Survey 1

Likelihood $n_1 | S \sim \text{Poisson}(A_1 S)$

Fisher information $I_1(S) = \frac{A_1}{S}$

Survey 2

actual # stars $m | S \sim \text{Poisson}(A_2 S)$

detected # star $n_2 | m \sim \text{Binomial}(m, \boxed{p})$

$$S \longrightarrow m \longrightarrow n_2$$

$$\mathcal{L}(n_2 | S) = \sum_{m=n_2}^{\infty} P(n_2 | m)\, P(m | S)$$ — Law of total probability.

$$= \sum_{m=n_2}^{\infty} {}^{m}C_{n_2}\, p^{n_2}(1-p)^{m-n_2} \cdot \frac{(A_2 S)^m e^{-A_2 S}}{m!}$$

$$= \sum_{m=n_2}^{\infty} \frac{p^{n_2}(1-p)^{m-n_2}(A_2 S)^m e^{-A_2 S}}{n_2!\,(m-n_2)!}$$

$$= \frac{(A_2 p S)^{n_2} e^{-A_2 S}}{n_2!} \underbrace{\sum_{\mu=0}^{\infty} \frac{\left((1-p) A_2 S\right)^{\mu}}{\mu!}}$$

where $\mu = m - n_2$

$$e^{x} = \sum_{\mu=0}^{\infty} \frac{x^{\mu}}{\mu!}$$

$$= \frac{(A_2 p S)^{n_2} e^{-A_2 p S}}{n_2!}$$

$$n_2 \mid S \sim \text{Poisson}(A_2 p S)$$

Fisher info $\quad I_2(S) = \frac{A_2 p}{S}$

2 better than 1 if $A_2 p > A_1$

Having found Bayesian posterior $\hat{P}(\theta_n | x)$ what can we do?

where $\hat{P}(\theta_n | x) = \mathcal{L}(x | \theta_n)\, \pi(\theta_n)$

<u>Summary Statistics</u>

measure $\theta = 1 \pm 0.2$ Horrible!

<u>Marginal distributions</u>

let θ_n be n-dimension params.

useful + nuisance params, which we don't care about.

e.g. focus θ_1 1-dim marginal posterior

$$\hat{P}(\theta_1 | x) = \int d\theta_2 \int d\theta_3 \ldots \int d\theta_n \; \hat{P}(\theta_n | x)$$

e.g. θ_1 and θ_2 2-dim marginal posterior

$$\hat{P}(\theta_1, \theta_2 | x) = \int d\theta_3 \int d\theta_4 \cdots \int d\theta_n \, \hat{P}(\theta_n | x)$$

