



Lecture 8

Imaging the Universe with

Radio Interferometers

Lecturer: Dr Haoyang Ye (hy297)



In the next one hour

- Join the quiz/discussion
- Ready to code in Python to solve a tiny problem (use QR code to download jupyter notebook)
- Ask questions loud if you need clarification
- You can discuss with each other



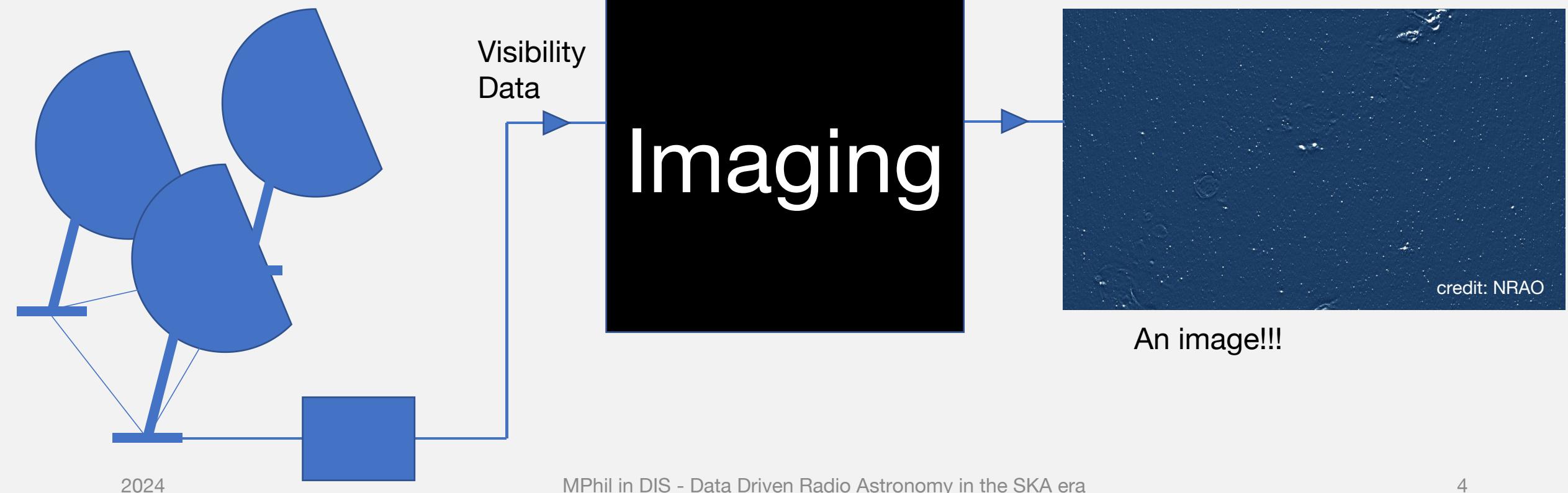


Overview

- Introduction to Big Data Radio Astronomy and Key Science Projects
- Instrument simulations and design tools
- Science Data Processing
 - Lecture 7: Calibration of radio observations
 - **Lecture 8: Imaging techniques**
 - Lecture 9: Advanced imaging techniques
 - Lecture 10: Time-domain radio astronomy
- Computing infrastructure
- Advanced ML and Bayesian methods for data analysis and science extraction

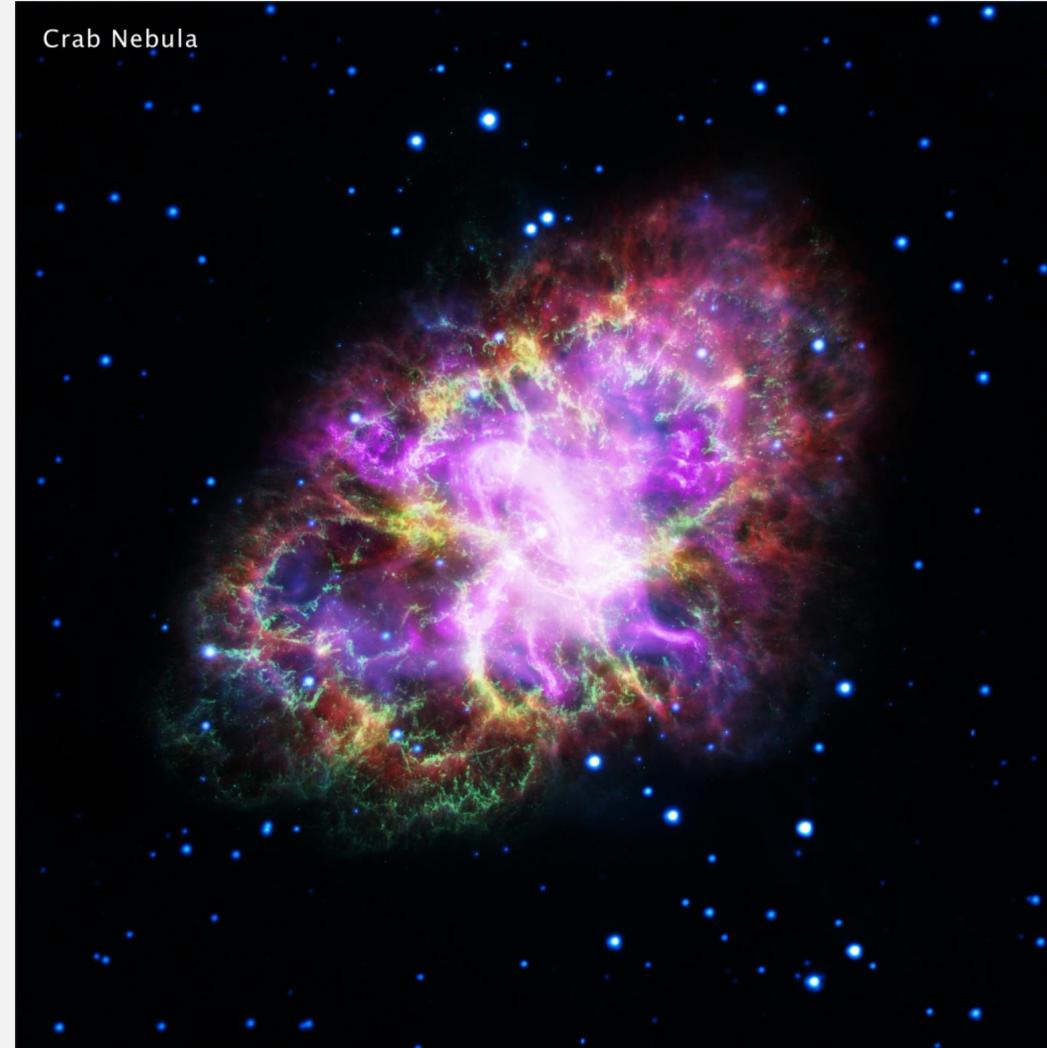


Today you will learn:



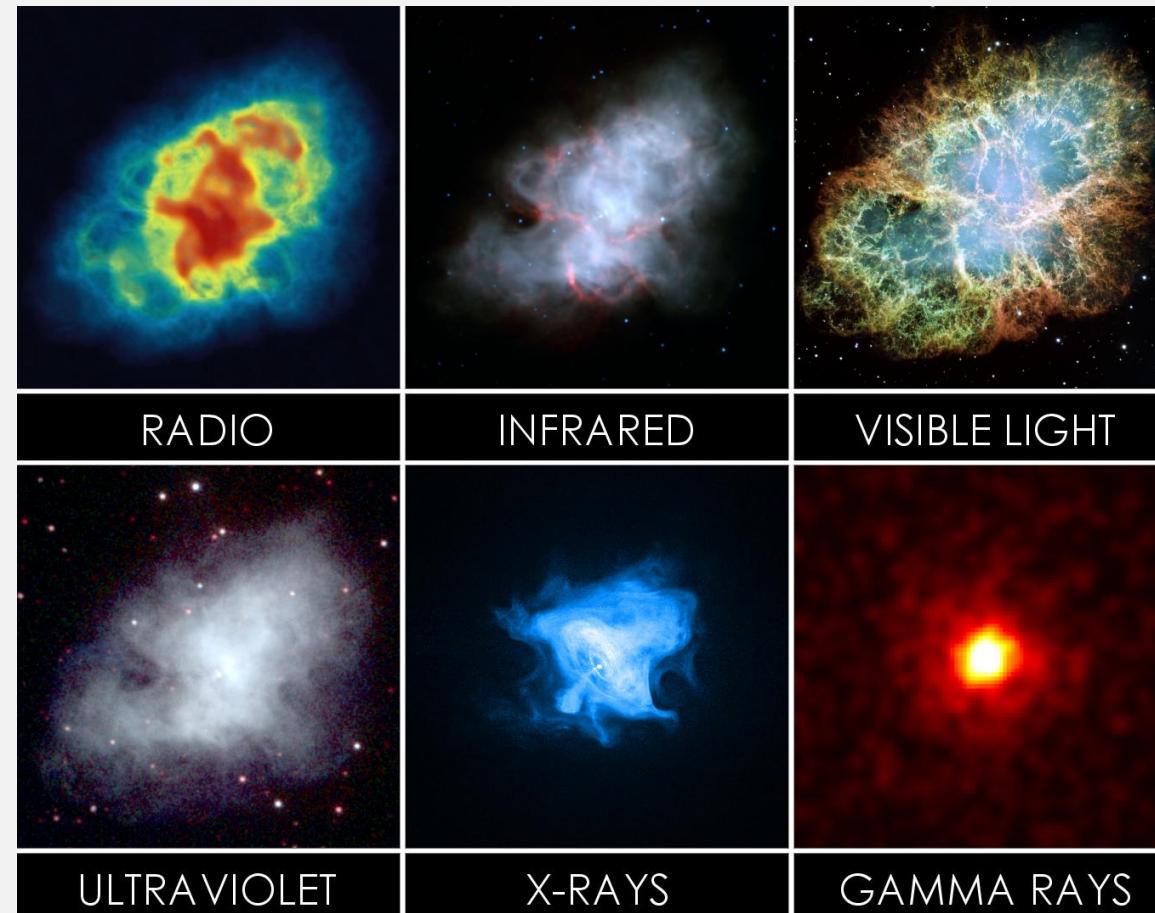


We love images!!!



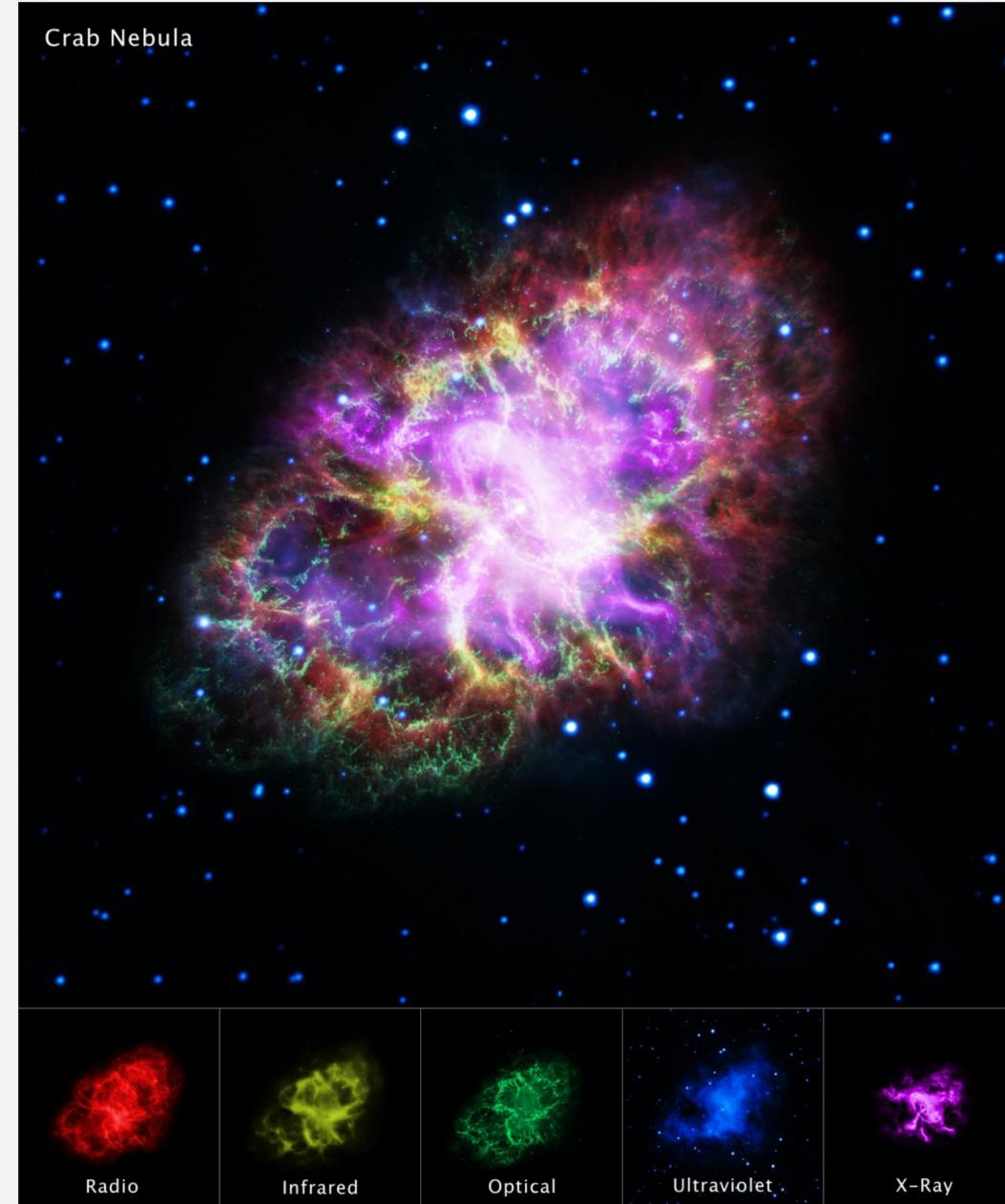


Crab Nebula imaged in Multiple Wavelengths



Credit: NRAO

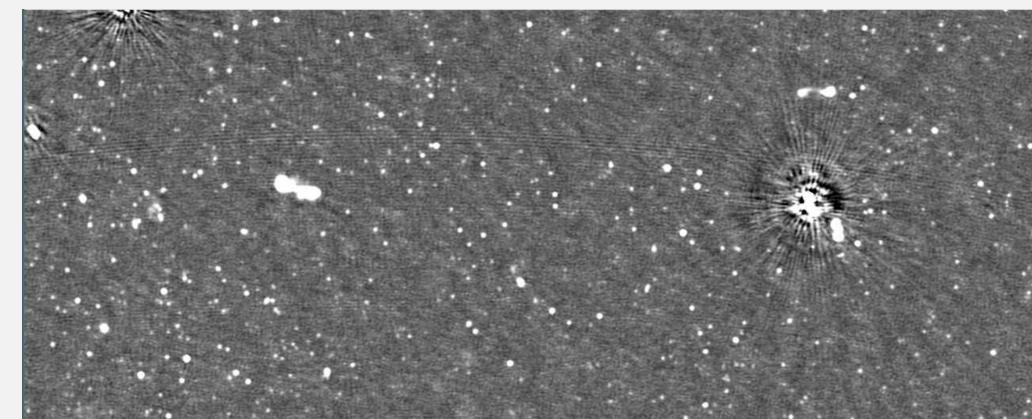
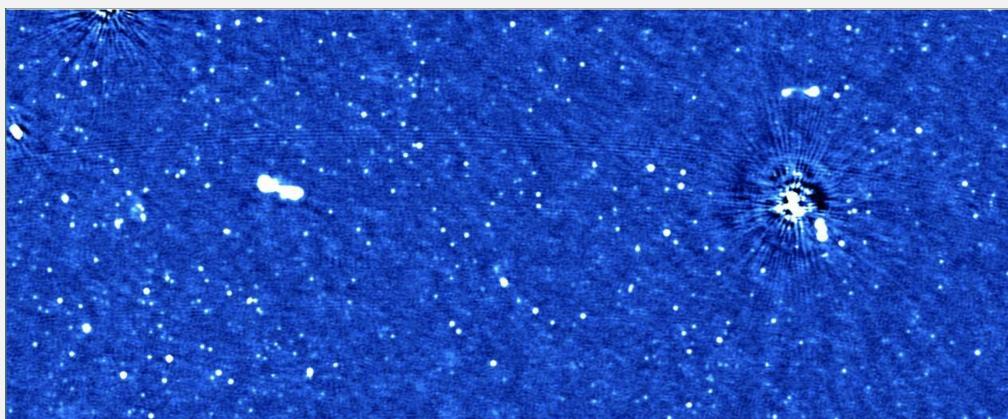
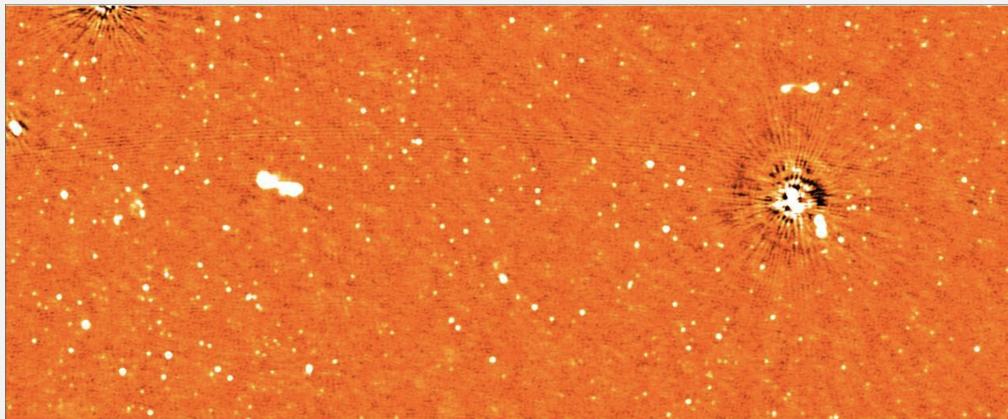
Crab Nebula imaged in Multiple Wavelengths



Credit: NRAO



Images are not perfect



And you can choose different color schemes



Question Time

Why not perfect?



About me

- Bachelor: Electronic Engineering
- PhD: Astrophysics - developing imaging and data analysis algorithms for radio interferometers
- Postdoc: worked with FAST*, LOFAR*, now working for SKA*

*FAST, LOFAR, SKA are radio telescope names



About you



Quiz Time



What was your major during your undergraduate studies?

Waiting for responses ...

 **Mentimeter**



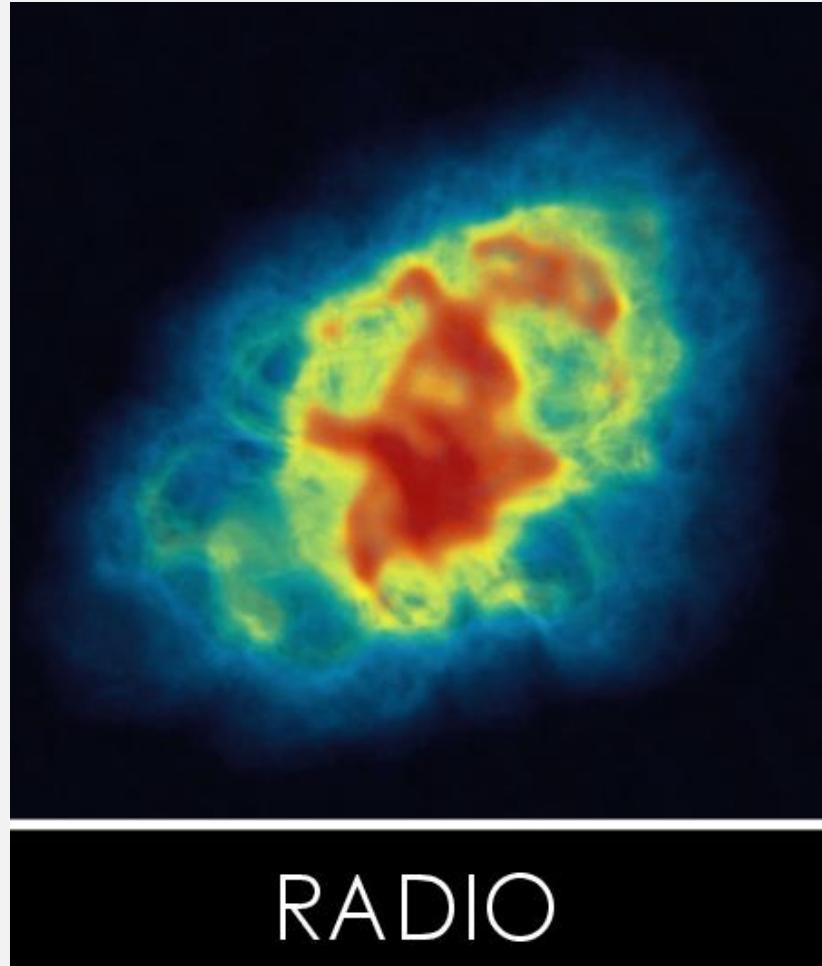
Quiz Time



Will I consider doing astronomy related jobs/studies?

0	0	0	0
I am applying	Maybe	Maybe not	No





Credit: NRAO

Sky Brightness Distribution

$$I(\theta, \phi, \nu)$$

θ and ϕ are the angular coordinates (e.g., right ascension and declination in celestial coordinates)
 ν is the frequency

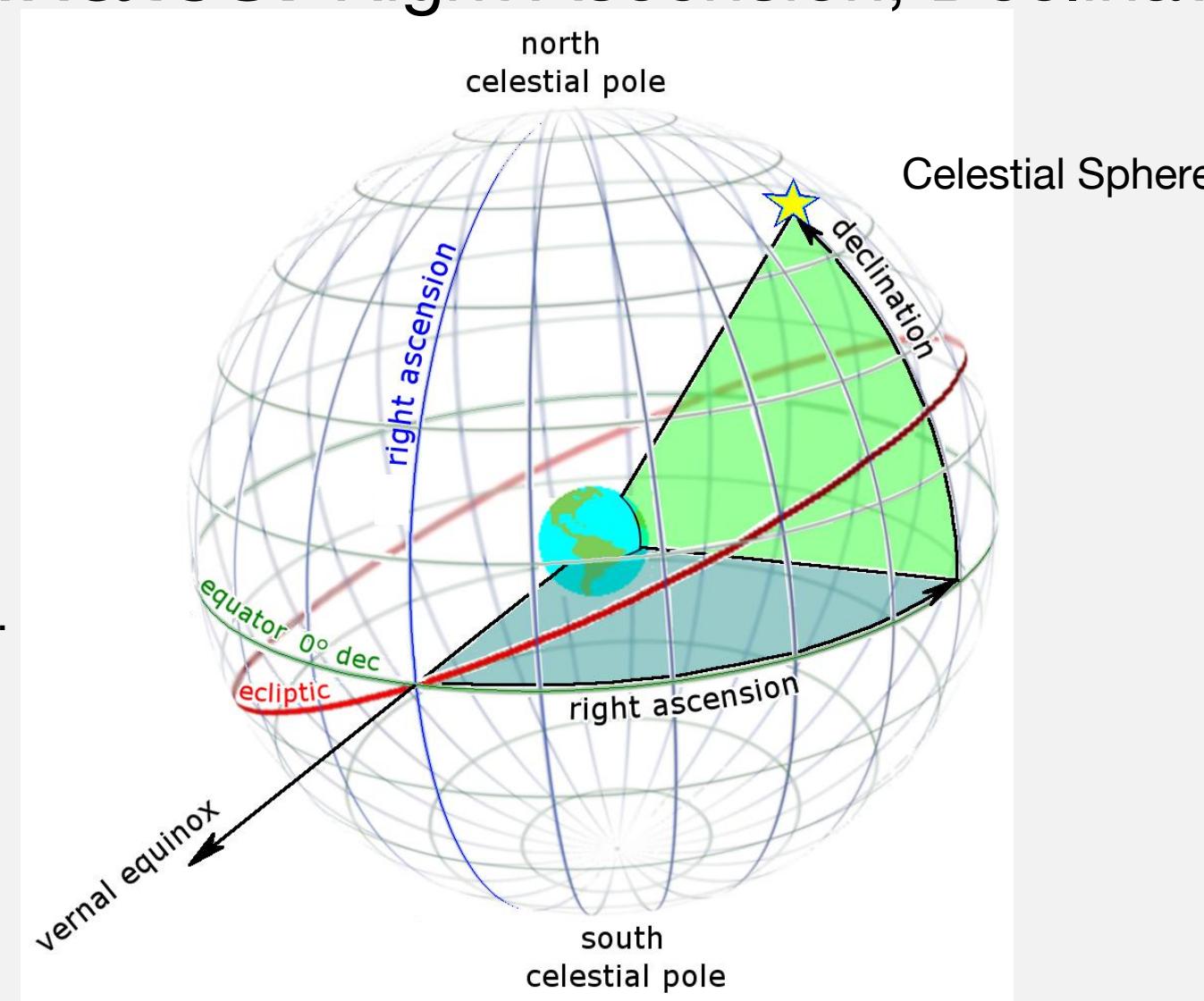
It describes how the intensity of radio emission varies across different directions in the sky at a specific wavelength or frequency.

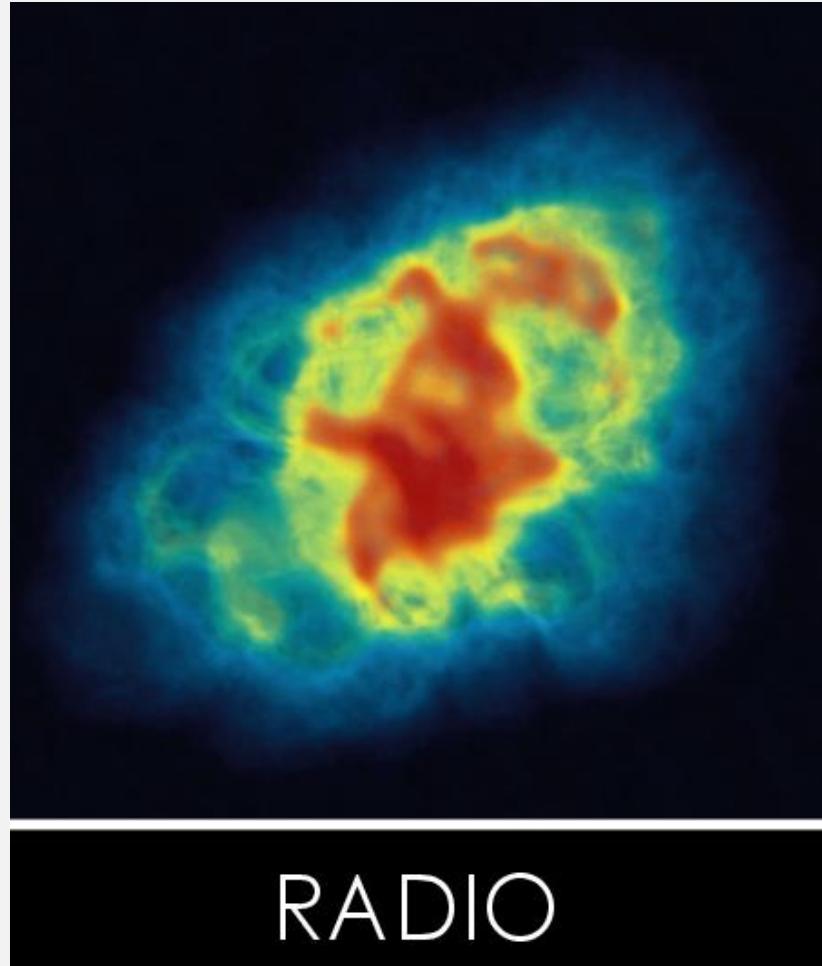


Sky Coordinates: Right Ascension, Declination

$$I(\theta, \phi)$$

- θ Right Ascension (RA) is the celestial equivalent of longitude.
- ϕ Declination (DEC) is the celestial equivalent of latitude.





