Metropolis-Hastings

[Correction]

current point (known)

step $x \longrightarrow x'$

new point, r.v. dist on this.

$$P(x'|x) = \rho(x',x)$$

$$\rho(x',x) = \alpha(x',x)\, Q(x'|x) + \delta(x'-x)\int dy \left(1-\alpha(y,x)\right) \times Q(y|x)$$

where $\alpha(x',x) = \min\left(1, \frac{P(x')}{P(x)} \frac{Q(x|x')}{Q(x'|x)}\right)$

Check

$$\int dx'\, \rho(x',x) \stackrel{?}{=} 1 = \cancel{\int dx'\, \alpha(x',x)\, Q(x'|x)} \cancel{- \int dy\, \alpha(y,x)\, Q(y|x)} + \underbrace{\int dy\, Q(y|x)}_{=1}$$

Q is normalised

compare MH & Rejection Sampling.

Both require us to choose proposal Q.

If we have bad Q, algorithm extremely slow.

No, algorithmic way to choose a good Q.

Special case

$$\alpha(y, x) = \cancel{P(x)} \frac{P(y)}{P(x)} \frac{Q(x|y)}{Q(y|x)}$$

choose Q s.t. $Q(x|y) = Q(y|x)$ symmetric proposal.

$$\Rightarrow \quad \alpha(y, x) = \frac{P(y)}{P(x)} = \frac{f(y)}{f(x)}$$

we just need $f(x) \propto P(x)$

Metropolis algorithm. $\leftarrow$ see demo.

demo

proposal $Q = N(x, \Sigma)$

Σ_{small} Σ_{large} Σ_{corr}

#1 #2 #3.

$x_{i+1} \mid x_i \sim N(x_i, \Sigma)$