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# Lecture 6
# Mutual coupling in antenna arrays II : Solving “million +” Method-of-Moments problem on a personal computer

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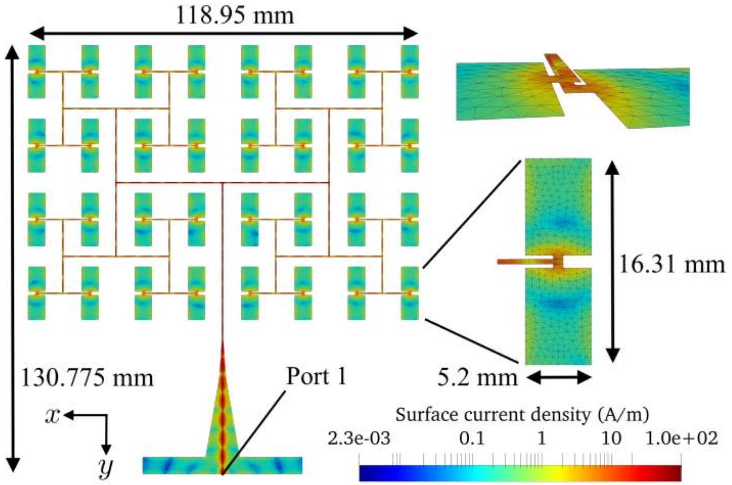
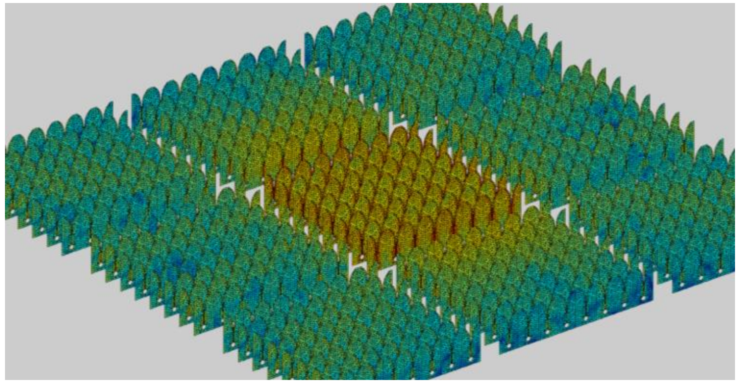
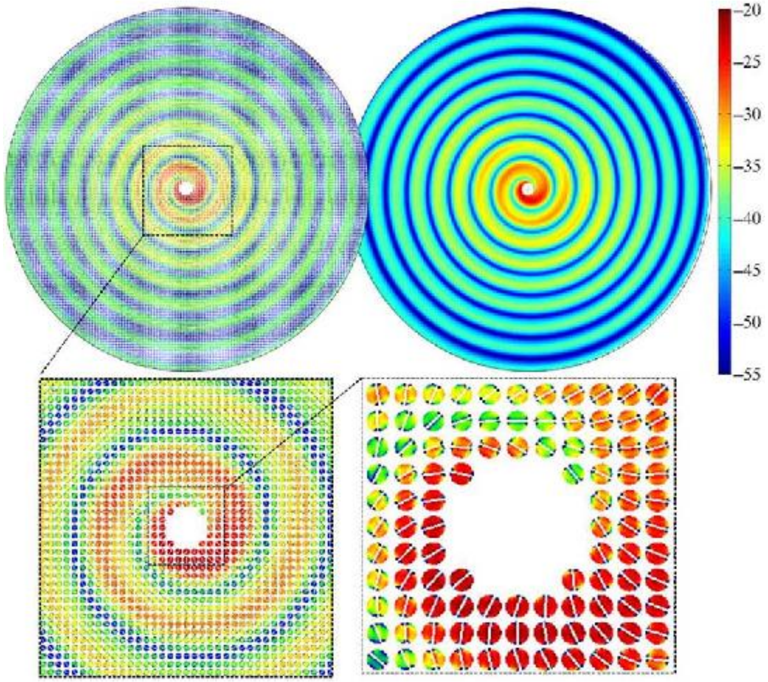
# Computational Electromagnetics (CEM)

S. Sharma, AIMx: An Extended Adaptive Integral Method for the Fast Electromagnetic Modeling of Complex Structures, 2020



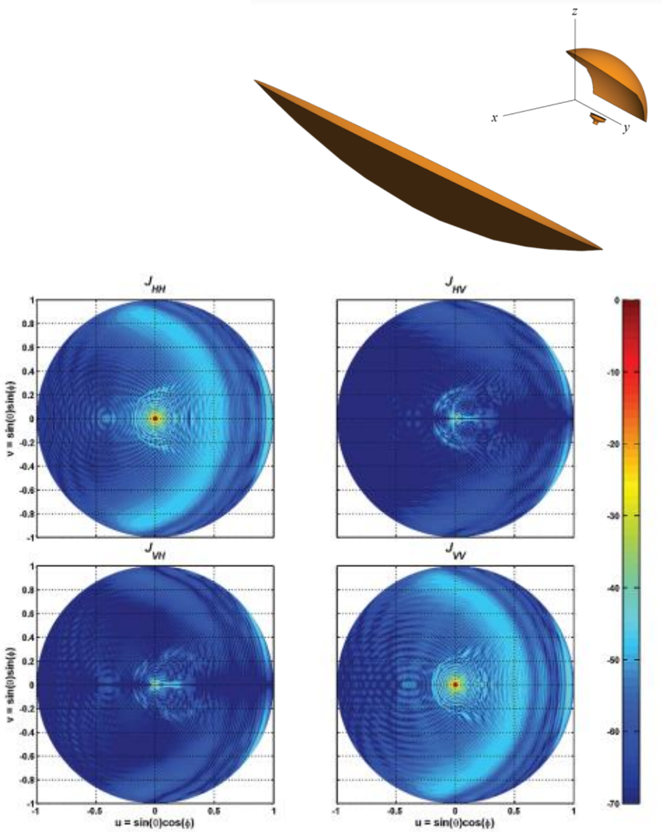



R. Maaskant, Large Antenna Arrays simulations, https://www.astron.nl/, 2007




D. Gonzalez, Gaussian Ring Basis Functions for the Analysis of Modulated Metasurface Antennas, 2015.

D.B. Davidson, Current capabilities for the full-wave electromagnetic modelling of dishes for SKA, 2013



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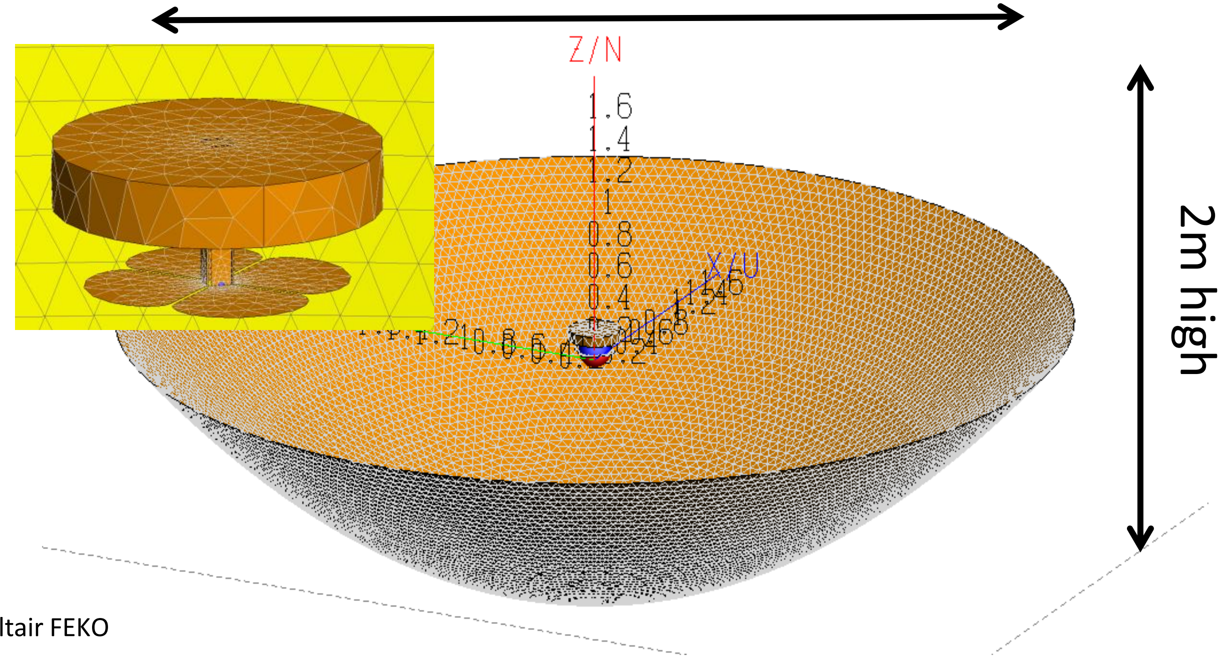
# CEM – Hydrogen Intensity and Real-time Analysis eXperiment (HIRAX)