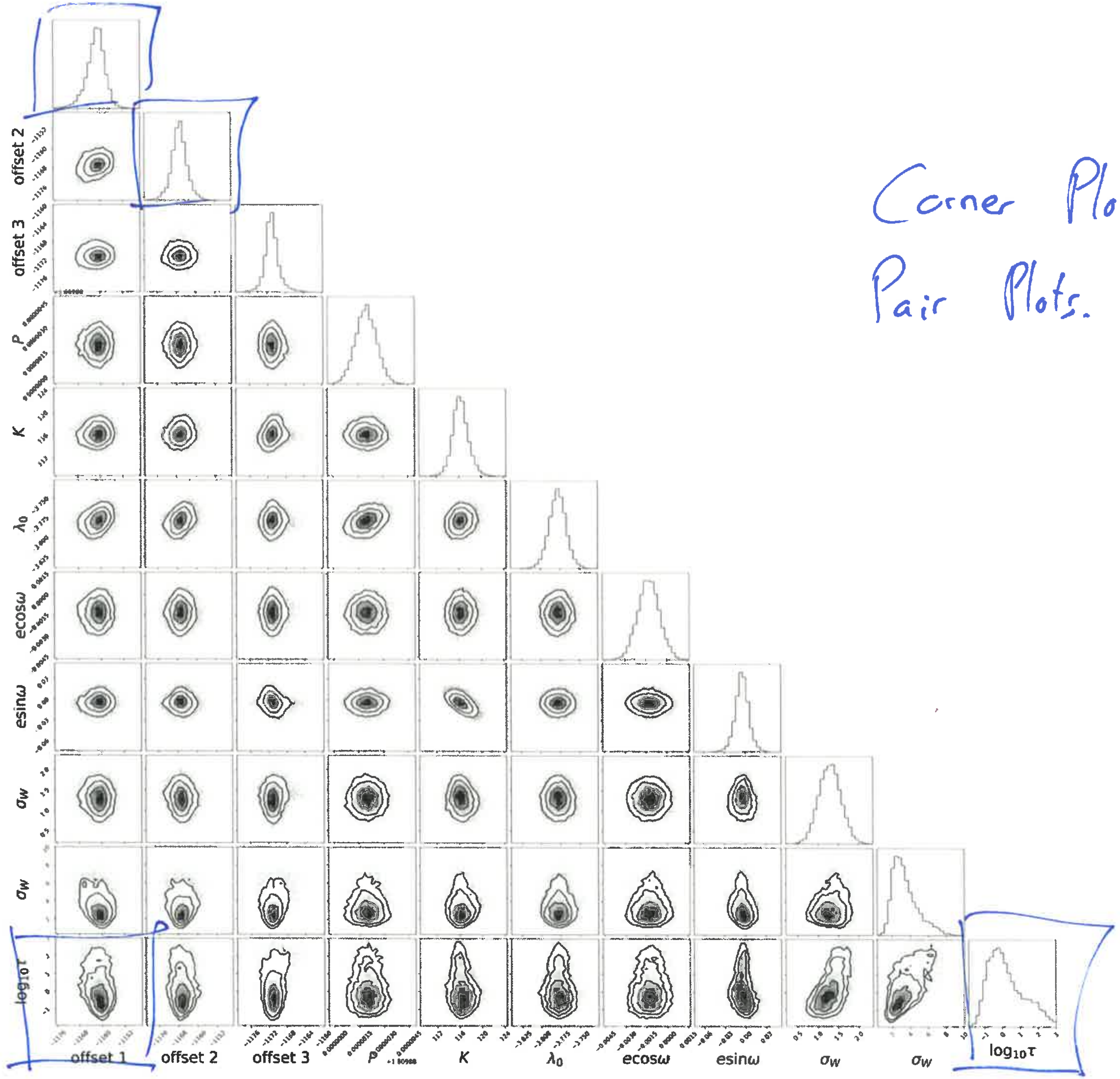
what happens if we marginalise over <u>all</u> n params?

$$Z = P(x)$$ — evidence

or Marginal Likelihood.

Corner Plots

Pair Plots.



Say we are interested in θ_1

Point Estimate what is best guess for θ_1 ?

- mean $\langle \theta_1 \rangle = E[\theta_1] = \int d^n\theta_\mu \; \theta_1 \, P(\theta_\mu | x)$

$$= \int d\theta_1 \; \theta_1 \, P(\theta_1 | x)$$

- MAP (mode) $\hat{\theta}_\mu$

$$\frac{\partial}{\partial \theta_\mu} \Big|_{\theta_\mu = \hat{\theta}_\mu} P(\theta_\mu | x) = 0 .$$

This should be calculated from full n-dim posterior.

- median of the 1-d marg dists.

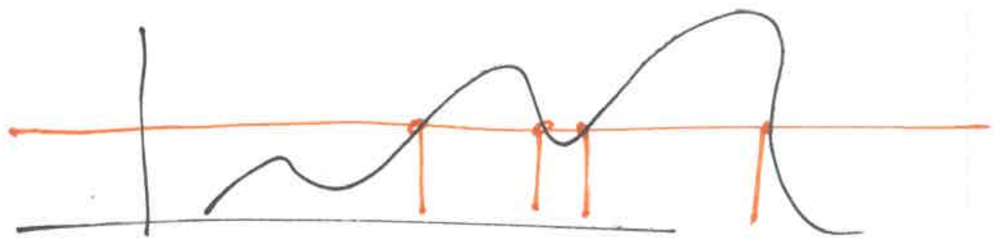
Credible Intervals

"error bar"

