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Composite disturbance-observer-based output feedback control and passive control for Markovian jump systems with multiple disturbances

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Abstract: This study addresses the problems of the composite disturbance-observer-based (DOB) output feedback control and passive control for Markovian jump systems with non-linearity and multiple disturbances. The multiple disturbances include two kinds: one is supposed to be a norm-bounded vector; the other is described by an exogenous system with perturbations. The purpose of the problem addressed is to design a DOB output feedback controller such that (i) the resulting composite system is passive; and (ii) different types of the above disturbances can be attenuated and rejected, respectively. The characterisation of the gains of the desired disturbance observer and the matrices of the expected output feedback controller are derived based on the solution to a convex optimisation problem that can be easily solved by standard numerical software. Finally, a simulation example is employed to show the effectiveness of the composite control scheme proposed in this study.

1 Introduction

Owing to increasing demands for high performance of the complex control systems, the anti-disturbance control has been an eternal topic since the beginning of the control theory. And now, it has attracted extensive interests and attention from both academia and engineer [1–8]. Several elegant schemes have been proposed to combat the above problem, such as non-linear disturbance-observer-based control (DOBC) theory, non-linear regulation theory, non-linear $\mathscr{H}_\infty$ theory and so on. A practically motivated way of handling the rejection of disturbance problems is non-linear DOBC approach [2, 9–11], by which the disturbance with some known information can be compensated completely. DOBC has also found its applications in the robotic systems [2], table drive systems [11], missile system [12] and so on. The basic idea of DOBC scheme is to construct an observer to estimate the disturbance and then based on the output of the observer, a feed-forward compensator plus conventional control law are applied to reject the disturbance and achieve the desired performance. When the system encounter multiple disturbances, hierarchical/composite control strategies consisting of DOBC and another control scheme are proposed to achieve the anti-disturbance performance, such as robust control [13], sliding mode control [14], adaptive control [12] and so on. Noted that, the control scheme in the above mentioned literature, are disturbance-observer-based (DOB) state-feedback, however, when the states or the estimation of the states are not available, such control scheme does not work any more. Consequently, DOB output feedback control strategy can be adopted while encountering the above situation.

On the other hand, Markovian jump linear systems (MJLs) has a strong practical background, since in practice many physical systems are subjected to abrupt variations in their structures because of random failures or repairs of components, sudden environmental disturbances, changing subsystem interconnections and abrupt variations in the operating point of a non-linear plant. MJLs have found wide applications in such systems as manufacturing systems, power systems, economics systems, communication systems and network-based control systems. Hence, MJLs have been drawing a continual interest from control theorists for decades. And many elegant works have been published on MJLs (see, e.g. to just name a few, [15–21] and the references therein).

In this paper, we are motivated to deal with the DOB output feedback control problem for MJLs with multiple disturbances and non-linearity. The main contributions of this paper can be highlighted as follows:

1. A new hierarchical/composite control methodology, which is DOBC plus passive control, for the controlled plant with multiple disturbances is proposed.
2. A new structure of the non-linear disturbance observer is constructed based on the information of the control input, measurement output and the derivative of the measurement output.

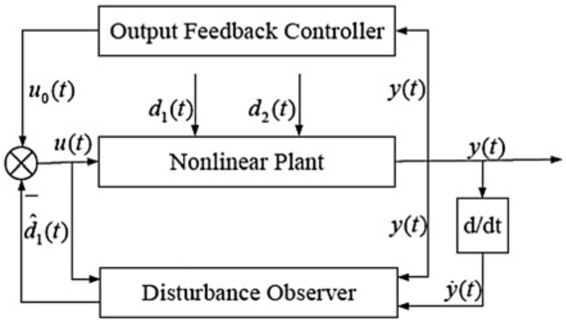

Fig. 1 *DOB output feedback control problem with multiple disturbances*

3. DOB output feedback controller is proposed to take place of DOB state feedback controller, which is under the assumption that the system states or the estimation of them are available.
4. The conditions of the existence of the above composite DOB output feedback controllers and passive controllers are proposed for both MJLs and linear systems with non-linearity, both of which have not been presented yet, up to now.

Notations. The notations used throughout the paper are standard. The superscript 'T' stands for matrix transposition; $\mathbb{R}^n$ denotes the n-dimensional Euclidean space; the notation $P > 0$ means that P is real symmetric and positive definite; I and 0 represent the identity matrix and a zero matrix, respectively; diag$\{\ldots\}$ stands for a block-diagonal matrix; $\|\cdot\|$ denotes the Euclidean norm of a vector and its induced norm of a matrix; and the signals that are square integrable over $[0, \infty)$ are represented by $\mathscr{L}_2[0, \infty)$ with the norm $\|\cdot\|_2$. $\mathbf{E}\{\cdot\}$ denotes the expectation operator with respect to probability measure $\mathcal{P}$. In addition, in symmetric block matrices or long matrix expressions, a star ($\star$) is used to represent a term that is induced by symmetry. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations.

2 Problem formulation and preliminaries

The DOB output feedback control problem subject to multiple disturbances is shown in Fig. 1. In this figure, the physical plant is modelled as Markovian jump non-linear systems, and the disturbance-observer-based output feedback controller is applied. In this section, we model the whole problem mathematically.

2.1 Non-linear plant

Fix a probability space $(\Omega, \mathcal{F}, \mathcal{P})$, where Ω is the sample space, $\mathcal{F}$ is the σ-algebra of subsets of the sample space and $\mathcal{P}$ is the probability measure on $\mathcal{F}$. Under this probability space, we consider the following MJLs with non-linearity

$$\dot{x}(t) = A(r_t)x(t) + F(r_t)f(x(t), t) + G(r_t)[u(t) + d_1(t)] + H(r_t)d_2(t) \quad (1a)$$

$$y(t) = D(r_t)x(t) \quad (1b)$$

$$z(t) = C(r_t)x(t) \quad (1c)$$

where $x(t) \in \mathbb{R}^n$ is the state vector; $u(t) \in \mathbb{R}^m$ is the control input; $y(t) \in \mathbb{R}^s$ is the output measurement; $z(t) \in \mathbb{R}^u$ is the controlled output; $f(x(t), t) \in \mathbb{R}^p$ are non-linear vector functions; and $d_1(t) \in \mathbb{R}^m$ is supposed to satisfy bounded conditions described as Assumption 1, which can represent the constant and harmonic noises. $d_2(t) \in \mathbb{R}^q$ is another disturbance which is assumed to be an arbitrary signal in $\mathscr{L}_2[0, \infty)$; the matrices $A_i \triangleq A(r_t = i)$, $F_i \triangleq F(r_t = i)$, $G_i \triangleq G(r_t = i)$, $H_i \triangleq H(r_t = i)$, $C_i \triangleq C(r_t = i)$ and $D_i \triangleq D(r_t = i)$, are known real constant matrices of appropriate dimensions; $\{r_t\}$ is a continuous-time Markov process with right continuous trajectories and taking values in a finite set $\mathcal{S} = \{1, 2, \ldots, \mathcal{N}\}$ with transition probability matrix $\Pi \triangleq \{\pi_{ij}\}$ given by

$$\Pr\{r_{t+\Delta} = j | r_t = i\} = \begin{cases} \pi_{ij}\Delta + o(\Delta) & \text{if } j \neq i \\ 1 + \pi_{ii}\Delta + o(\Delta) & \text{if } j = i \end{cases} \quad (2)$$

where $\Delta > 0$, $\lim_{\Delta \to 0}(o(\Delta)/\Delta) = 0$, $\pi_{ij} \geq 0$ is the transition rate from i at time t to j at time $t + \Delta$, and $\pi_{ii} = -\sum_{j=1, j\neq i}^{\mathcal{N}} \pi_{ij}$.

