Statistics coursework

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Abstract

1 Introduction

2 Geometric relationship

This section corresponds to the question **i**. I am going to compute the geometric relationship between α, β, θ and x. *alpha* and β are the coordinates of the lighthouse on the map. i.e. the lighthouse is at position *alpha* along a straight coastline and a distance β out to sea.

Given that the angle which the light emits are uniformly distributed between $-\pi/2$ and $\pi/2$, we can write the probability density function of the angle as:

$$f(\theta) = \frac{1}{\pi} \quad \text{for} \quad -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2} \tag{1}$$

The geometric relationship between α, β, θ and x is shown in the figure 1.

$$\tan\theta = \frac{\alpha - x}{\beta}$$

from which we derive the infinitesimal relationship between θ and x as:

$$dx = \beta \sec^2\theta d\theta \tag{2}$$

We know the relationship between θ and x is given by:

$$p_\theta(\theta)d\theta = p_x(x)dx \tag{3}$$

Therefore

$$\begin{aligned} p_x(x) &= p_\theta(\theta)\frac{d\theta}{dx} \\ &= \frac{1}{\pi}\frac{1}{\beta\sec^2\theta} \\ &= \frac{1}{\pi\beta}\cos^2\theta \\ &= \frac{1}{\pi\beta}\frac{\beta^2}{\beta^2 + (x-\alpha)^2} \\ &= \frac{\beta}{\pi(\beta^2 + (x-\alpha)^2)} \end{aligned}$$

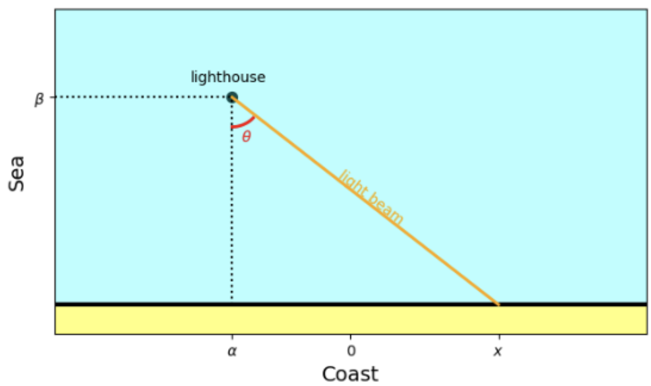

Figure 1: Geometric relationship between α, β, θ and x

3 Best estimator for α

This section corresponds to the question **iii**. First, it could be shown that the