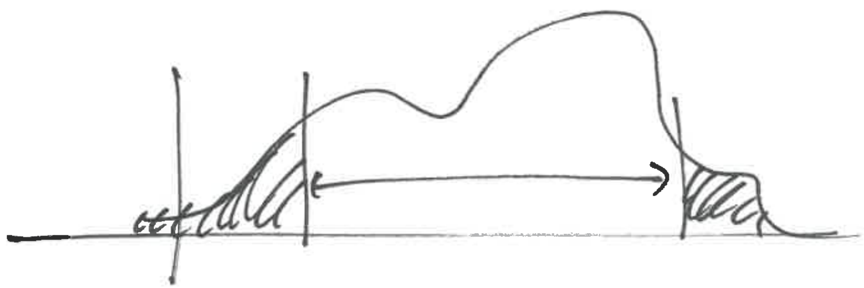
- narrowest possible credible interval.

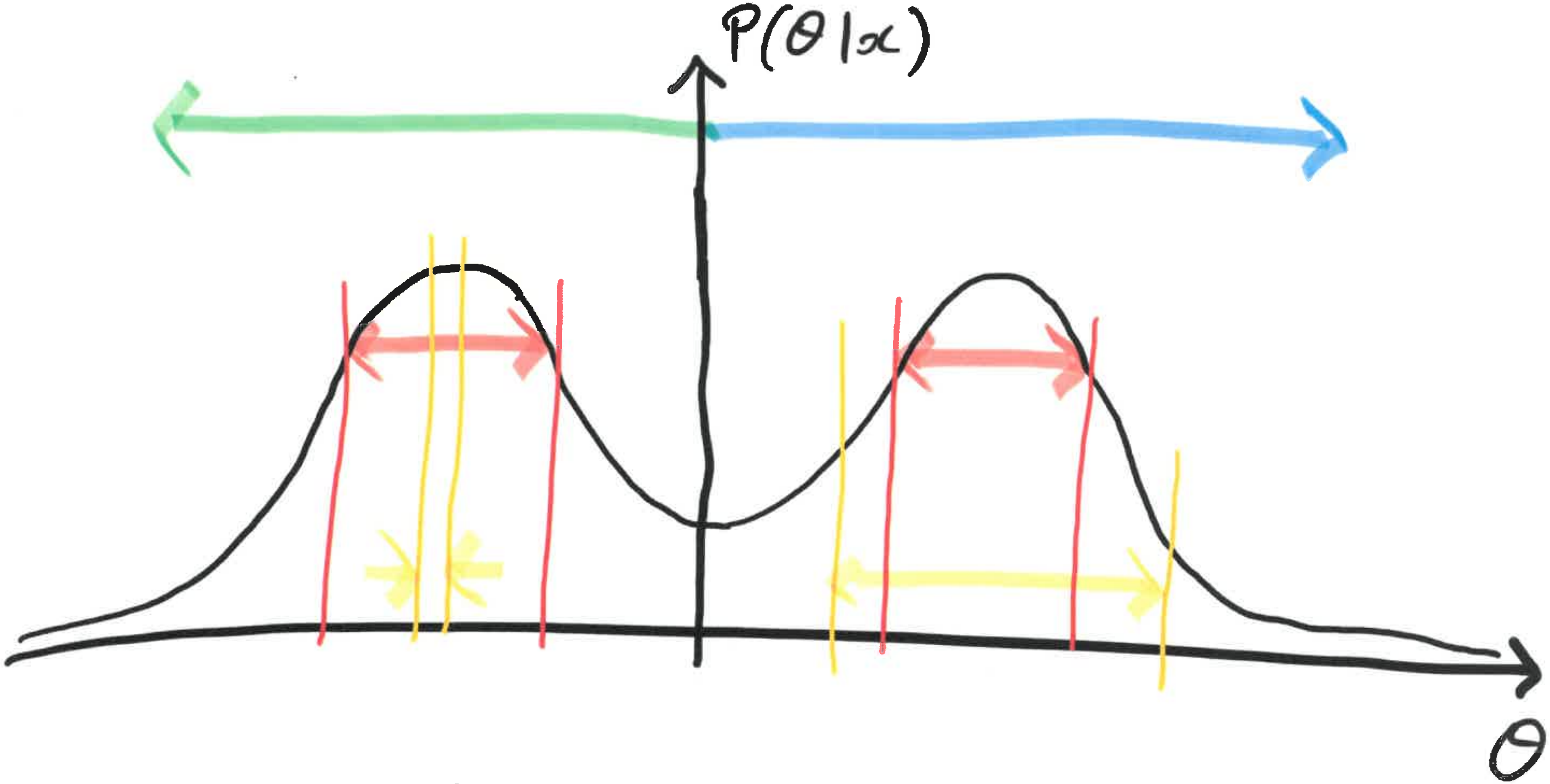
  DRAWBACKS
  - not unique
  - not invariant under ~~the~~ change of params.