6m = 16 wavelengths at 800 MHz




Altair FEKO

- orientable dish
- electrically large antenna
- soil underneath

- 256 to 1024 antennas
- Densely packed (0.5m min. spacing)
- Regular, hexagonal or pseudo random layouts

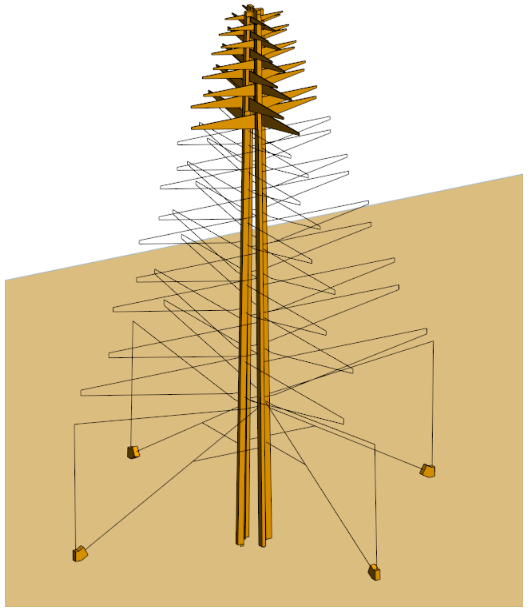
- 1MHz sampling (400MHz band)
- 3-4 digits accuracy on the impulse response

Single antenna mesh with ~100.000 unknowns

# SKA-low simulations in Cambridge

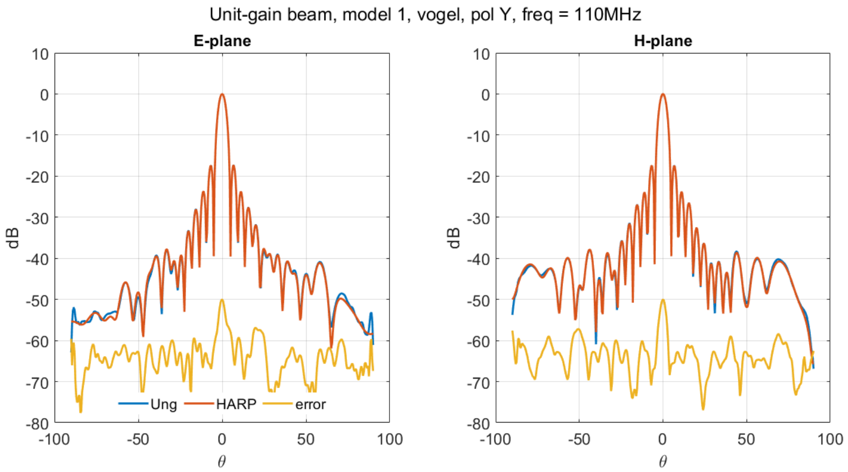
## Sub-wavelength

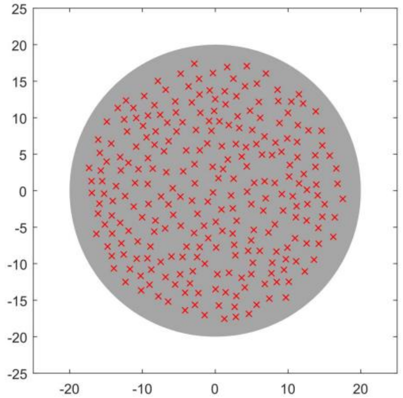
SKALA4.1, 10.000 basis functions, 0.2 wavelength footprint at 50 MHz



## Intermediate size

1 station, 256 irregular-spaced antennas

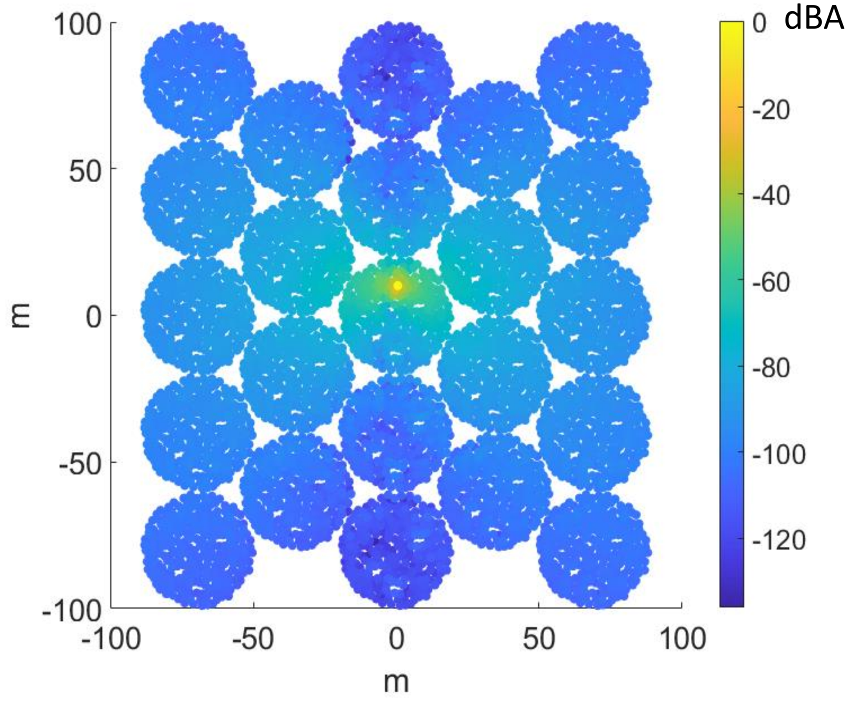


Finite ground plane and infinite soil

## Electrically-large

23 stations, 5888 irregular-spaced antennas



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# Speed-ups

## Intra-station MC

COMPUTATIONAL TIME OBTAINING EMBEDDED ELEMENT PATTERNS OF A SKA1-LOW STATION AT 110 MHz

| | CST | WIPL-D | HARP |
|---|---|---|---|
| Simulation Time | 96 hours | 97 hours | 0.5 min. |

commercial softs    in-house code

10.000 times faster

## Inter-station MC

in-house code 1    in-house code 2

| Solver | HARP | | FDS | | |
|---|---|---|---|---|---|
| Nb. of stations | 1 | 7 | 1 | 7 | 23 |
| Peak mem (GB) | 4.4 | 585 | 0.8 | 27 | 160 |
| Facto./Fill time (mins) | 0.6 | 201 | 0.5 | 6.4 | 53 |
| Solve time (mins) | 0.4 | 33 | 0.03 | 3.6 | 51 |
| Total time (mins) | 1 | 234 | 0.53 | 10 | 104 |

+20 times faster and less mem.

