: Fig. 4 plots the average of the states of composite system (28) over 200 switching sequences, which shows the stochastic stability of the composite system (28); Fig. 5 describes the disturbance $d_1(t)$, its estimation $\hat{d}_1(t)$ and the estimation error $d_1(t) - \hat{d}_1(t)$ along a single switching sequence, which indicates the proposed disturbance observer works well; Fig. 6 denotes the reference output $z(t)$ in (29) by the proposed composite control methodology, and by the single $\mathscr{H}_\infty$ control strategy, respectively, which expresses that our scheme is able to reject and attenuate the multiple disturbances, and the single $\mathscr{H}_\infty$ control strategy is not suitable to handle systems with multiple disturbances.

Remark 6. Single $\mathscr{H}_\infty$ control means that only $\mathscr{H}_\infty$ control strategy is used for system (1) with $d_1(t)$ and $d_2(t)$. For $d_2(t) \in \mathscr{L}_2[0, \infty)$, $\mathscr{H}_\infty$ control is effective to attenuate it. For $d_1(t)$ described in (3), if we choose $W_i = \begin{bmatrix} 0 & c \\ -c & 0 \end{bmatrix}$ with $c > 0$ as in the above numerical example, then $d_1(t)$ represents the periodic disturbance. Thus, $\mathscr{H}_\infty$ control fails to attenuate it.

Based on the above analysis, the main difference between the single $\mathscr{H}_\infty$ control strategy and the composite control method is

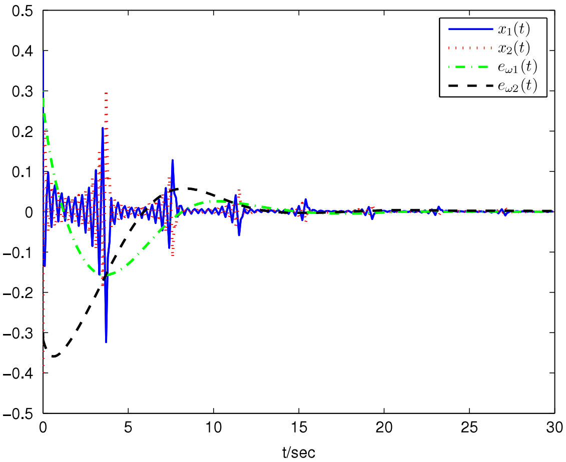

Fig. 4. The average of the states of the composite system (28) by Monte Carlo simulations.

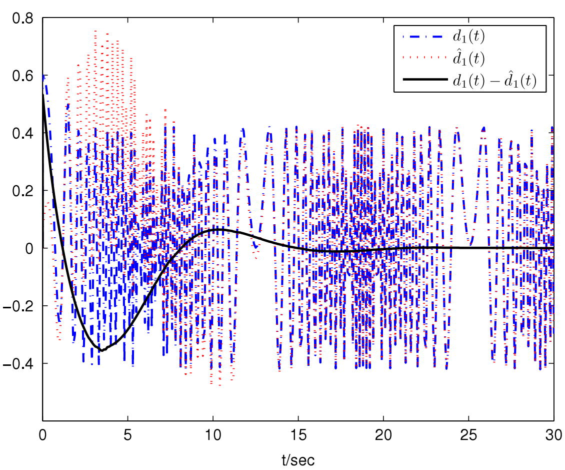

Fig. 5. Disturbance estimation error along an individual switching sequence for unknown nonlinearity.

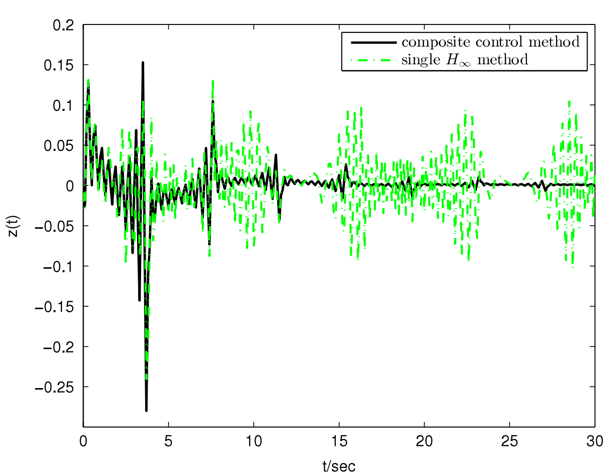

Fig. 6. Reference output along an individual switching sequence for unknown nonlinearity.

as follows: in the single $\mathscr{H}_\infty$ control, the disturbance observer for $d_1(t)$ is useless, and the control input is set as $u(t) = K_i x(t)$ instead of $u(t) = -\hat{d}_1(t) + K_i x(t)$. Consequently, single $\mathscr{H}_\infty$ control cannot attenuate or reject $d_1(t)$, thus, $d_1(t)$ still influences the output $z(t)$, which may be not stable from Figs. 3 and 6.

6. Conclusion

In this work, the composite anti-disturbance control problems have been investigated for a class of nonlinear system with jump parameters and multiple disturbances. The Lyapunov stability approach and the LMI technique have been applied to the analysis and the design of the disturbance observer and controller for the concerned system. The designed observer and controller ensure a prescribed performance level of the resulting composite system. A numerical example has been provided to demonstrate the efficiency of the proposed method.

Acknowledgments

The authors would like to acknowledge the Editor, the Associate Editor and the anonymous reviewers for their valuable comments and suggestions which helped to considerably improve the paper.

This work was supported in part by the Major State Basic Research Development Program of China (973 Program) under Grant 2012CB720003, the National Natural Science Foundation of China under Grants 61203041, 61127007, 60925012, 61121003 and 9101600, the Chinese National Post-doctor Science Foundation under Grants 2011M500217 and 2012T50036, and the Doctoral Fund of Ministry of Education of China under Grant 20120036120013.

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