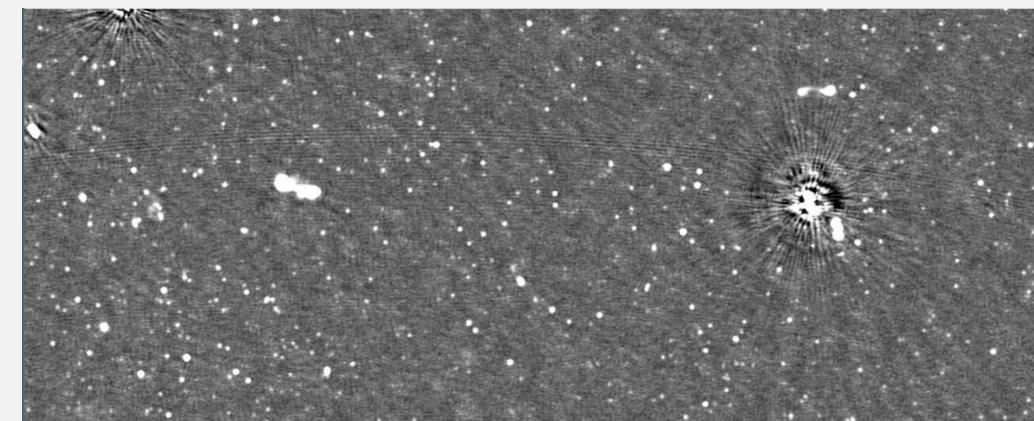
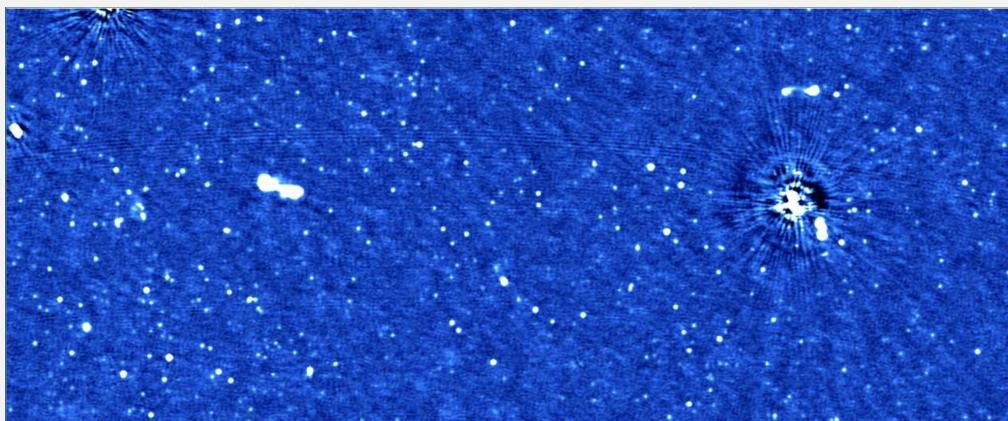
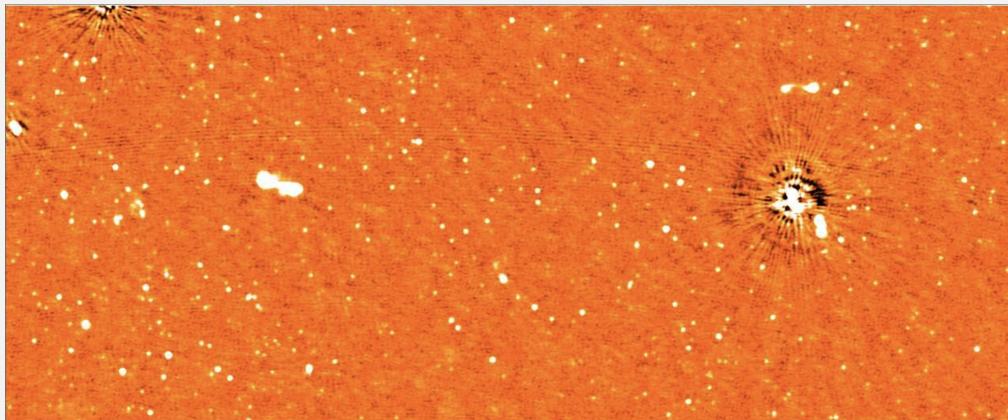
Credit: NRAO

Sky Brightness Distribution $I(\theta, \phi, \nu)$

Which area has higher
radio emission intensity?



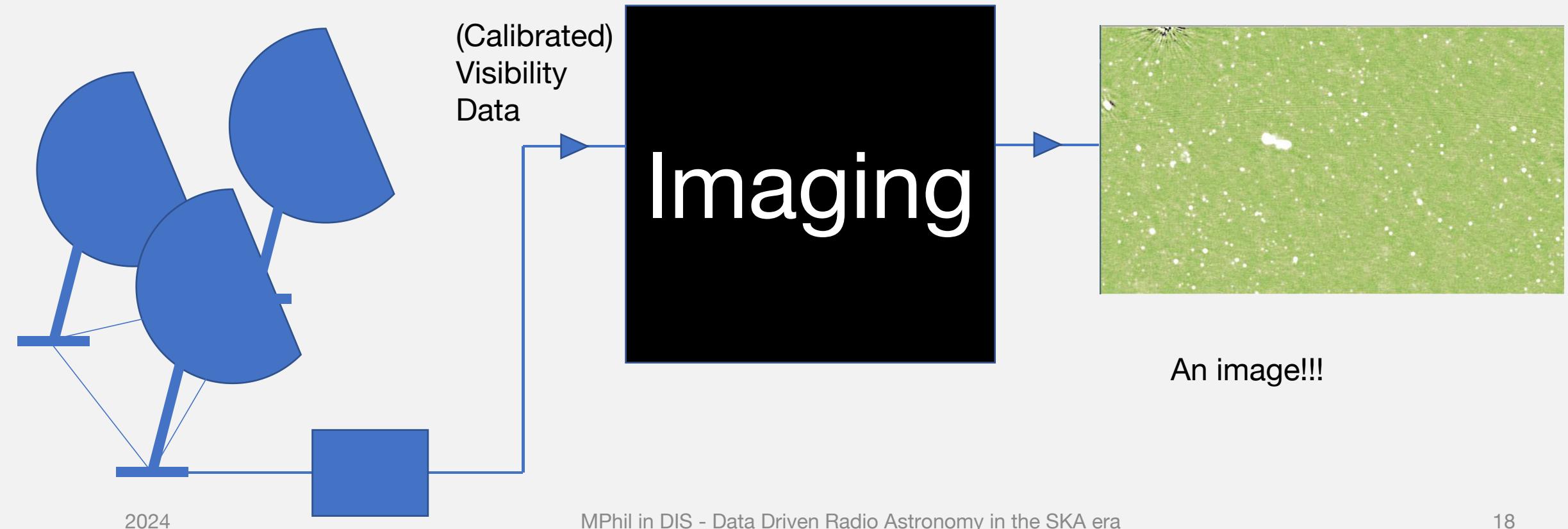
Display examples



Which area has higher radio emission intensity?



But we start from visibility data!!!!



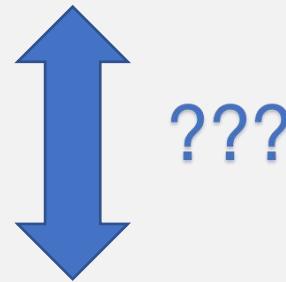
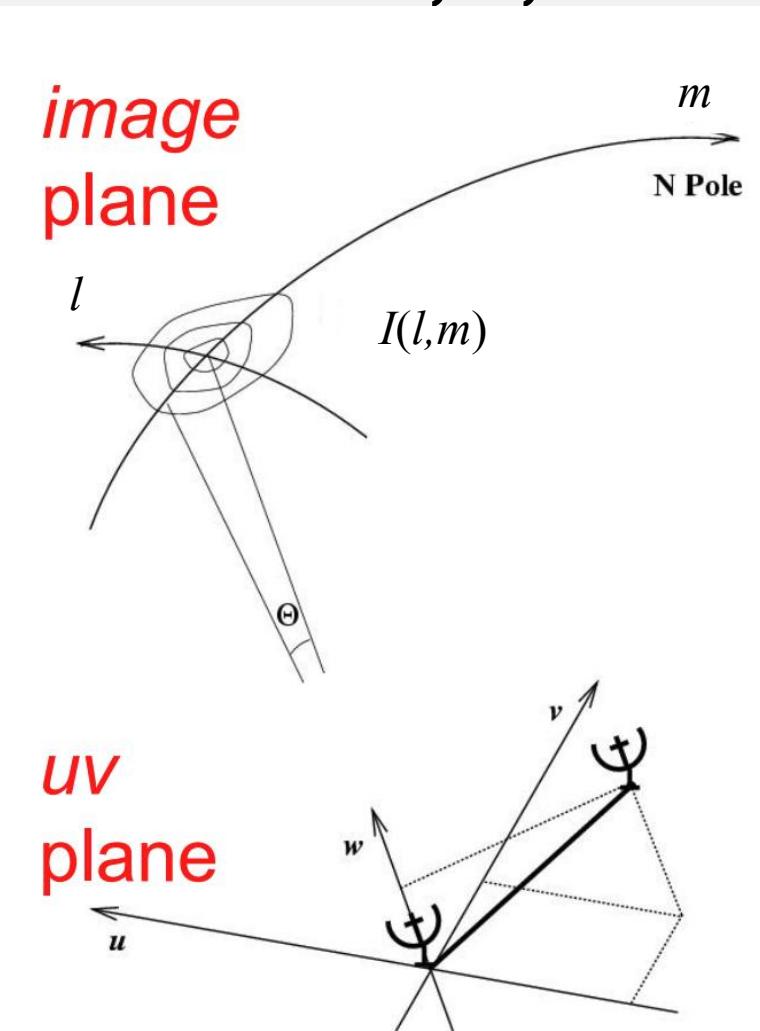


Visibility data $V(u, v, w)$ - read the data

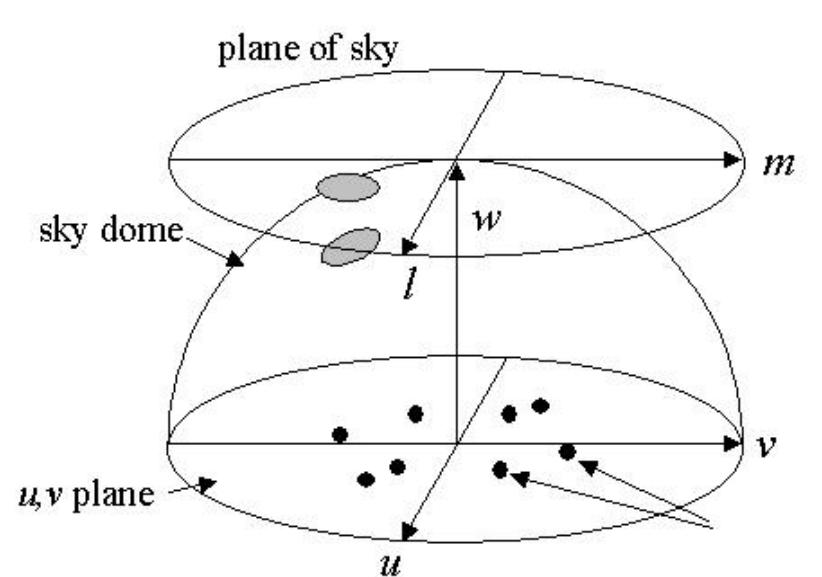
$V(u, v, w)$ is usually stored inside a .ms file,
short for "Measurement Set".



Baseline Coordinates: u,v,w

 $V(u, v, w)$

 $I(l, m)$


Check more about coordinates: https://science.nrao.edu/science/meetings/2018/16th-synthesis-imaging-workshop/talks/Perley_Geometry_new.pdf





Visibility data $V(u, v, w)$ - read the data

$V(u, v, w)$ is usually stored inside a .ms file,
short for "Measurement Set".

```
>>> import pyrap.tables as pt  
  
>>> MS = '/PATH/to/DATA.ms' # measurement set  
  
>>> maintable = pt.table(MS) # read measurement set  
  
>>> maintable.colnames() # Check column names
```

```
>>> MS = '/home/hye/Desktop/complist_only1.vla.a.ms'  
>>> maintable = pt.table(MS)  
Successful readonly open of default-locked table /home/hye/Desktop/complist_only1.vla.a.ms: 24 columns, 28080 rows  
>>> maintable.colnames()  
['UVW', 'FLAG', 'FLAG_CATEGORY', 'WEIGHT', 'SIGMA', 'ANTENNA1', 'ANTENNA2', 'ARRAY_ID', 'DATA_DESC_ID', 'EXPOSURE', 'FEED1',  
 'FEED2', 'FIELD_ID', 'FLAG_ROW', 'INTERVAL', 'OBSERVATION_ID', 'PROCESSOR_ID', 'SCAN_NUMBER', 'STATE_ID', 'TIME', 'TIME_CEN-  
 TROID', 'DATA', 'MODEL_DATA', 'CORRECTED_DATA']
```



Visibility data $V(u, v, w)$ - inspect the data

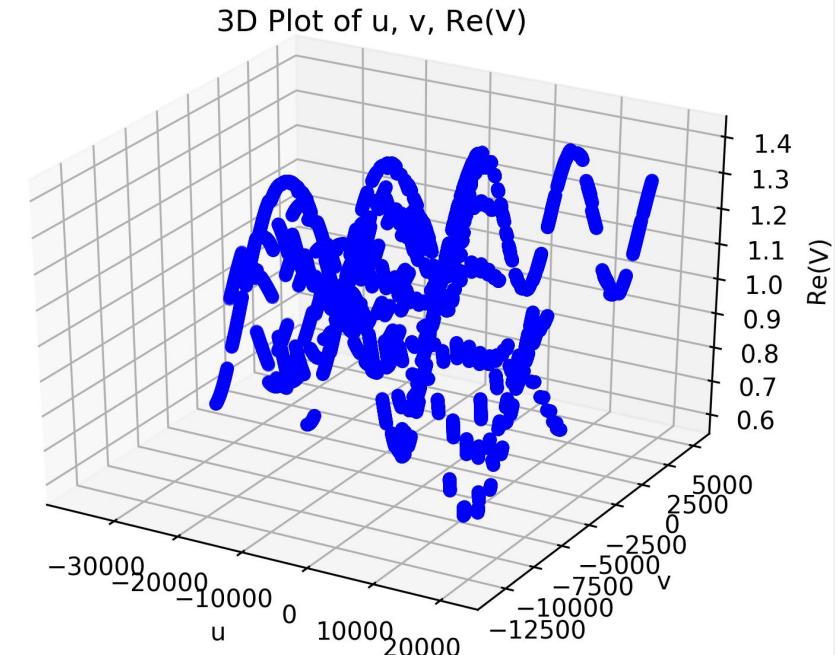
```
>> import pyrap.tables as pt
>> MS = '/PATH/to/DATA.ms' # measurement set
>> maintable = pt.table(MS) # read measurement set
>> maintable.colnames() # Check column names
>> uvw_column = maintable.getcol("UVW") # get coordinates u,v,w
>> udata, vdata, wdata=uvw_column[:,0], uvw_column[:,1], uvw_column[:,2]
>>> uvw_column = maintable.getcol("UVW")
>>> vdata=uvw_column[:,1]
>>> udata=uvw_column[:,0]
>>> udata.shape
(28080,)
>>> vdata.shape
(28080,)
>>> udata
array([ 922.82439341, 2256.7422902 , 3954.86445702, ...,
       346.07608855,
       724.5101658 , 378.43407725])
```



Visibility data $V(u, v, w)$ - inspect the data

```
>> import pyrap.tables as pt  
  
>> MS = '/PATH/to/DATA.ms' # measurement set  
  
>> maintable = pt.table(MS) # read measurement set  
  
>> maintable.colnames() # Check column names  
  
>> vis = maintable.getcol("DATA") # get visibility data
```

```
>>> vis = maintable.getcol("DATA")  
>>> vis.shape  
(28080, 1, 2)  
>>> vis[:, :, 0]  
array([[1.3615973+2.0712614e-06j],  
       [1.2061307+2.6077032e-07j],  
       [1.0166397-9.7304583e-06j],  
       ...,  
       [1.3638289-5.6065619e-06j],  
       [1.2508852-6.7204237e-06j],  
       [1.3567369-5.9716403e-06j]], dtype=complex64)
```





Coding Time

Given that each pair of antennas (a baseline) generates one visibility data per frequency channel per second,

Do complete this Python function to calculates the total number of visibility data generated by an array of antennas.



Coding Time

Given that each pair of antennas (a baseline) generates one visibility data per frequency channel per second,

Do complete this Python function to calculate the total number of visibility data generated by an array of antennas.

```
def calculate_visibility_num(antennas, observation_time, frequency_channel):
    """
    Calculate the total number of visibility data generated by an array of antennas.

    Parameters:
        antennas (int): Number of antennas in the array.
        observation_time (int): Observation time in seconds.
        frequency_channel (int): Number of frequency channels.

    Returns:
        int: Total number of visibility data generated by the array.
    """
    # Calculate the number of pairs of antennas
    if antennas < 2:
        baseline_num = 0 # No baselines if there are less than 2 antennas
    else:
        baseline_num = ???????

    # Calculate the total visibility data
    visibility_num = baseline_num * observation_time * frequency_channel

    return visibility_num
```



Coding Time - unit test

Given that each pair of antennas (a baseline) generates one visibility data per frequency channel per second,

Do complete this Python function to calculates the total number of visibility data generated by an array of antennas.

```
import unittest

class TestCalculateVisibilityData(unittest.TestCase):

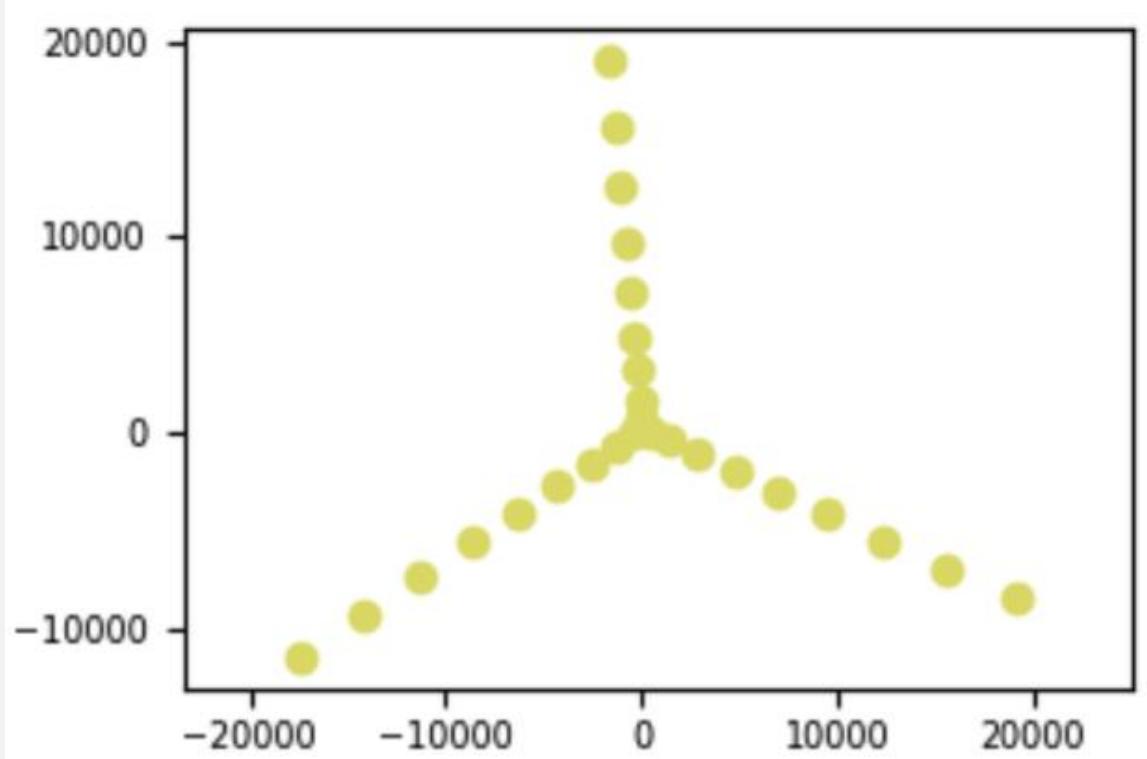
    def test_visibility_data_calculation(self):
        # Test case with 4 antennas, 60 seconds observation time, and frequency
        # channel 10
        result = calculate_visibility_num(4, 60, 10)
        self.assertEqual(result, 3600) # Expected result: 6 pairs of antennas * 60
        # seconds * 10 frequency channels = 3600

        # Test case with 2 antennas, 30 seconds observation time, and frequency
        # channel 5
        result = calculate_visibility_num(2, 30, 5)
        self.assertEqual(result, 150) # Expected result: 1 pair of antennas * 30 seconds
        # * 5 frequency channels = 150

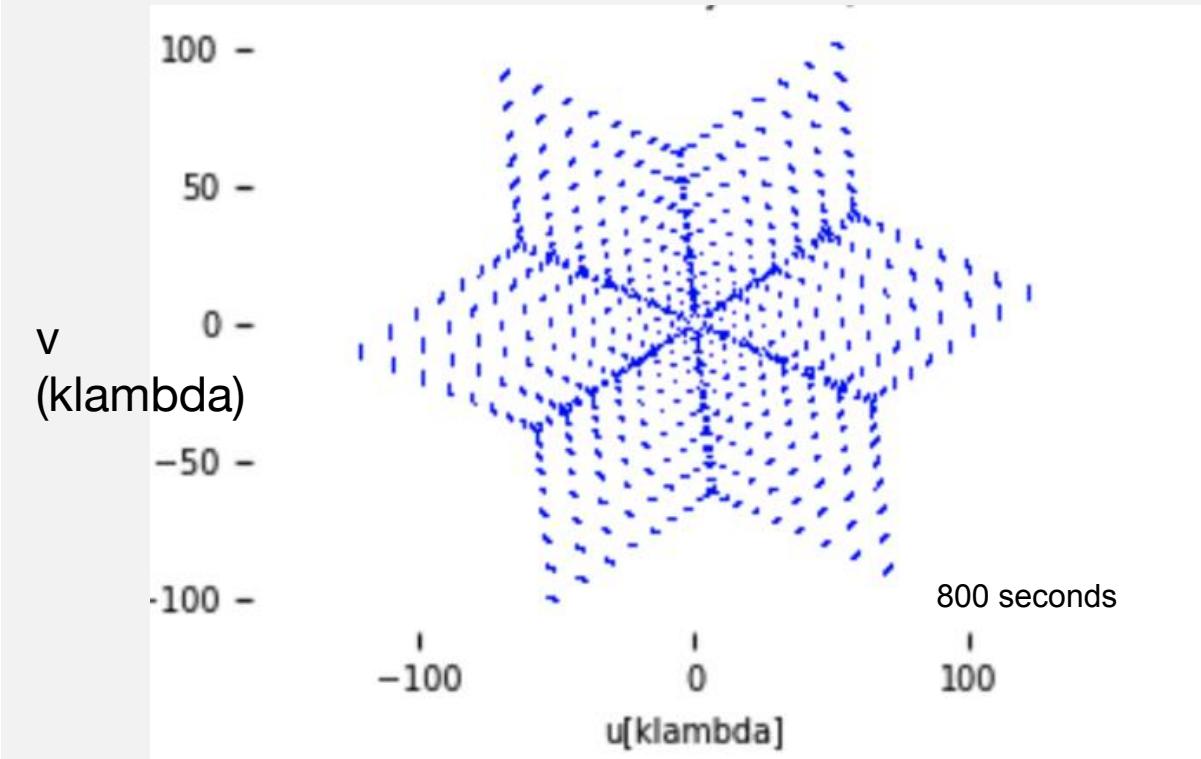
        # Test case with 1 antenna, 10 seconds observation time, and frequency
        # channel 1
        result = calculate_visibility_num(1, 10, 1)
        self.assertEqual(result, 0) # Expected result: No pairs of antennas
```

Visibility data $V(u, v, w)$ - plot the data

Antenna location



uv-coverage

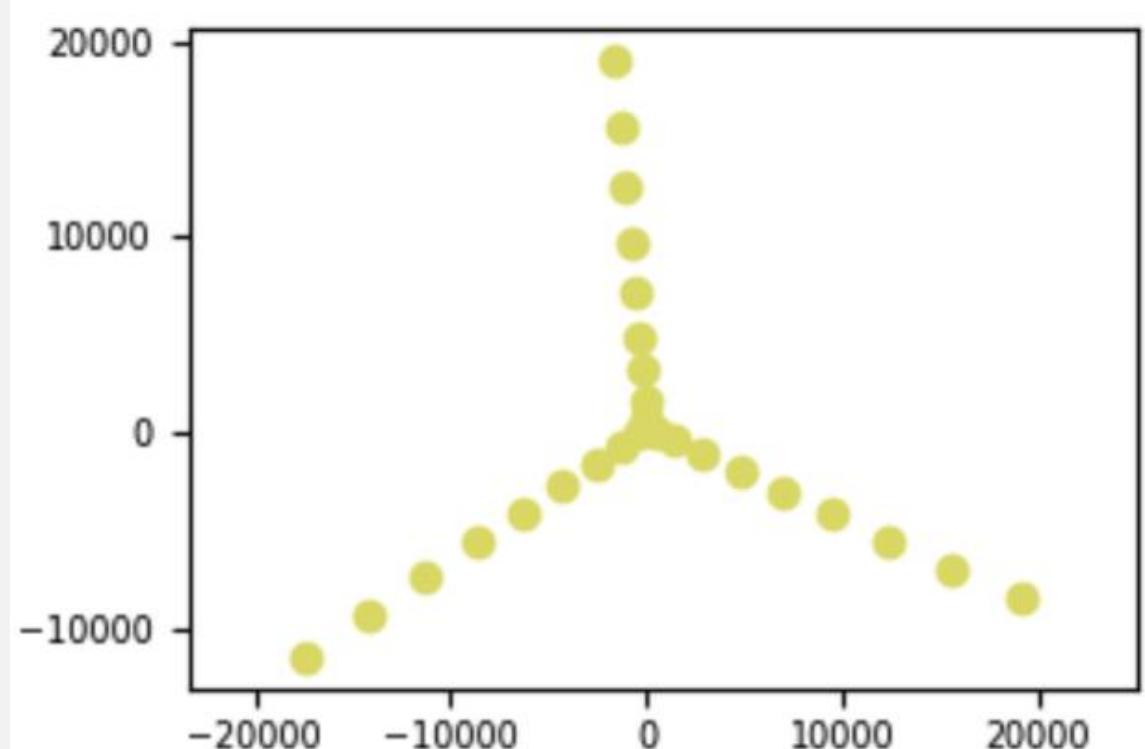


Generated using CASA

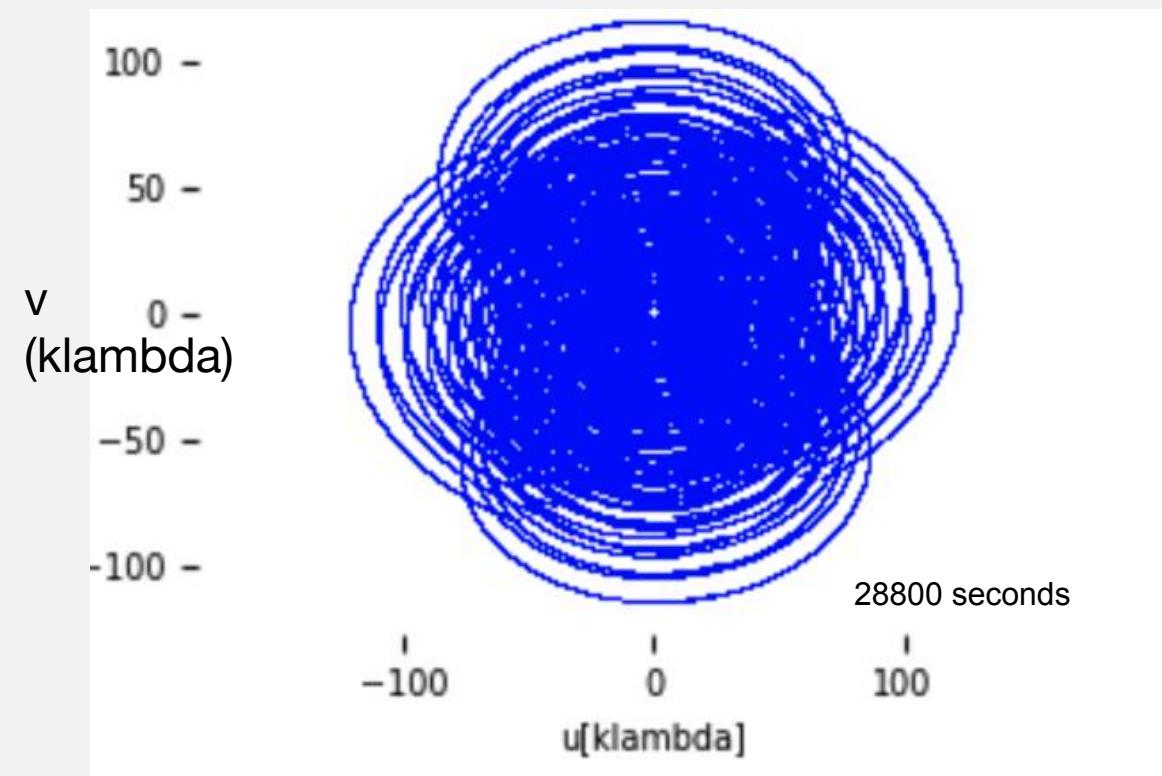


Visibility data $V(u, v, w)$ - plot the data

Antenna location



uv-coverage



Generated using CASA



Question Time

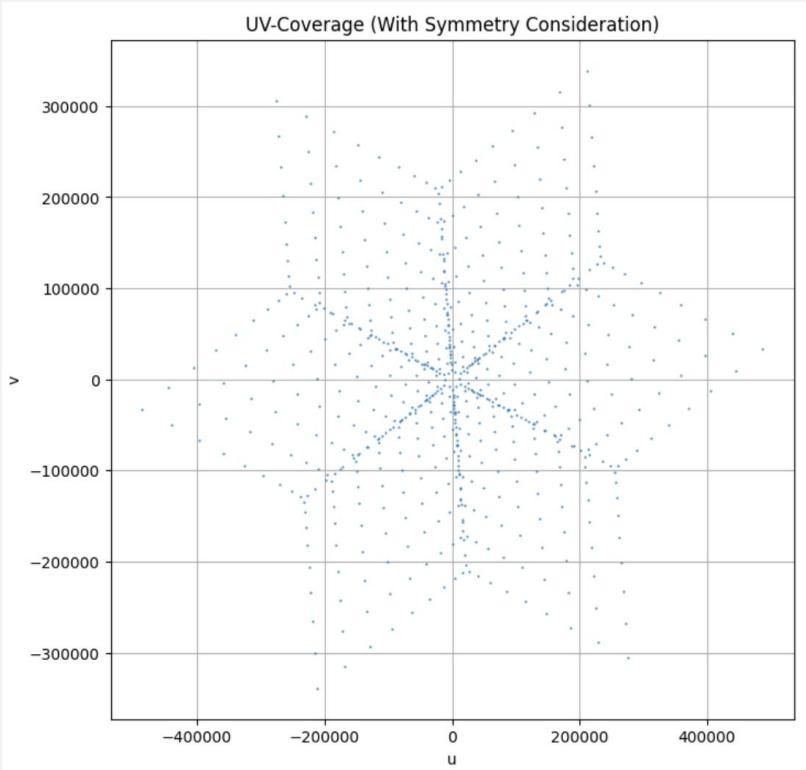
What changed?



