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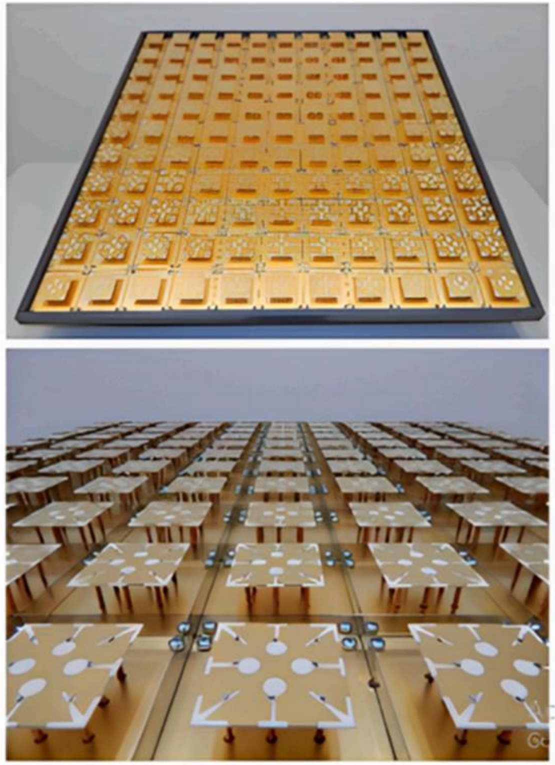
Lecture 5
Mutual coupling in antenna arrays I : Fundamentals of antenna theory and challenges in radio astronomy

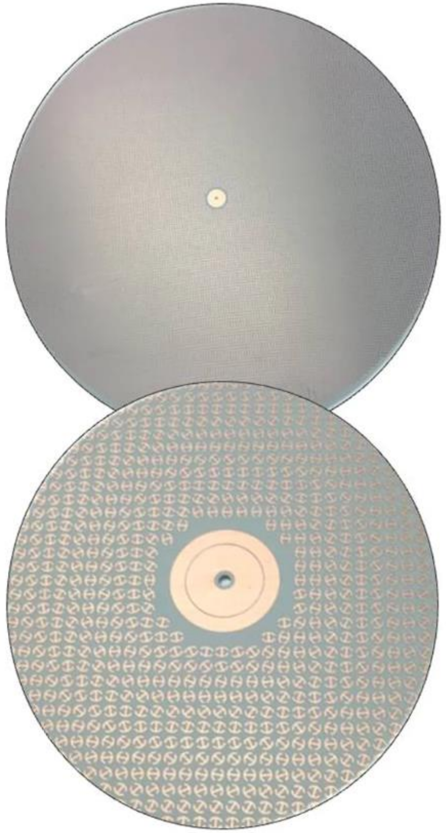
Lecturer: **Dr Quentin Gueuning** (qdg20)

MC in today’s antenna technologies

(11 × 11) massive MIMO array of multimode elements with 484 antenna ports, S. K. Ibrahim, *Design, Challenges and Developments for 5G Massive MIMO Antenna Systems at Sub 6-GHz Band: A Review*, 2023.

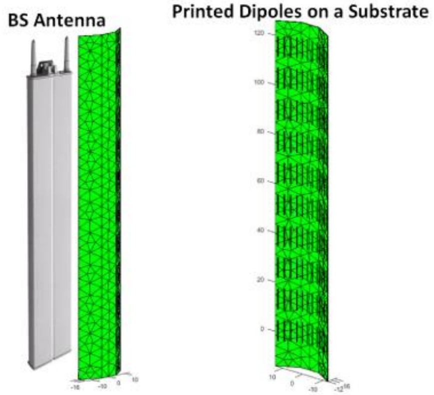


MTS prototype at ~29 GHz, M. Faezi, *Metasurface Antennas: New Models, Applications and Realizations*, Nature, 2019.



EMBRACE, SKA-mid prototype in Nancay, connected array of 4608 Vivaldi antennas, S.A. Torchinsky, 2016.