## Finite ground plane and soil

| Step | Required Time |
|---|---|
| Computation of poles, residues for (22) | 15 sec / MBF |
| Tabulation of the field radiated by every MBF on large rectangular grids using (12) and (22) | 30 sec/MBF |
| Pre-computation of the Fourier transform of the currents on one angular sector of the ground plane for (29) | 90 sec |
| Ground plane meshing and calculation of $Z_{gg}$ | 50 min |

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# How do we solve problems faster ?

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# Top ten algorithms of the (past) century

## Top Ten algorithms of the Century

Provenant de diverses sources sur le Web

- Fast Fourier transform
- Krylov subspace iteration methods
- The Fortran optimizing compiler
- Integer relation Detection
- Quicksort Algorithm For Sorting
- Metropolis algorithm for Monte Car...
- QR algorithm for computing eigenv...
- Simplex method for linear program...
- The decompositional approach to ...
- Fast multipole method

# Structure

1. Green’s function, potential vector and fields
2. Method-of-Moments
3. Macro-Basis functions
4. Multipole and Interpolatory methods

# Green's function

In the frequency domain, the Helmholtz equation for the vector potential **A** assuming a **"point"** source current **J** is written as

$$\nabla^2 \mathbf{A}(\mathbf{r}) + k^2 \mathbf{A}(\mathbf{r}) = \mathbf{J}(\mathbf{r}) \qquad \mathbf{J}(\mathbf{r}) = \delta(x)\delta(y)\delta(z)\hat{\mathbf{z}}$$

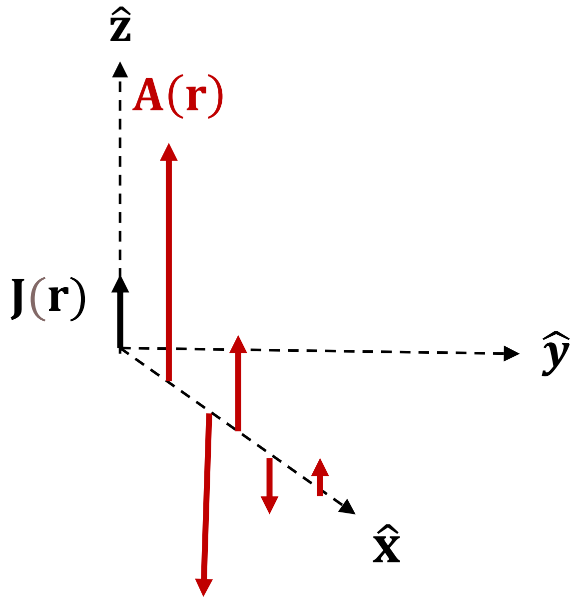
The solution is the 3D free-space Green's function,

with the distance $R = \sqrt{x^2 + y^2 + z^2}$ and free-space wavenumber $k$

$$\mathbf{A}(\mathbf{r}) = \frac{e^{-jkR}}{4\pi R}\hat{\mathbf{z}}$$

oscillatory when kR is large

singular when kR is small



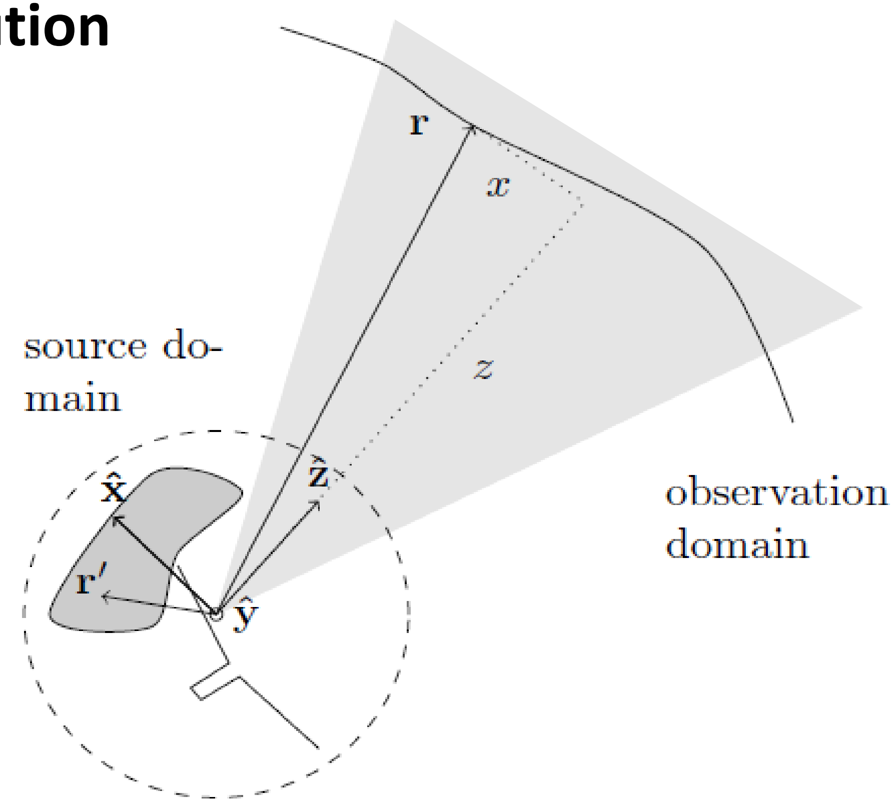
# Fields and Currents

The potential vector $\mathbf{A}$ is obtained from a **convolution product** between the electric current density $\mathbf{J}$ (in A/m^3) and the Green's function G,

$$\mathbf{A}(\mathbf{r}) = \iiint \mathbf{J}(\mathbf{r}')\, G(k, R)\, dV'$$

with

$$R = \sqrt{(x' - x)^2 + (y' - y)^2 + (z' - z)^2}$$



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# Fields and Currents

In free-space, the scattered magnetic and electric fields are given from spatial derivatives of the vector potential,

$$\mathbf{H}(\mathbf{r}) = \nabla \times \mathbf{A}(\mathbf{r}) \qquad \mathbf{E}(\mathbf{r}) = -\mathrm{j}k\eta\left(\mathbf{A}(\mathbf{r}) - \frac{1}{k^2}\nabla\nabla\cdot\mathbf{A}(\mathbf{r})\right)$$

This yields,

$$\mathbf{E}_{sc}(\mathbf{r}) = -\mathrm{j}k\eta \iiint \mathbf{J}(\mathbf{r}')\cdot \underline{\underline{\mathbf{G}}}_e(k,R)\,dV'$$

$$\mathbf{H}_{sc}(\mathbf{r}) = \iiint \mathbf{J}(\mathbf{r}')\cdot \underline{\underline{\mathbf{G}}}_h(k,R)\,dV'$$

The 3x3 matrices $\underline{\underline{\mathbf{G}}}_e$, $\underline{\underline{\mathbf{G}}}_h$ are called dyadic Green's functions