Coding Time

Given a visibility data $V(u, v, w)$,

Do plot uv-coverage.



```
# Load the data
Vis = np.loadtxt('onesource.csv', usecols=[0, 1, 2, 3, 4])
u = ****
v = ****

# Plot the uv-coverage
plt.figure(figsize=(8, 8))
plt.plot(u, v, '.', markersize=1)
plt.xlabel('u')
plt.ylabel('v')
plt.title('UV-Coverage (With Symmetry Consideration)')
plt.grid(True)
plt.show()
```



From the basics



Quiz



Is it a radio interferometer? Or not?



Frequency range:
12 to 18 GHz



Frequency range:
10 to 240 GHz



Frequency range:
119 to 374 THz



Frequency range:
12 to 999 THz





Discussion



Is it a radio interferometer? Or not?



Frequency range:
12 to 18 GHz

AMI



Frequency range:
10 to 240 GHz

LOFAR
(low: 10-80 MHz
High: 120-
240MHz)



Frequency range:
119 to 374 THz

Hubble



Frequency range:
12 to 999THz

VLT





Discussion

To qualify as a radio interferometer, how many antennas must it possess?

0 ————— 50

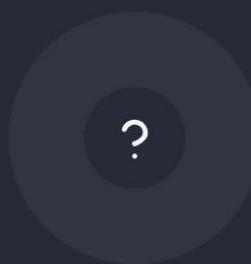


Discussion

Truth or Lie: Do I understand this jargon?



Visibility data (or
visibilities)



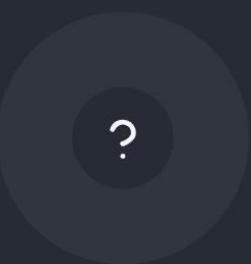
FFT



Gridding



UV coverage

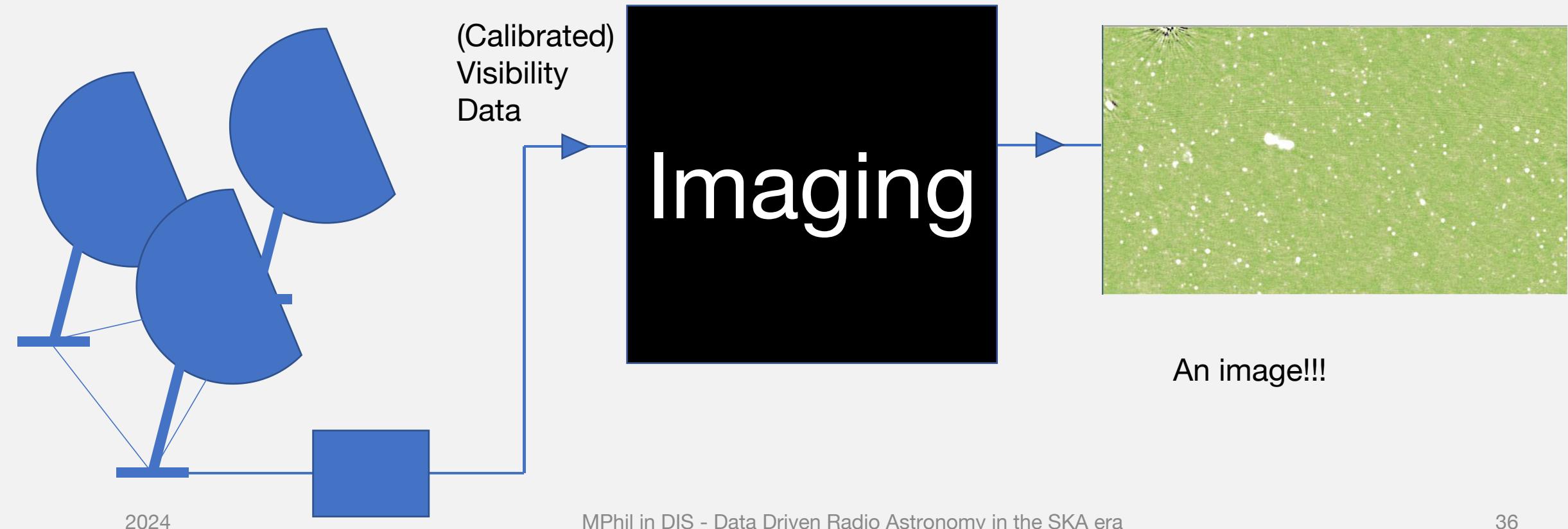


Brightness





And how do we convert visibility data to an image???





How to convert visibility data into imaging?

Visibility Plane

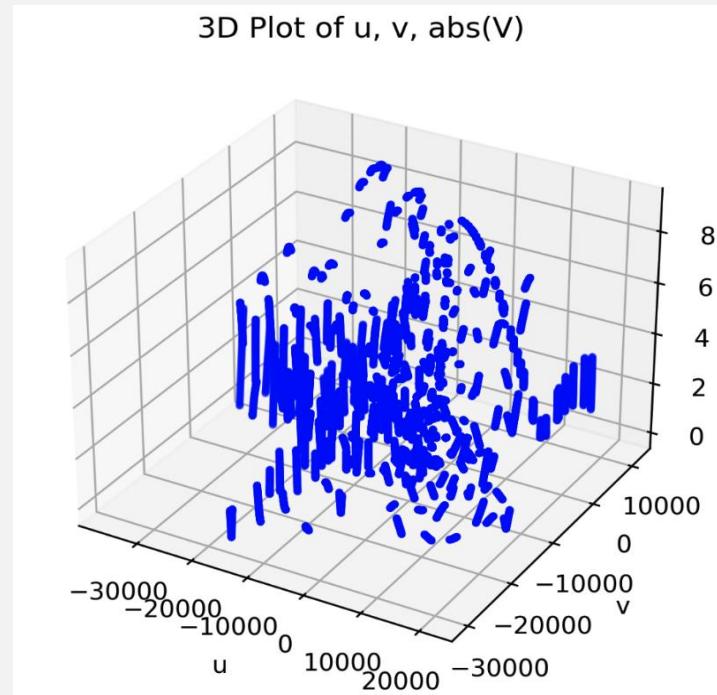
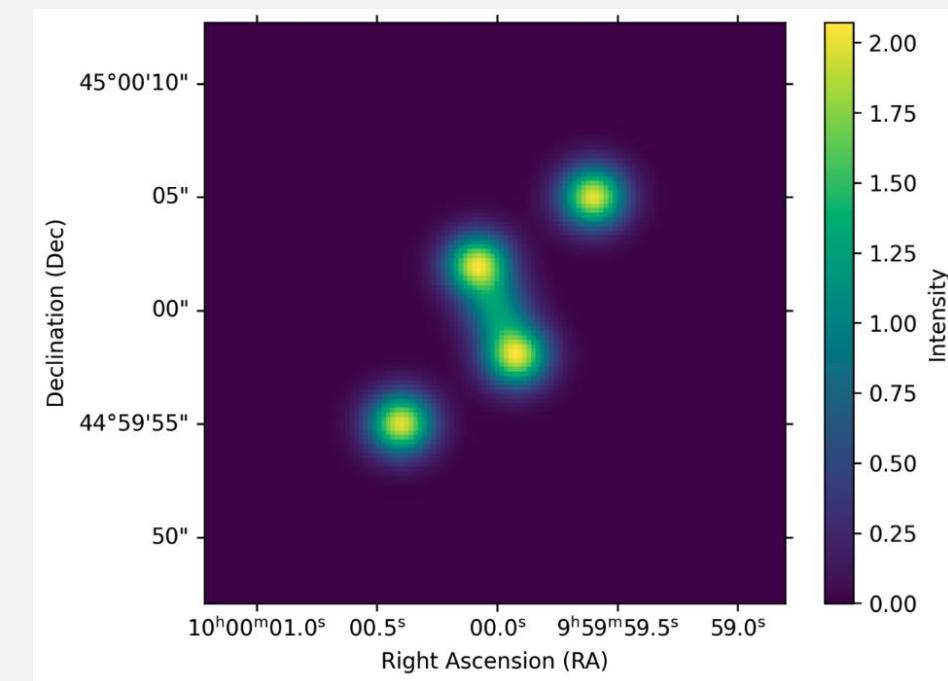
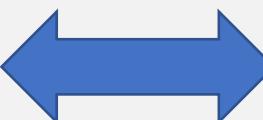


Image Plane



Visibility Data

$$V(u, v, w)$$



Sky Brightness Distribution

$$I(l, m)$$



How to convert visibility data into imaging?

$$V(u, v, w) = \iint \frac{I(l, m)}{\sqrt{1 - l^2 - m^2}} e^{-2\pi i [ul + vm + w(\sqrt{1 - l^2 - m^2} - 1)]} dl dm$$

Detailed explanation can be found at <https://www.aspbooks.org/publications/6/11.pdf>



How to convert visibility data into imaging?

$$V(u, v, w) = \iint \frac{I(l, m)}{\sqrt{1 - l^2 - m^2}} e^{-2\pi i [ul + vm + w(\sqrt{1 - l^2 - m^2} - 1)]} dl dm$$

$$(\sqrt{1 - l^2 - m^2} - 1)w \approx 0$$

$$V(u, v, w) = \iint I(l, m) e^{-2\pi i (ul + vm)} dl dm$$

How to convert visibility data into imaging?

$$\left(\sqrt{1 - l^2 - m^2} - 1 \right) w \approx 0$$

$$V(u, v) = \int \int I(l, m) e^{-2\pi i (ul + vm)} dl dm$$



$$I(l, m) = \int \int V(u, v) e^{2\pi i (ul + vm)} du dv$$



How to convert visibility data into imaging?

when the field of view is small...

$$\left(\sqrt{1-l^2-m^2} - 1\right)w \approx -\frac{1}{2}(l^2+m^2)w \approx 0$$

If you are interested, figure it out and tell me how you reached this!

Or when w is negligible

$$\left(\sqrt{1-l^2-m^2} - 1\right)w \approx 0$$



How to convert visibility data into imaging?

$$I(l,m) =$$

Inverse Fourier Transform of $V(u,v)$

*when the field of view is small...

or when w is negligible

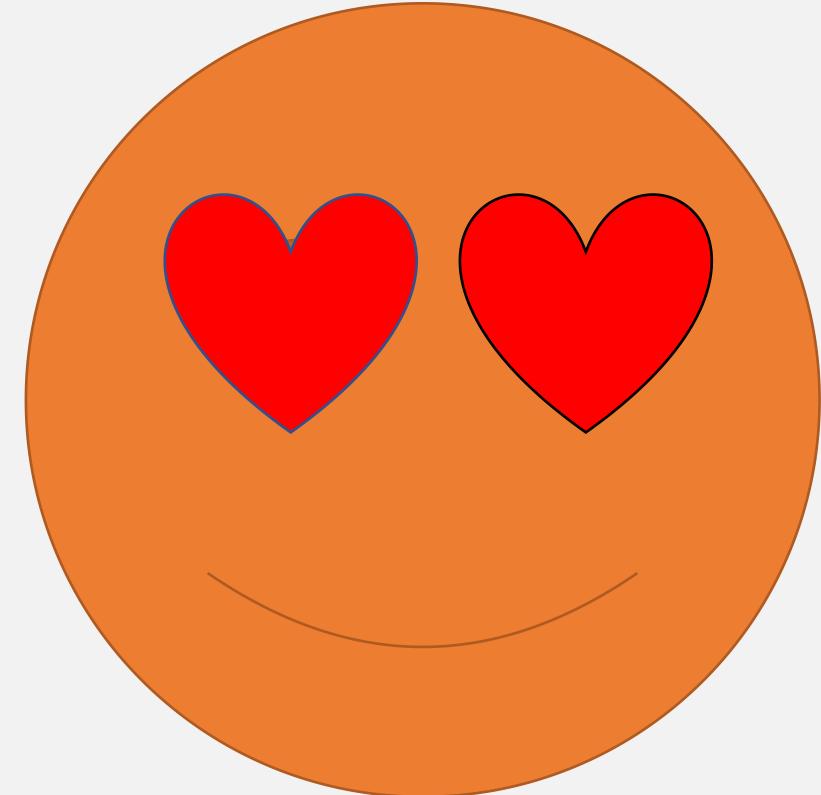


How to convert visibility data into imaging?

Perfect!!!!

We just need one row of code!!!!

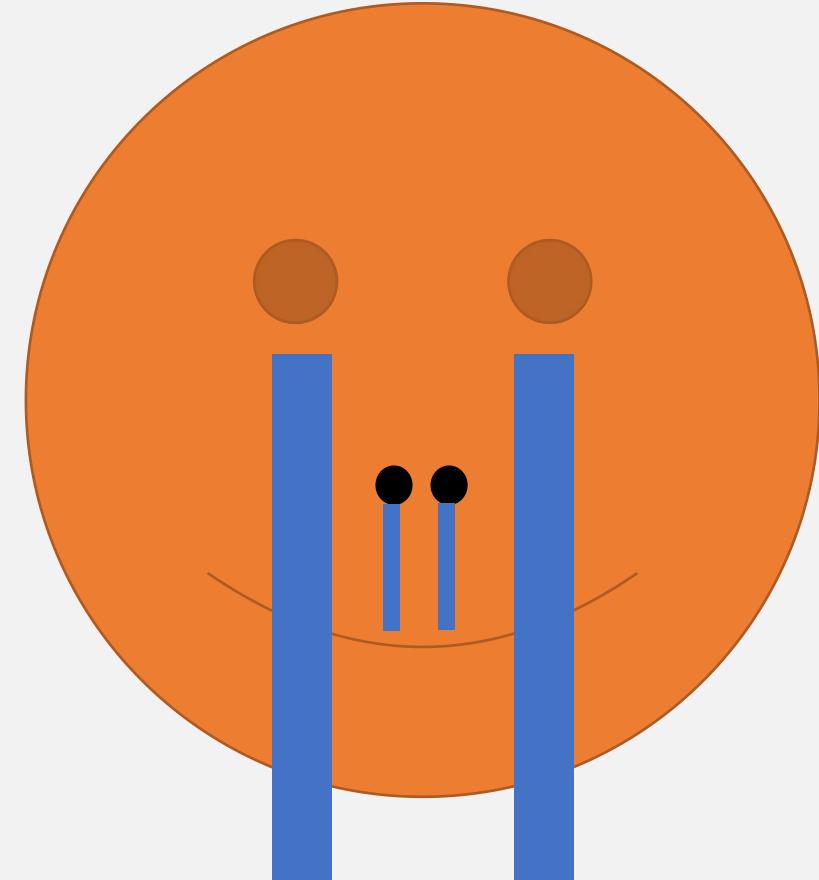
Hooray!!!!





How to convert visibility data into imaging?

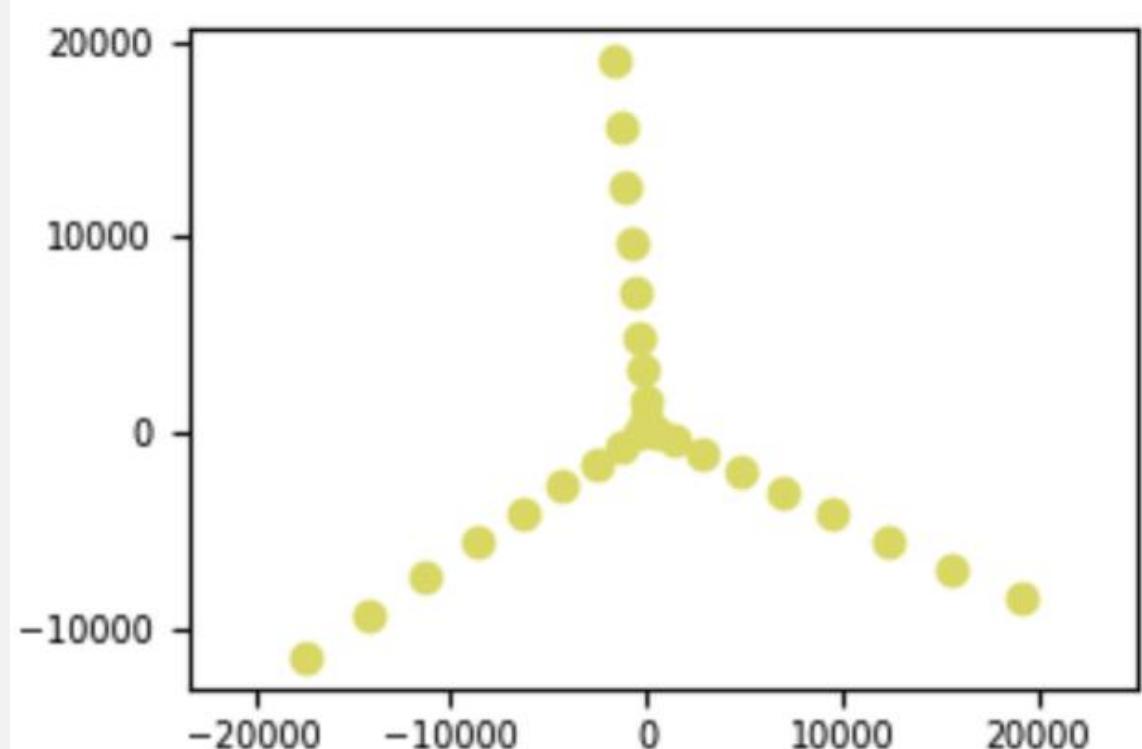
Wait.....



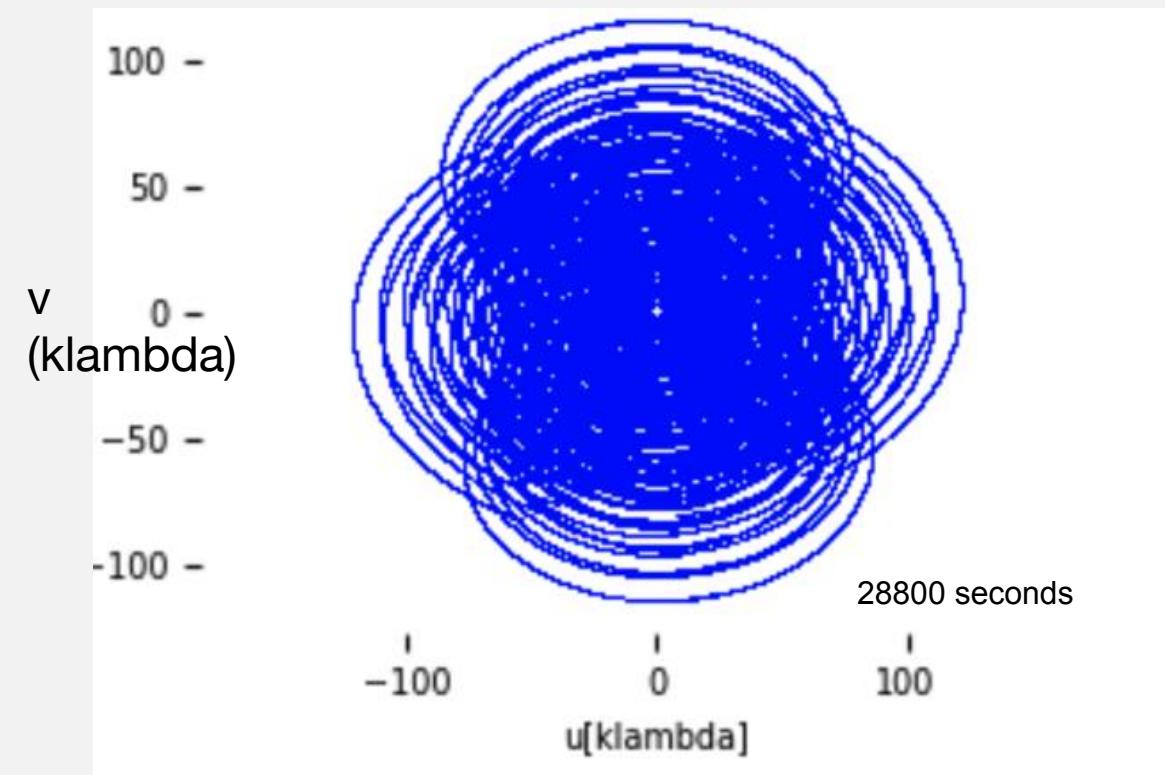


Visibility data $V(u, v, w)$ - plot the data

Antenna location



uv-coverage

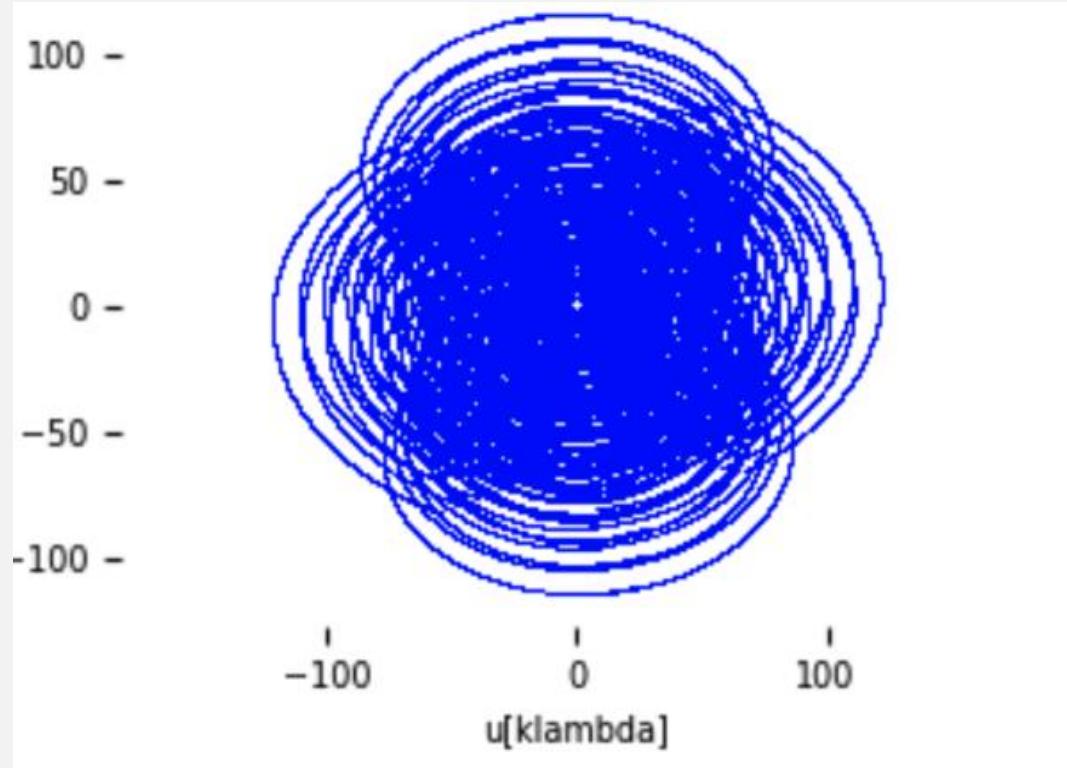


Generated using CASA

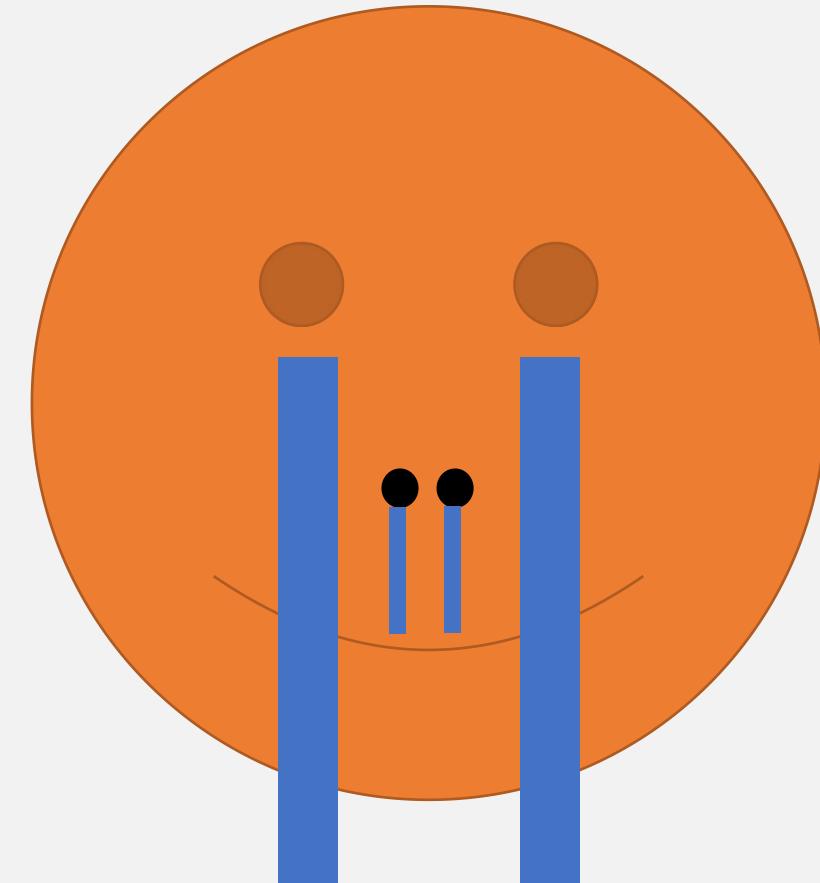


How to convert visibility data into imaging?

uv-coverage



We don't have all uv values sampled!!!!!!!