- highest density region

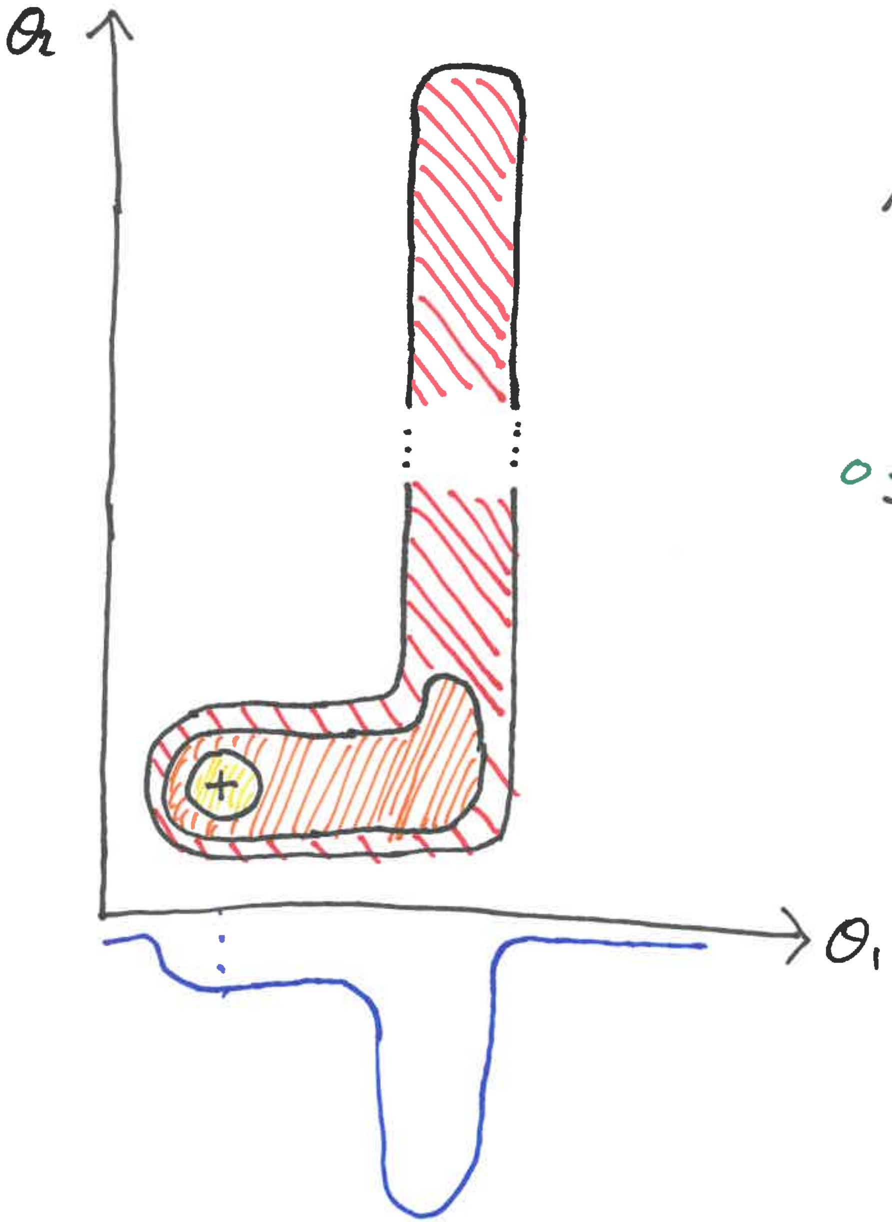