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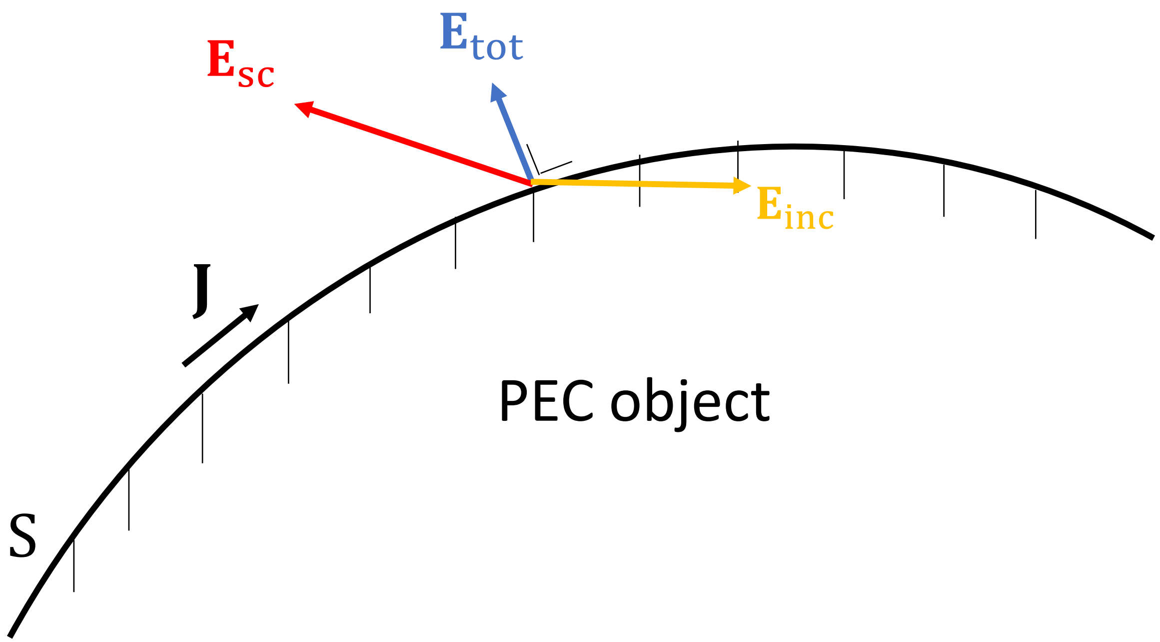
# The Electric Field Integral Equation (EFIE)

Tangential components of the total electric field $\mathbf{E}_t$ (incident + scattered) must cancel out on the surface of a perfectly electric conductor (PEC)

$$\hat{\mathbf{n}} \times \mathbf{E}_{tot} = \hat{\mathbf{n}} \times (\mathbf{E}_{inc} + \mathbf{E}_{sc}) = 0$$

Expressing $\mathbf{E}_{sc}$ as function of $\mathbf{J}$ leads to the EFIE

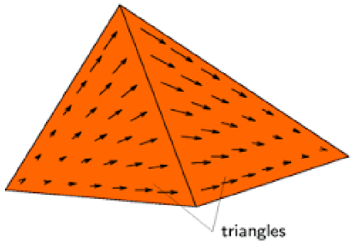
$$-\hat{\mathbf{n}} \times \mathbf{E}_{inc}(\mathbf{r}) = \hat{\mathbf{n}} \times -jk\eta \iint \mathbf{J}(\mathbf{r}') \cdot \underline{\underline{\mathbf{G}}}_e(k, R)\, dS'$$



# The Method-of-Moments (MoM)

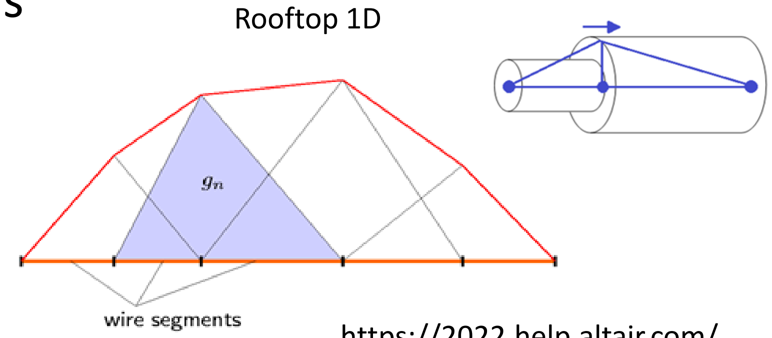
The surface current is expanded into **elementary** basis functions

Rao-Wilton-Glisson (RWG)




$$\mathbf{J}(\mathbf{r}) \approx \sum_{n=1}^{N_b} \mathrm{i_n}\, \mathbf{B}_\mathrm{n}(\mathbf{r})$$

Rooftop 1D




https://2022.help.altair.com/

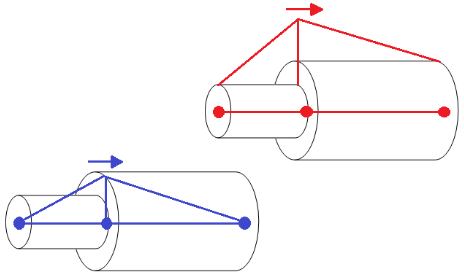
Inserting this expansion into the EFIE and using a Galerkin testing leads to the MoM equations,

$$-\iint \mathbf{B}_m(\mathbf{r}) \cdot \mathbf{E}_\mathrm{inc}(\mathbf{r})\, \mathrm{dS_t} = -\mathrm{jk\eta} \sum_{n=1}^{N_b} \mathrm{i_n} \iint \mathbf{B}_m(\mathbf{r}) \cdot \iint \underline{\underline{\mathbf{G}}}_\mathrm{e}(\mathrm{k},\mathrm{R}) \cdot \mathbf{B}_\mathrm{n}\,(\mathbf{r}')\, \mathrm{dS_b dS_t}$$

Incident field

Unknown coefficient

Testing function



Basis function

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# The Method-of-Moments (MoM)

$$\mathbf{Zi} = \mathbf{v}$$

Impedance matrix — Unknown coefficients — Excitation vector

$$\begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1N} \\ Z_{21} & Z_{22} & \cdots & Z_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ Z_{N1} & Z_{N2} & \cdots & Z_{NN} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ \vdots \\ i_N \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_N \end{bmatrix}$$

$$v_i = -\iint \mathbf{E}_{inc}(\mathbf{r}) \cdot \mathbf{B}_i(\mathbf{r})\, dS_i$$

Integral reaction between basis function j and testing function i

$$Z_{ij} = -jk\eta \iint \iint G(k, R_{ij}) \left( \underbrace{\mathbf{B}_i(\mathbf{r}) \cdot \mathbf{B}_j(\mathbf{r}')}_{\text{“Current” interaction}} - \underbrace{\frac{1}{k^2} \nabla \cdot \mathbf{B}_i(\mathbf{r})\, \nabla \cdot \mathbf{B}_j(\mathbf{r})}_{\text{“Charge” interaction}} \right) dS_i dS_j$$

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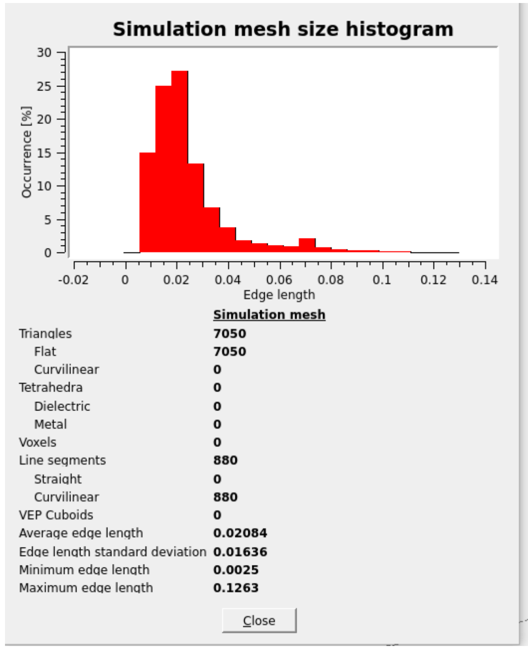
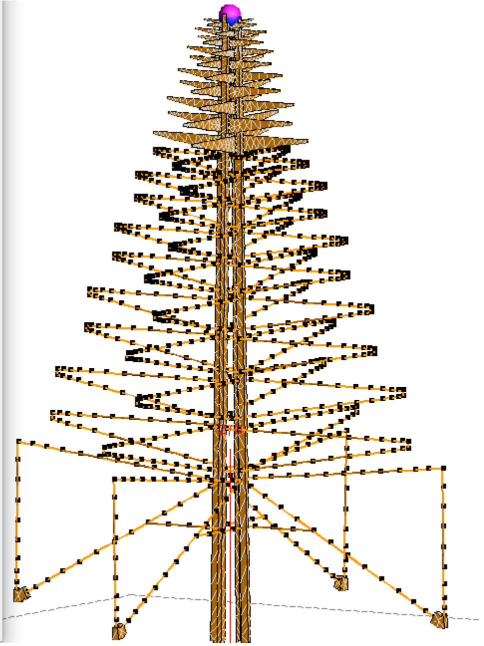
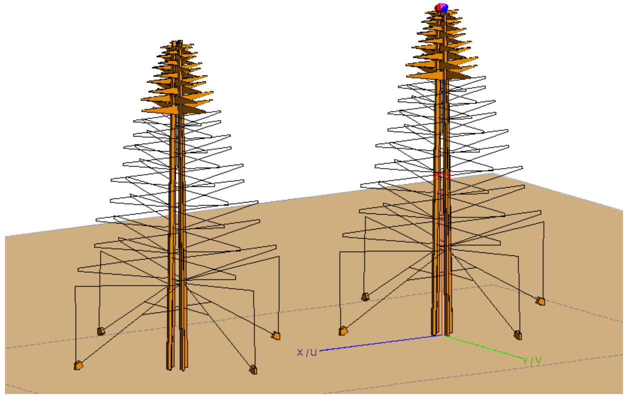
# MoM, scaling