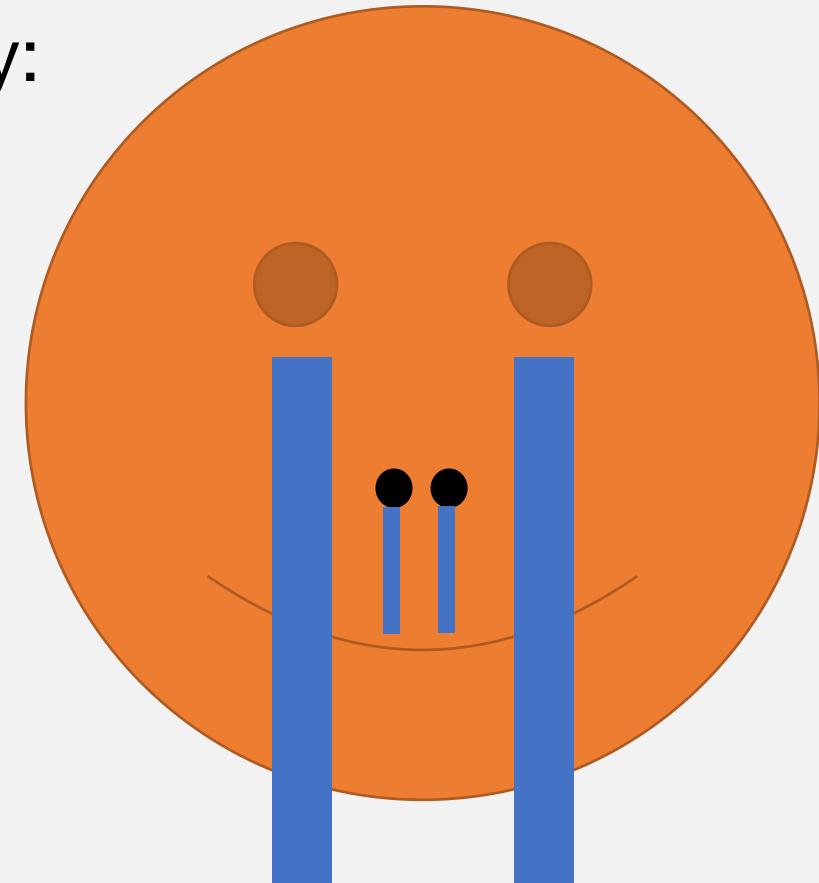


How to convert visibility data into imaging?

The visibility data we observed is actually:

$$\begin{aligned} V^s(u, v) &= V(u, v) S(u, v) \\ &= \sum_{k=1}^M \delta(u - u_k, v - v_k) V(u_k, v_k) \end{aligned}$$

$$, \text{ where } S(u, v) = \sum_{k=1}^M \delta(u - u_k, v - v_k)$$



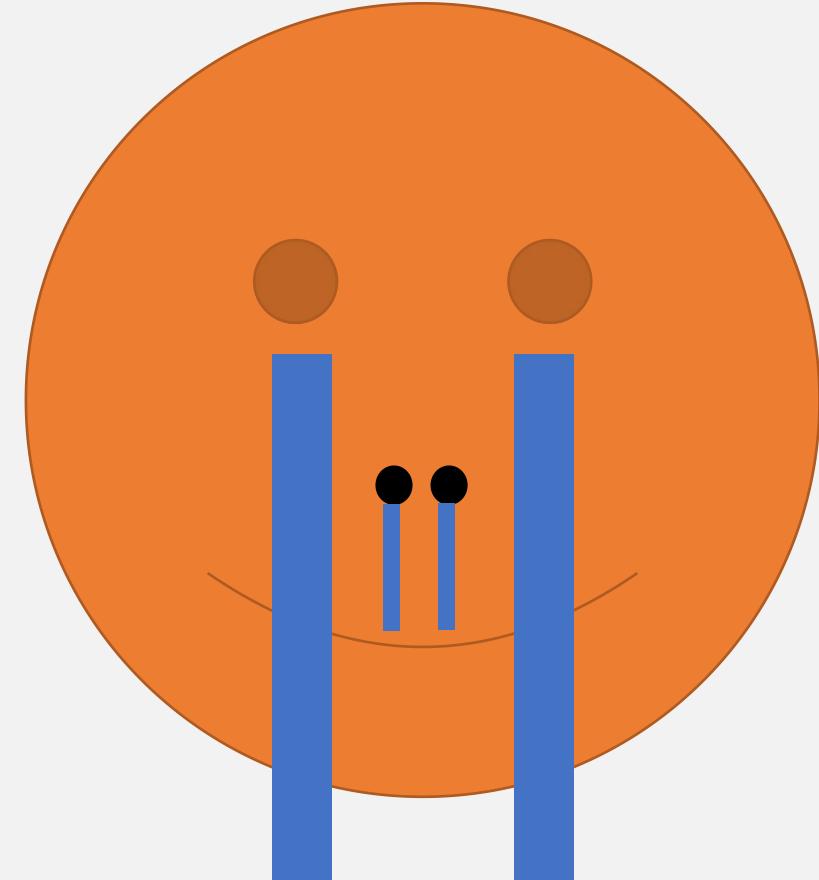


How to convert visibility data into imaging?

The Fourier Transform of

$$V(u,v)S(u,v)$$

would become???????





How to convert visibility data into imaging?

The Fourier Transform of

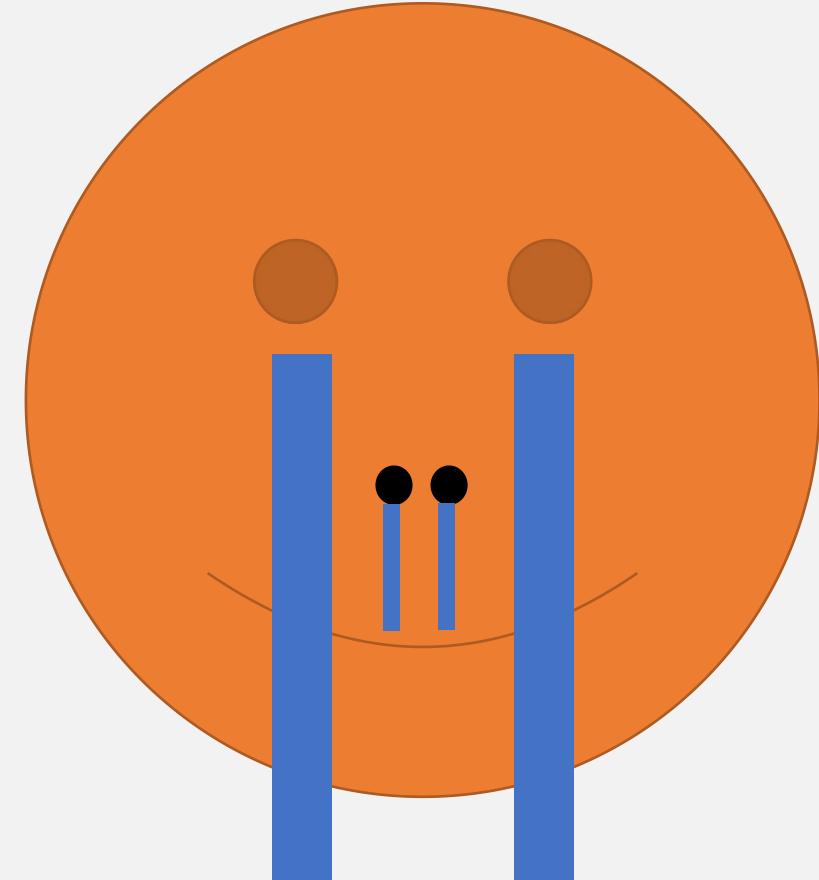
$$\mathcal{F}(V(u,v)S(u,v)) = \mathcal{F}(V(u,v))^* \mathcal{F}(S(u,v))$$

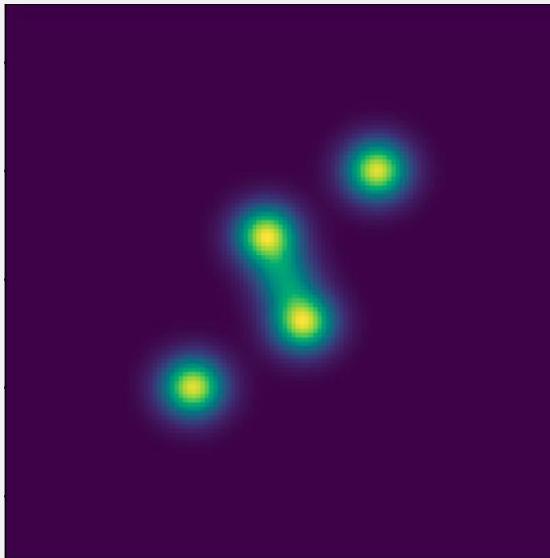
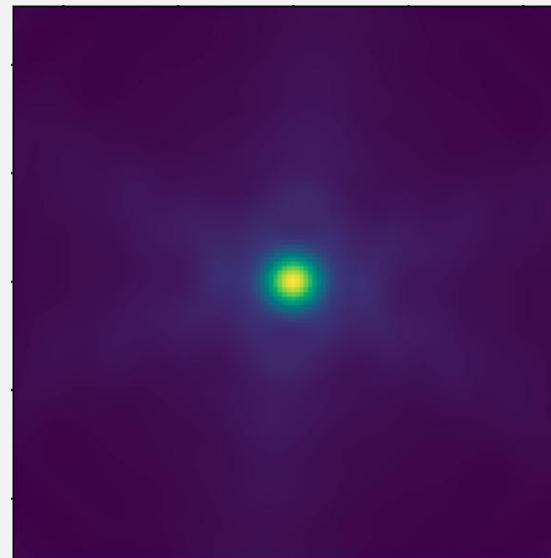
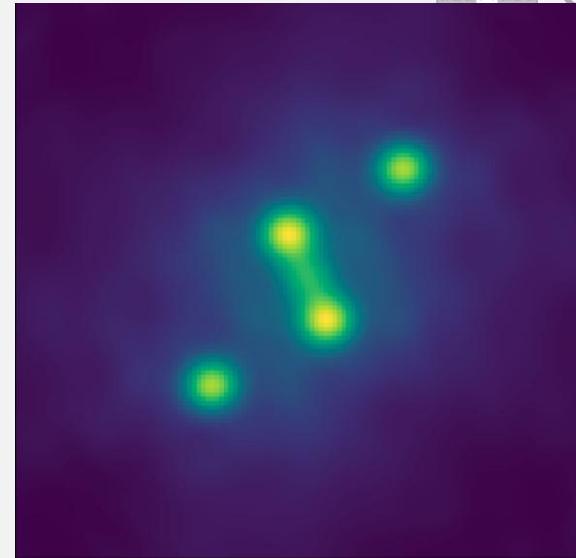
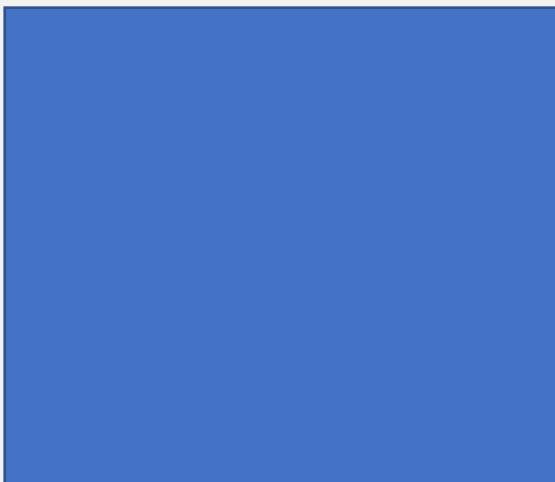
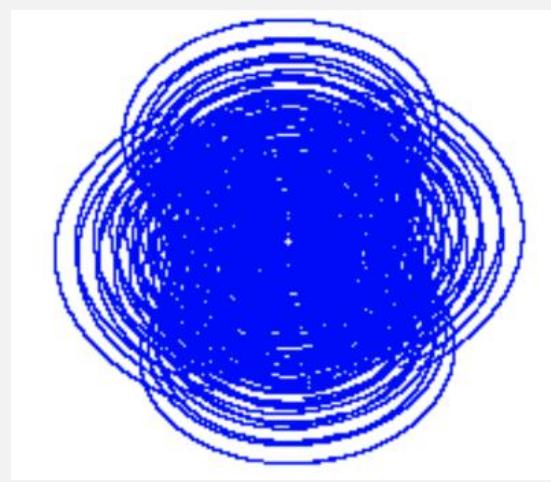
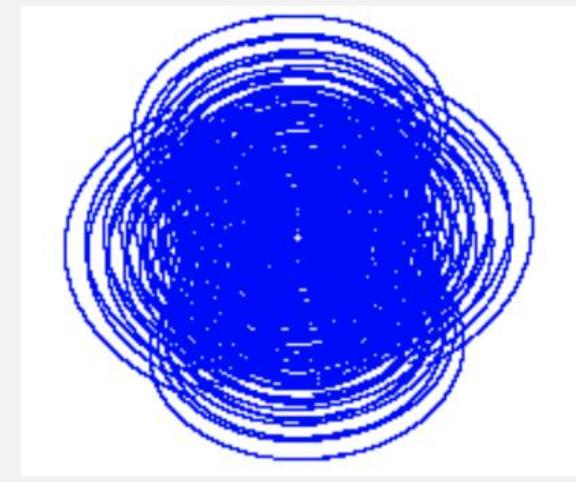
We want this!!!!

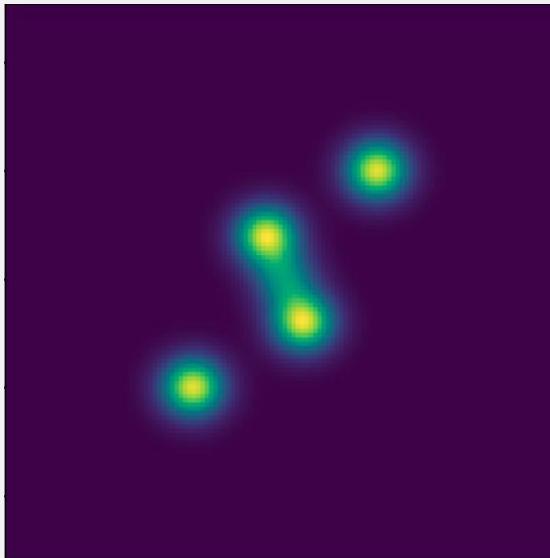
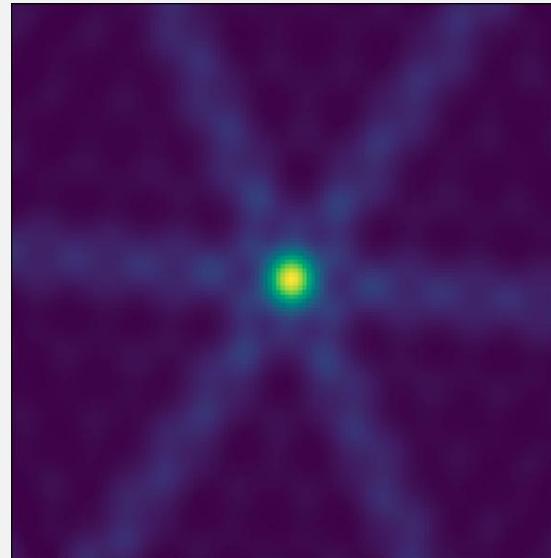
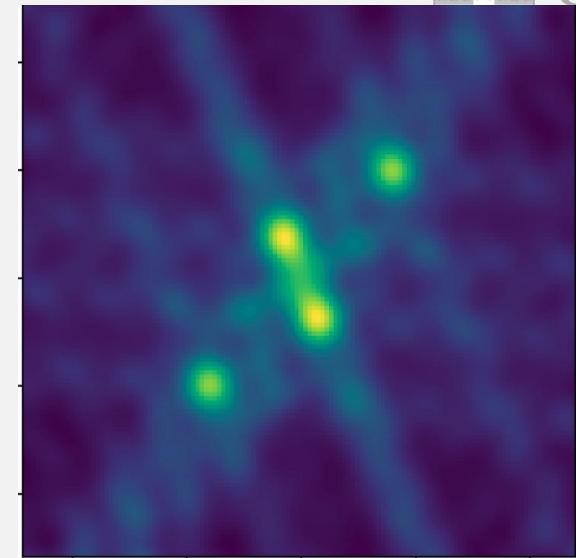
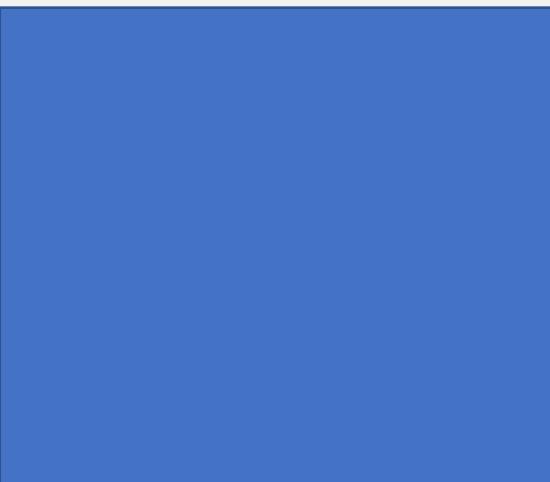
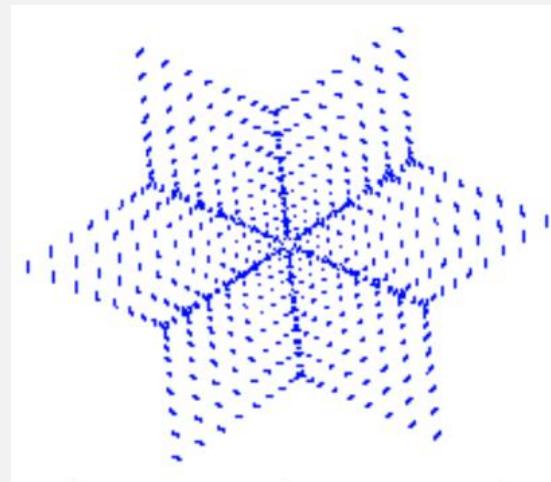
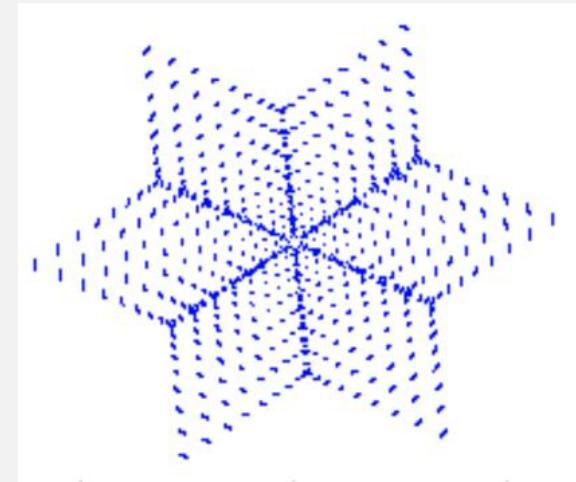
$$I_D(l,m) = I(l,m)^* B(l,m)$$

‘Dirty image’

Dirty beam



 $I(l,m)$  $B(l,m)$  $B(l,m)^*I(l,m)$  $V(u,v)$  $S(u,v)$  $S(u,v)V(u,v)$

 $I(l,m)$  $B(l,m)$  $B(l,m)^*I(l,m)$  $V(u,v)$  $S(u,v)$  $S(u,v)V(u,v)$



How to convert visibility data into imaging?

Well, it is not that ideal, but I can at least get an image





Coding Time

Given the sampling function
 $S(u,v,w)$,

Do image the dirty beam.

When the field of view is narrow, we have

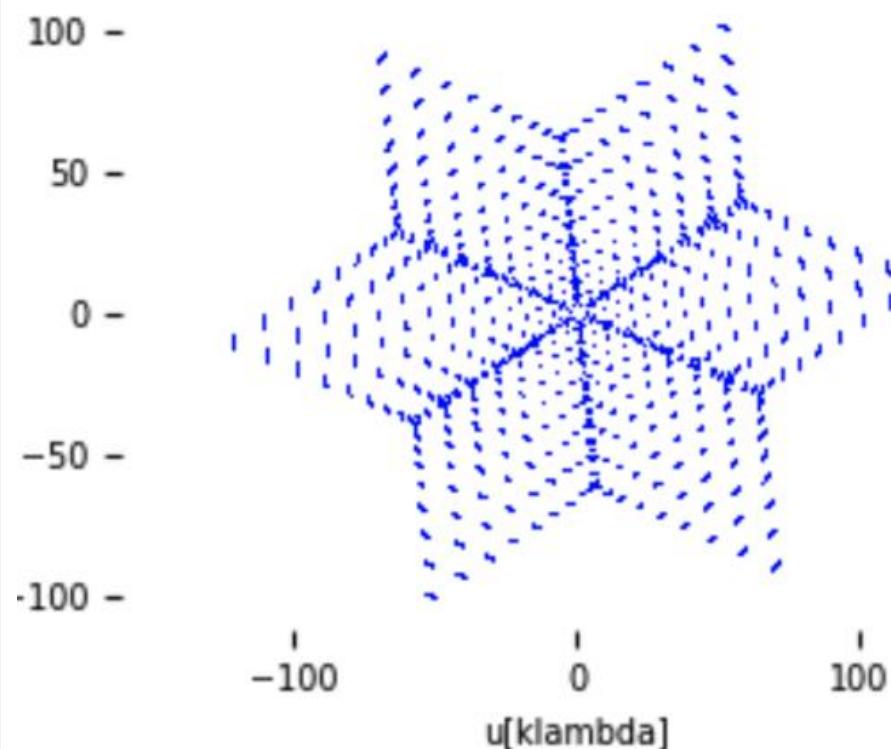
$$\text{Dirty image: } I_D(\mathbf{x}) = \Re \left[\sum_{k=1}^M w_k V_k \exp(i2\pi \mathbf{u}_k \cdot \mathbf{x}) \right]$$

$\mathbf{u}_k = (u_k, v_k)$ and $\mathbf{x} = (x, y)$.

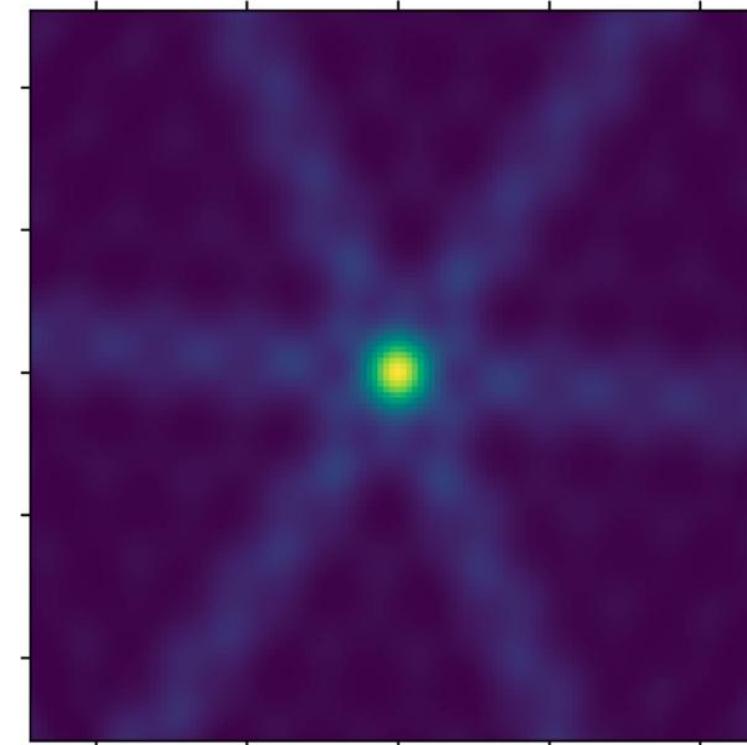
$$\text{Dirty beam: } B(\mathbf{x}) = \Re \left[\sum_{k=1}^M w_k \exp(i2\pi \mathbf{u}_k \cdot \mathbf{x}) \right]$$



Dirty Beam



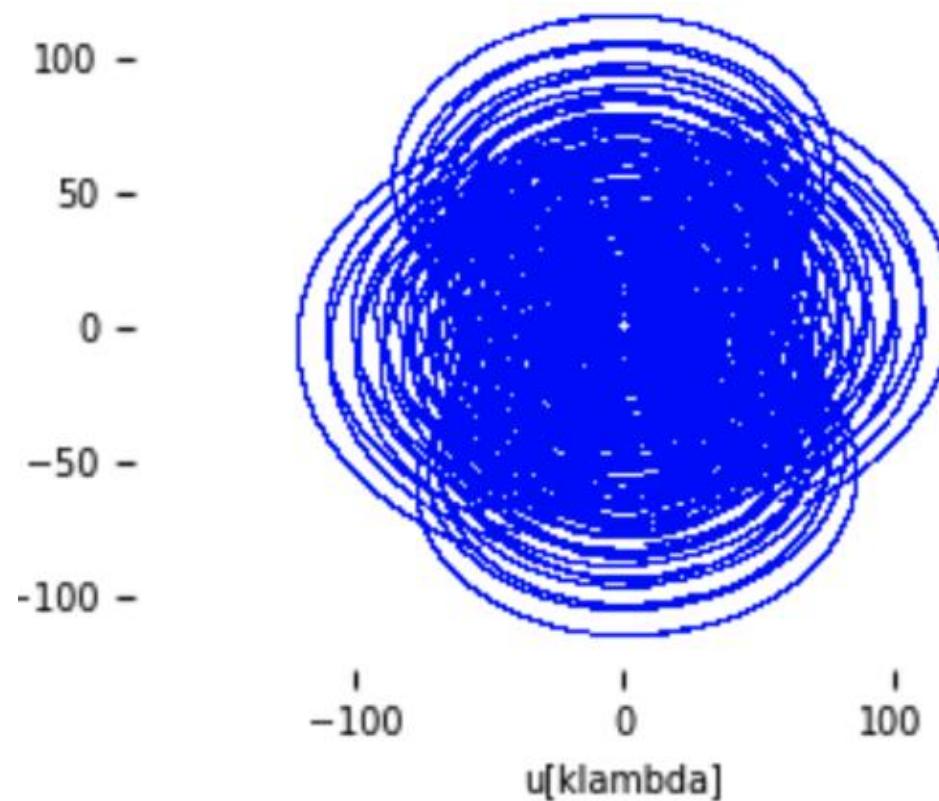
$$S(u, v)$$



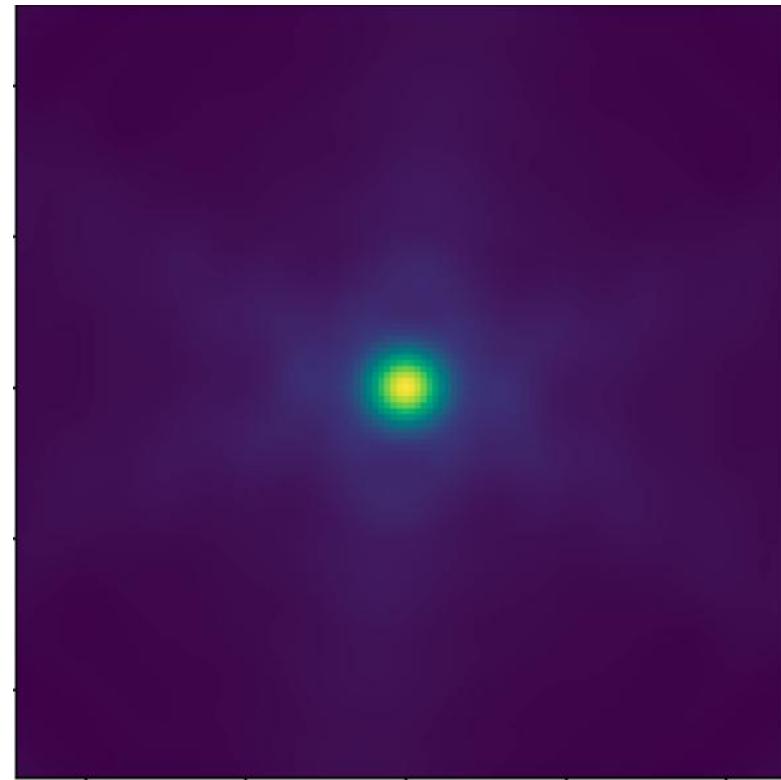
$$B(l, m)$$



Dirty Beam



$$S(u, v)$$

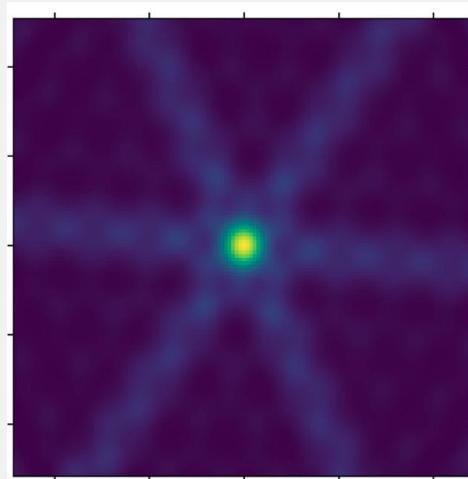


$$B(l, m)$$

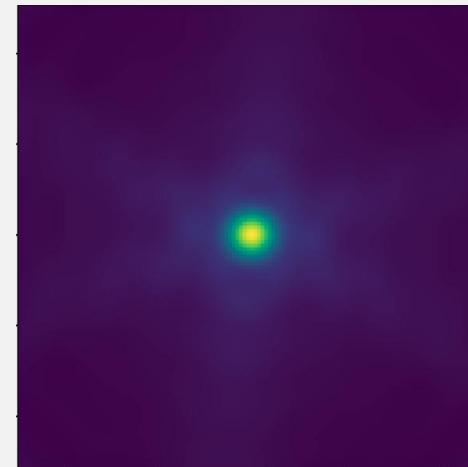


Dirty Beam

Which beam do you prefer? Why?