Assumption 1: The disturbance $d_1(t)$ is the control input path can be formulated by the following exogenous system

$$\begin{aligned} \dot{\omega}(t) &= W(r_t)\omega(t) + M(r_t)\delta(t) \\ d_1(t) &= V(r_t)\omega(t) \end{aligned} \quad (3)$$

where $W(r_t) \in \mathbb{R}^{r\times r}$, $M(r_t) \in \mathbb{R}^{r\times l}$ and $V(r_t) \in \mathbb{R}^{m\times r}$ are the proper known matrices. $\delta(t) \in \mathbb{R}^l$ is the additional disturbance which result from the perturbations and uncertainties in the exogenous system. It is also supposed that $\delta(t)$ belong to $\mathscr{L}_2[0, \infty)$. In many cases, system disturbance can be described as a dynamic system with unknown parameters and initial conditions, and can also be used to include the unmodeling error and system perturbations [10].

In this note, we make the following assumptions on the system (1).

Assumption 2: (i) $f(0, t) = 0$.
(ii) $\|f(x_1(t), t) - f(x_2(t), t)\| \leq \|U(x_1(t) - x_2(t))\|$ where U is given constant weighting matrix.

Assumption 3: (W_i, G_iV_i) is observable.

Assumption 4: The matrix G_i is full column rank (i.e. rank$\{G_i\} = m$).

2.2 DOB output feedback control

For the physical plant (1), we construct the following DOB output feedback control scheme

$$\hat{\omega}(t) = \nu(t) - L_i y(t) \quad (4a)$$

$$\hat{d}_1(t) = V_i\hat{\omega}(t) \quad (4b)$$

$$\dot{\nu}(t) = (W_i + L_iD_iG_iV_i)(\nu(t) - L_iy(t)) + L_iD_iG_iu(t) \quad (4c)$$

$$\dot{\hat{x}}(t) = \hat{A}_i\hat{x}(t) + \hat{B}_iy(t) \quad (4d)$$

$$u_0(t) = \hat{C}_i\hat{x}(t) + \hat{D}_iy(t) \quad (4e)$$

$$u(t) = u_0(t) - \hat{d}_1(t) \quad (4f)$$

where $\nu(t)$ is the state of the disturbance observer, $\hat{\omega}(t)$ is the estimation of $\omega(t)$, $\hat{d}_1(t) \in \mathbb{R}^r$ is the estimation of $d_1(t)$,

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$\hat{x}(t) \in \mathbb{R}^n$ is the state of the output feedback controller, $\hat{A}_i$, $\hat{B}_i$, $\hat{C}_i$ and $\hat{D}_i$ are the controller matrices to be determined and $L_i \in \mathbb{R}^{r\times n}$ is the observer gain to be determined.

Remark 1: DOB state-feedback control scheme is adopted in [10, 12–14], that is, $u(t) = -\hat{d}_1(t) + Kx(t)$ where K is the state-feedback gain. The disturbance $d_1(t)$ can be compensated through the $-\hat{d}_1(t)$, while $Kx(t)$ guarantees the dynamic system stable and achieving desired dynamic performances. In the above literatures, the states of the physical plant or the estimation of them are assumed available. However, it is not the case in practical. Thus, a new DOBC scheme, that is, DOB output feedback controller is presented in the form of (4).

Remark 2: From (4a), we can deduce that

$$\begin{aligned}\dot{\hat{\omega}}(t) &= \dot{v}(t) - L_i\dot{y}(t) = (W_i + L_iD_iG_iV_i)\hat{\omega}(t)\\ &\quad + L_iD_iG_iu(t) - L_i\dot{y}(t)\end{aligned} \tag{5}$$

Based on (4a)–(4c) and the structure of (5), we can conclude that the non-linear disturbance observer just include the information of the control input $u(t)$, the control output $y(t)$ and the derivative of the control output $\dot{y}(t)$. In the practical engineering, it is very difficult or costly to obtain the plant states, so the non-linear disturbance observer here we constructed can be easily applied in the practical engineering.

2.3 Composite system

Define the estimation error for the states of the disturbance $d_1(t)$ as

$$e_\omega(t) \triangleq \omega(t) - \hat{\omega}(t) \tag{6}$$

Combining (1), (3), (4) and (6), it is shown that composite system dynamics satisfies

$$\dot{\xi}(t) = \bar{A}_i\xi(t) + \bar{F}_if(\xi(t), t) + \bar{H}_id(t) \tag{7}$$

with $\xi(t) \triangleq [x(t)^{\mathrm{T}} \quad \hat{x}(t)^{\mathrm{T}} \quad e_\omega(t)^{\mathrm{T}}]^{\mathrm{T}}$, $d(t) \triangleq [d_2(t)^{\mathrm{T}} \quad \delta(t)^{\mathrm{T}}]^{\mathrm{T}}$, $f(\xi(t), t) = f(x(t), t)$ and

$$\bar{A}_i \triangleq \begin{bmatrix}\tilde{A}_{1i} & \tilde{A}_{2i}\\ \tilde{A}_{4i} & \tilde{A}_{3i}\end{bmatrix},\quad \bar{F}_i \triangleq \begin{bmatrix}\tilde{F}_i\\ L_iD_iF_i\end{bmatrix},\quad \bar{H}_i \triangleq \begin{bmatrix}\tilde{H}_{1i}\\ \tilde{H}_{2i}\end{bmatrix}$$

$$\tilde{A}_{1i} \triangleq \begin{bmatrix}A_i + G_i\hat{D}_iD_i & G_i\hat{C}_i\\ \hat{B}_iD_i & \hat{A}_i\end{bmatrix},\quad \tilde{A}_{2i} \triangleq \begin{bmatrix}G_iV_i\\ 0\end{bmatrix}$$

$$\tilde{A}_{3i} \triangleq W_i + L_iD_iG_iV_i,\quad \tilde{A}_{4i} \triangleq [L_iD_iA_i \quad 0],\quad \tilde{F}_i \triangleq \begin{bmatrix}F_i\\ 0\end{bmatrix}$$

$$\tilde{H}_{1i} \triangleq \begin{bmatrix}H_i & 0\\ 0 & 0\end{bmatrix},\quad \tilde{H}_{2i} \triangleq [L_iD_iH_i \quad M_i]$$

Remark 3: As $\delta(t) \in \mathscr{L}_2[0, \infty)$ and $d_2(t) \in \mathscr{L}_2[0, \infty)$, we can deduce $d(t) \in \mathscr{L}_2[0, \infty)$ defined in the composite system (7). Thus, system (7) becomes non-linear Markovian jump systems with norm-bounded disturbance. On the other hand, passivity, as a particular case of dissipativity, plays an important role in circuits, networks, systems and control theory. It offers a convenient tool for the stability analysis of uncertain or non-linear systems while presenting an efficient design approach to dealing with dynamic positioning of ships [22], formation control of autonomous underwater vehicles [23], fuzzy systems [24], networked control systems [25], neural networks [26] and so on. Passive control issue for network-based systems has also drawn research attention in recent years and and many results have been reported in [27, 28]. From the practical application point of view, many systems need to be passive in order to attenuate noises effectively. Thus, passive control scheme provides a nice tool for the analysis and synthesis of the composite systems (7) with norm-bounded disturbance $d(t)$.