~8000 unknowns per antenna



altair FEKO



For 2 antennas, per frequency,

| | |
|---|---|
| 120 CPUs | Matrix filling time : 1 min |
| 2TB RAM | Solve time : 12 secs |
| machine | Matrix size: 1 GB |

For 256 antennas, per frequency

| | |
|---|---|
| | Matrix filling time : 11 days |
| Estimated | Solve time : 291 days |
| | Matrix size: 65 TB |

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# How do we speed things up ?

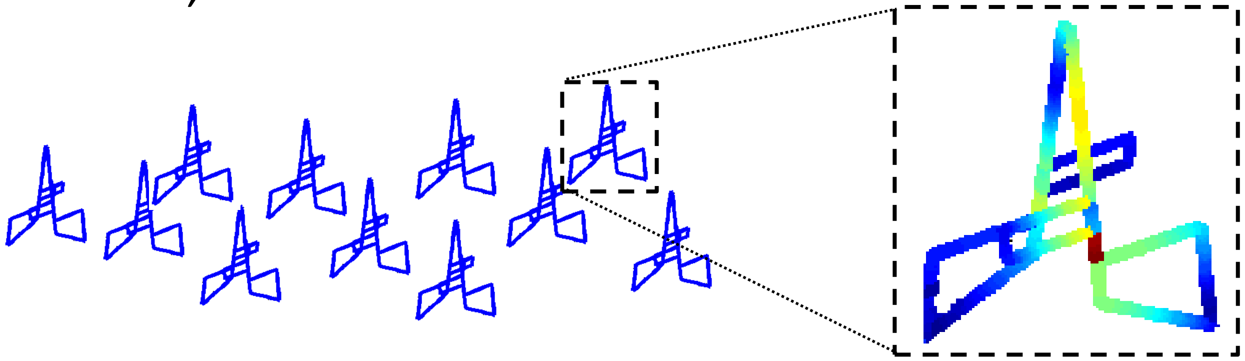
1. Reduce the number of unknowns

the current on each antenna in the array can be decomposed into a (small) set of vectors of the solution space, called MBF or « Macro Basis Functions », precomputed beforehand.

# Macro-Basis Functions (MBFs)

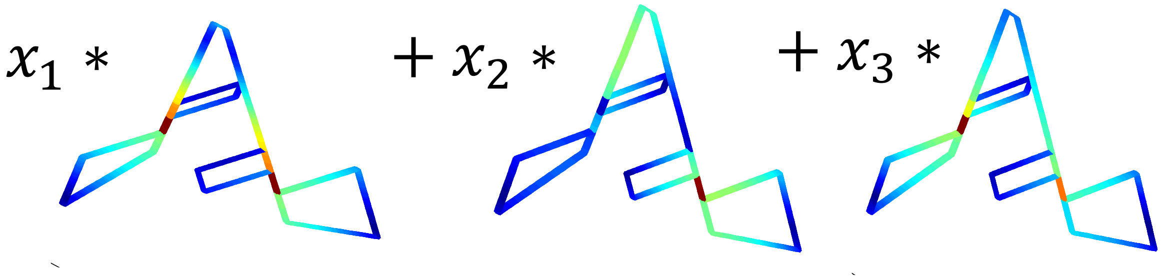
The current distribution $\mathbf{J}$ on each antenna is expanded into a small number of modes defined over the surface of an antenna,

$$\mathbf{J}(\mathbf{r}) \approx \sum_{n=1}^{N_{mbf}} x_n \, \mathbf{J}_n(\mathbf{r})$$



with each MBF $\mathbf{J}_n$ expressed in terms of elementary BF $\mathbf{B}_n$

$$\mathbf{J}_m(\mathbf{r}) = \sum_{n=1}^{N_b} q_{mn} \, \mathbf{B}_n(\mathbf{r})$$



# Reduced system of equations

By stacking the MBF coefficients into a $N_b \times N_m$ matrix $\mathbf{Q}$, we can compress the interactions between antenna “a” and “b” using

$$\mathbf{Z}_{\mathrm{r,ab}} = \mathbf{Q}^{\mathrm{H}} \, \mathbf{Z}_{\mathrm{ab}} \, \mathbf{Q}$$

$$\mathbf{v}_{r,a} = \mathbf{Q}^{H} \mathbf{v}_a$$

$$\begin{array}{cc} & \overset{\mathrm{N_a} \times \mathrm{N_{mbf}}}{\longleftrightarrow} \\ \mathrm{N_a} \times \mathrm{N_{mbf}} \updownarrow & \begin{bmatrix} \mathbf{Z}_{r,11} & \cdots & \mathbf{Z}_{r,1N_a} \\ \vdots & \ddots & \vdots \\ \mathbf{Z}_{r,N_a 1} & \cdots & \mathbf{Z}_{r,N_a N_a} \end{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ \vdots \\ \mathbf{x}_{N_a} \end{bmatrix} = \begin{bmatrix} \mathbf{v}_{r,a} \\ \vdots \\ \mathbf{v}_{r,a} \end{bmatrix} \end{array}$$

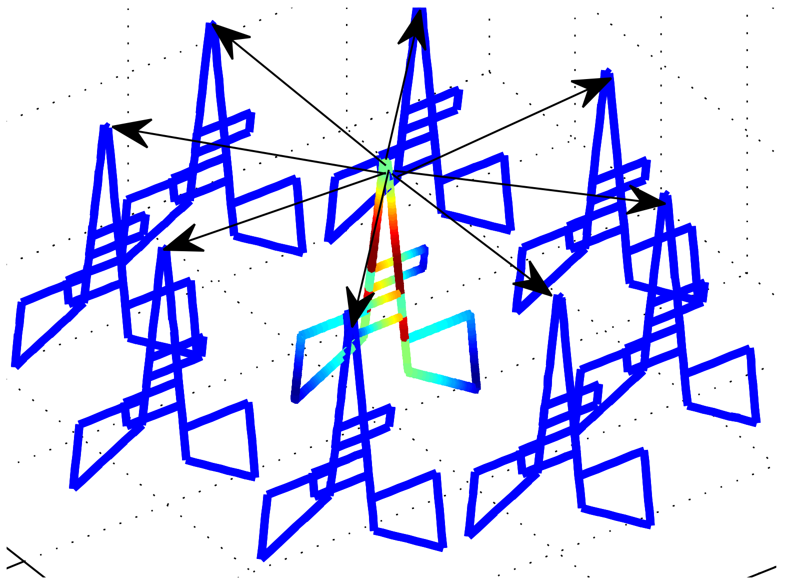
From there, we can obtain the elem. BF coefficients for each antenna using