A



B





Dirty Beam

Therefore the layout of the antennas

$$S(u,v) = \sum_{k=1}^M \delta(u - u_k, v - v_k)$$

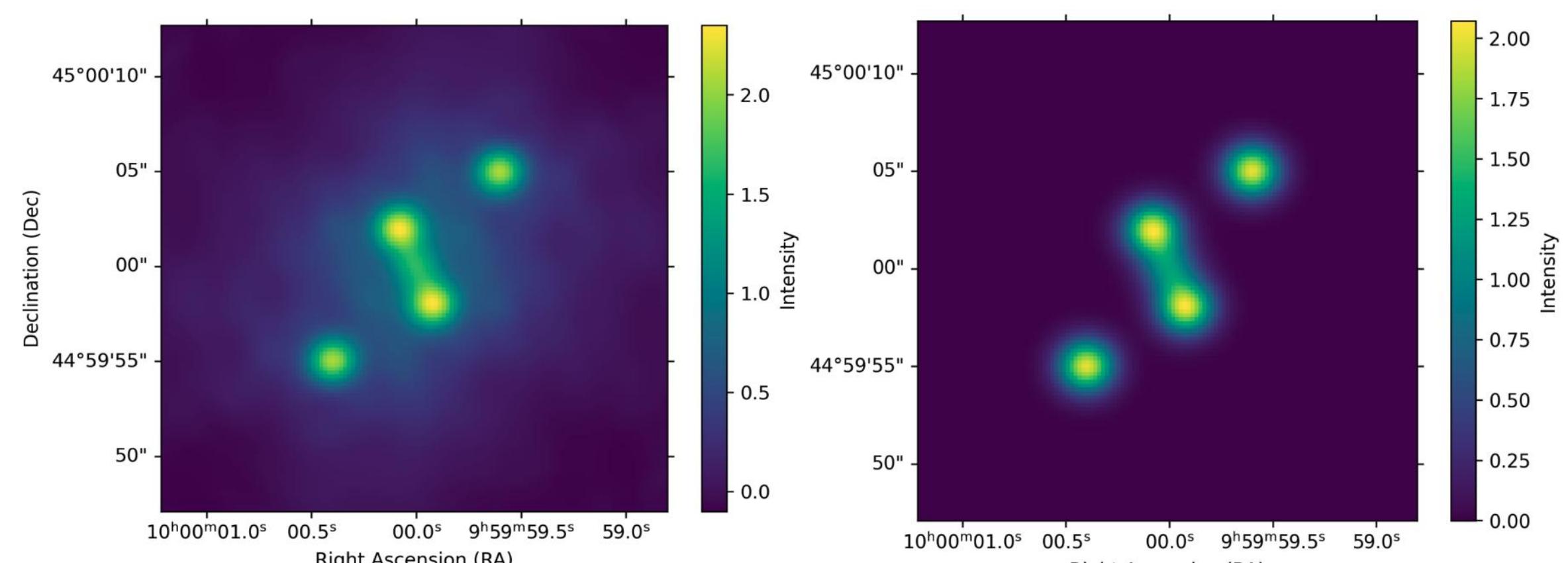
needs to be designed properly!



Dirty image

VS

Sky image

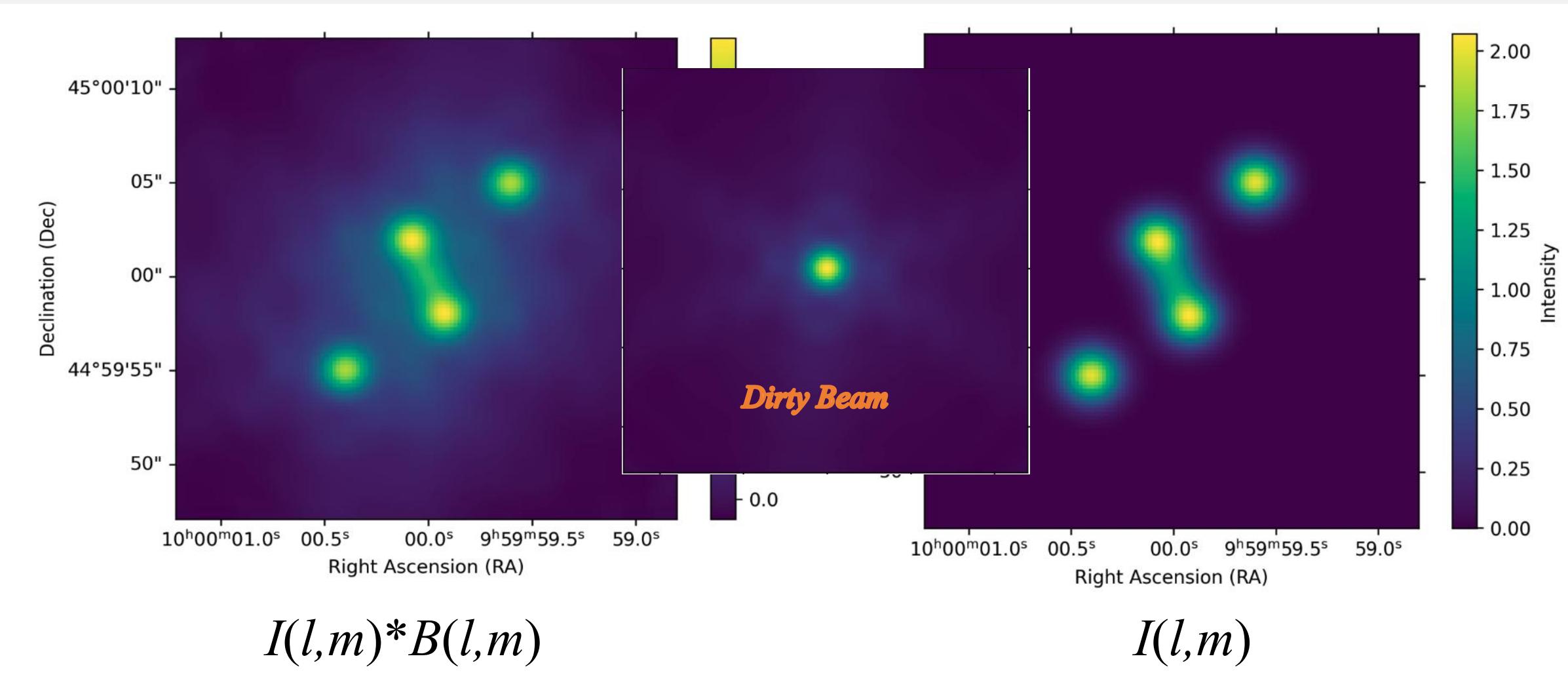


$$I(l,m)^*B(l,m)$$

$$I(l,m)$$



Dirty image VS Sky image

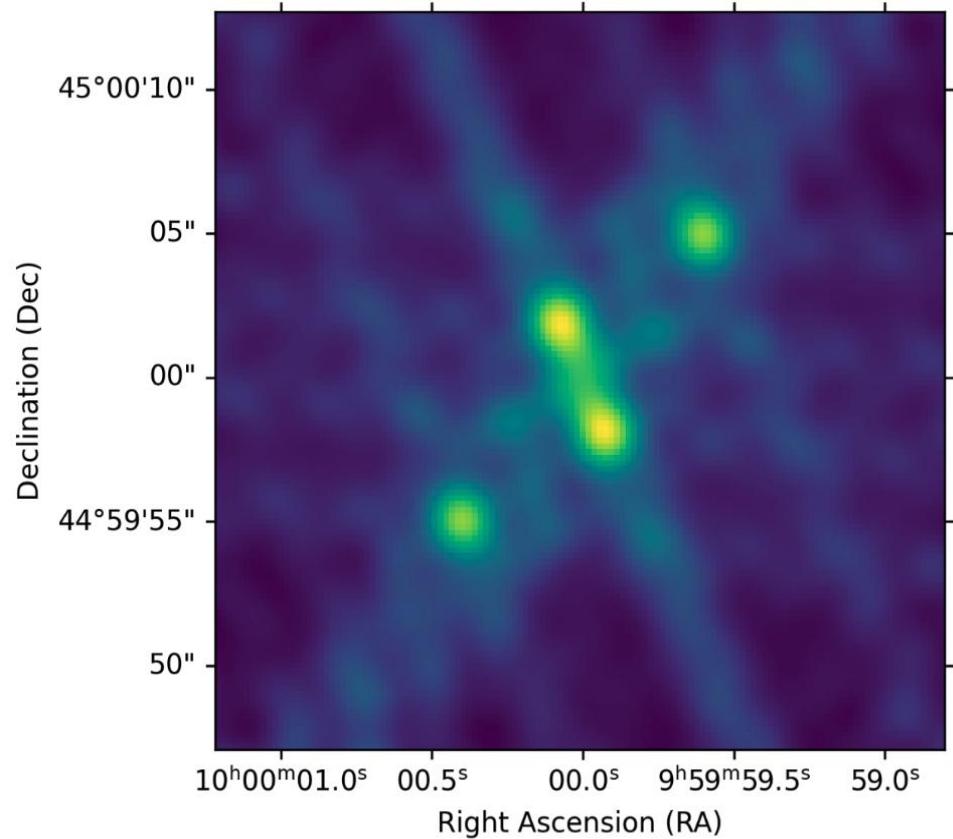




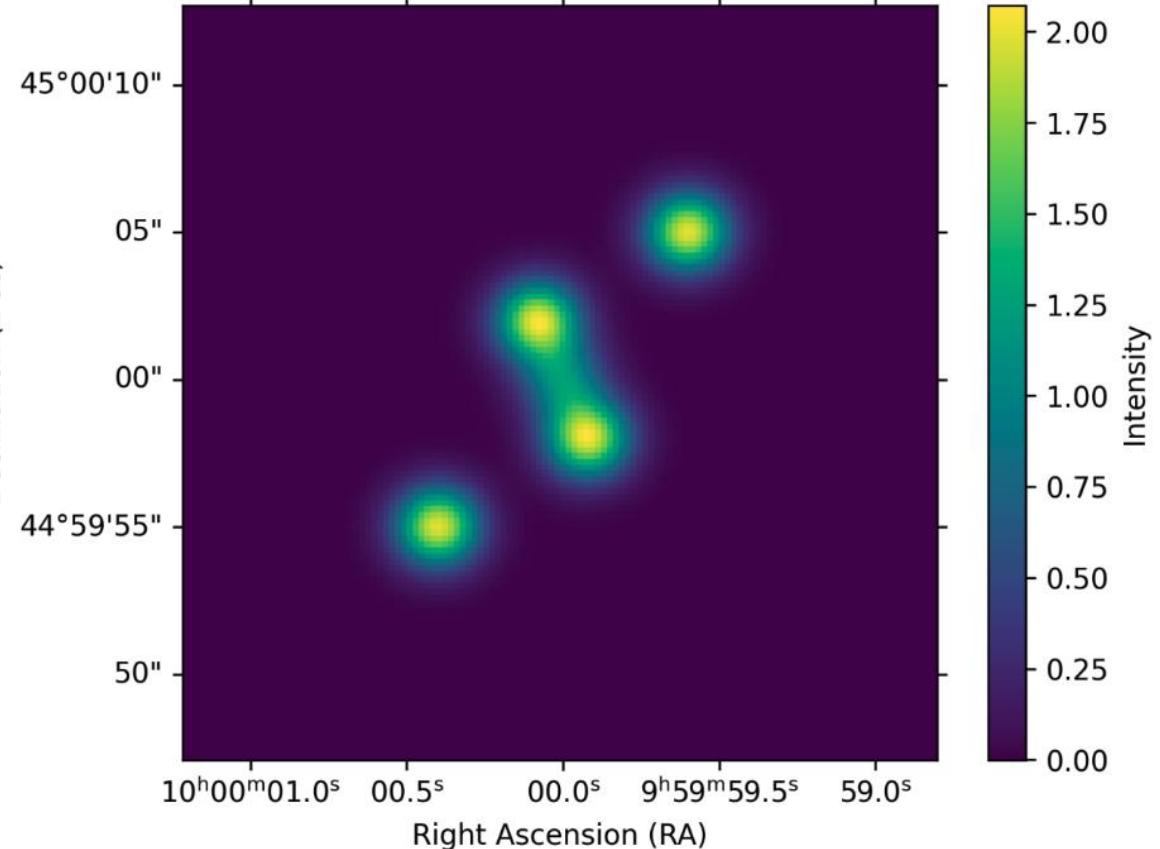
Dirty image

VS

Sky image



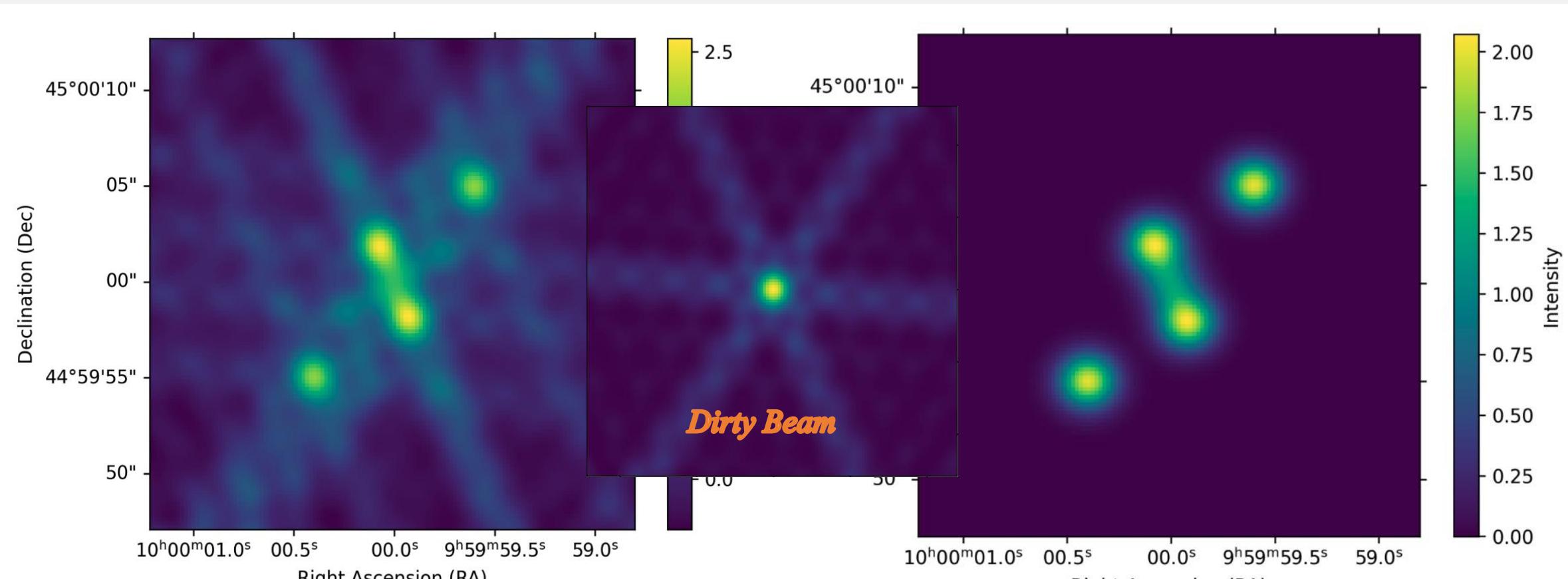
$$I(l,m)^*B(l,m)$$



$$I(l,m)$$



Dirty image VS Sky image

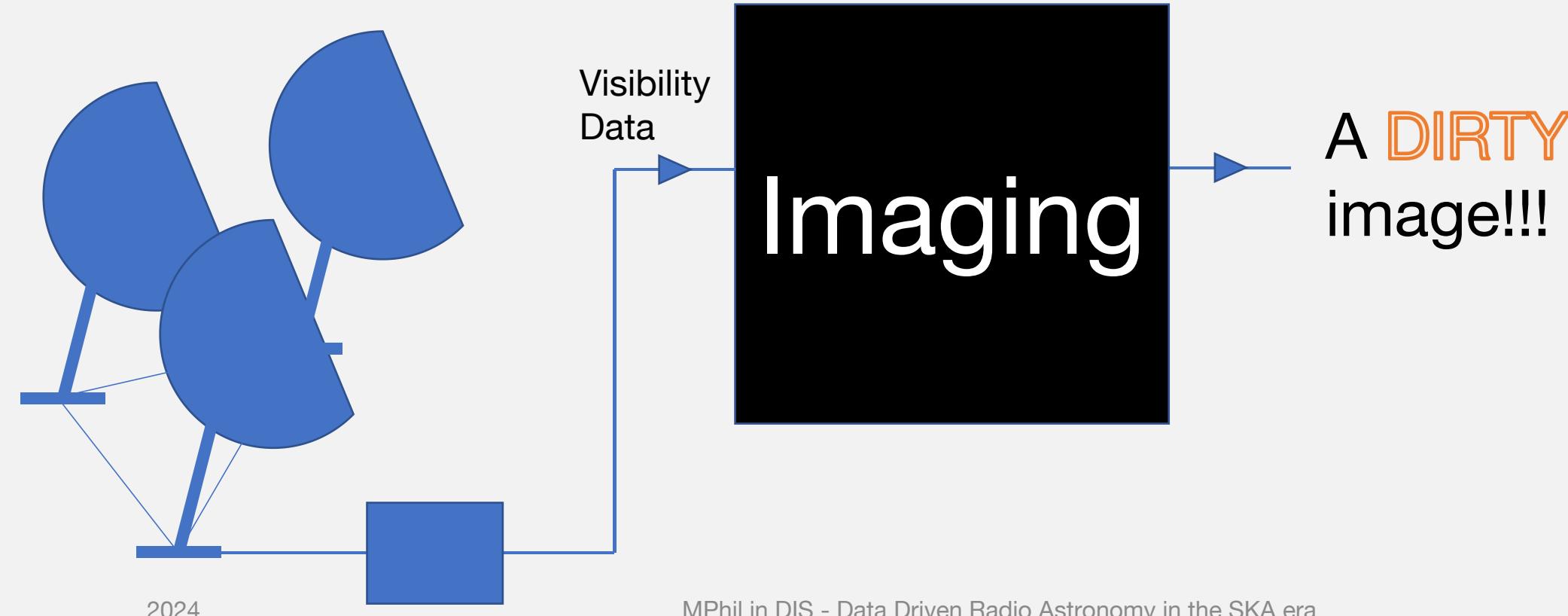


$$I(l,m)^*B(l,m)$$

$$I(l,m)$$

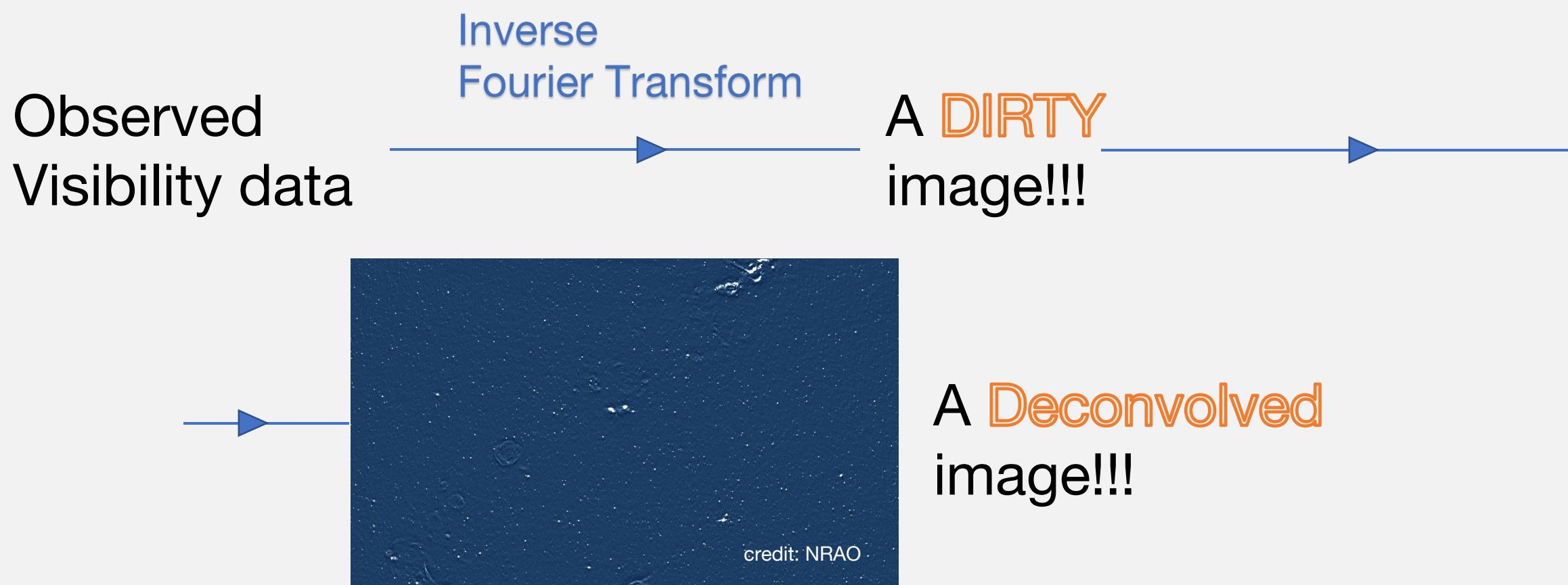


Today you have learned:





Deconvolution





Högbom CLEAN

Jan Arvid Högbom (born 3 October 1929) is a Swedish radio astronomer and astrophysicist. Högbom obtained his PhD in 1959 from the University of Cambridge with Martin Ryle.

1. Initialisation: Let $D(x, y)$ represent the dirty image, where (x, y) are the spatial coordinates, and set the residual image $R(x, y)$ equal to the dirty image: $R(x, y) = D(x, y)$.
2. Identify Peaks: Search for the location (x_p, y_p) of the brightest pixel (peak) in the residual image
3. Extract Component: Subtract a (scaled) Gaussian component $G(x, y)$ from the residual image at the location of the identified peak: $R(x, y) \leftarrow R(x, y) - aG(x_p, y_p)$, a is the scaled peak intensity at (x_p, y_p)
4. Update Clean Component List: Add the extracted component to the clean component list
5. Iterate: Repeat steps 2-4 iteratively until one or more termination criteria are met.
6. Final Image Reconstruction: $I(x, y)$ is reconstructed by combining the clean component list with the residual image



Högbom CLEAN

Jan Arvid Högbom (born 3 October 1929) is a Swedish radio astronomer and astrophysicist. Högbom obtained his PhD in 1959 from the University of Cambridge with Martin Ryle.

$$\begin{aligned} I_D(x,y) &= I(x,y)^* B(x,y) \\ &= [I_1(x,y) + I_2(x,y) + \dots + I_n(x,y)]^* B(x,y) \end{aligned}$$

$$= \sum_{i=1}^M I_i(x,y)^* B(x,y)$$

$$R(x,y) \equiv I_D(x,y) - \alpha I_{CLEAN}(x,y)^* B(x,y)$$

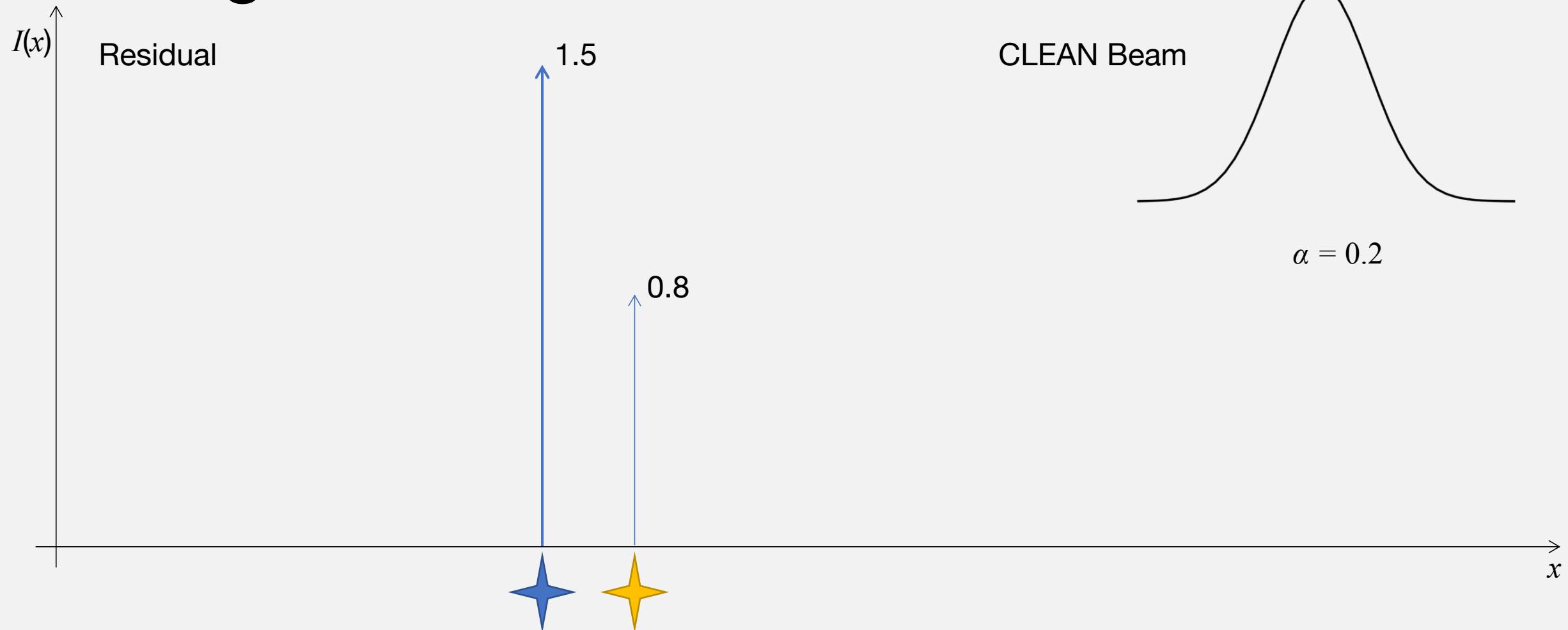
$$I_{reconstructed}(x,y) \equiv R(x,y) + I_{CLEAN}(x,y)$$

We usually don't use dirty beam $B(x,y)$, but use a Gaussian for simplicity, which is called "CLEAN BEAM"

α is a scaling factor, so it takes several loops to subtract the highest intensity - do you know why?