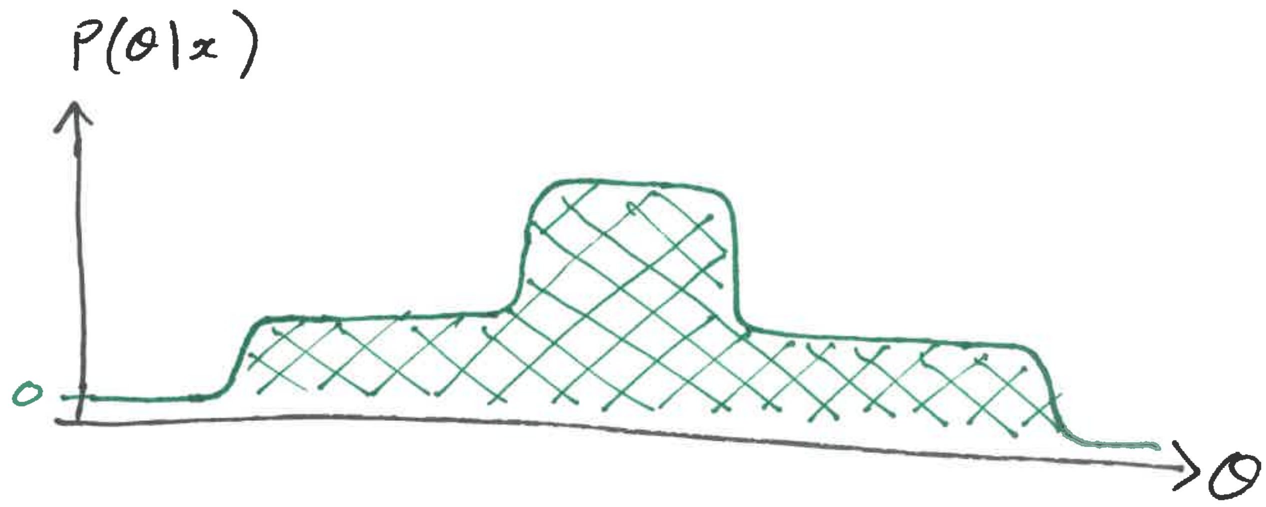
- (in 1 dim) equal-tailed.