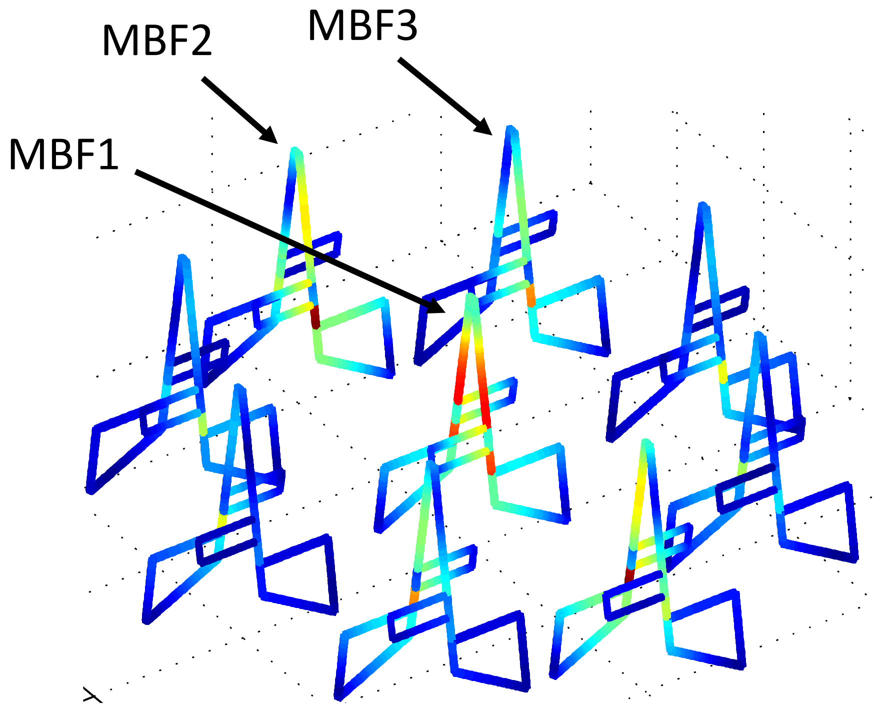
$$\mathbf{i}_a = \mathbf{Q} \, \mathbf{x}_a$$

For SKALA4.1, per antenna, we have $\mathrm{N_b} = 8000$ and $\mathrm{N_m} = 50–100$

# Generation of MBFs

MBFs are generated by solving a much smaller EM problem





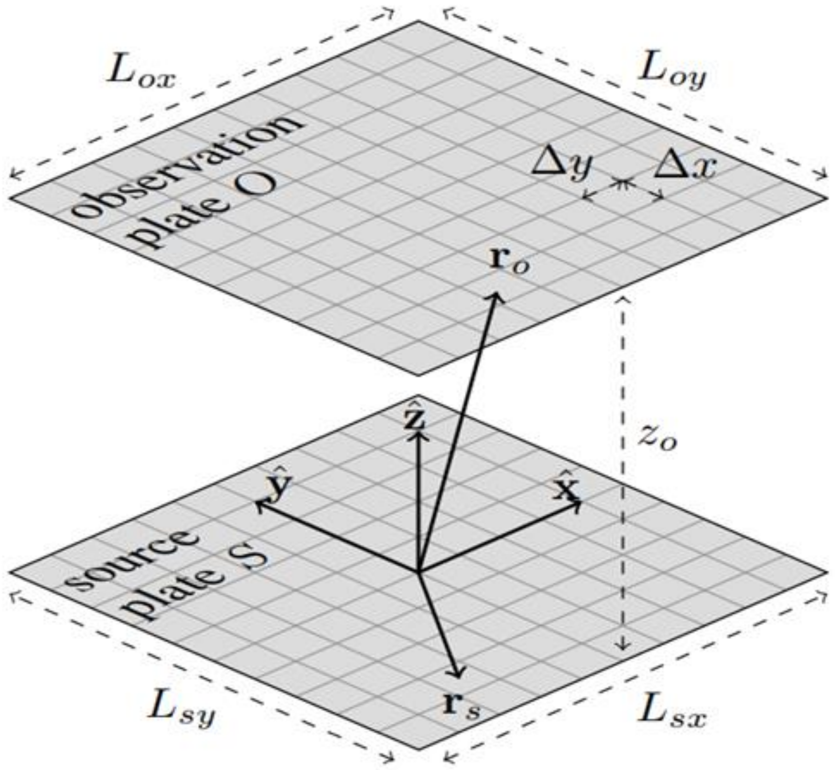

first-order bounces

# How do we speed things up ?

## 2. Accelerate the computation of the MoM interactions

Evaluate the interactions in a group-by-group manner, instead of computing interactions between all the basis functions.

# A simple planar geometry




The radiation integral is purely a 2D convolution,

$$\mathbf{A}(x_o, y_o) = \iint \mathbf{J}(x_s, y_s)\, G(k, x_o - x_s, y_o - y_s)\, dx_s dy_s$$

We can simply use the convolution theorem,

$$\begin{array}{cccccl} & \mathbf{J}(x,y) & & G(k,x,y) & & \text{Spatial domain} \\ & \downarrow & & \downarrow & & \textit{FFT } O(N\log N) \\ \tilde{\mathbf{A}}(k_x,k_y) = & \tilde{\mathbf{J}}(k_x,k_y) & * & \tilde{G}(k_x,k_y) & & \text{Spectral domain} \\ & & \downarrow & & & \textit{IFFT } O(N\log N) \\ & & \mathbf{A}(x,y) & & & \text{Spatial domain} \end{array}$$

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# Spectral-domain

The integral reaction in the spatial domain is expressed by

$$Z_{mn} = -jk\eta \iint \mathbf{B}_m(\mathbf{r}_t) \cdot \iint \underline{\underline{\mathbf{G}}}_e(k, R) \cdot \mathbf{B}_n\,(\mathbf{r}_b)\, dS_b dS_t$$

In the spectral domain, i.e. $\mathbf{k} = k_x\,\hat{\mathbf{x}} + k_y\,\hat{\mathbf{y}} + k_z\,\hat{\mathbf{z}}$ with $k_z = \left(k^2 - k_x^2 - k_y^2\right)^{1/2}$ by

$$Z_{mn} = \frac{-jk\eta}{(2\pi)^2} \iint_{-\infty}^{\infty} \widetilde{\mathbf{B}}_m(\mathbf{k}) \cdot \widetilde{\mathbf{B}}_n\,(-\mathbf{k})\, G(k_z)\, dk_x dk_y$$

Complex pattern of BF "m"

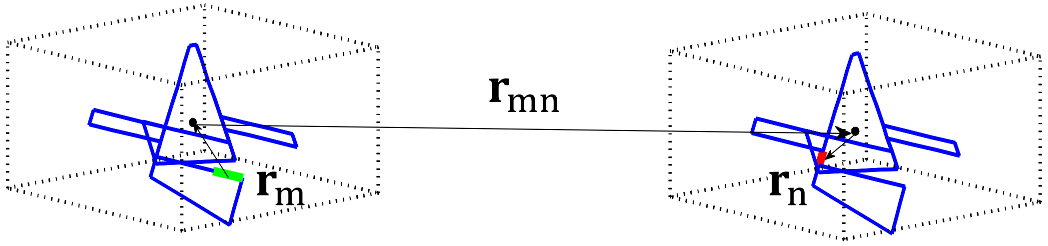
$$\widetilde{\mathbf{B}}_m(\mathbf{k}) = \iint (\mathbf{B}_m(\mathbf{r}) - (\mathbf{k} \cdot \mathbf{B}_m(\mathbf{r}))\,\mathbf{k})\, e^{j\mathbf{k}\cdot\mathbf{r}}\, dS$$