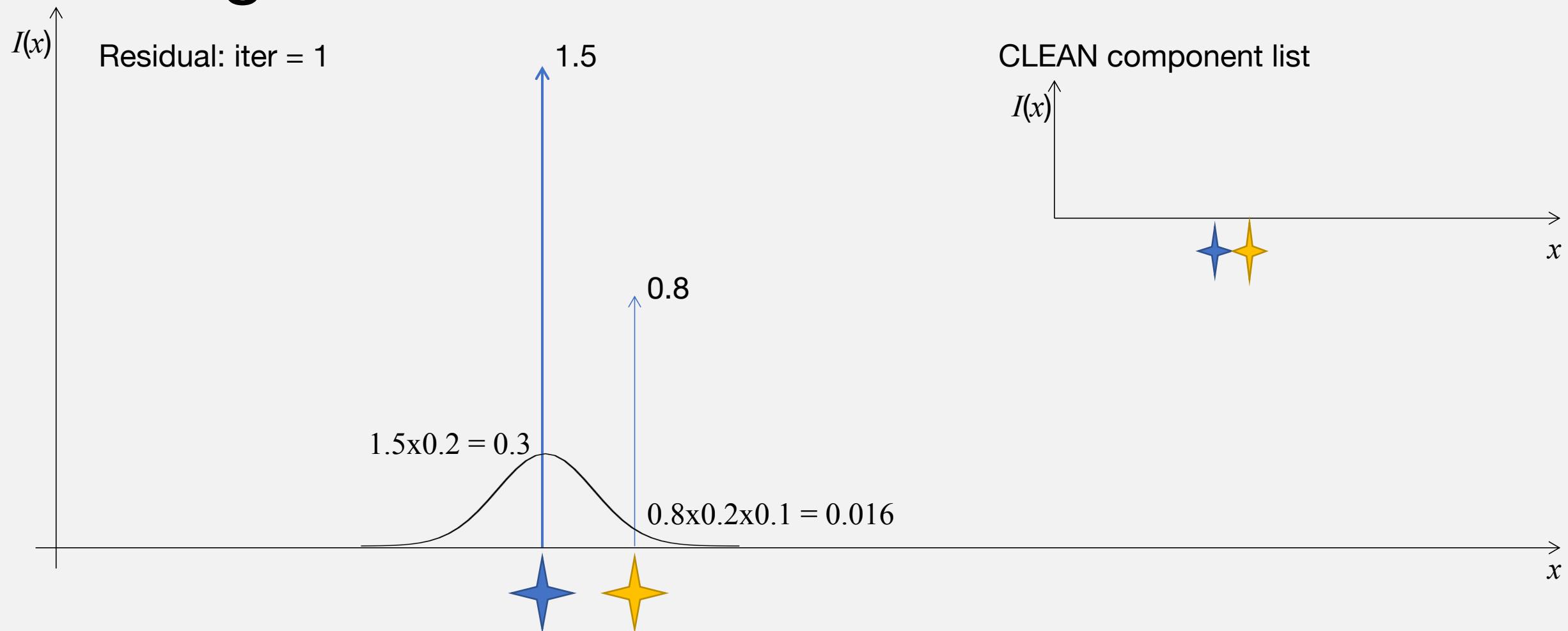


Högbom CLEAN - 1D Demo



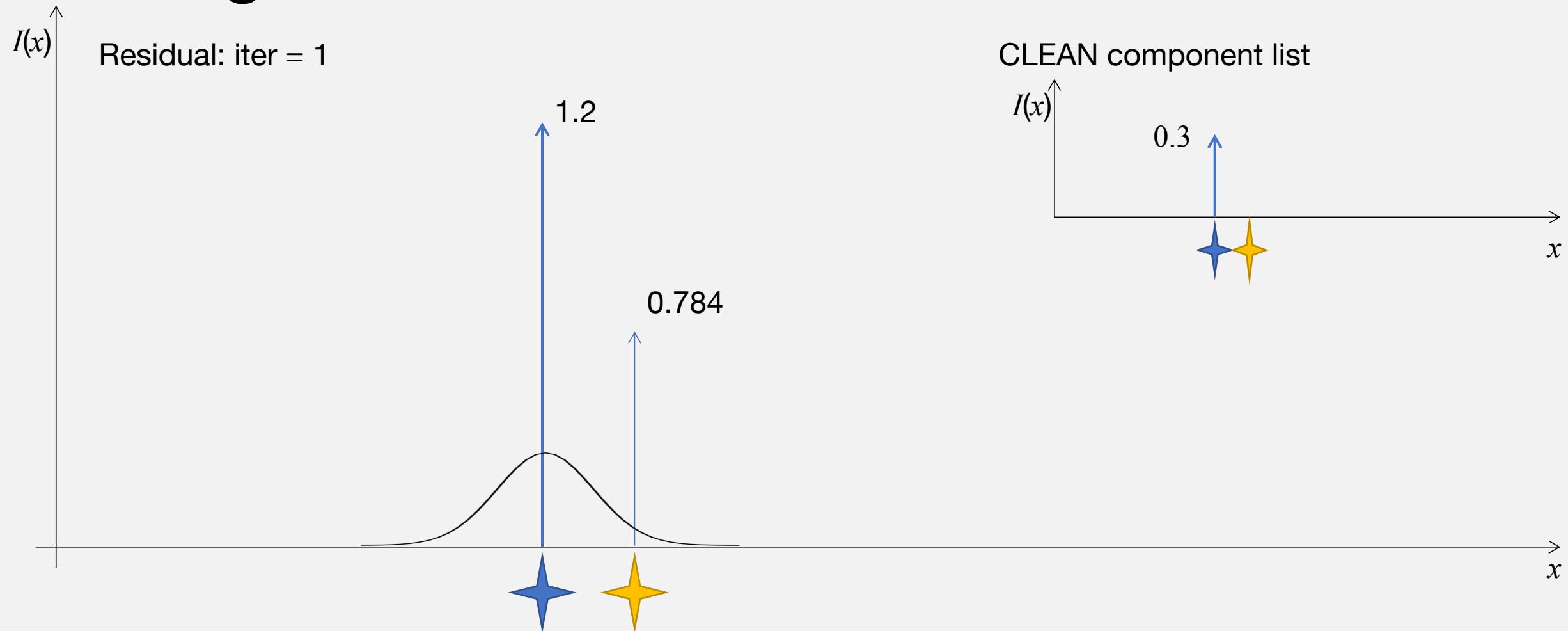


Högbom CLEAN - 1D Demo



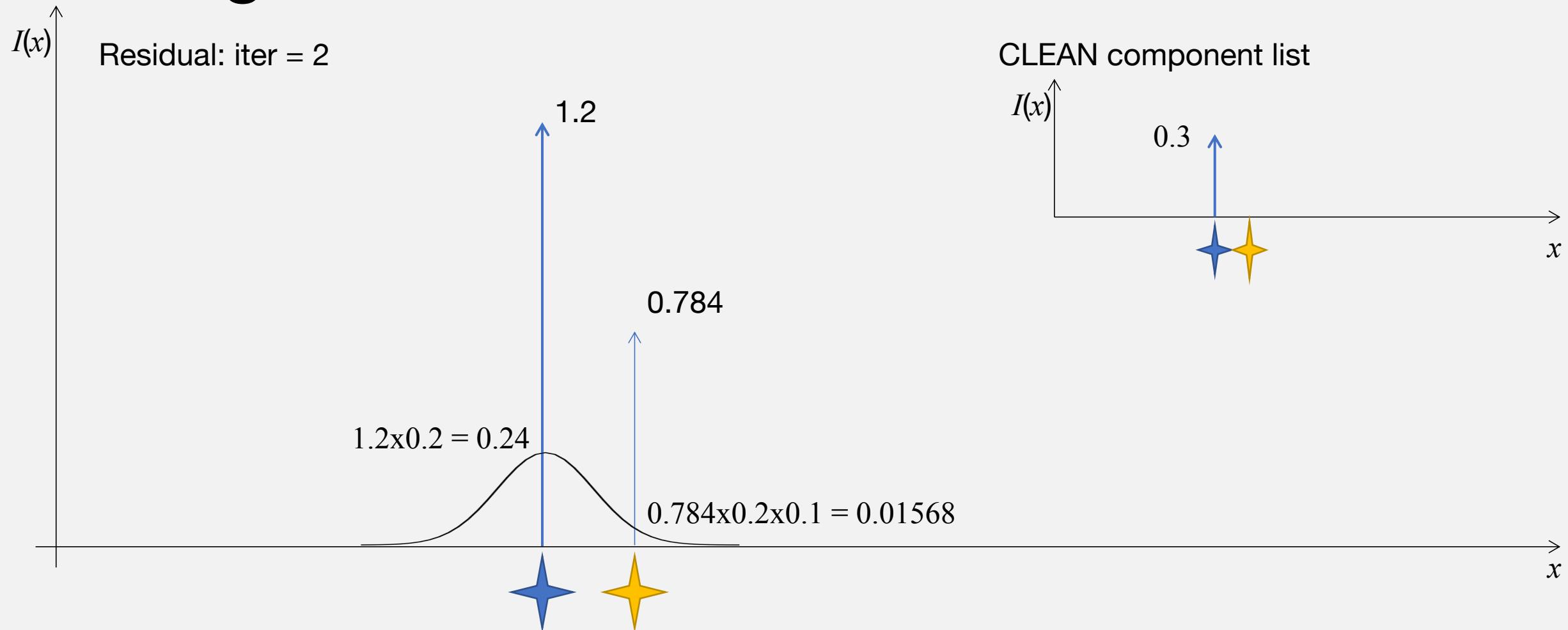


Högbom CLEAN - 1D Demo



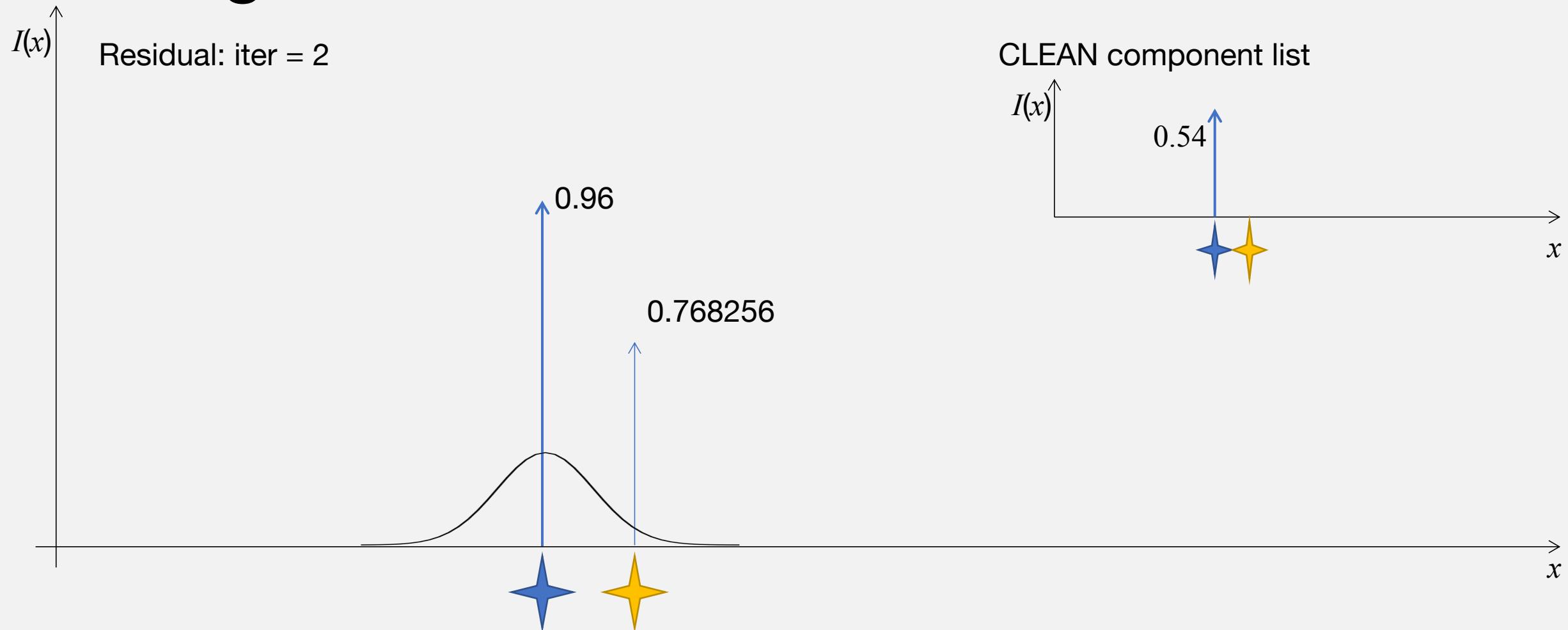


Högbom CLEAN - 1D Demo



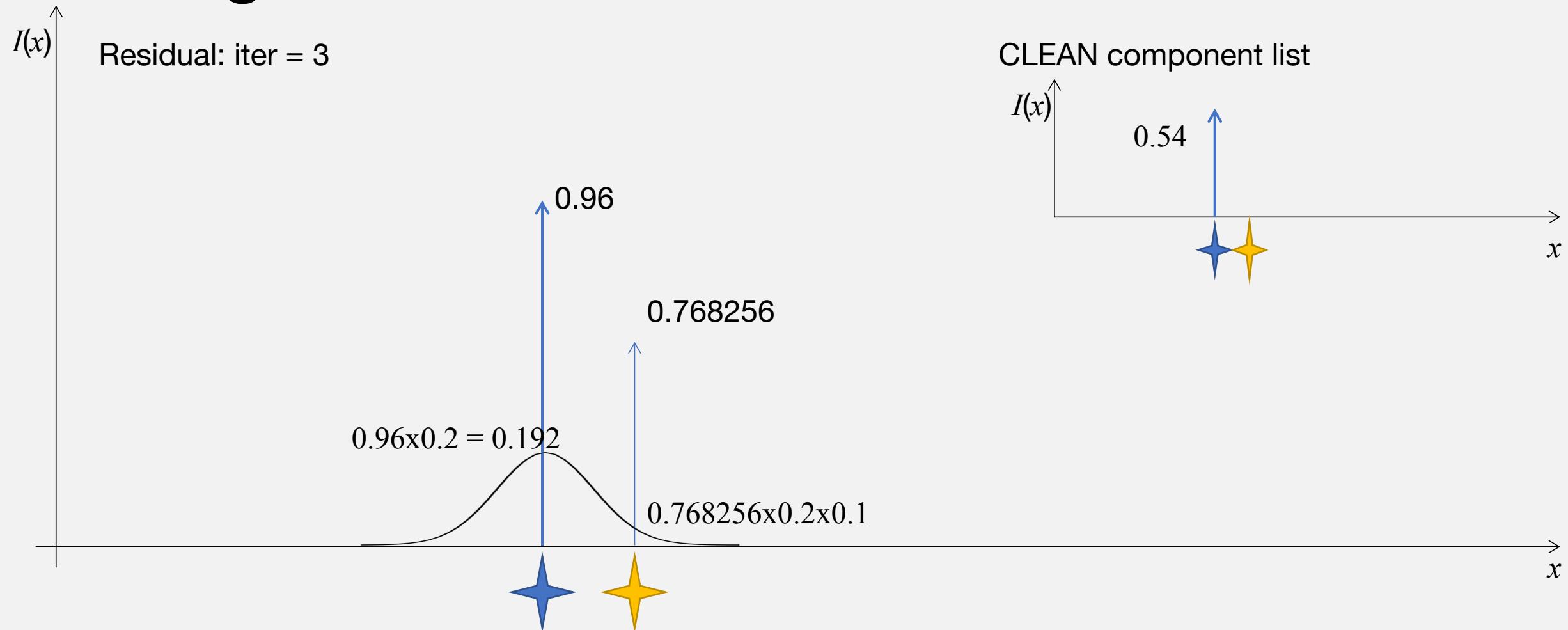


Högbom CLEAN - 1D Demo



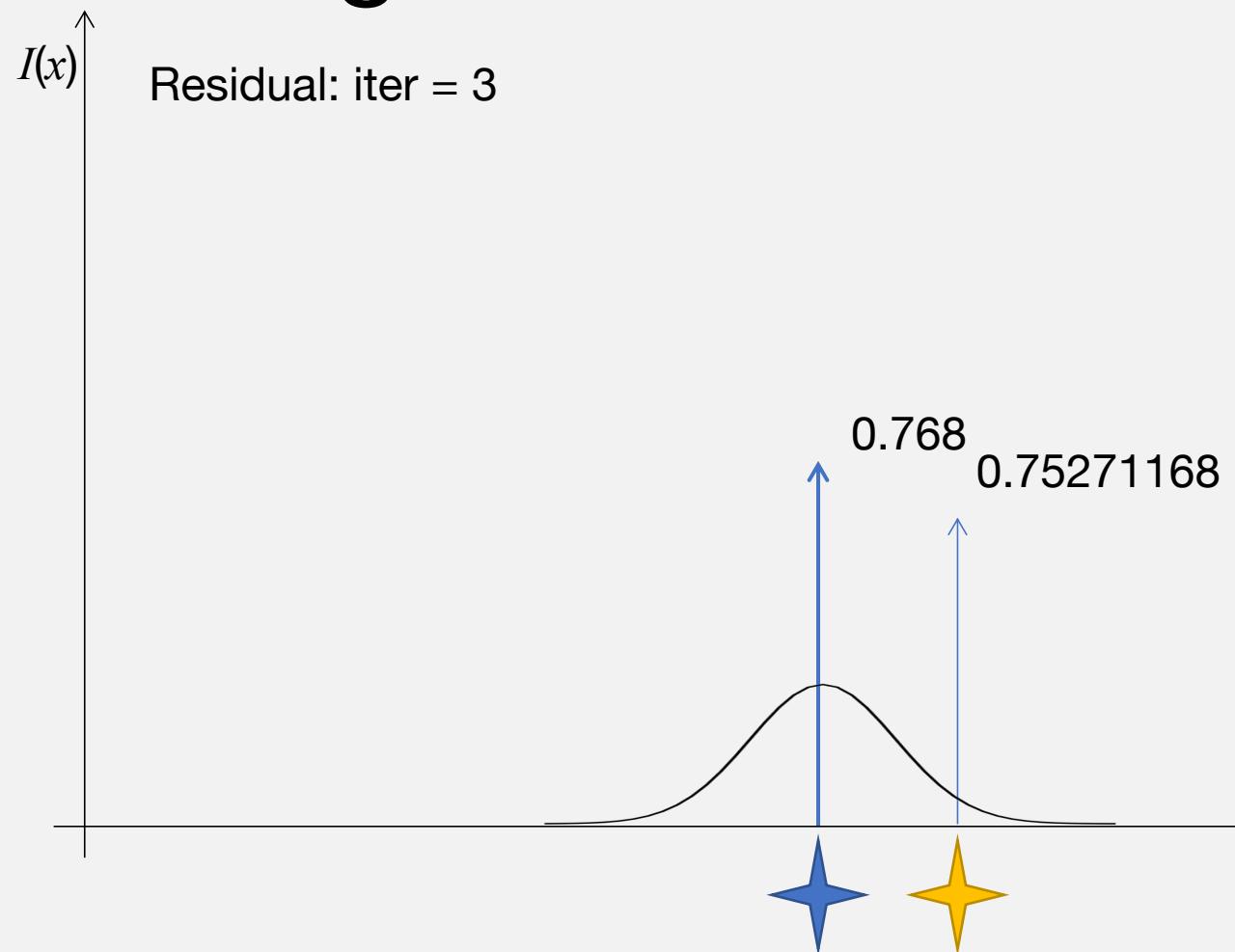


Högbom CLEAN - 1D Demo

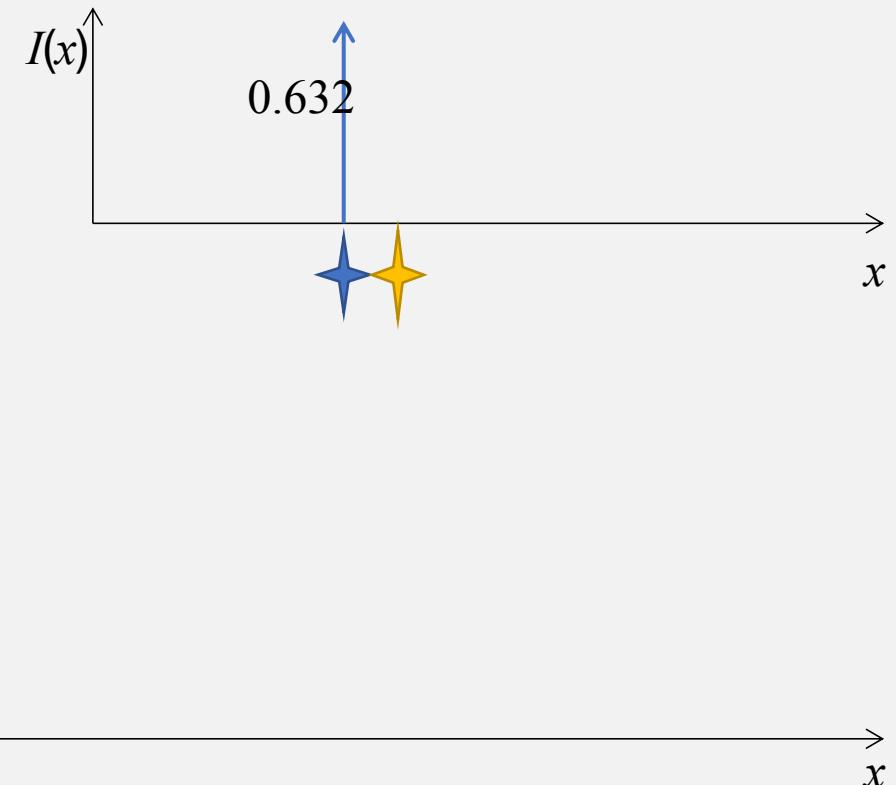




Högbom CLEAN - 1D Demo

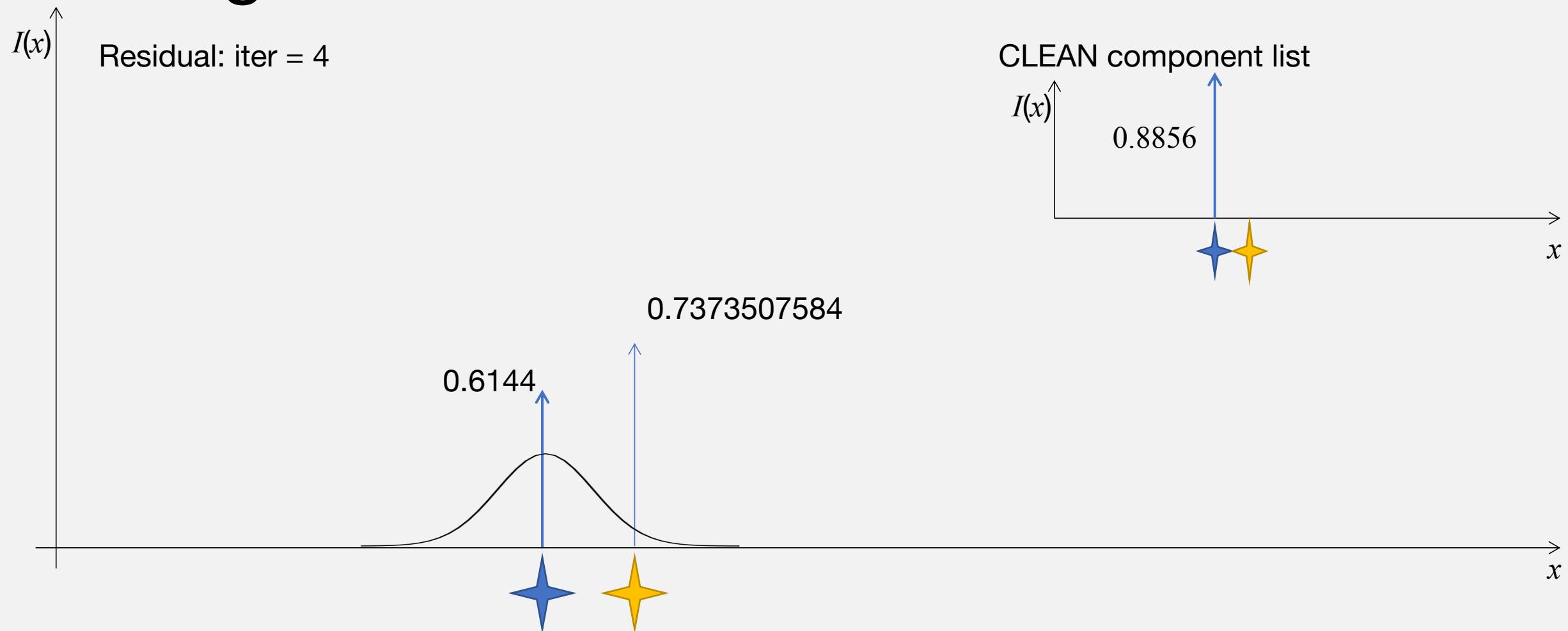


CLEAN component list



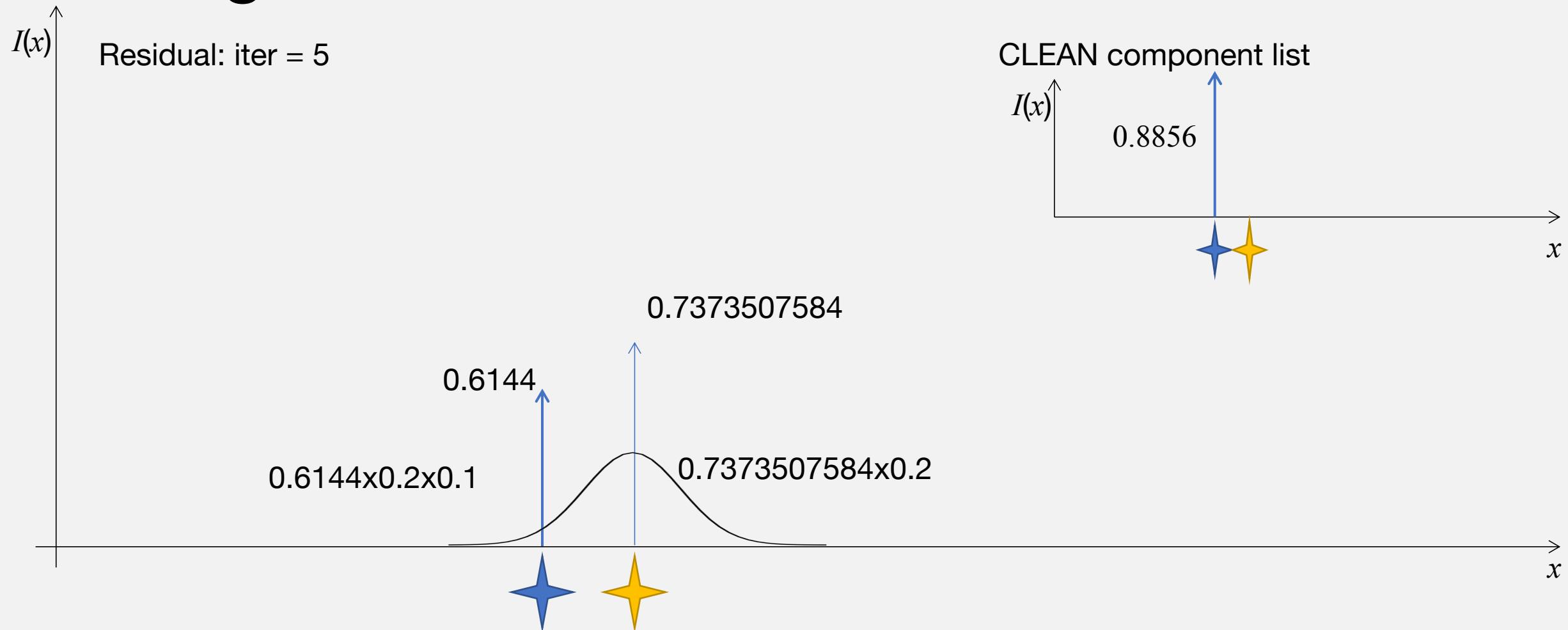


Högbom CLEAN - 1D Demo



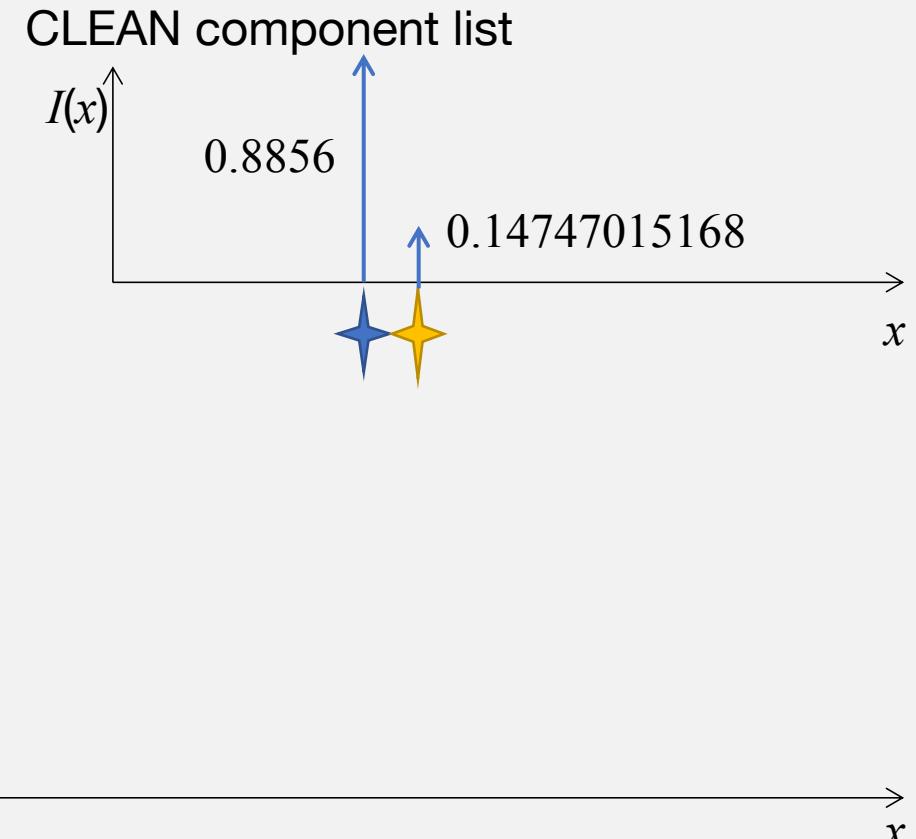
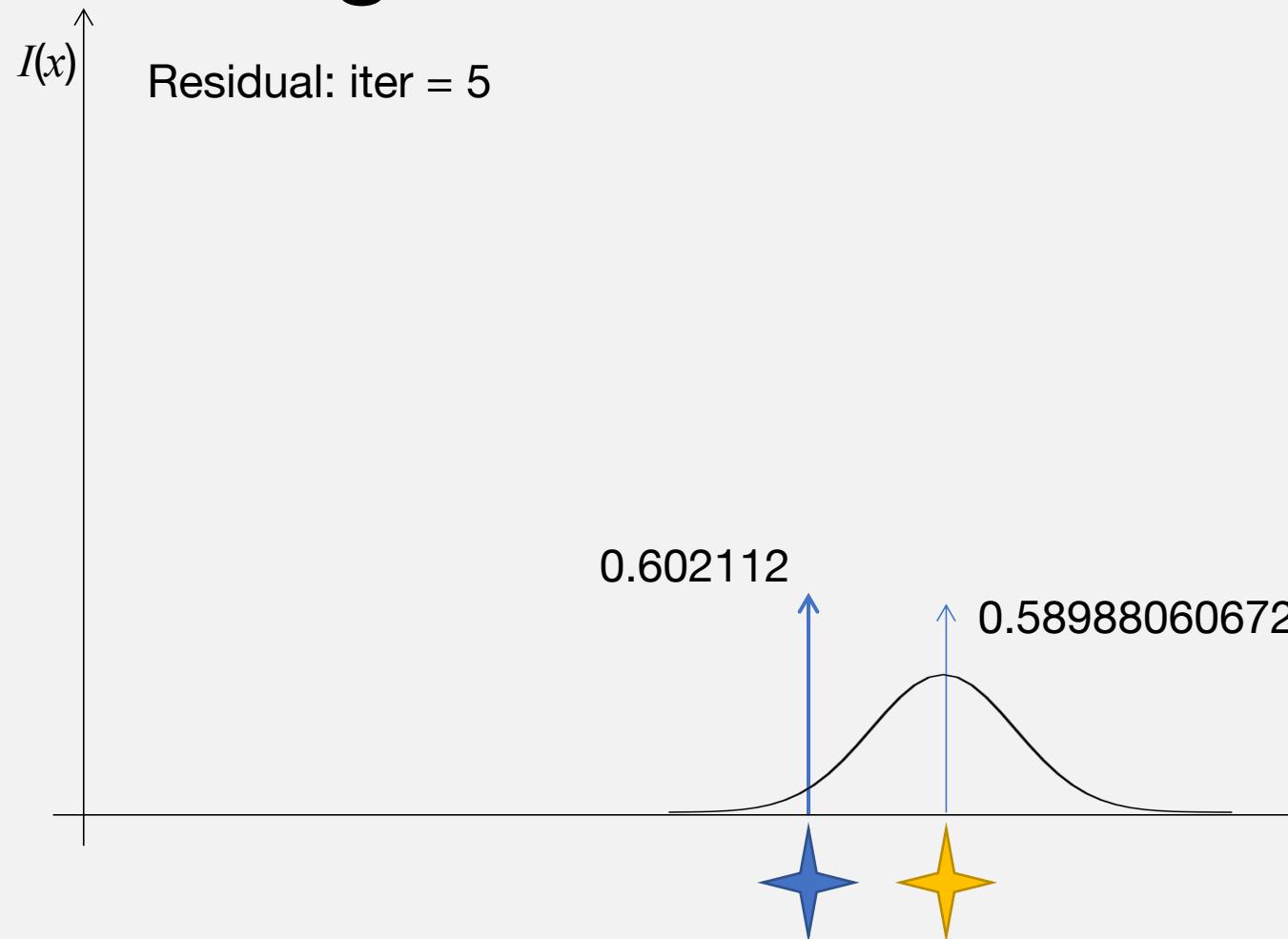


Högbom CLEAN - 1D Demo