Here, based on the above analysis in Remarks 1 and 3, we can summarise the notion of the composite DOB output feedback control and passive control scheme as follows.

Definition 1: For physical plant (1) with two types of disturbances: $d_1(t)$ and $d_2(t)$. $d_2(t)$ is supposed to be $\mathscr{H}_2$ norm-bounded vector; $d_1(t)$ is described by an exogenous system with $\mathscr{H}_2$ norm-bounded perturbations $\delta(t)$. Estimate the disturbance $d_1(t)$ with the disturbance observer (4a)–(4c), then based on the output of the disturbance observer $\hat{d}_1(t)$, construct the special form of output feedback controller in (4d)–(4f), we obtain the composite system (7) with only one type of energy-bounded disturbance. And hence, the analysis and synthesis of the problem can be deduced to passive control problem for the composite system (7). We name such scheme as 'composite DOB output feedback control and passive control' scheme.

Before formulating the main problem, we first give the following definitions [29].

Definition 2: The composite MJLs in (7) and (1c) is said to be passive if there exists a scalar $\gamma > 0$ such that

$$2\mathbf{E}\left\{\int_0^T \mathrm{d}^{\mathrm{T}}(t)z(t)\,\mathrm{d}t\right\} \geq -\gamma\mathbf{E}\left\{\int_0^T \mathrm{d}^{\mathrm{T}}(t)\,\mathrm{d}(t)\,\mathrm{d}t\right\} \tag{8}$$

for all $T > 0$ under zero initial conditions.

Remark 4: According to Definition 2, $z(t)$ and $\mathrm{d}(t)$ should be with the same dimensions. Thus, we should choose $u = q + l$.

Then, the problems to be addressed in this paper can be expressed as follows.

Problem 1 (passivity analysis): Under Assumptions 1–3, consider the non-linear MJLs (1) with two kinds of disturbances $d_1(t)$ and $d_2(t)$. Given the output feedback controller matrices $\hat{A}_i$, $\hat{B}_i$, $\hat{C}_i$, $\hat{D}_i$ in (4d) and (4e), and the gains L_i of the disturbance observer in (4a)–(4c), determine under what condition the resulting composite MJLs in (7) and (1c) is passive in the sense of Definition 2.

Problem 2 (Passification): Under Assumptions 1–3, consider the non-linear MJLs (1) with two kinds of disturbances $d_1(t)$ and $d_2(t)$. Determine the output feedback controller matrices $\hat{A}_i$, $\hat{B}_i$, $\hat{C}_i$, $\hat{D}_i$ in (4d) and (4e), and the gains L_i of the disturbance observer in (4a)–(4c), such that the resulting composite MJLs in (7) and (1c) is passive in the sense of Definition 2.

3 Passivity analysis

Under Assumptions 1–3 and suppose $\hat{A}_i$, $\hat{B}_i$, $\hat{C}_i$, $\hat{D}_i$ in (4d) and (4e), and the gains L_i of the disturbance observer in (4a)–(4c) are given, we first present a sufficient condition in terms of linear matrix inequalities (LMIs) for the composite MJLs (7), by which the closed-loop system is passive.

Theorem 1: Consider the composite system (7) under Assumptions 1–3. Given the controller matrices $\hat{A}_i$, $\hat{B}_i$, $\hat{C}_i$, $\hat{D}_i$ and the disturbance observer gains L_i, parameters $\lambda > 0$ and $\gamma > 0$, the composite system in (7) and (1c) is passive in the sense of Definition 2 if there exist matrices $P_{1i} > 0$ and $Q_i > 0$ such that the following LMIs hold for $i = 1, \ldots, \mathcal{N}$,

$$\begin{bmatrix} \Phi_{1i} & \Phi_{3i} & P_{1i}\tilde{F}_i & P_{1i}\tilde{H}_{1i} - \tilde{C}_i^{\mathrm{T}} & \tilde{U}^{\mathrm{T}} \\ \star & \Phi_{2i} & Q_iL_iD_iF_i & Q_i\tilde{H}_{2i} & 0 \\ \star & \star & -\frac{1}{\lambda^2}I & 0 & 0 \\ \star & \star & \star & -\gamma I & 0 \\ \star & \star & \star & \star & -\lambda^2 I \end{bmatrix} < 0 \quad (9)$$

with

$$\Phi_{1i} \triangleq P_{1i}\tilde{A}_{1i} + \tilde{A}_{1i}^{\mathrm{T}}P_{1i} + \bar{P}_{1i}$$
$$\Phi_{2i} \triangleq Q_i\tilde{A}_{3i} + \tilde{A}_{3i}^{\mathrm{T}}Q_i + \bar{Q}_i$$
$$\Phi_{3i} \triangleq P_{1i}\tilde{A}_{2i} + \tilde{A}_{4i}^{\mathrm{T}}Q_i$$
$$\tilde{U} \triangleq [U \quad 0], \quad \tilde{C}_i \triangleq [C_i \quad 0]$$
$$\bar{P}_{1i} \triangleq \sum_{j=1}^{\mathcal{N}} \pi_{ij}P_{1j}, \quad \bar{Q}_i \triangleq \sum_{j=1}^{\mathcal{N}} \pi_{ij}Q_j$$

and $\tilde{A}_{1i}$, $\tilde{A}_{2i}$, $\tilde{A}_{3i}$, $\tilde{F}_i$, $\tilde{H}_{1i}$ and $\tilde{H}_{2i}$ are defined in (7).

Proof: Define a Lyapunov functional candidate as follows

$$V(\xi(t), r_t, t) \triangleq V_1(\xi(t), r_t, t) + V_2(\xi(t), r_t, t) \quad (10)$$

with

$$\begin{cases} V_1(\xi(t), r_t, t) \triangleq \xi^{\mathrm{T}}(t)P_i\xi(t) \\ V_2(\xi(t), r_t, t) \triangleq \frac{1}{\lambda^2}\int_0^t \|Ux(\tau)\|^2 - \|f(x(\tau), \tau)\|^2 \mathrm{d}\tau \end{cases}$$

where $P_i > 0$, $i \in \mathcal{S}$.

Let $\mathcal{A}$ be the weak infinitesimal generator of the random process $\{\xi(t), r_t\}$. Then, for each $r_t = i$, $i \in \mathcal{S}$, it can be shown that

$$\begin{aligned} \mathcal{A}V_1(\xi(t), i, t) = {} & \xi^{\mathrm{T}}(t)(P_i\bar{A}_i + \bar{A}_i^{\mathrm{T}}P_i)\xi(t) + \xi^{\mathrm{T}}(t)\bar{P}_i\xi(t) \\ & + 2\xi^{\mathrm{T}}(t)P_i\bar{F}_if(x(t), t) + 2\xi^{\mathrm{T}}(t)P_i\bar{H}_id(t) \end{aligned} \quad (11)$$

$$\mathcal{A}V_2(\xi(t), i, t) = \frac{1}{\lambda^2}x^{\mathrm{T}}(t)U^{\mathrm{T}}Ux(t) - \frac{1}{\lambda^2}f^{\mathrm{T}}(x(t), t)f(x(t), t) \quad (12)$$