Spectral Green's function

$$G(k, k_\rho) = \frac{e^{-jk_z z_o}}{2jk_z}$$

# Multipoles expansion

The multipole expansion of the free-space Green's function,




$$\frac{e^{-jkR}}{4\pi R} = \frac{-jk}{(4\pi)^2} \int_0^{2\pi} \int_0^{\pi} e^{j\mathbf{k}.\mathbf{r}_m}\, T(\mathbf{r}_{mn}, \mathbf{k}) e^{-j\mathbf{k}.\mathbf{r}_n} \sin\theta \, d\theta d\phi$$

relative distance between two groups

3D multipoles translation function

$$T(\mathbf{r}_{mn}, \mathbf{k}) = \sum_{l=0}^{L} (2l+1) j^{-l} h_l^{(2)}(k r_{mn}) P_l(\hat{\mathbf{k}} \,.\, \hat{\mathbf{r}}_{mn})$$

$$\hat{\mathbf{k}} = \sin\theta\cos\phi\,\hat{\mathbf{x}} + \sin\theta\sin\phi\,\hat{\mathbf{y}} + \cos\theta\,\hat{\mathbf{z}}$$

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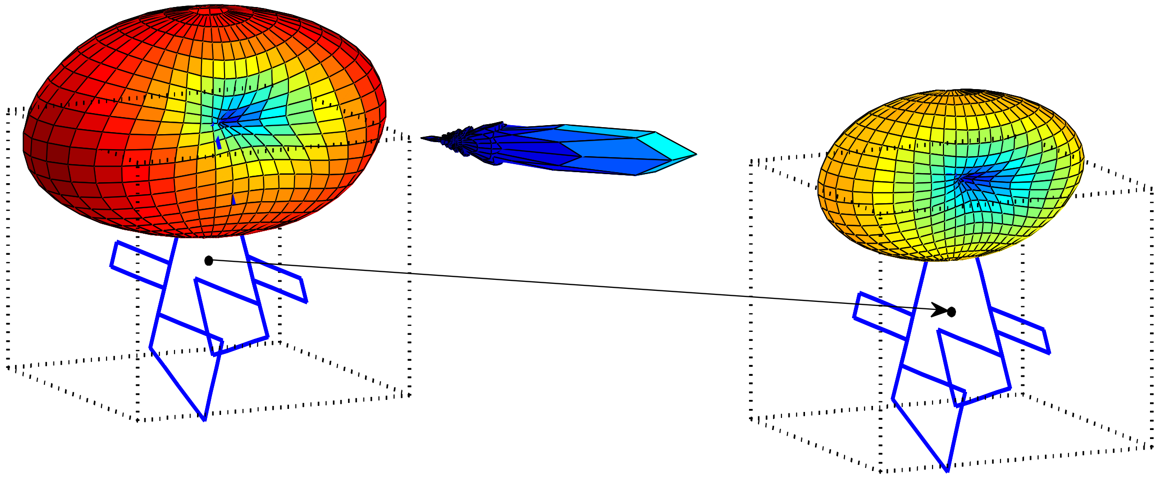
# Multipoles expansion

Multipoles decomposition of a **far** interaction between two Macro Basis Functions :



$$Z_{mn} = c\int_0^{2\pi}\int_0^{\pi} \underbrace{\widetilde{\mathbf{B}}_{\mathrm{m}}(-\mathbf{k})}_{\text{Radiation pattern of testing function}} \cdot \underbrace{\widetilde{\mathbf{B}}_{\mathrm{n}}(\mathbf{k})}_{\text{Radiation pattern of source function}} \underbrace{\mathrm{T}(\mathbf{r}_{mn},\mathbf{k})}_{\text{3D multipoles translation function}} \sin\theta\, \mathrm{d}\theta\mathrm{d}\phi$$

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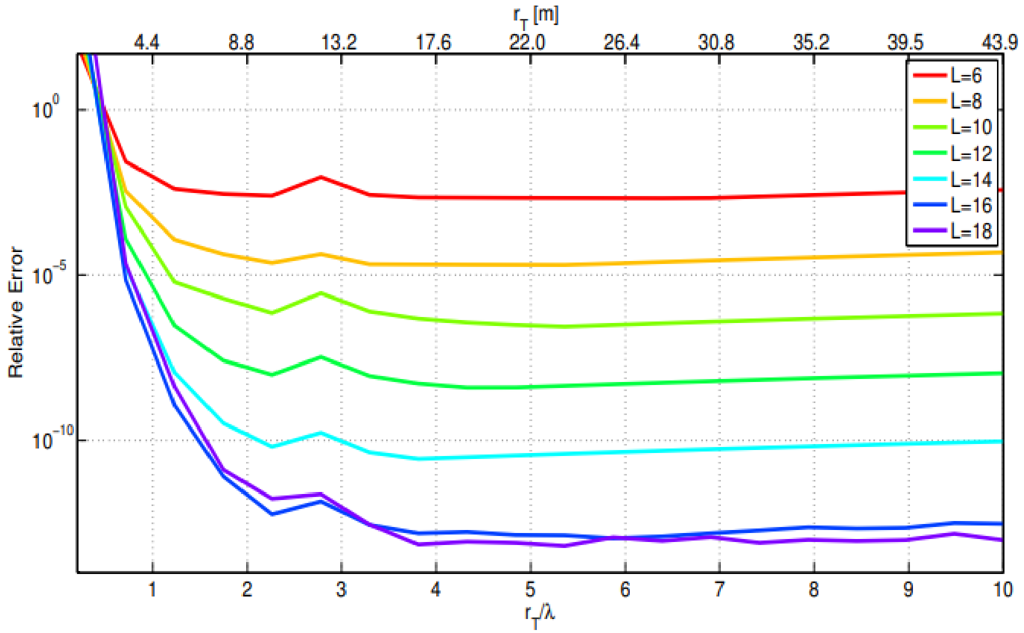
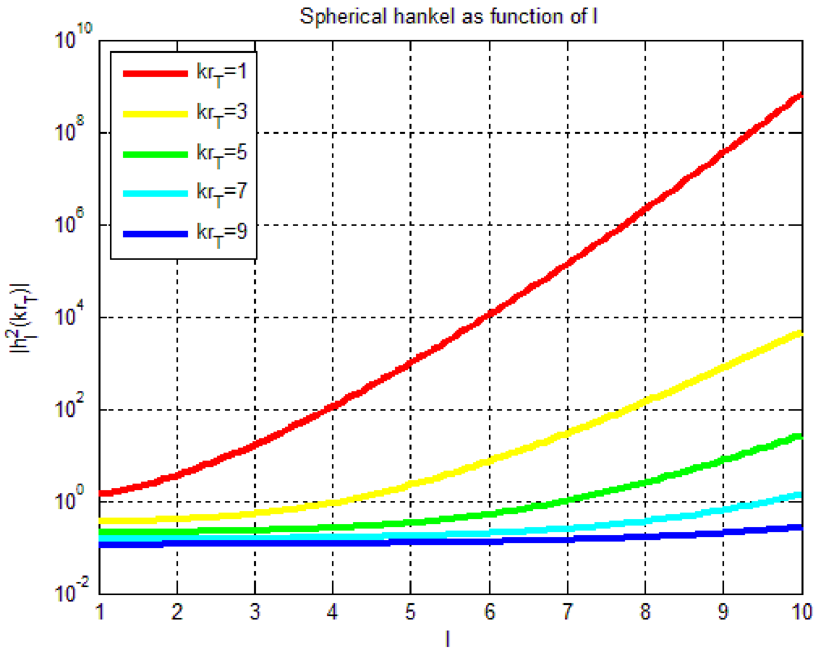
# Multipoles expansion

$$L \cong 2kr_B + 1.8d_0^{\frac{2}{3}}(2kr_B)^{\frac{1}{3}}$$




Trying to express near-field from far-field information -> ill-conditioned !