Unintended interaction of antennas on a repeater tower, 2010s. Source: Trevor S. Bird, Mutual coupling between antennas



Reconfigurable Cellular Base Station Antenna Consisting of Parasitic Radiators, Ali Kayani et al., IEEE TIE, 2019

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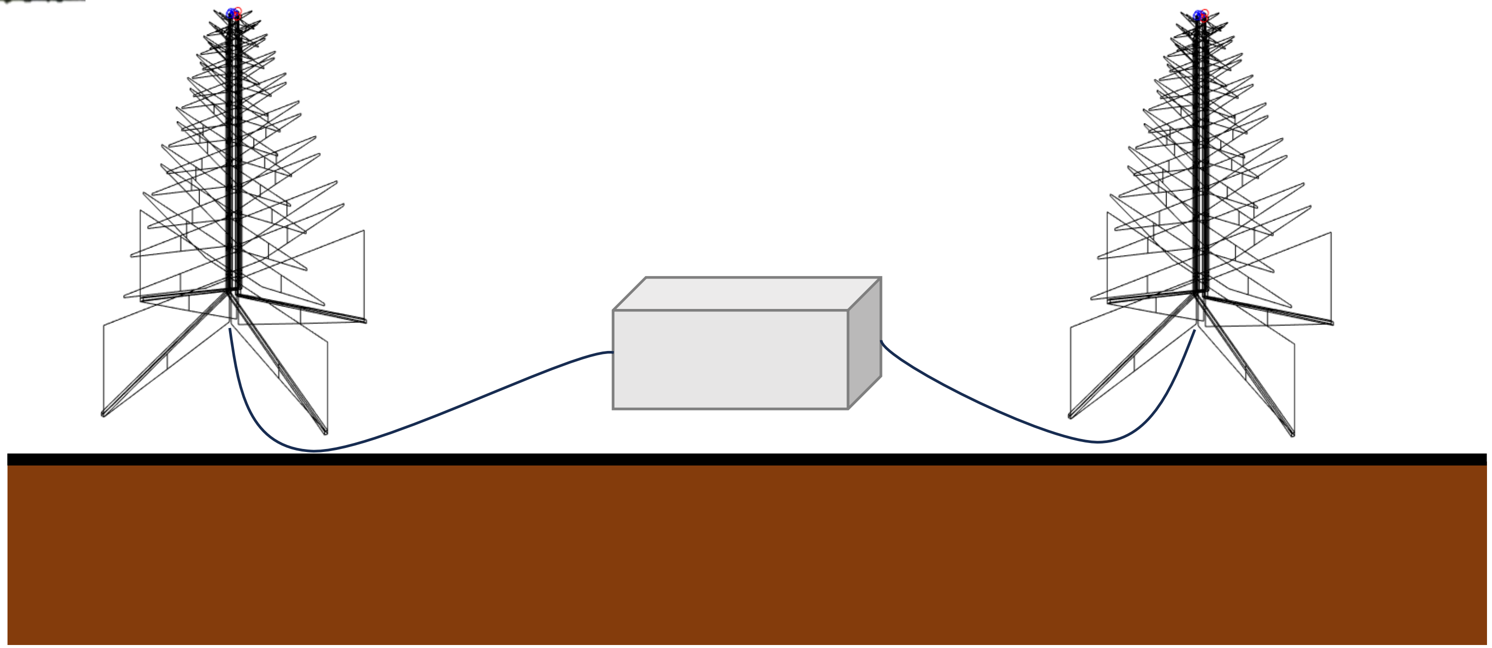
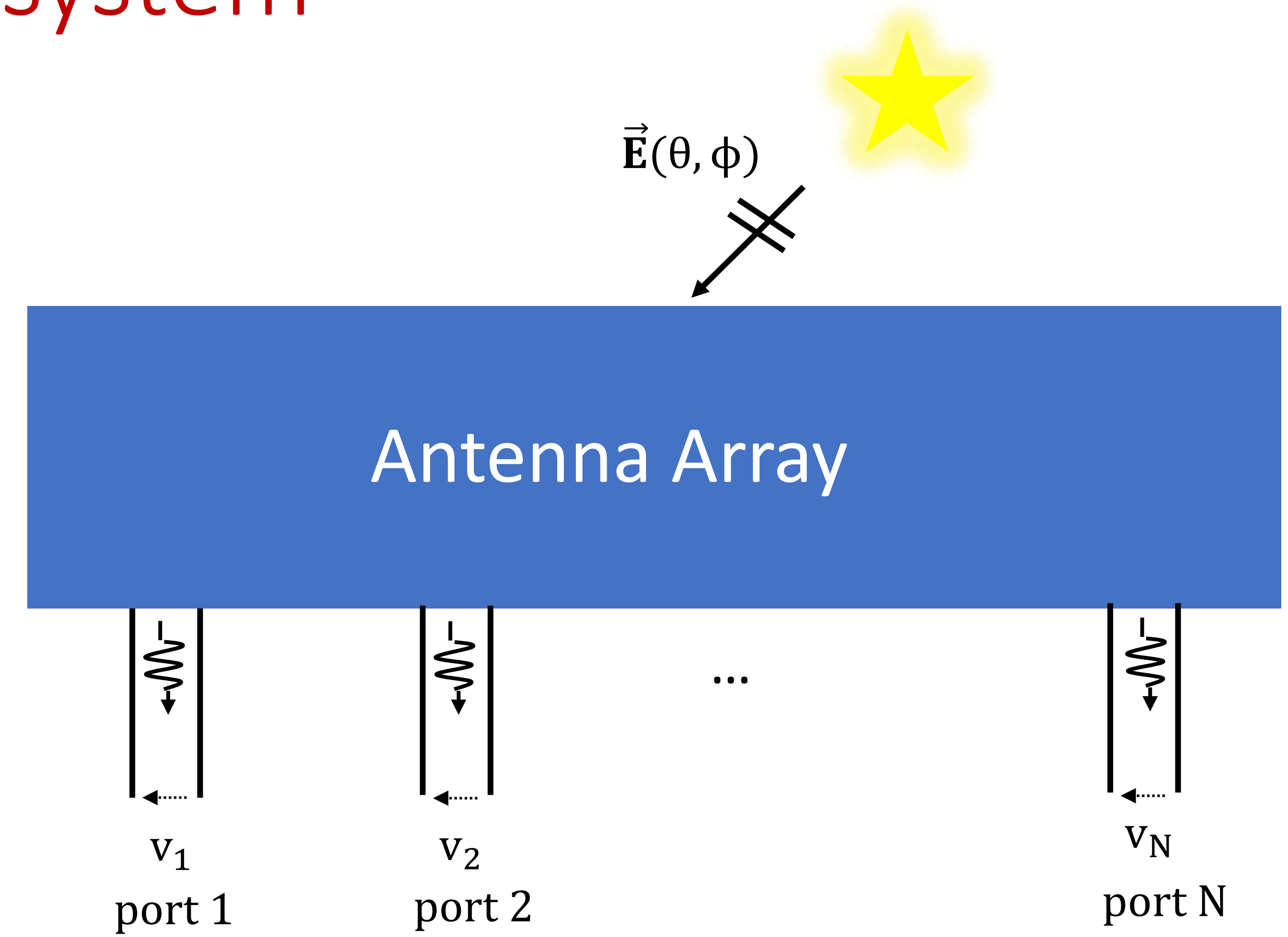
SKA-low context …