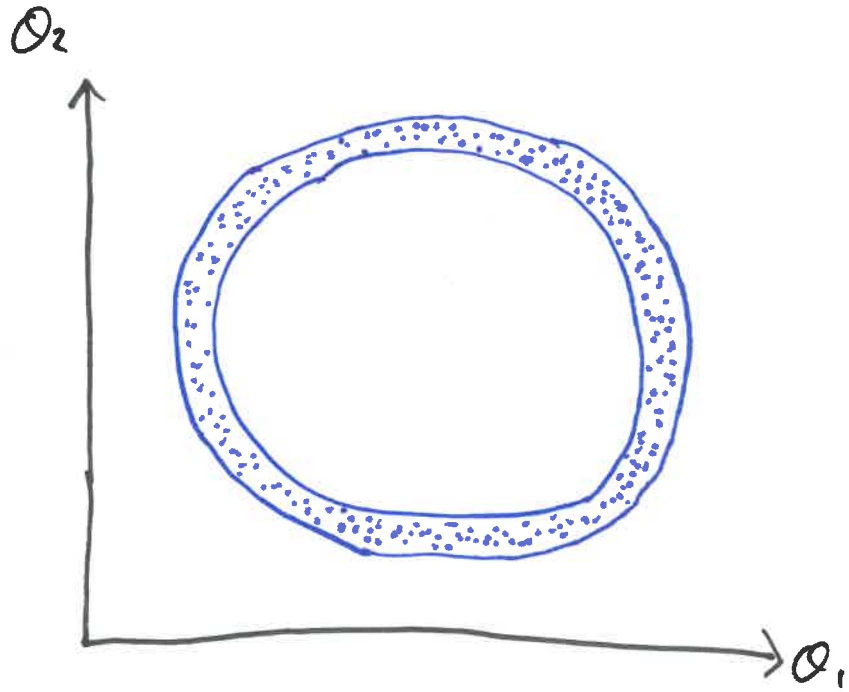


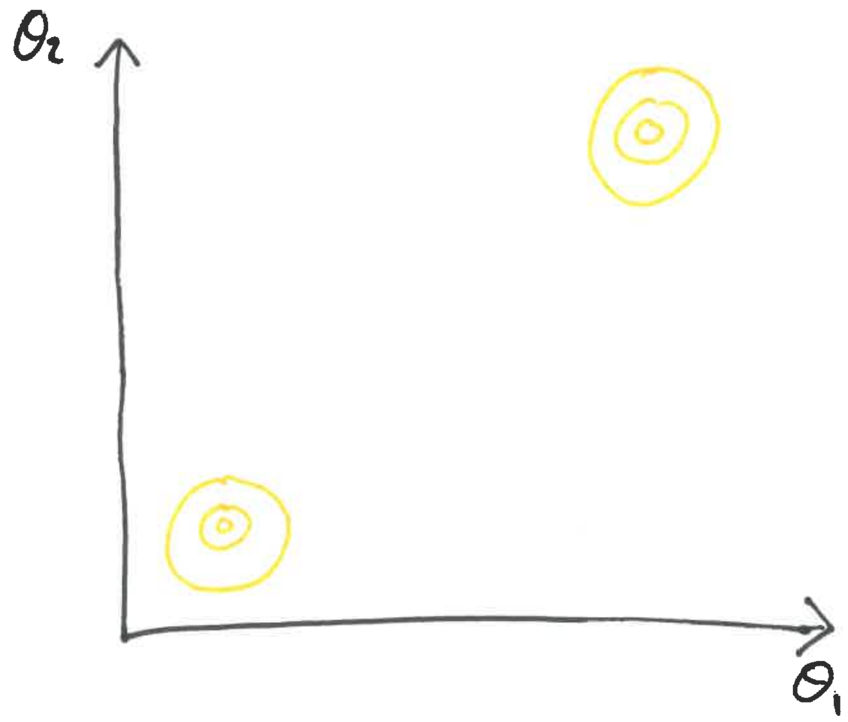


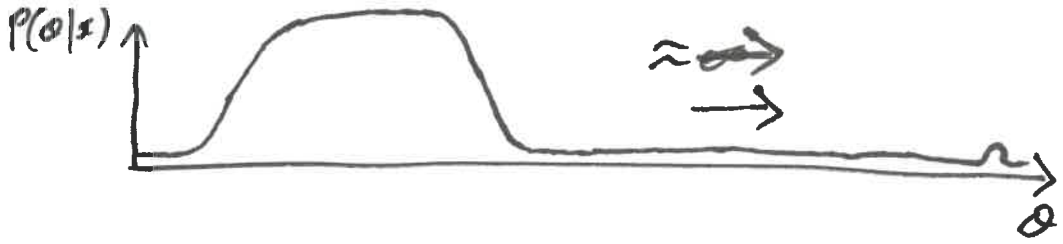
50% credible interval – not unique.