**Over-exponential** behaviour as function of **order l** -> Low-order terms of the translation lost in **numerical noise** of high order terms

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# Interpolatory method

This technique interpolates the MBF interactions in the "baselines" domain $\mathbf{r} = \mathrm{r}\cos\alpha\,\hat{\boldsymbol{x}} + \mathrm{r}\sin\alpha\,\hat{\boldsymbol{y}}$ by means of an empirical model which is harmonic in α and polynomial in $(\mathrm{kr})^{-1}$

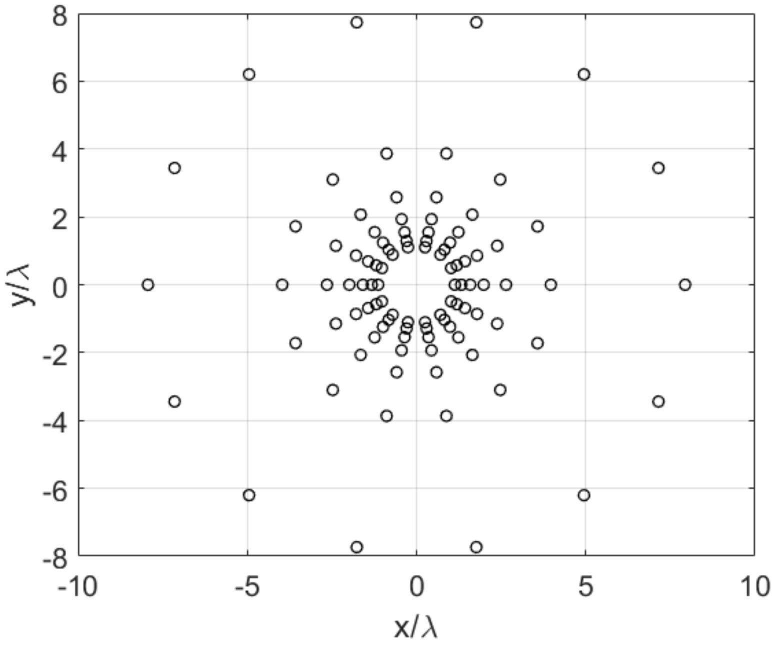
Pre-computed coefficients

$$\mathrm{Z_{mn}}(\mathrm{r},\alpha) = \mathrm{e}^{-\mathrm{jkr}} \sum_{\mathrm{p}=-\mathrm{P}}^{\mathrm{P}} \mathrm{e}^{\mathrm{jp}\alpha} \sum_{\mathrm{q}=0}^{\mathrm{Q}} \mathrm{c_{pq}} \left(\frac{1}{\mathrm{kr}}\right)^{\mathrm{q}}$$

phase factor

Harmonic

Polynomial



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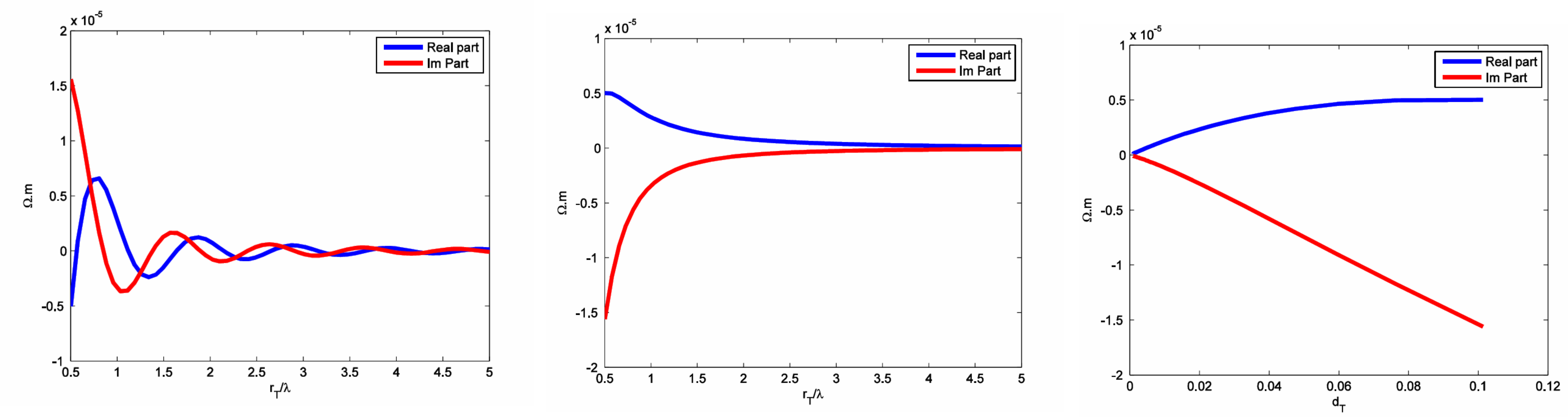
# Interpolatory method

phase extraction — d=1/r change of variables

$$Z_{mn}(r) \dashrightarrow e^{jkr} Z_{mn}(r) \dashrightarrow e^{jk/d} Z_{mn}(1/d)$$



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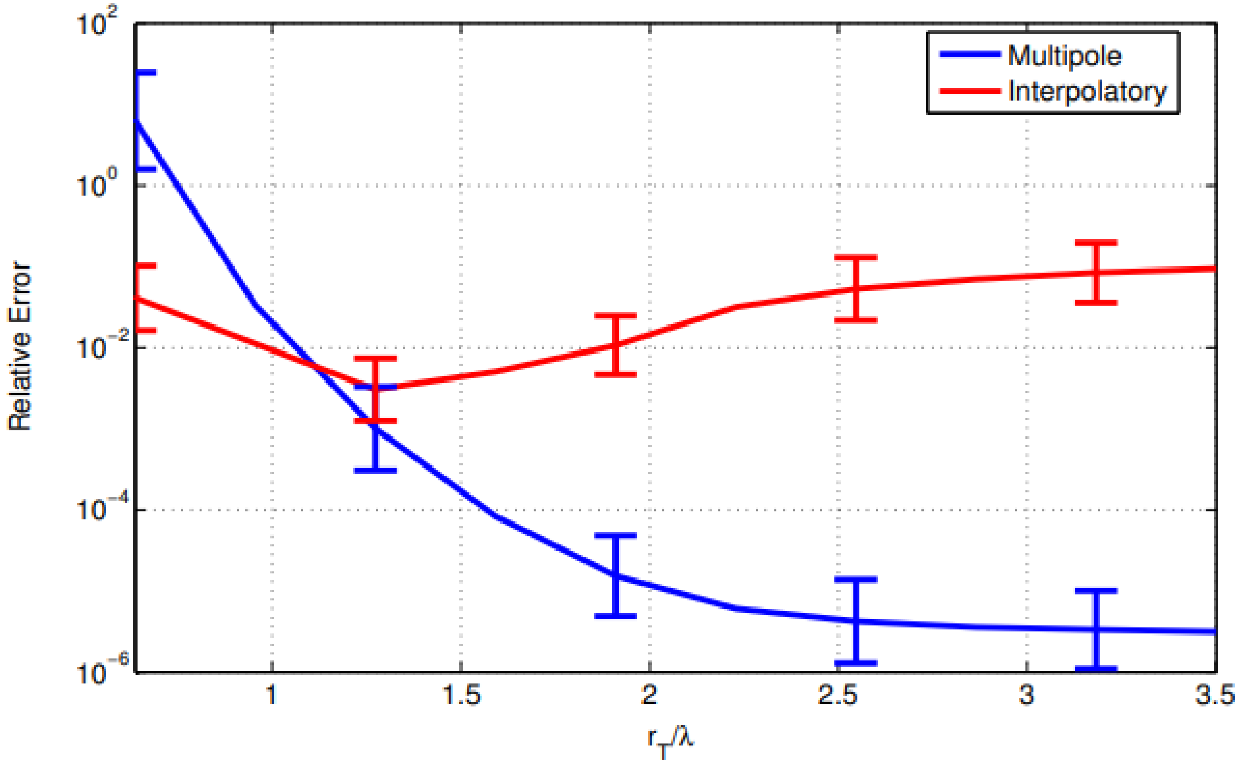
# Interpolatory method



# Thank you

qdg20@cam.ac.uk