Linear system



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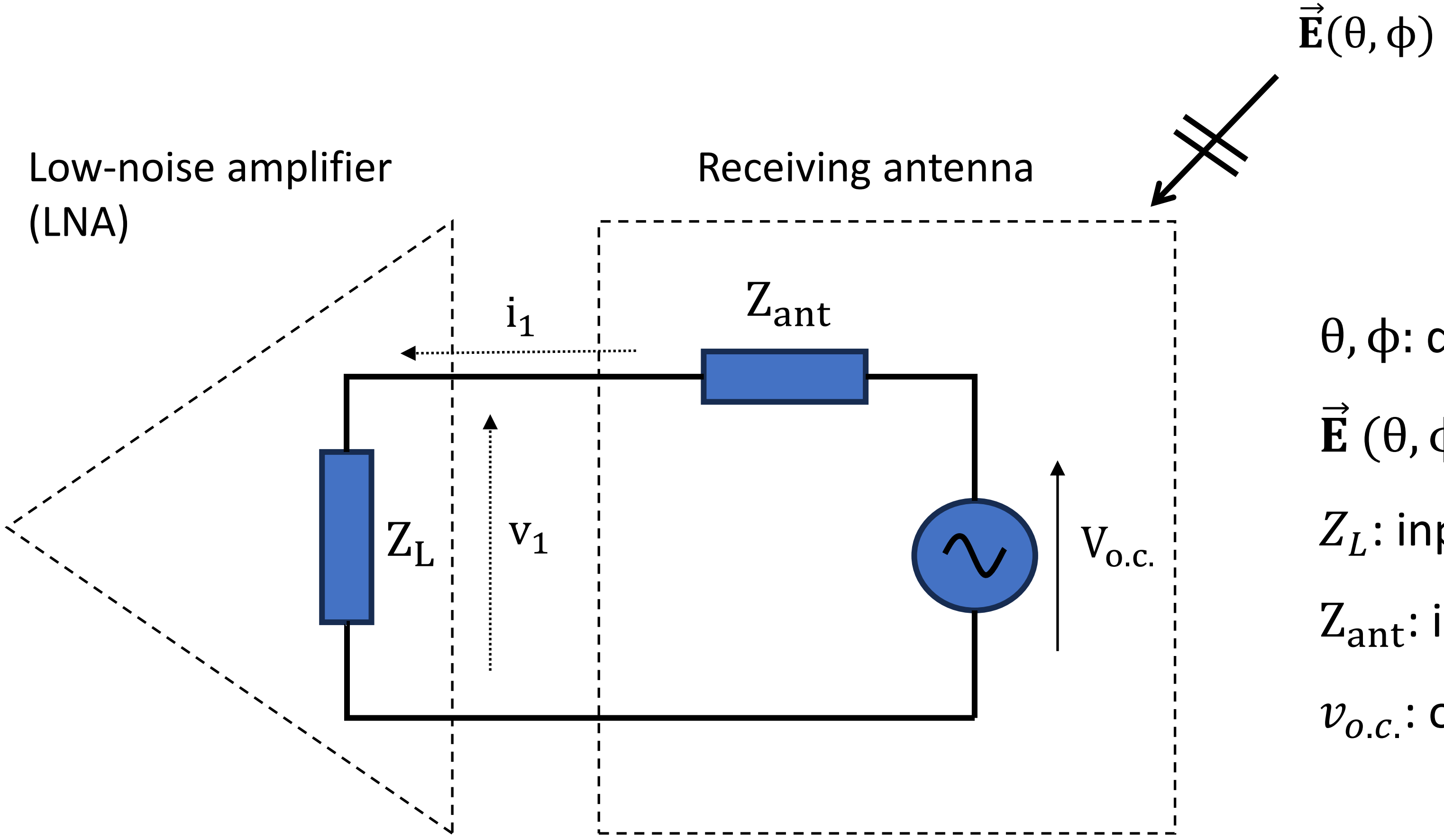
Circuit model of a single antenna



θ, ϕ: direction of incidence

$\vec{\mathbf{E}}\,(\theta, \phi)$: incident electric field

Z_L: input impedance of the amplifier

Z_{ant}: input impedance of the antenna

$v_{o.c.}$: open circuit antenna voltage

P.S. Kildal, *Foundations of Antenna Engineering: A Unified Approach for Line-of-Sight and Multipath*, Artech, 2015.