Högbom CLEAN - 1D Demo

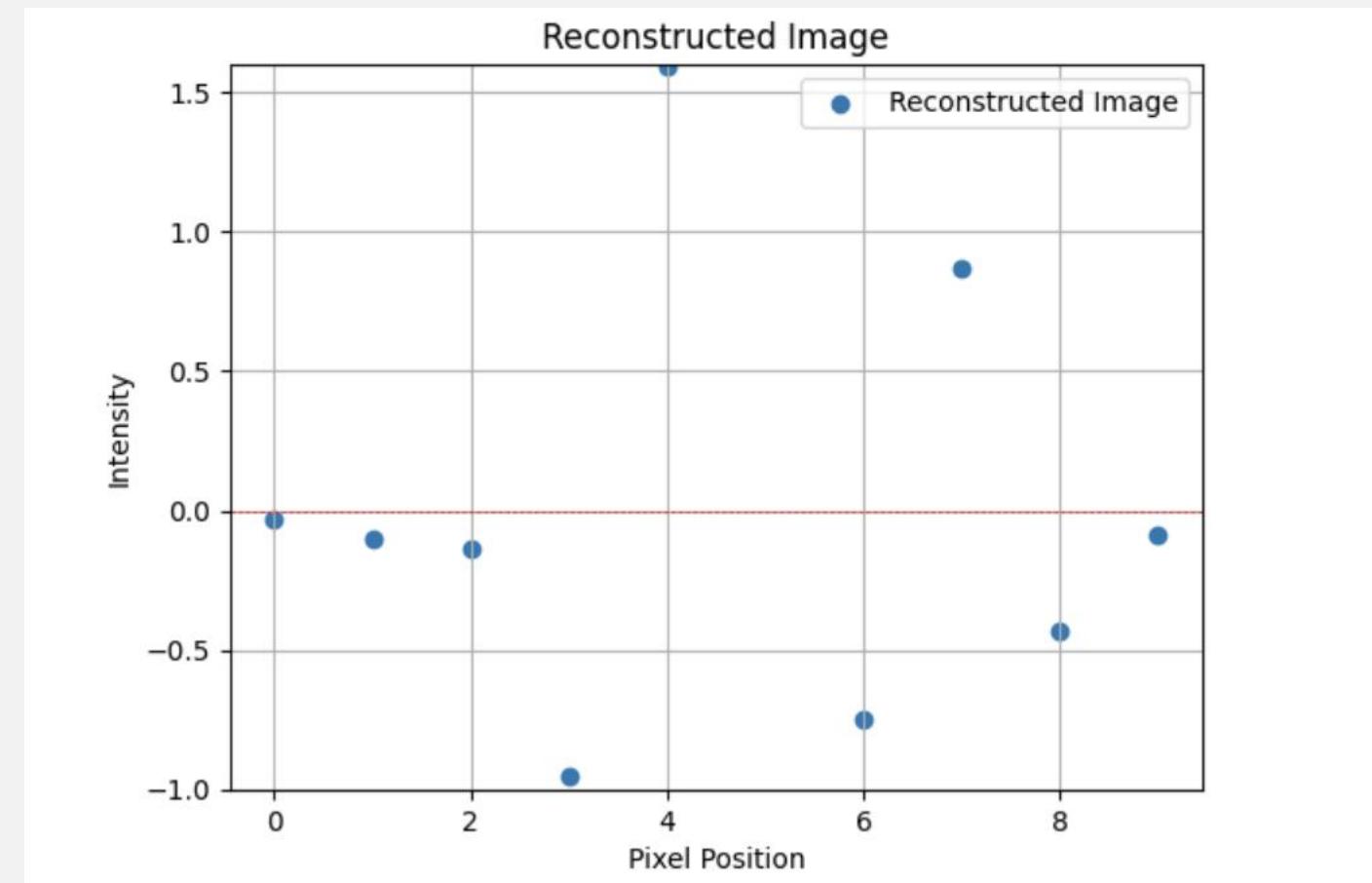
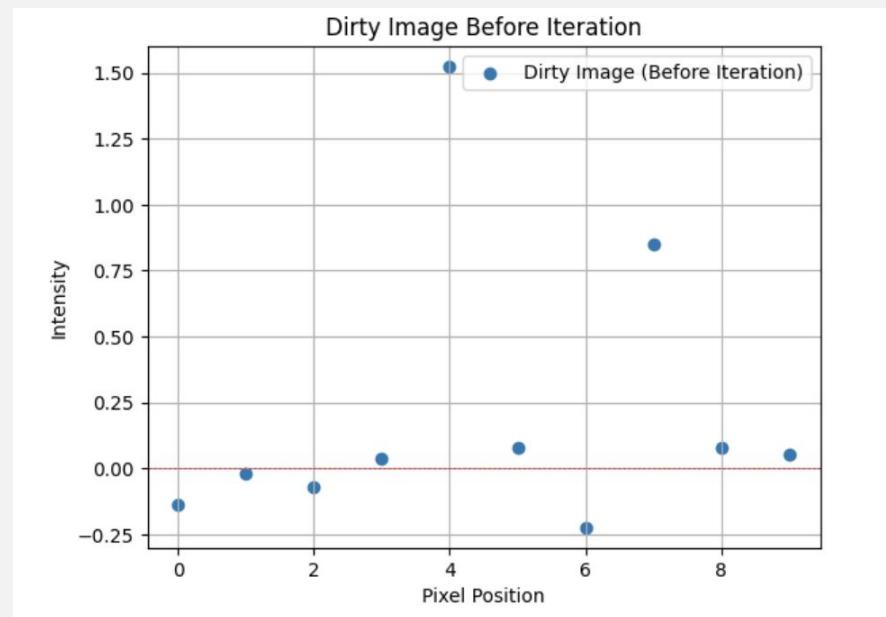




Coding Time

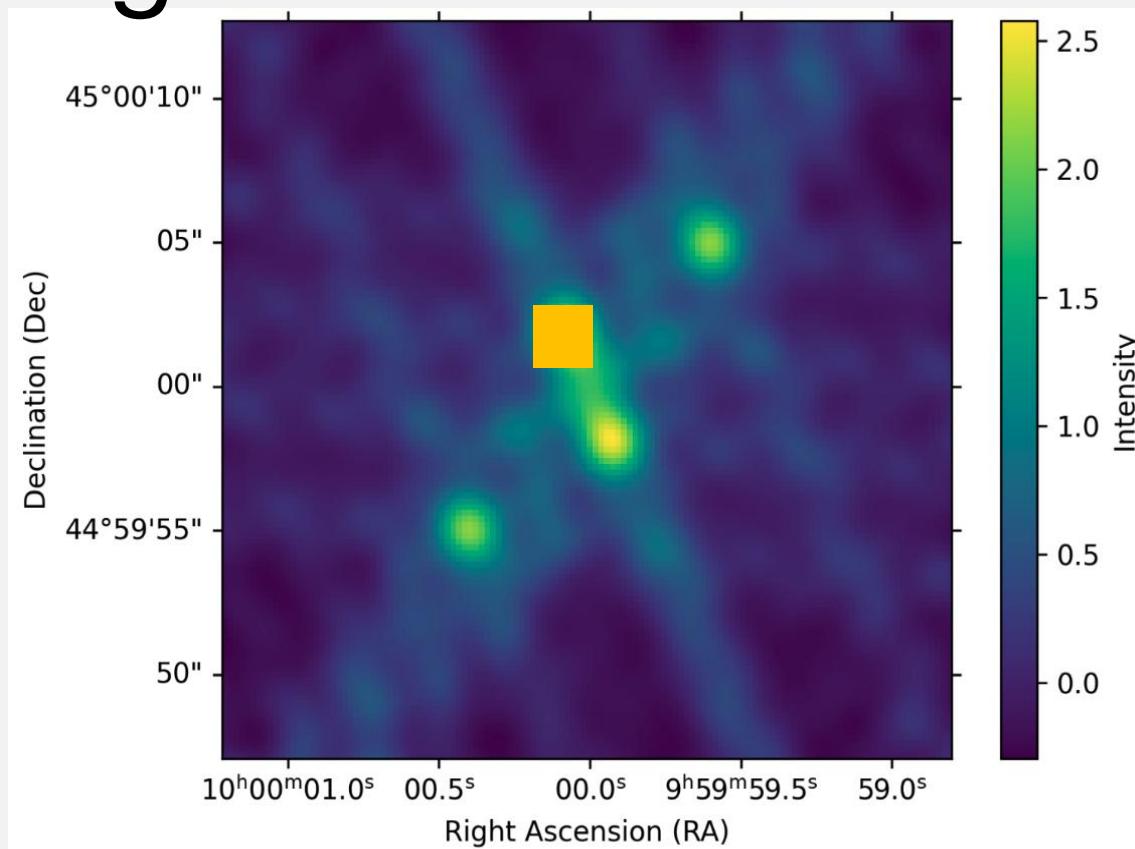
Given the 1-D Hogbom CLEAN algorithm example

Do complete the codes





Högbom CLEAN



CLEAN Component list

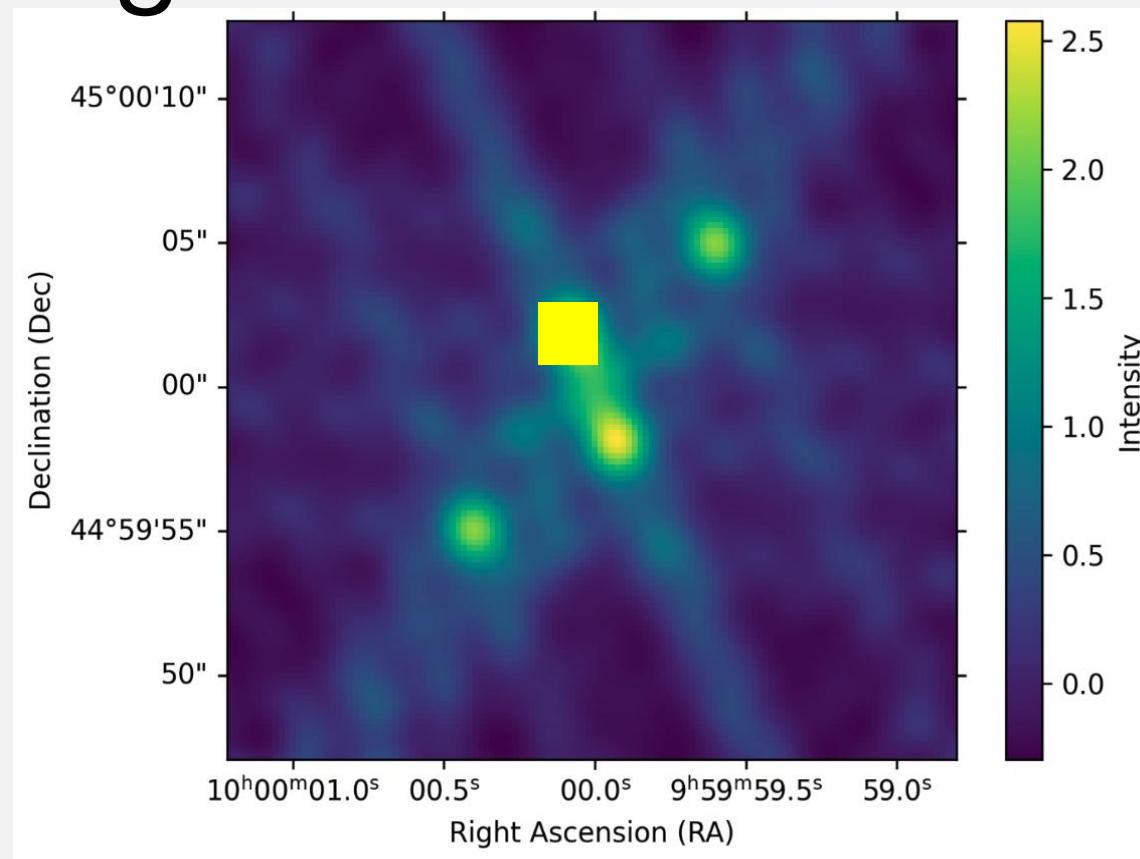


2. Identify Peak

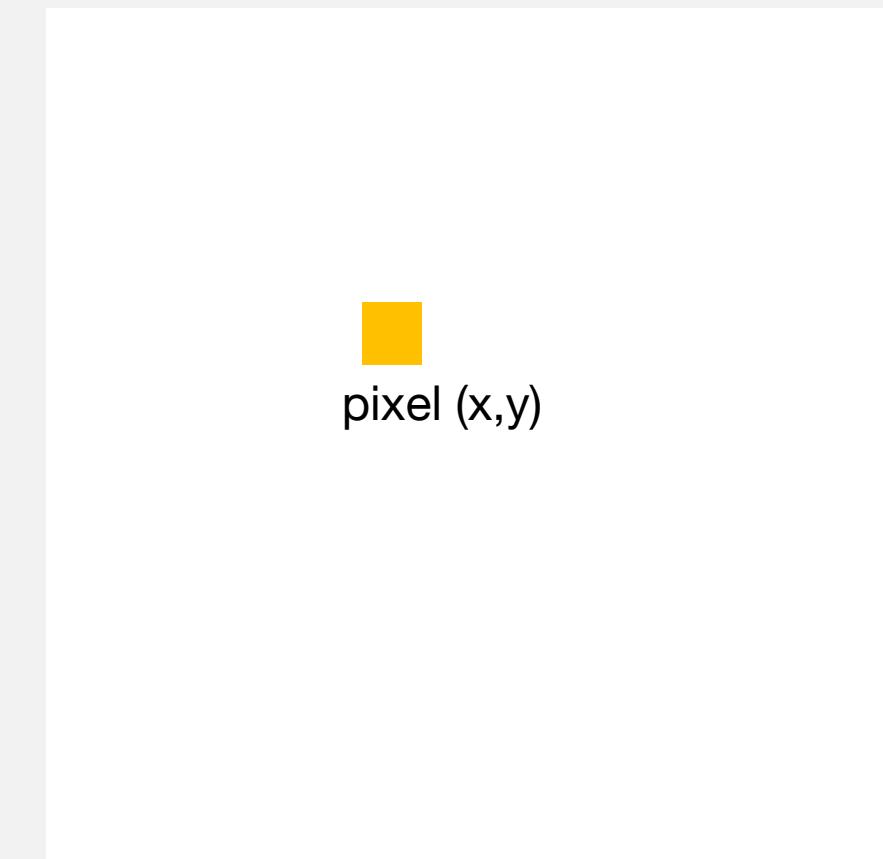
Find the brightest pixel !!!!



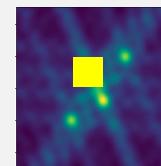
Högbom CLEAN



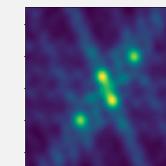
CLEAN Component list



3. Subtract Model



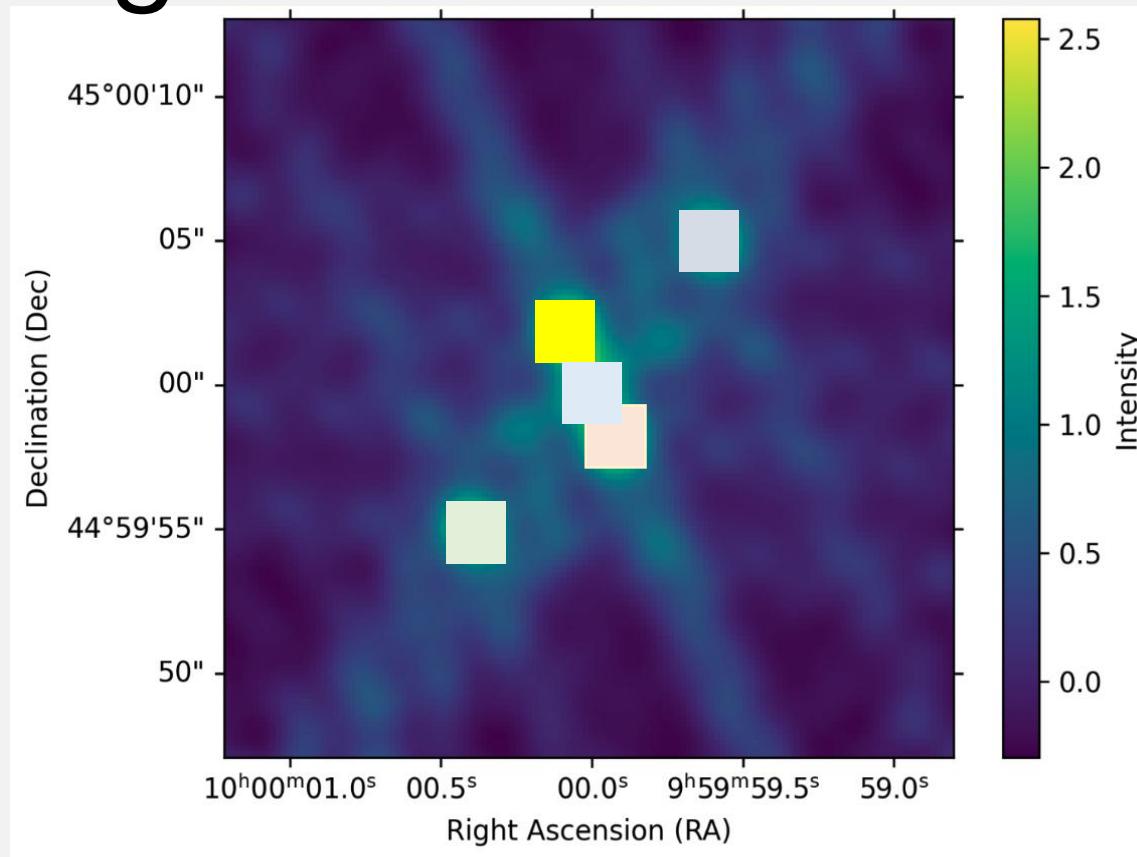
=



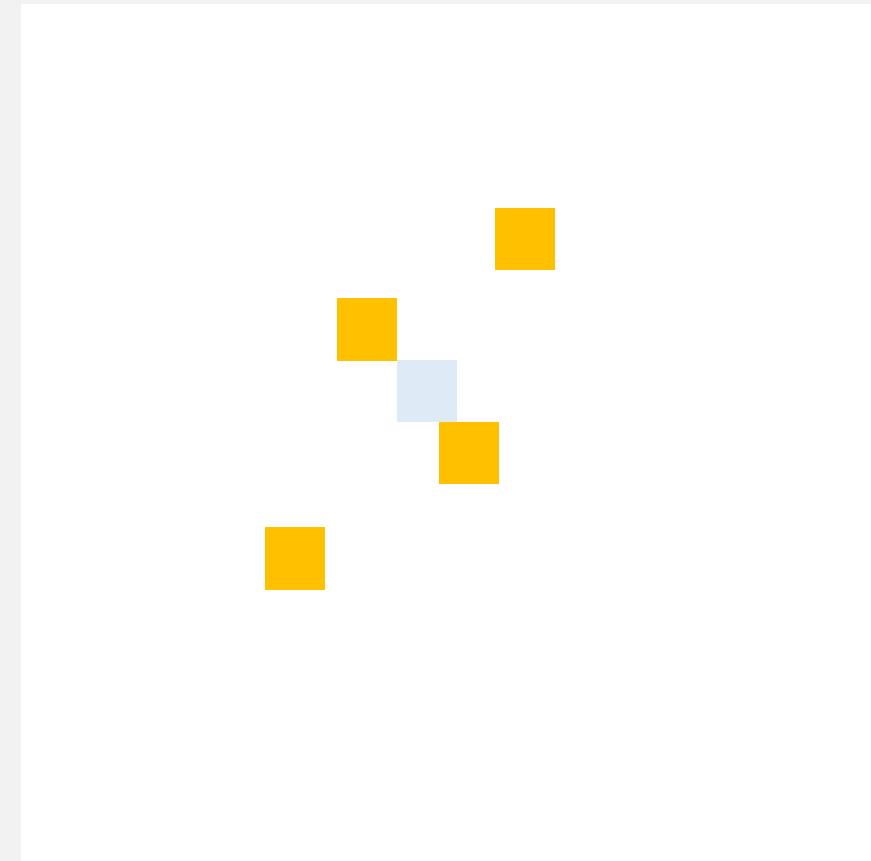
- Gaussian \times scaling factor \times intensity at pixel (x,y)