Combining (11) and (12), we can derive

$$\begin{aligned} \mathcal{A}V(\xi(t), i, t) &= \mathcal{A}V_1(\xi(t), i, t) + \mathcal{A}V_2(\xi(t), i, t) \\ &= \eta^{\mathrm{T}}(t)\begin{bmatrix} \bar{\Phi}_{1i} & P_i\bar{F}_i & P_i\bar{H}_i \\ \star & -\frac{1}{\lambda^2}I & 0 \\ \star & \star & 0 \end{bmatrix}\eta(t) \end{aligned} \quad (13)$$

with $\eta(t) \triangleq [\xi^{\mathrm{T}}(t) \quad f^{\mathrm{T}}(x(t), t) \quad d^{\mathrm{T}}(t)]^{\mathrm{T}}$ and

$$\bar{\Phi}_{1i} \triangleq P_i\bar{A}_i + \bar{A}_i^{\mathrm{T}}P_i + \bar{P}_i + \frac{1}{\lambda^2}\bar{U}^{\mathrm{T}}\bar{U}$$
$$\bar{U} \triangleq [U \quad 0 \quad 0], \quad \bar{P}_i \triangleq \sum_{j=1}^{\mathcal{N}} \pi_{ij}P_j$$

Consider the following index

$$J(T) \triangleq \mathbf{E}\left\{\int_0^T [\mathcal{A}V(x_t, r_t = i) - 2d^{\mathrm{T}}(t)z(t) - \gamma d^{\mathrm{T}}(t)d(t)]\,\mathrm{d}t\right\}$$

Then, under the zero initial conditions, it follows from (13) that

$$J(T) = \mathbf{E}\left\{\int_0^{\mathrm{T}} \eta^{\mathrm{T}}(t)\Theta_i\eta(t)\,\mathrm{d}t\right\} \quad (14)$$

with

$$\Theta_i \triangleq \begin{bmatrix} \bar{\Phi}_{1i} & P_i\bar{F}_i & P_i\bar{H}_i - \bar{C}_i^{\mathrm{T}} \\ \star & -\frac{1}{\lambda^2}I & 0 \\ \star & \star & -\gamma I \end{bmatrix}$$

where $\bar{C}_i \triangleq [C_i \quad 0 \quad 0]$.

Therefore, if $\Theta_i < 0$, then $J(T) \le 0$, and we can obtain

$$\mathbf{E}\left\{\int_0^T 2\,\mathrm{d}^{\mathrm{T}}(t)z(t) + \gamma\,\mathrm{d}^{\mathrm{T}}(t)\,\mathrm{d}(t)]\,\mathrm{d}t\right\} \ge \mathbf{E}V(x_t, i, t) \ge 0 \quad (15)$$

that is

$$2\mathbf{E}\left\{\int_0^T \mathrm{d}^{\mathrm{T}}(t)z(t)\,\mathrm{d}t\right\} \ge -\gamma\mathbf{E}\left\{\int_0^T \mathrm{d}^{\mathrm{T}}(t)\,\mathrm{d}(t)\,\mathrm{d}t\right\}$$

which implies the composite system (7) is passive.

Now, we are in the position to show condition (9) guarantee $\Theta_i < 0$.

Define

$$P_i \triangleq \begin{bmatrix} P_{1i} & 0 \\ 0 & Q_i \end{bmatrix} \quad (16)$$

with $P_{1i} \ge 0$ and $Q_i \ge 0$.

Then, take P_i defined in (16) into Θ_i, we can derive

$$\Theta_i = \begin{bmatrix} \tilde{\Phi}_{1i} & \Phi_{3i} & P_{1i}\tilde{F}_i & P_{1i}\tilde{H}_{1i} - \tilde{C}_i^{\mathrm{T}} \\ \star & \Phi_{2i} & Q_iL_iD_iF_i & Q_i\tilde{H}_{2i} \\ \star & \star & -\frac{1}{\lambda^2}I & 0 \\ \star & \star & \star & -\gamma I \end{bmatrix} \quad (17)$$

with $\tilde{\Phi}_{1i} \triangleq \Phi_{1i} + \frac{1}{\lambda^2}\tilde{U}^{\mathrm{T}}\tilde{U}$. Using Schur complement to (9), we can readily derive $\Theta_i < 0$. The proof is completed. □

4 Passification

This section is devoted to solving the passification problem formulated in Section 2.

Theorem 2: Consider non-linear Markovian jump systems (1) with Assumptions 1–3. Given parameters $\lambda > 0$ and $\gamma > 0$, there exist DOB output feedback controllers in the form of (4) such that the composite system in (7) and (1c) is passive if there exist matrices $P_{2i} > 0$, $Q_i > 0$, Y_i, $S_i > 0$,

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A_{ci}, B_{ci}, C_{ci}, D_{ci}, S_{ij}, $V_{ij} > 0$ $(j \neq i, j = 1, \dots, \mathcal{N})$ such that for $i = 1, 2, \dots, \mathcal{N}$

$$\begin{bmatrix} \Lambda_{1i} + \Lambda_{2i} & \Lambda_{3i} & \Lambda_{4i} & \Lambda_{5i} - \Lambda_{6i} & \Lambda_{7i} & \check{X}_i & \check{Y}_i^{\mathrm{T}} \\ \star & \Lambda_{8i} & Y_i D_i F_i & \Lambda_{9i} & 0 & 0 & 0 \\ \star & \star & -\frac{1}{\lambda^2} I & 0 & 0 & 0 & 0 \\ \star & \star & \star & -\gamma I & 0 & 0 & 0 \\ \star & \star & \star & \star & -\lambda^2 I & 0 & 0 \\ \star & \star & \star & \star & \star & -I & 0 \\ \star & \star & \star & \star & \star & \star & -I \end{bmatrix} < 0 \tag{18a}$$

$$\begin{bmatrix} P_{2i} & S_i \\ \star & S_i \end{bmatrix} > 0 \tag{18b}$$

with

$$\Lambda_{1i} \triangleq \pi_{ii} \begin{bmatrix} P_{2i} & S_i^{\mathrm{T}} \\ S_i & S_i \end{bmatrix} + \sum_{j=1, j \neq i}^{\mathcal{N}} \pi_{ij} \begin{bmatrix} P_{2j} & S_{ij}^{\mathrm{T}} \\ S_{ij} & V_{ij} \end{bmatrix}$$

$$\Lambda_{2i} \triangleq \begin{bmatrix} P_{2i} A_i + B_{ci} D_i & A_{ci} \\ S_i A_i + B_{ci} D_i & A_{ci} \end{bmatrix} + \begin{bmatrix} P_{2i} A_i + B_{ci} D_i & A_{ci} \\ S_i A_i + B_{ci} D_i & A_{ci} \end{bmatrix}^{\mathrm{T}}$$

$$\Lambda_{3i} \triangleq \begin{bmatrix} P_{2i} G_i V_i + (Y_i D_i A_i)^{\mathrm{T}} \\ S_i G_i V_i \end{bmatrix}, \quad \Lambda_{4i} \triangleq \begin{bmatrix} P_{2i} F_i \\ S_i F_i \end{bmatrix}$$

$$\Lambda_{5i} \triangleq \begin{bmatrix} P_{2i} H_i & 0 \\ S_i H_i & 0 \end{bmatrix}, \quad \Lambda_{6i} \triangleq \begin{bmatrix} C_i^{\mathrm{T}} \\ 0 \end{bmatrix}, \quad \Lambda_{7i} \triangleq \begin{bmatrix} U^{\mathrm{T}} \\ 0 \end{bmatrix}$$

$$\Lambda_{8i} \triangleq Q_i W_i + Y_i D_i G_i V_i + (Q_i W_i + Y_i D_i G_i V_i)^{\mathrm{T}} + \bar{Q}_i$$

$$\Lambda_{9i} \triangleq \begin{bmatrix} Y_i D_i H_i & Q_i M_i \end{bmatrix}$$

$$\check{X}_i \triangleq \begin{bmatrix} P_{2i} & 0 \\ 0 & S_i \end{bmatrix}, \quad \check{Y}_i \triangleq \begin{bmatrix} G_i D_{ci} D_i & G_i C_{ci} \\ G_i D_{ci} D_i & G_i C_{ci} \end{bmatrix}$$