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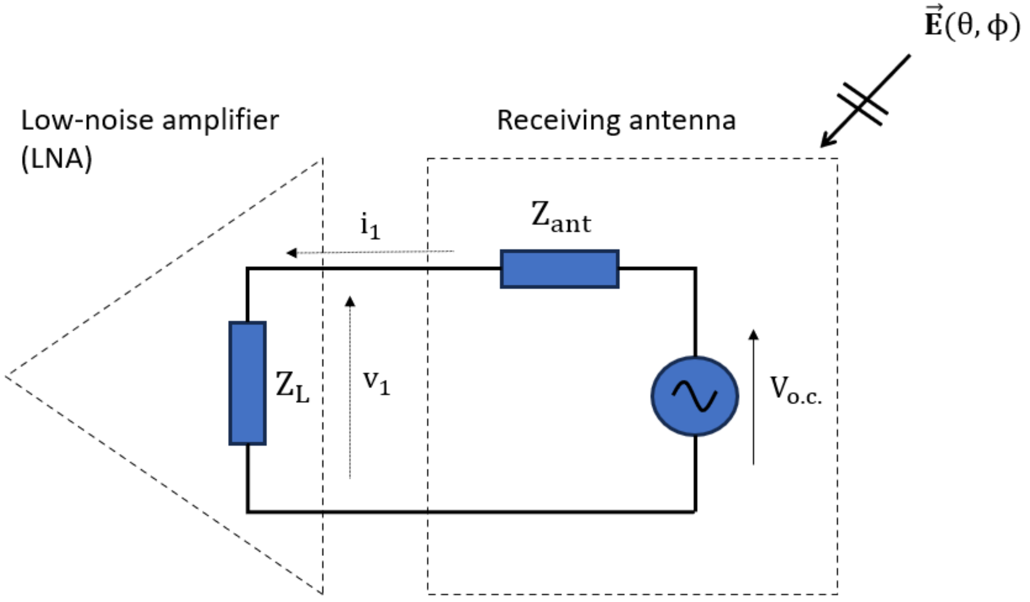
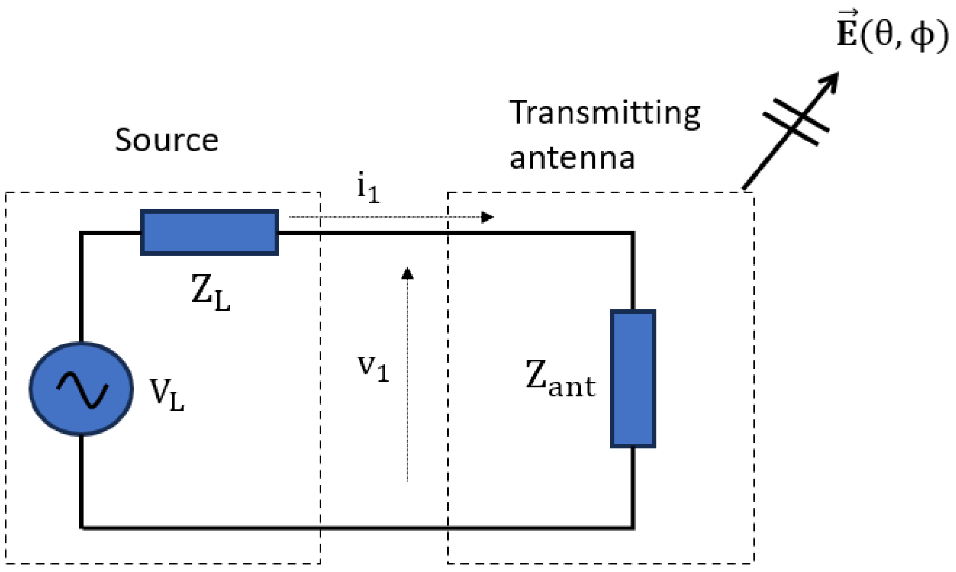
Reciprocity

