Param $X \sim \mathcal{N}(\mu, \Sigma^2)$ $\quad \pi(X) = \dfrac{\exp\left(-\frac{1}{2}\frac{(X-\mu)^2}{\Sigma^2}\right)}{\sqrt{2\pi}\,\Sigma}$

N measurements $\{x_i\}$ iid Gaussian errors

$$\mathcal{L}(\{x_i\}|X) = \prod_{i=1}^{N} \frac{\exp\left(-\frac{1}{2}\frac{(x_i - X)^2}{\sigma^2}\right)}{\sqrt{2\pi}\,\sigma}$$

Bayes' $\Rightarrow P(X|\{x_i\}) = \frac{1}{Z}\mathcal{L}(\{x_i\}|X)\,\pi(X)$

$$P(X|\{x_i\}) = Z^{-1}(2\pi)^{-\left(\frac{N+1}{2}\right)}\sigma^{-N}\Sigma^{-1}\exp\left(-\frac{1}{2}\left[\frac{\sum_i (X-x_i)^2}{\sigma^2} + \frac{(X-\mu)^2}{\Sigma^2}\right]\right)$$

$$= Z^{-1}(2\pi)^{-\left(\frac{N+1}{2}\right)}\sigma^{-N}\Sigma^{-1}\exp\left(-\frac{1}{2}\left[\frac{N\Sigma^2+\sigma^2}{\sigma^2\Sigma^2}X^2 - 2X\left(\frac{\sum_i x_i}{\sigma^2} + \frac{\mu}{\Sigma^2}\right) + \left(\frac{\sum_i x_i^2}{\sigma^2} + \frac{\mu^2}{\Sigma^2}\right)\right]\right)$$

$$= Z^{-1}(2\pi)^{-\left(\frac{N+1}{2}\right)}\sigma^{-N}\Sigma^{-1}\exp\left(-\frac{1}{2}\frac{N\Sigma^2+\sigma^2}{\sigma^2\Sigma^2}\left[X^2 - 2X\frac{\Sigma^2\sum_i x_i + \mu\sigma^2}{N\Sigma^2+\sigma^2} + \frac{\Sigma^2\sum_i x_i^2 + \sigma^2\mu^2}{N\Sigma^2+\sigma^2}\right]\right)$$

$\boxed{1}$ $$= Z^{-1}(2\pi)^{-\left(\frac{N+1}{2}\right)}\sigma^{-N}\Sigma^{-1}\exp\left(-\frac{1}{2}\frac{(X-\mu')^2}{\Sigma'^2} + \frac{1}{2}\frac{\mu'^2}{\Sigma'^2} - \frac{1}{2}\frac{\mu^2}{\Sigma^2} - \frac{\sum_i x_i^2}{2\sigma^2}\right)$$

where $\mu' = \dfrac{\Sigma^2\sum_i x_i + \mu\sigma^2}{N\Sigma^2+\sigma^2}$ $\boxed{3}$ and $\Sigma'^2 = \dfrac{\sigma^2\Sigma^2}{N\Sigma^2+\sigma^2}$. $\boxed{4}$

But, we recognise this as a Gaussian PDF. Therefore, we have

$$P(X|\{x_i\}) = \frac{\exp\left(-\frac{1}{2}\frac{(X-\mu')^2}{\Sigma'^2}\right)}{\sqrt{2\pi}\,\Sigma'} \quad \boxed{2}$$

We have two expressions for the posterior. Comparing $\boxed{1}$ & $\boxed{2}$ gives

$$Z = (2\pi)^{-N/2}\sigma^{-N}\frac{\Sigma'}{\Sigma}\exp\left(-\frac{1}{2}\left[\frac{\mu^2}{\Sigma^2} - \frac{\mu'^2}{\Sigma'^2} + \frac{\sum_i x_i^2}{\sigma^2}\right]\right)$$

where μ' and Σ' are functions of μ, Σ, N, σ and $\{x_i\}$, given above in equations $\boxed{3}$ and $\boxed{4}$.