Moreover, if the above conditions are feasible, the gains of desired disturbance observer (4a)–(4c) are $L_i = Q_i^{-1} Y_i$, and the desired matrices of the output feedback controller (4d) and (4e) are

$$\begin{bmatrix} \hat{A}_i & \hat{B}_i \\ \hat{C}_i & \hat{D}_i \end{bmatrix} = \begin{bmatrix} S_i^{-1} & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} A_{ci} & B_{ci} \\ C_{ci} & D_{ci} \end{bmatrix} \tag{19}$$

Proof: To prove the first part, we partition P_{1i} in Theorem 1 as

$$P_{1i} \triangleq \begin{bmatrix} P_{2i} & P_{3i} \\ \star & P_{4i} \end{bmatrix} > 0 \tag{20}$$

By invoking a small perturbation if necessary, we can assume that P_{3i} and P_{4i} are non-singular. Thus, we can introduce the following invertible matrix

$$J_i \triangleq \begin{bmatrix} I & 0 \\ 0 & P_{4i}^{-1} P_{3i}^{\mathrm{T}} \end{bmatrix}$$

Also, we define

$$S_i \triangleq P_{3i} P_{4i}^{-1} P_{3i}^{\mathrm{T}}, \quad A_{ci} \triangleq P_{3i} \hat{A}_i P_{4i}^{-1} P_{3i}^{\mathrm{T}}, \quad B_{ci} \triangleq P_{3i} \hat{B}_i$$
$$C_{ci} \triangleq \hat{C}_i P_{4i}^{-1} P_{3i}^{\mathrm{T}}, \quad D_{ci} \triangleq \hat{D}_i, \quad Y_i \triangleq Q_i L_i \tag{21}$$

and for $j \neq i, j = 1, 2, \dots, \mathcal{N}$

$$S_{ij} \triangleq P_{3i} P_{4i}^{-1} P_{3j}^{\mathrm{T}}, \quad V_{ij} \triangleq P_{3i} P_{4i}^{-1} P_{4j} P_{4i}^{-1} P_{3i}^{\mathrm{T}} \tag{22}$$

Then, performing a congruence transformation by diag$\{J_i, I, I, I, I\}$ to (9), and considering (21) and (22), we readily obtain

$$\begin{bmatrix} \bar{\Lambda}_{1i} & \Lambda_{3i} & \Lambda_{4i} & \Lambda_{5i} - \Lambda_{6i} & \Lambda_{7i} \\ \star & \Phi_{2i} & Q_i L_i D_i F_i & Q_i \tilde{H}_{2i} & 0 \\ \star & \star & -\frac{1}{\lambda^2} I & 0 & 0 \\ \star & \star & \star & -\gamma I & 0 \\ \star & \star & \star & \star & -\lambda^2 I \end{bmatrix} < 0 \tag{23}$$

with $\bar{\Lambda}_{1i} \triangleq \Lambda_{1i} + \Lambda_{2i} + \check{X}_i \check{Y}_i + \check{Y}_i^{\mathrm{T}} \check{X}_i$. Note that $\check{X}_i$ and $\check{Y}_i$ are with the same dimensions, thus we have

$$\check{X}_i \check{Y}_i + \check{Y}_i^{\mathrm{T}} \check{X}_i \leq \check{X}_i \check{X}_i + \check{Y}_i^{\mathrm{T}} \check{Y}_i \tag{24}$$

By Schur complement, we readily obtain (18a).

In addition, from $J_i^{\mathrm{T}} P_{1i} J_i > 0$, we have (18b).

Now we consider the second part. From (21), we have

$$\begin{bmatrix} \hat{A}_i & \hat{B}_i \\ \hat{C}_i & \hat{D}_i \end{bmatrix} = \begin{bmatrix} P_{3i}^{-1} & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} A_{ci} & B_{ci} \\ C_{ci} & D_{ci} \end{bmatrix} \begin{bmatrix} P_{3i}^{-T} P_{4i} & 0 \\ 0 & I \end{bmatrix}$$

Define $u_1(t) \triangleq \hat{C}_i \hat{x}(t) + \hat{D}_i y(t)$. And according to (4d)–(4e), the transfer function [30, 31] from control signal $u_1(t)$ to measured output $y(t)$ can be described by

$$\begin{aligned} T_{u_1 y} &= \hat{C}_i (sI - \hat{A}_i)^{-1} \hat{B}_i + \hat{D}_i \\ &= C_{ci} P_{3i}^{-T} P_{4i} (sI - P_{3i}^{-1} A_{ci} P_{3i}^{-T} P_{4i})^{-1} P_{3i}^{-1} B_{ci} + D_{ci} \\ &= C_{ci} (sI - S_i^{-1} A_{ci})^{-1} S_i^{-1} B_{ci} + D_{ci} \end{aligned} \tag{25}$$

Therefore we can conclude from (25) that the parameters of the output feedback controller to be specified in (4d) and (4e) can be constructed by (19), which completes the proof. □

Remark 5: The conditions (18) of Theorem 2 may be conservative, due to the inequalities (24) was introduced.

By noting $\bar{\Lambda}_{1i} \triangleq \Lambda_{1i} + \Lambda_{2i} + \Gamma_i + \Gamma_i^{\mathrm{T}}$ with

$$\Gamma_i = \begin{bmatrix} P_{2i} G_i \\ S_i G_i \end{bmatrix} X_i \bar{D}_i, \quad X_i \triangleq [D_{ci} \quad C_{ci}], \quad \bar{D}_i \triangleq \begin{bmatrix} D_i & 0 \\ 0 & I \end{bmatrix} \tag{26}$$

Letting

$$P_{2i} G_i = G_i N_{1i}, \quad S_i G_i = G_i N_{2i}, \quad \begin{bmatrix} N_{1i} \\ N_{2i} \end{bmatrix} X \triangleq \bar{X} \tag{27}$$

where N_{1i}, N_{2i} and $\bar{X}$ are new variables.

Based on Remark 5 and following along the similar lines in Theorem 2, we readily give the following theorem.