Högbom CLEAN



CLEAN Component list



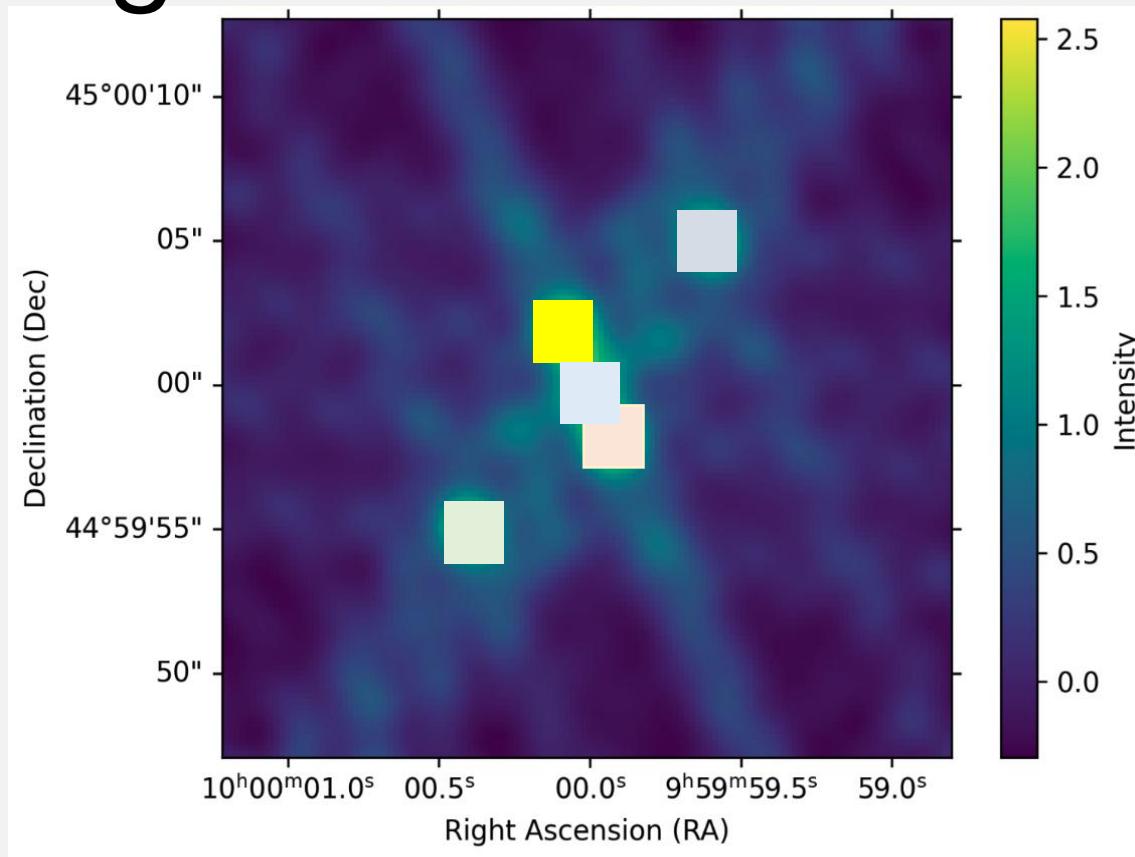
Repeat 2 and 3



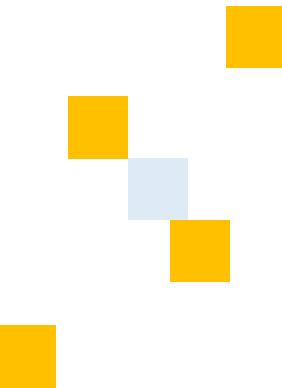
Residual image



Högbom CLEAN



CLEAN Component list

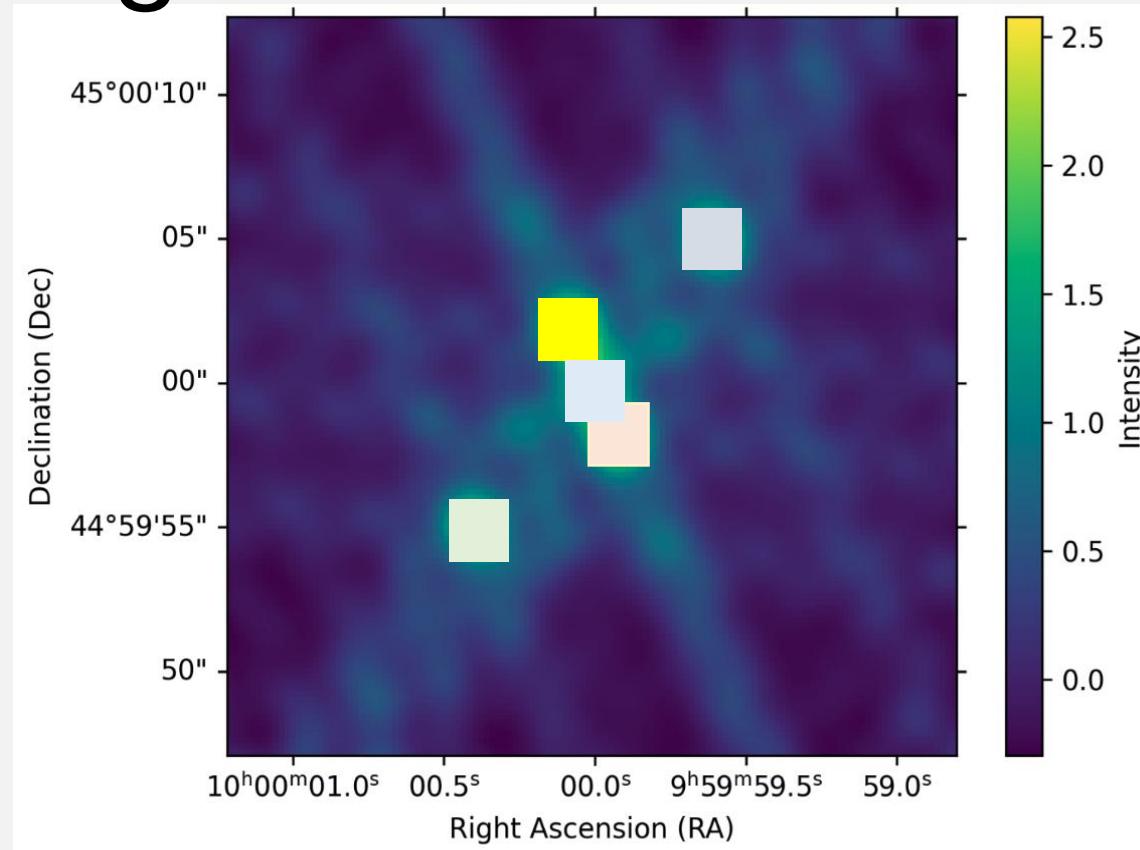


Iteration STOPS when

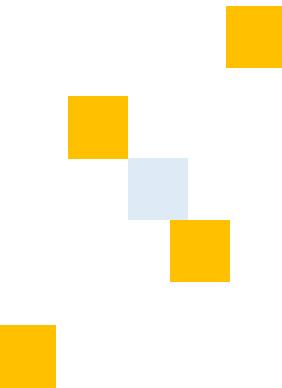
peak (Residual image) < given value or
≈
iteration cycle > given cycle number



Högbom CLEAN



CLEAN Component list



Final image =

Residual image + CLEAN component list



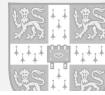
Coding Time

If my miniproject got selected, you will be able to try to write a full CLEAN algorithm in python :D



Reading

- Fourier Synthesis Imaging by Prof. Dale E. Gary:
<https://web.njit.edu/~gary/728/Lecture6.html>
- Synthesis Imaging in Radio Astronomy: A Collection of Lectures from the Third NRAO Synthesis Imaging Summer School:
https://www.aspbooks.org/a/volumes/table_of_contents/?book_id=118
- Imaging and Deconvolution by David J. Wilner:
https://science.nrao.edu/science/meetings/2018/16th-synthesis-imaging-workshop/talks/Wilner_Imaging.pdf



UV-coverage

<https://www.aspbooks.org/publications/6/83.pdf>

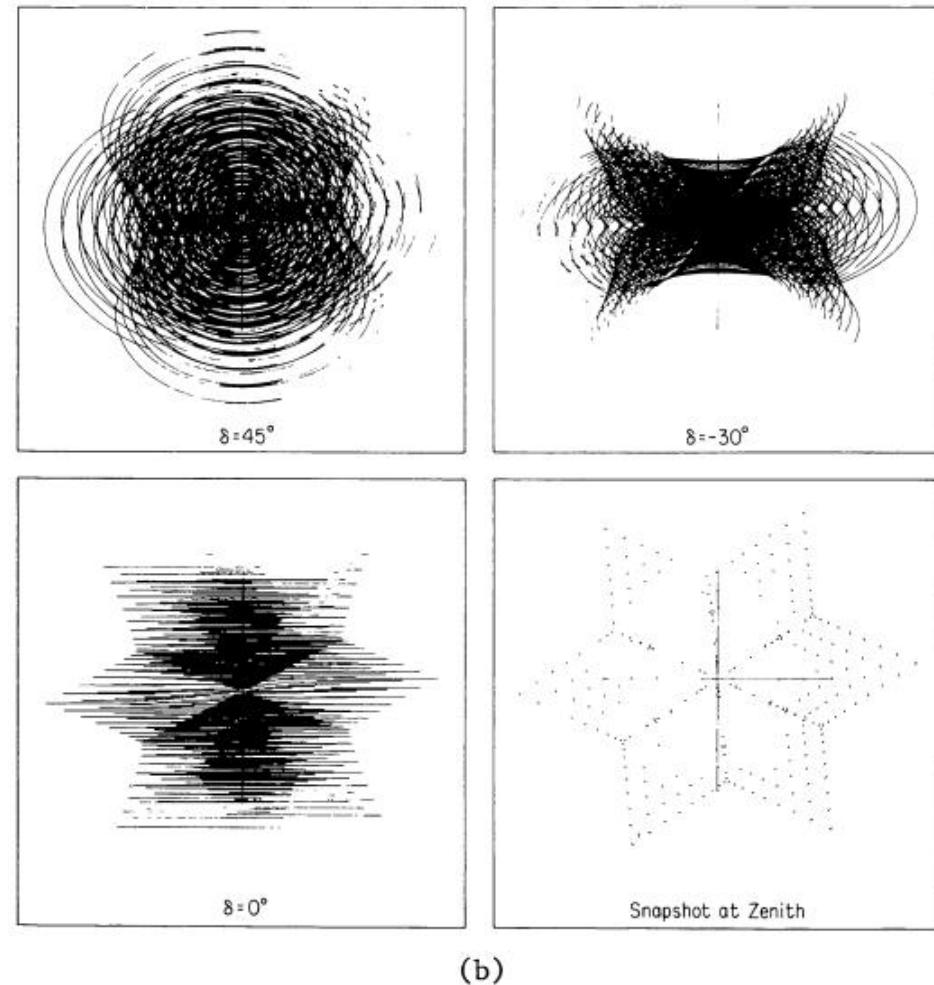
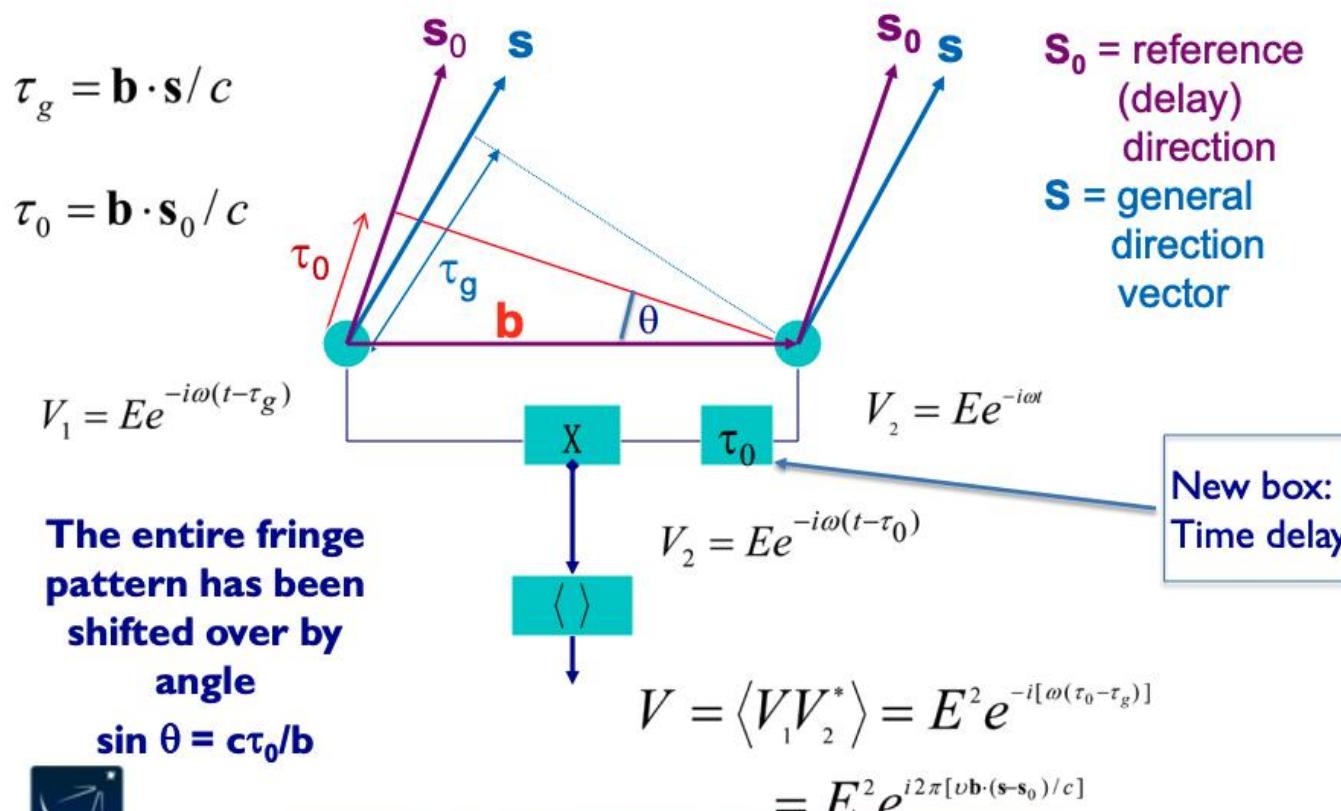


Figure 2-14. (a) The configuration of the 27 antennas of the VLA. (b) The transfer functions for four declinations with observing durations of $\pm 4^h$ for $\delta = 0^\circ$ and 45° , $\pm 3^h$ for $\delta = -30^\circ$, and $\pm 5^m$ for the snapshot. [From Napier, Thompson and Ekers (© 1983 IEEE).]



Correlation

Adding Time Delay

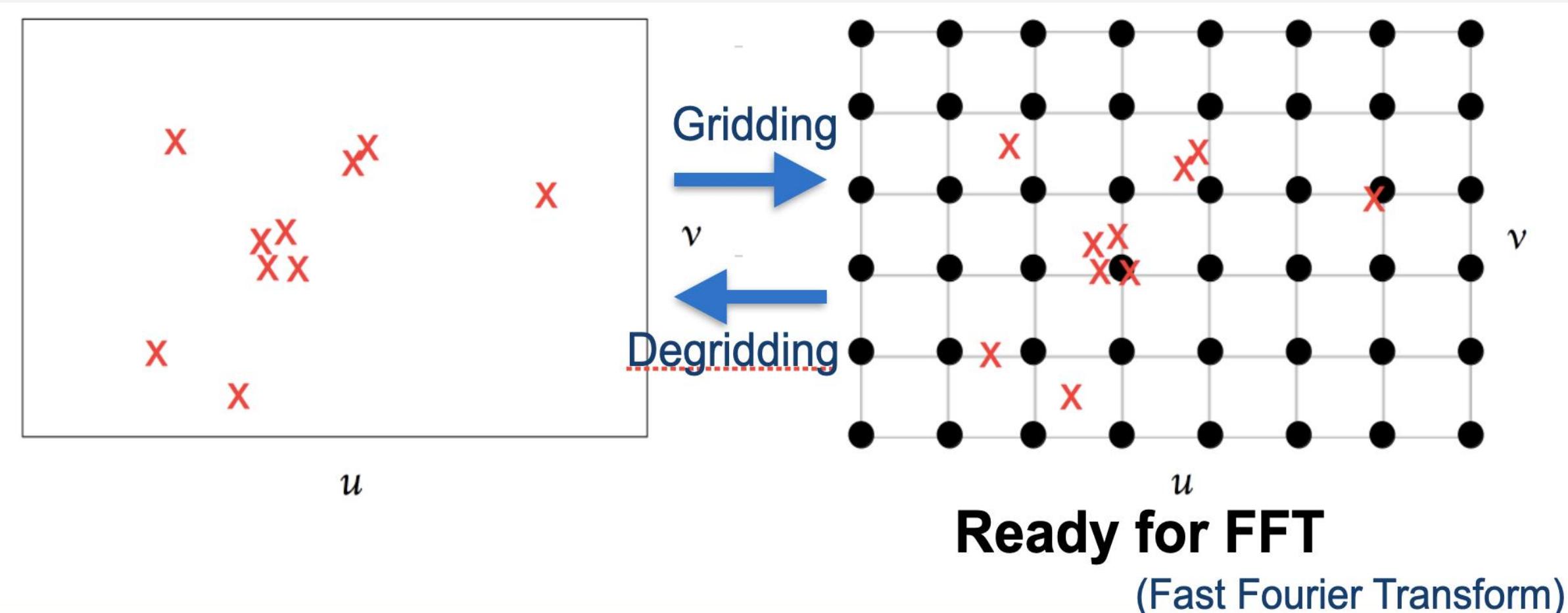


https://science.nrao.edu/science/meetings/2018/16th-synthesis-imaging-workshop/talks/Perley_Advanced.pdf





GRIDDING





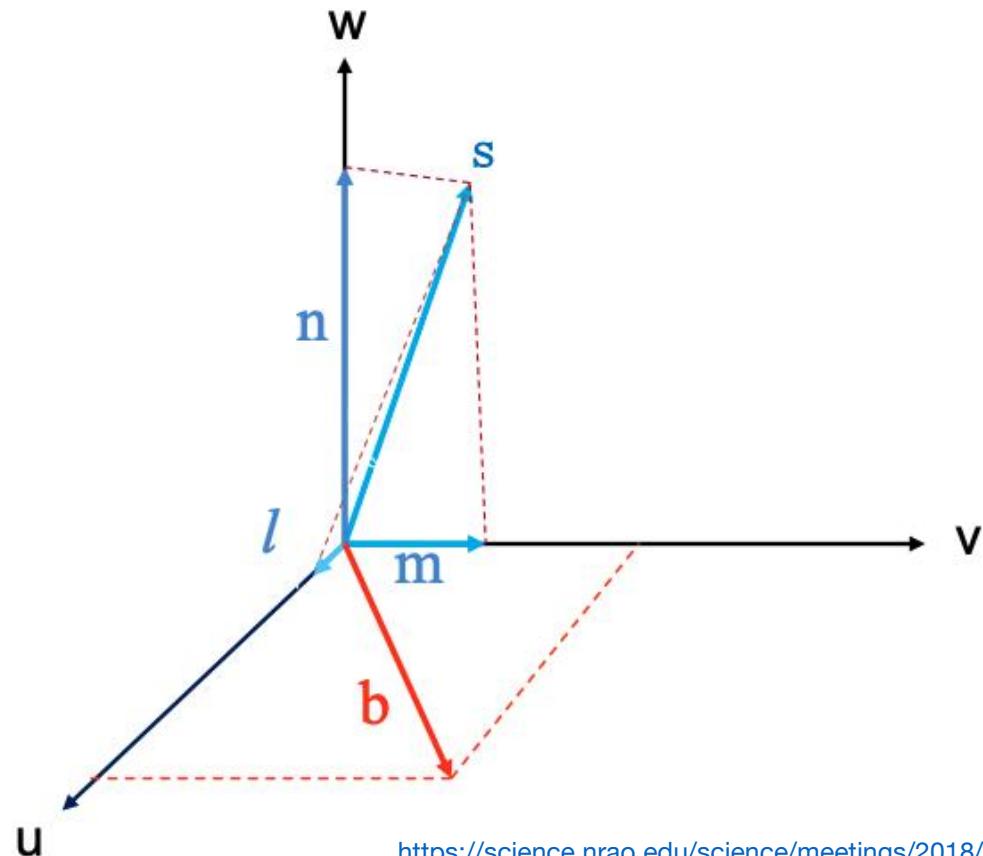
Direction Cosines – describing the source

The unit direction vector \mathbf{s} is defined by its projections (l, m, n) on the (u, v, w) axes. These components are called the **Direction Cosines**.

$$l = \cos(\alpha)$$

$$m = \cos(\beta)$$

$$n = \cos(\theta) = \sqrt{1 - l^2 - m^2}$$



https://science.nrao.edu/science/meetings/2018/16th-synthesis-imaging-workshop/talks/Perley_Geometry_new.pdf

The angles, α , β , and θ are between the direction vector and the three axes.





Högbom CLEAN

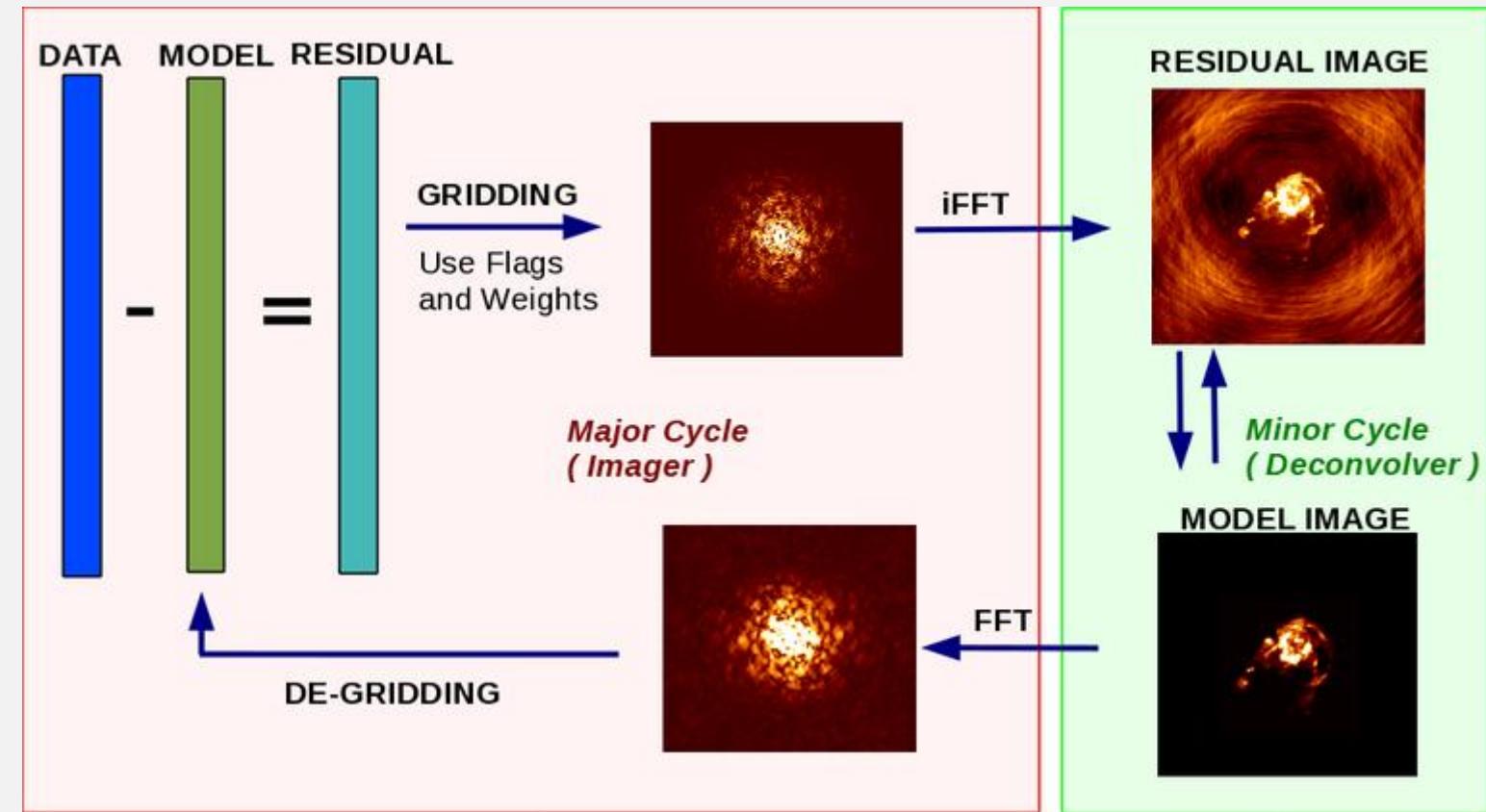
Jan Arvid Högbom (born 3 October 1929) is a Swedish radio astronomer and astrophysicist. Högbom obtained his PhD in 1959 from the University of Cambridge with Martin Ryle.

1. Initialisation: Start with an empty image grid.
2. Identify Peak: Find the brightest pixel (peak) in the dirty image, which represents the location of a strong source.
3. Subtract Image: Subtract a scaled and shifted PSF (usually a Gaussian) centered at the location of the identified peak from the dirty image. This creates a residual image.
4. Update Model: Add the identified peak to the corresponding location in the model image.
5. Iterate: Repeat steps 2-4 until either a maximum number of iterations is reached or the residual image falls below a specified threshold level.

The final output is the model image + the residual image.



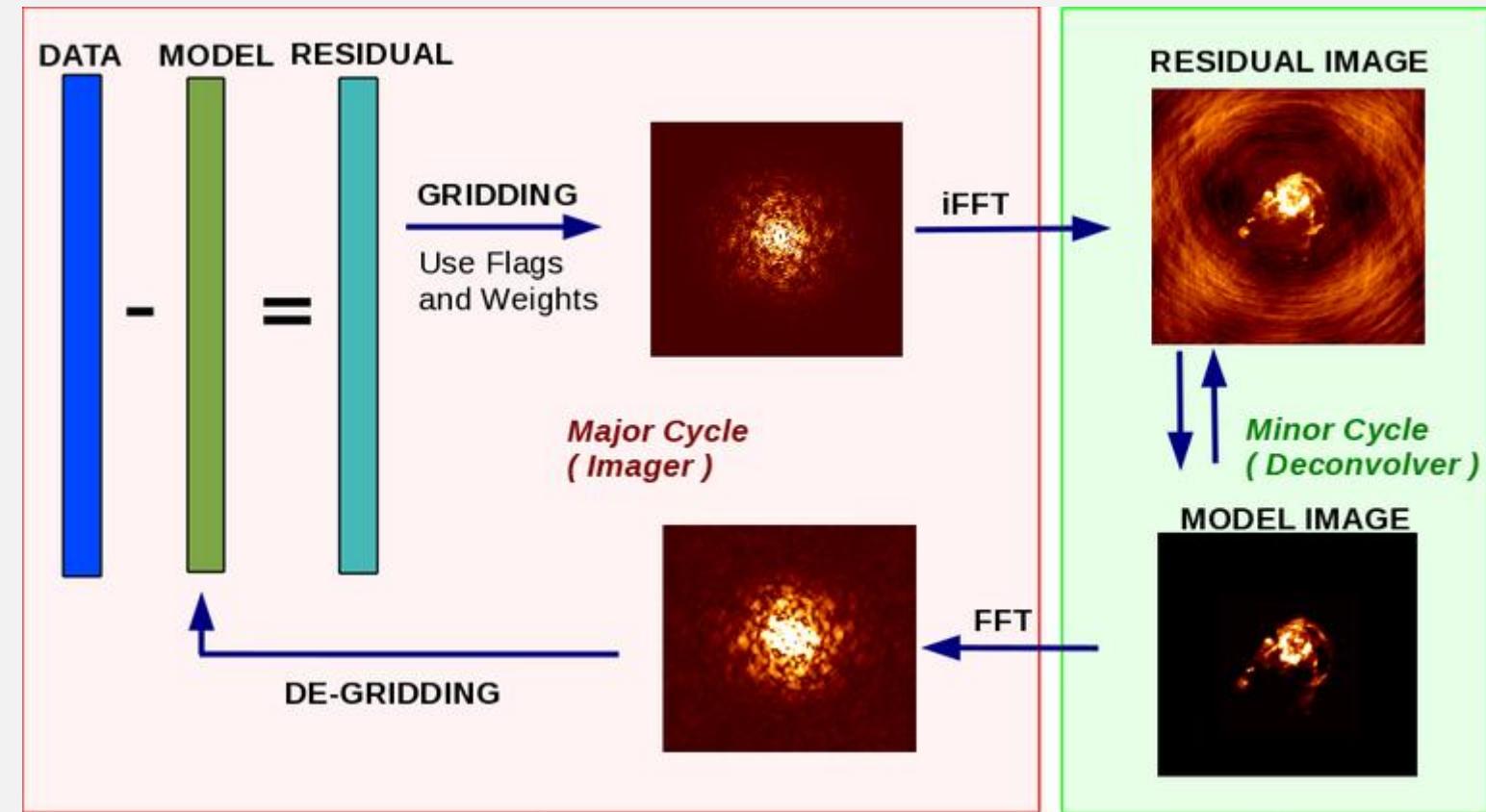
Schwab-Cotton CLEAN



Resource: https://casadocs.readthedocs.io/en/stable/notebooks/synthesis_imaging.html#Introduction



Maximum Entropy



Resource: https://casadocs.readthedocs.io/en/stable/notebooks/synthesis_imaging.html#Introduction