Passive mode



$$v_1 = \frac{2\lambda}{j\eta}\frac{Z_{ant}}{Z_{ant}+Z_L}\frac{\vec{F}(\theta,\phi)\,.\,\vec{E}(\theta,\phi)}{i_0}$$

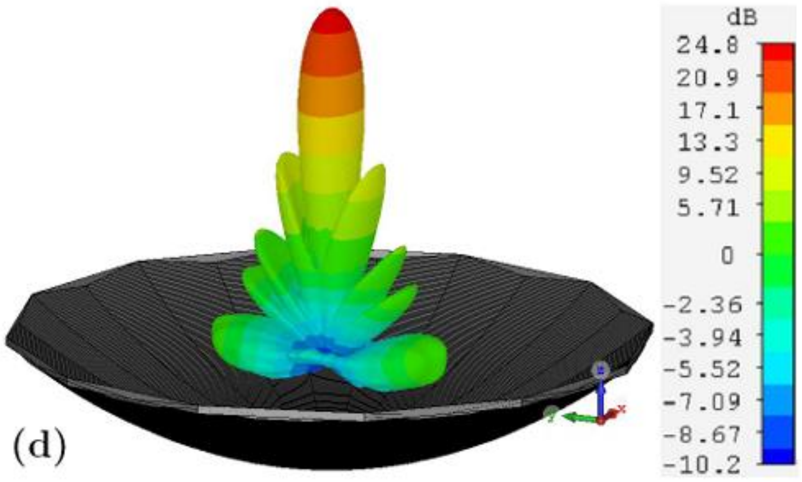
Active mode

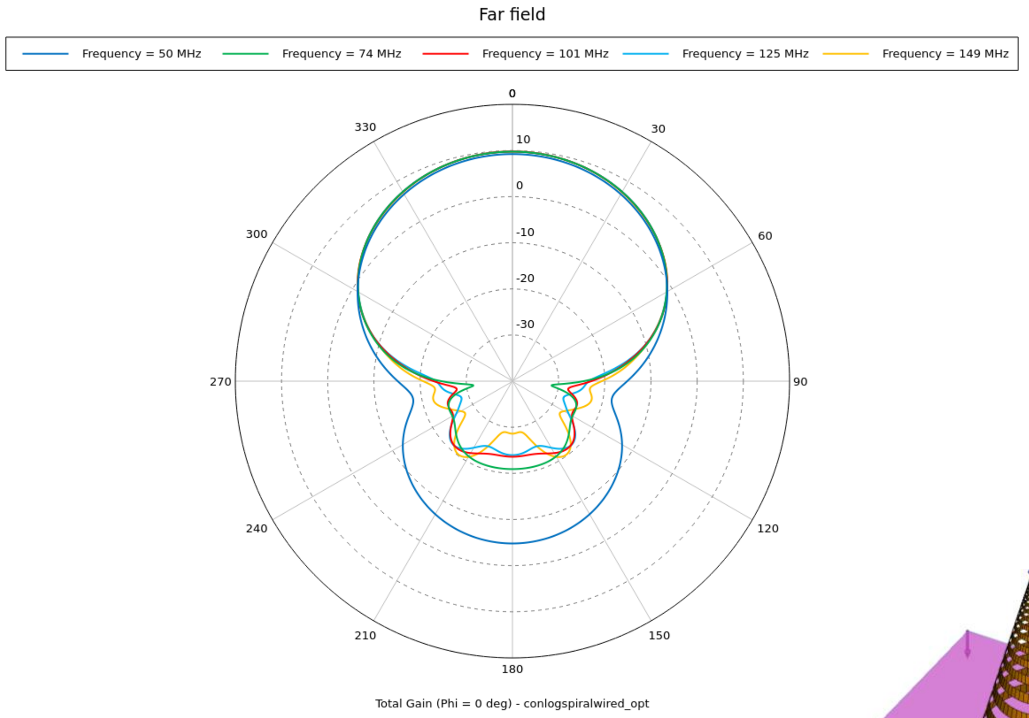


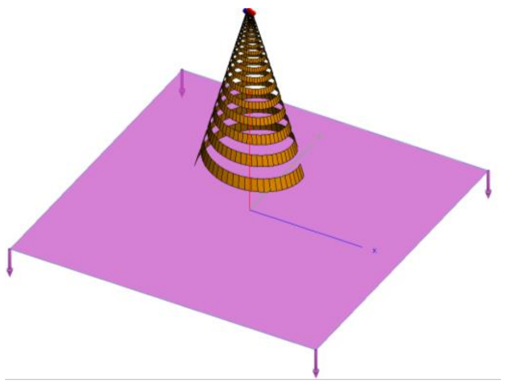
$\vec{F}$ in Volts is the radiation pattern

$$\vec{E}(\theta,\phi) = \frac{\vec{F}(\theta,\phi)}{i_0}\frac{v_1}{Z_L+Z_{ant}}\frac{e^{-jkR}}{R}$$

Examples of radiation patterns







N. Fagnoni et al., “Understanding the HERA Phase I receiver system with simulations and