Theorem 3: Consider non-linear Markovian jump systems (1) with Assumptions 1–4. Given parameters $\lambda > 0$ and $\gamma > 0$, there exist DOB output feedback controllers in the form of (4) such that the composite system in (7) and (1c) is passive if there exist matrices $P_{2i} > 0$, $Q_i > 0$, Y_i, $S_i > 0$, A_{ci}, B_{ci}, N_{1i}, N_{2i}, $\bar{X}_i$, S_{ij}, $V_{ij} > 0$ $(j \neq i, j = 1, \dots, \mathcal{N})$ such that for

$i = 1, 2, \ldots, \mathcal{N}$

$$\begin{bmatrix} \Lambda_{1i} + \Lambda_{2i} + \bar{\Gamma}_i & \Lambda_{3i} & \Lambda_{4i} & \Lambda_{5i} - \Lambda_{6i} & \Lambda_{7i} \\ \star & \Lambda_{8i} & Y_i D_i F_i & \Lambda_{9i} & 0 \\ \star & \star & -\frac{1}{\lambda^2} I & 0 & 0 \\ \star & \star & \star & -\gamma I & 0 \\ \star & \star & \star & \star & -\lambda^2 I \end{bmatrix} < 0 \tag{28a}$$

$$\begin{bmatrix} P_{2i} & S_i \\ \star & S_i \end{bmatrix} > 0 \tag{28b}$$

$$P_{2i} G_i = G_i N_{1i} \tag{28c}$$

$$S_i G_i = G_i N_{2i} \tag{28d}$$

with $\bar{\Gamma}_i \triangleq \left[\begin{smallmatrix} G_i & 0 \\ 0 & G_i \end{smallmatrix}\right] \bar{X}_i \bar{D}_i + \left(\left[\begin{smallmatrix} G_i & 0 \\ 0 & G_i \end{smallmatrix}\right] \bar{X}_i \bar{D}_i\right)^{\mathrm{T}}$.

Moreover, if the above conditions are feasible, $[D_{ci} \quad C_{ci}] = \left[\begin{smallmatrix} N_{1i} \\ N_{2i} \end{smallmatrix}\right]^{+} \bar{X}_i$ where '+' means the generalised inversion of the matrix. The gains of desired disturbance observer (4a)–(4c) are $L_i = Q_i^{-1} Y_i$, and the desired matrices of the output feedback controller (4d) and (4e) are

$$\begin{bmatrix} \hat{A}_i & \hat{B}_i \\ \hat{C}_i & \hat{D}_i \end{bmatrix} = \begin{bmatrix} S_i^{-1} & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} A_{ci} & B_{ci} \\ C_{ci} & D_{ci} \end{bmatrix} \tag{29}$$

Now, we consider the case: system (1) under Assumptions 1–4 is without jumping parameters (i.e. $N = 1$)

$$\dot{x}(t) = Ax(t) + Ff(x(t), t) + G[u(t) + d_1(t)] + Hd_2(t) \tag{30a}$$

$$y(t) = Dx(t) \tag{30b}$$

$$z(t) = Cx(t) \tag{30c}$$

with the disturbance d_1 described as

$$\dot{\omega}(t) = W\omega(t) + M\delta(t)$$
$$d_1(t) = V\omega(t) \tag{31}$$

and thus the corresponding DOB output feedback controller is constructed in the following form

$$\hat{\omega}(t) = v(t) - Ly(t) \tag{32a}$$

$$\hat{d}_1(t) = V\hat{\omega}(t) \tag{32b}$$

$$\dot{v}(t) = (W + LDGV)(v(t) - Ly(t)) + LDGu(t) \tag{32c}$$

$$\dot{\hat{x}}(t) = \hat{A}\hat{x}(t) + \hat{B}y(t) \tag{32d}$$

$$u_0(t) = \hat{C}\hat{x}(t) + \hat{D}y(t) \tag{32e}$$

$$u(t) = u_0(t) - \hat{d}_1(t) \tag{32f}$$

where $\hat{A}$, $\hat{B}$, $\hat{C}$ and $\hat{D}$ are the output feedback controller matrices to be determined, and L is the observer gain to be determined.

For such a case, the composite system becomes a class of non-linear system effectively operating at one of the subsystems all the time, and it can be described by

$$\dot{\xi}(t) = \bar{A}\xi(t) + \bar{F}f(\xi(t), t) + \bar{H}d(t) \tag{33a}$$

$$z(t) = \bar{C}\xi(t) \tag{33b}$$

$$\bar{A} \triangleq \begin{bmatrix} A + G\hat{D}D & G\hat{C} & GV \\ \hat{B}D & \hat{A} & 0 \\ LDA & 0 & W + LDGV \end{bmatrix}, \quad \bar{F} \triangleq \begin{bmatrix} F \\ 0 \\ LDF \end{bmatrix}$$

$$\bar{H} \triangleq \begin{bmatrix} H & 0 \\ 0 & 0 \\ LDH & M \end{bmatrix}, \quad \bar{C} \triangleq [C \quad 0 \quad 0]$$

Corollary 1: Consider non-linear system (30) with (31) under Assumptions 2–4. Given parameters $\lambda > 0$ and $\gamma > 0$, there exist a DOB output feedback controller in the form of (32) such that the composite system (33) is passive if there exist matrices $P_2 > 0$, $Q > 0$, Y, $S > 0$, A_c, B_c, N_1, N_2 and $\bar{X}$ such that

$$\begin{bmatrix} \Omega_1 & \Omega_2 & \Omega_3 & \Omega_4 & \Omega_5 \\ \star & \Omega_6 & Y_i D_i F_i & \Omega_7 & 0 \\ \star & \star & -\frac{1}{\lambda^2} I & 0 & 0 \\ \star & \star & \star & -\gamma I & 0 \\ \star & \star & \star & \star & -\lambda^2 I \end{bmatrix} < 0 \tag{34a}$$

$$\begin{bmatrix} P_2 & S \\ \star & S \end{bmatrix} > 0 \tag{34b}$$

$$P_2 G = GN_1 \tag{34c}$$

$$SG = GN_2 \tag{34d}$$

with

$$\Omega_1 \triangleq \begin{bmatrix} P_2 A + B_c D & A_c \\ SA + B_c D & A_c \end{bmatrix} + \begin{bmatrix} P_2 A + B_c D & A_c \\ SA + B_c D & A_c \end{bmatrix}^{\mathrm{T}} + \begin{bmatrix} G & 0 \\ 0 & G \end{bmatrix} \bar{X}\bar{D} + \left(\begin{bmatrix} G & 0 \\ 0 & G \end{bmatrix} \bar{X}\bar{D} \right)^{\mathrm{T}}$$

$$\Omega_2 \triangleq \begin{bmatrix} P_2 GV + (YDA)^{\mathrm{T}} \\ SGV \end{bmatrix}, \quad \Omega_3 \triangleq \begin{bmatrix} P_2 F \\ SF \end{bmatrix}$$

$$\Omega_4 \triangleq \begin{bmatrix} P_2 H & 0 \\ SH & 0 \end{bmatrix} - \begin{bmatrix} C^{\mathrm{T}} \\ 0 \end{bmatrix}, \quad \Omega_5 \triangleq \begin{bmatrix} U^{\mathrm{T}} \\ 0 \end{bmatrix}$$

$$\Omega_6 \triangleq QW + YDGV + (QW + YDGV)^{\mathrm{T}}$$

$$\Omega_7 \triangleq [YDH \quad QM], \quad \bar{D} \triangleq \mathrm{diag}\{D, I\}$$

Moreover, if the above conditions are feasible, $[D_c \quad C_c] = \left[\begin{smallmatrix} N_1 \\ N_2 \end{smallmatrix}\right]^{+} \bar{X}$ where '+' means the generalised inversion of the matrix. The gain of desired disturbance observer (32a)–(32c) are $L = Q^{-1}Y$, and the desired matrices of the output feedback controller (32d) and (32e) are

$$\begin{bmatrix} \hat{A} & \hat{B} \\ \hat{C} & \hat{D} \end{bmatrix} = \begin{bmatrix} S^{-1} & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} A_c & B_c \\ C_c & D_c \end{bmatrix} \tag{35}$$

Remark 6: To the best of the authors' knowledge, this is the first time that the DOB output feedback control strategy is applied to the control problem for linear system with Lipschitz non-linearity and multiple disturbances.