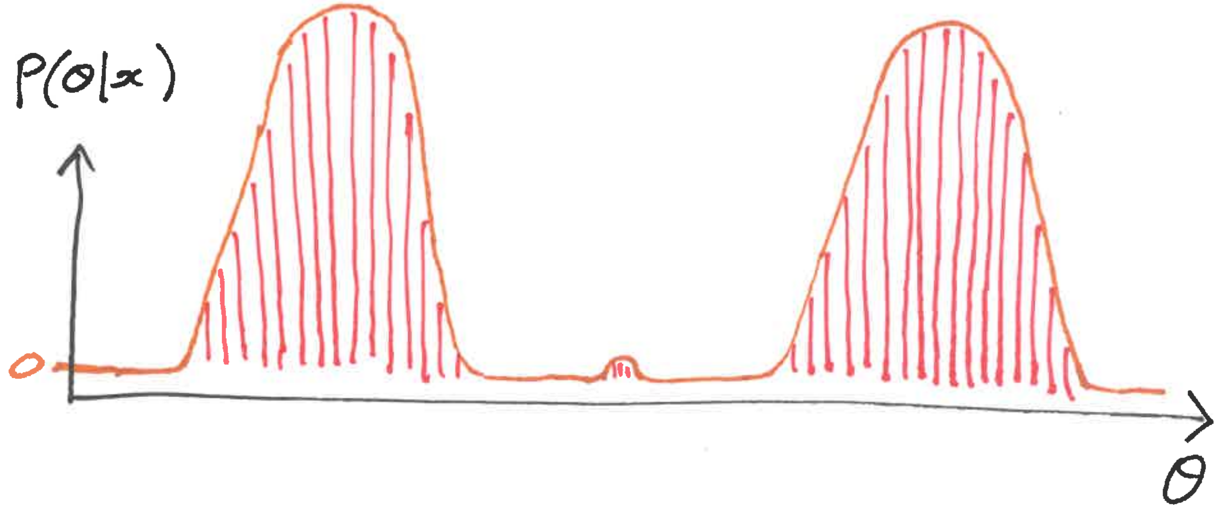

θ2
θ1

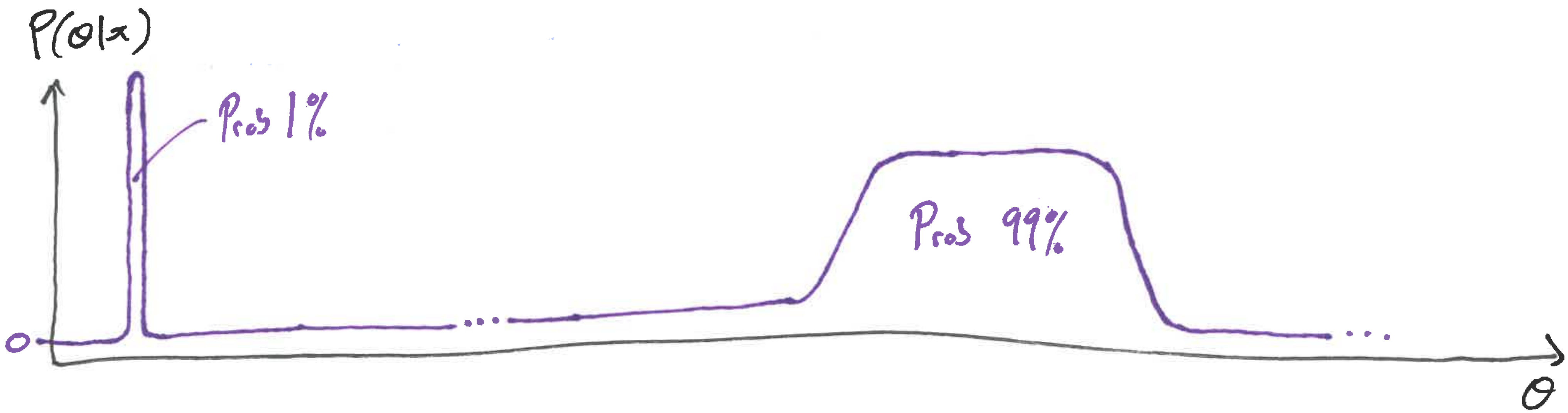

P(θ|x)
0
θ


θ2
θ1


θ_2
θ_1


$P(\theta|x)$
θ


$P(\theta|x)$
0
θ


$P(\theta|x)$
Prob 1%
Prob 99%
0
θ