5 Illustrative example

In this section, we will present an illustrative example to demonstrate the effectiveness of the proposed approaches. Consider the systems in (1) and (3), which involves two modes, and the parameters of the systems are given

as follows:
Mode 1:

$$A_1 = \begin{bmatrix} 2.2 & -0.3 \\ 0.1 & -5.0 \end{bmatrix}, \quad F_1 = \begin{bmatrix} 0.01 \\ 0.01 \end{bmatrix}, \quad G_1 = \begin{bmatrix} -1.1 \\ 0.1 \end{bmatrix}$$
$$C_1 = \begin{bmatrix} 1.0 & 0.6 \\ 0.5 & 0.1 \end{bmatrix}, \quad D_1 = \begin{bmatrix} 0.2 & 0.1 \end{bmatrix}, \quad H_1 = \begin{bmatrix} 0.2 \\ 0.1 \end{bmatrix}$$
$$W_1 = \begin{bmatrix} 0 & 1.0 \\ -1.0 & 0 \end{bmatrix}, \quad V_1 = \begin{bmatrix} 0.5 & 0 \end{bmatrix}, \quad M_1 = \begin{bmatrix} 0.2 \\ 0.4 \end{bmatrix}$$

Mode 2:

$$A_2 = \begin{bmatrix} 1.2 & -0.3 \\ 2.1 & -3.0 \end{bmatrix}, \quad F_2 = \begin{bmatrix} 0.05 \\ 0.01 \end{bmatrix}, \quad G_2 = \begin{bmatrix} -0.8 \\ 0.2 \end{bmatrix}$$
$$C_2 = \begin{bmatrix} 2.0 & 0.6 \\ 1.5 & 2.1 \end{bmatrix}, \quad D_2 = \begin{bmatrix} 0.15 & 0.15 \end{bmatrix}, \quad H_2 = \begin{bmatrix} 2.0 \\ 1.0 \end{bmatrix}$$
$$W_2 = \begin{bmatrix} 0 & 1.0 \\ -1.0 & 0 \end{bmatrix}, \quad V_2 = \begin{bmatrix} 0.1 & 0 \end{bmatrix}, \quad M_2 = \begin{bmatrix} 0.2 \\ 0.2 \end{bmatrix}$$

The transition probability matrix is assumed to be $\Pi = \left[\begin{smallmatrix} -1 & 1 \\ 2 & -2 \end{smallmatrix}\right]$. Assume $\lambda = \gamma = 1$ and $U = \left[\begin{smallmatrix} 0 & 0 \\ 0 & 1.0 \end{smallmatrix}\right]$. Suppose $f(x(t), t) = x_2(t)\sin(\mathrm{t})$, we can find $\|f(x(t), t)\| \le \|Ux(t)\|$.

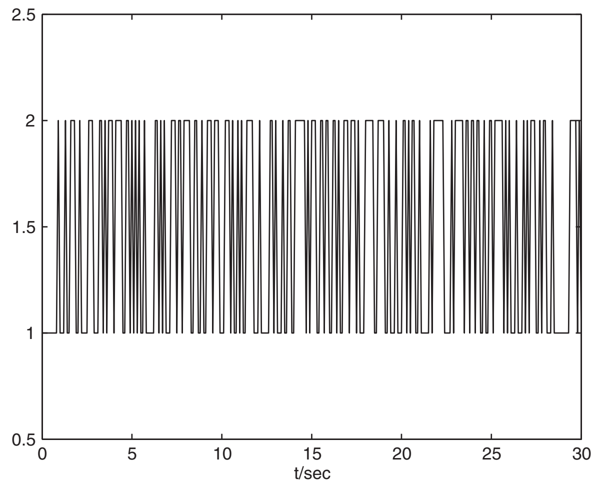

Fig. 2 *Switching signal*

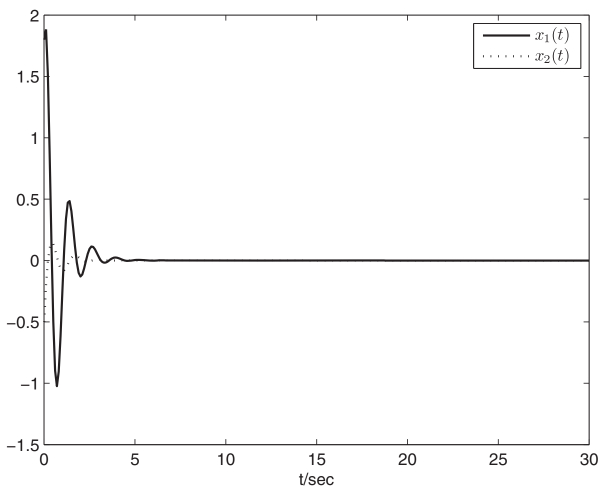

Fig. 3 *States of the system (1) with the DOB output feedback controller (4)*

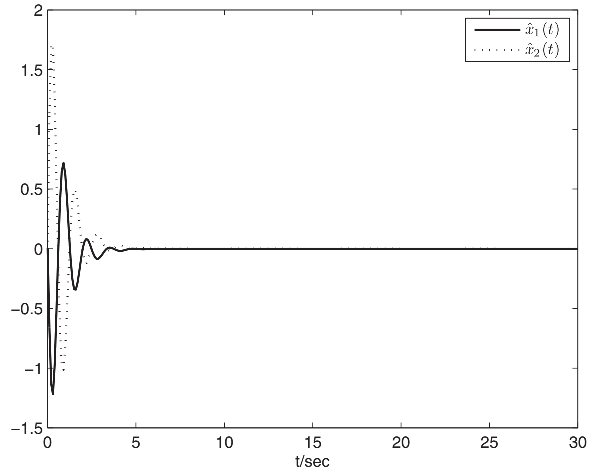

Fig. 4 *States of the output feedback controller in (4d) and (4e)*

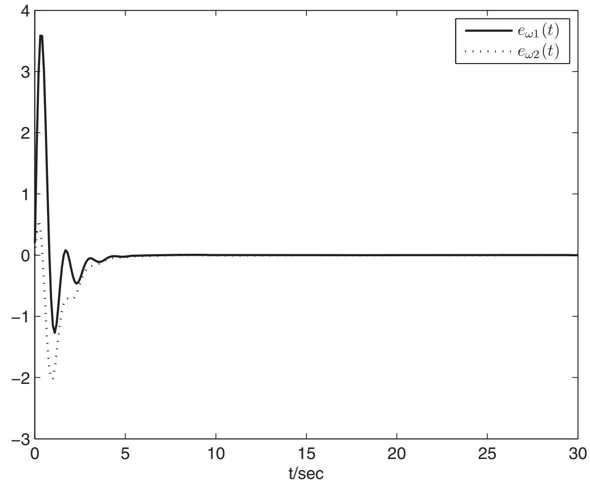

Fig. 5 *Estimation error $e_\omega(t)$ of the states of the disturbance $d_1(t)$*

Assume $d_2(t) = (1/5 + 10t)$ and $\delta(t) = (\sin(\mathrm{t})/1 + t^2)$. Given the initial condition as $x(0) = [1.8 \quad -0.5]^{\mathrm{T}}$, $\hat{x}(0) = [0 \quad 0]^{\mathrm{T}}$ and $e_\omega(0) = [0.2 \quad -0.1]^{\mathrm{T}}$.

Our intention here is to design the DOB output feedback controller in the form of (4) such that the composite system in (7) and (1c) is passive. We resort to the LMI Toolbox in Matlab to solve the LMIs in (28), and the gains of the desired output feedback controller and disturbance observer are given by

$$\hat{A}_1 = \begin{bmatrix} -9.1078 & -2.7607 \\ 0.6636 & -6.0615 \end{bmatrix}, \quad \hat{B}_1 = \begin{bmatrix} -21.2809 \\ 36.6158 \end{bmatrix}$$
$$\hat{A}_2 = \begin{bmatrix} -29.8322 & -2.2106 \\ -3.4608 & -32.2830 \end{bmatrix}, \quad \hat{B}_2 = \begin{bmatrix} -191.6202 \\ -151.2263 \end{bmatrix}$$
$$\hat{C}_1 = \begin{bmatrix} -2.3135 & 1.2147 \end{bmatrix}, \quad \hat{C}_2 = \begin{bmatrix} 0.4170 & -1.0293 \end{bmatrix}$$
$$L_1 = \begin{bmatrix} 14.6649 \\ 4.6497 \end{bmatrix}, \quad L_2 = \begin{bmatrix} 0.0557 \\ 0.0386 \end{bmatrix}$$
$$\hat{D}_1 = 9.4092, \quad \hat{D}_2 = 41.6411$$

Fig. 2 plots the switching signal, where '1' and '2' represent, respectively, the first and the second subsystem. Fig. 3 represents states of the system (1) with the DOB output feedback controller (4), whereas Fig. 4 displays the states of the

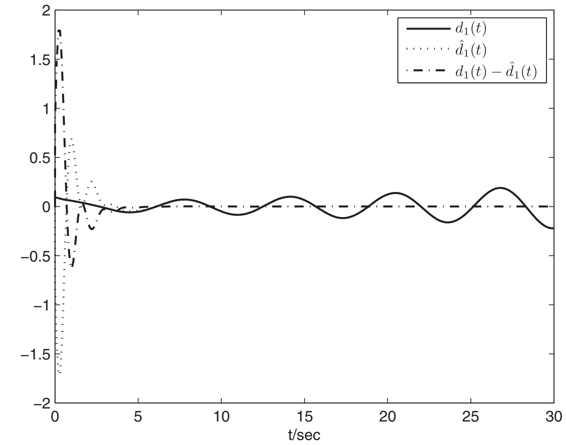

Fig. 6 *$d_1(t)$, $\hat{d}_1(t)$ and $d_1(t) - \hat{d}_1(t)$*

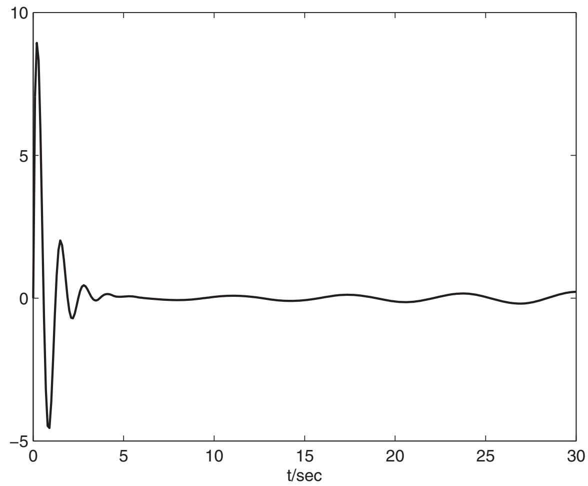

Fig. 7 *Control input u(t)*

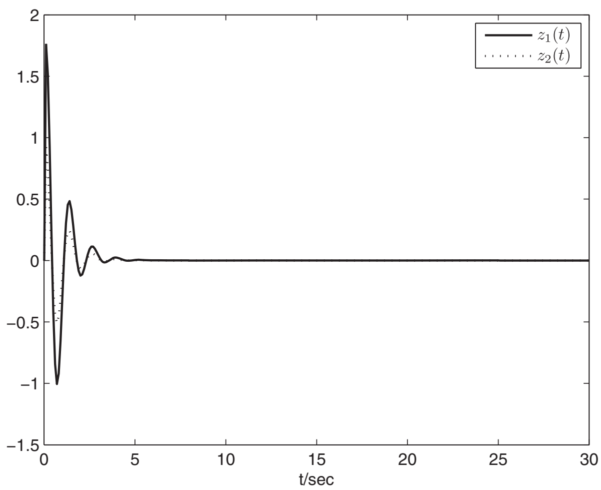

Fig. 8 *Controlled output in (1c) of the system (1) with DOBC and passive control*

output feedback controller in (4d) and (4e). Fig. 5 depicts the estimation error $e_\omega(t)$ of the states of the disturbance $d_1(t)$, while disturbance $d_1(t)$, $\hat{d}_1(t)$ and the estimation error $d_1(t) - \hat{d}_1(t)$ are described in Fig. 6. Fig. 7 is the control input in (4f).

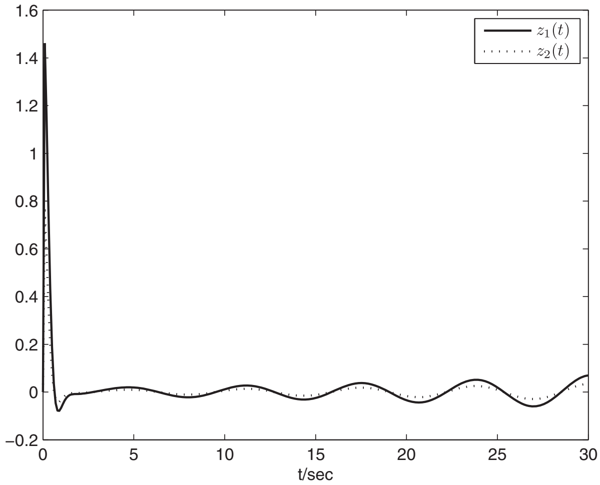

Fig. 9 *Controlled output in (1c) of the system (1) with single passive control*

Fig. 8 denotes controlled output in (1c) of the system (1) based on composite DOBC and passive control methodologies, whereas Fig. 9 depicts the output $z(t)$ by single passive control strategy.

Remark 7: From Figs. 8 and 9, we can draw a conclusion that the composite system in (7) and (1c) is stable and able to reject and attenuate the multiple disturbances ($d_1(t)$ and $d_2(t)$) under the DOB output feedback controller obtained above, and single passive control methodology is not suitable to handle systems with multiple disturbances. For single passive control method, $d_1(t)$ can not be rejected and still influence $z(t)$.

6 Conclusion

In this work, the composite DOB output feedback control and passive control problems have been investigated for a class of non-linear system with jump parameters and multiple disturbances. The Lyapunov stability approach and the LMI technique have been applied to the analysis and the design of the disturbance observer and controller for the concerned system. The designed observer and controller ensure a prescribed performance level of the resulting composite system. A numerical example has been provided to demonstrate the efficiency of the proposed method.

7 Future work

Composite control problem for Markovian jump systems with multiple disturbances has been solved in a model-based framework. That is, the model of the Markovian jump systems is known. However, it is not easy to obtain the physical model for modern complicated systems [32]. Thus, to achieve more practical oriented results, it is better to deal with the control problem for the Markovian jump systems in the data-driven framework [33]. In the future, we will study the control problem based on the data-driven framework by assuming the model of the plant is unknown and the model of the disturbance $d_1(t)$ is known.

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