its impact on the detectability of the EoR delay power spectrum”, *MNRAS*, 2021

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Received power

$$\mathrm{P_r} = \mathrm{A_{eff}}(\theta, \phi)\ S$$ plane wave intensity in $\mathrm{W/m^2}$

Effective area in m^2

$$\mathrm{A_{eff}}(\theta, \phi) = \frac{\lambda^2}{4\pi}\ (1 - |\Gamma|^2)\ D(\theta, \phi)\ R(\theta, \phi)\ S$$

Reflection coefficient

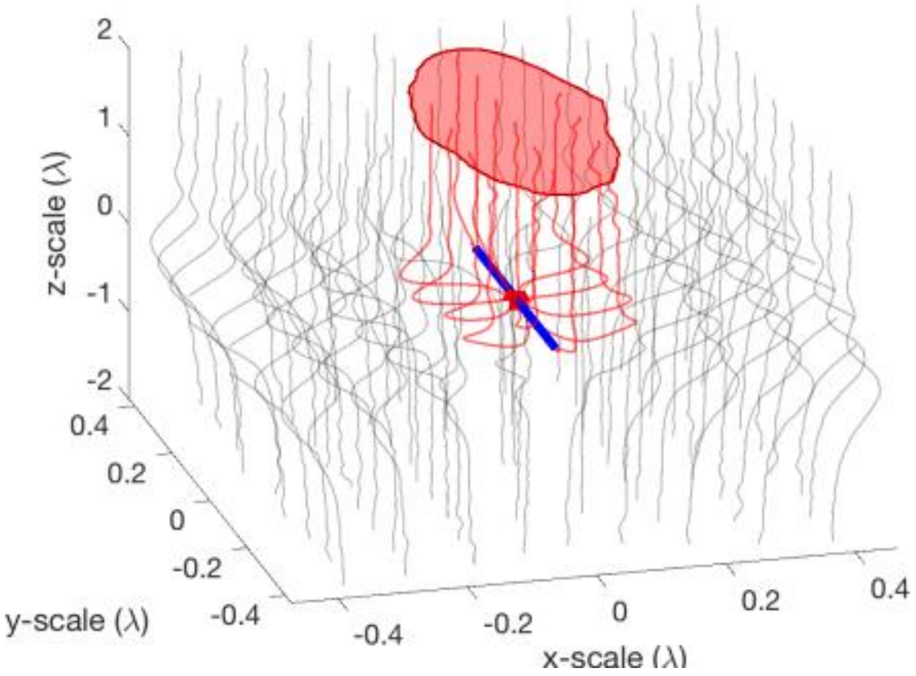
$$\Gamma = \frac{\mathrm{Z}_L - Z_{ant}}{\mathrm{Z}_L + Z_{ant}}$$

Polarisation Loss Factor

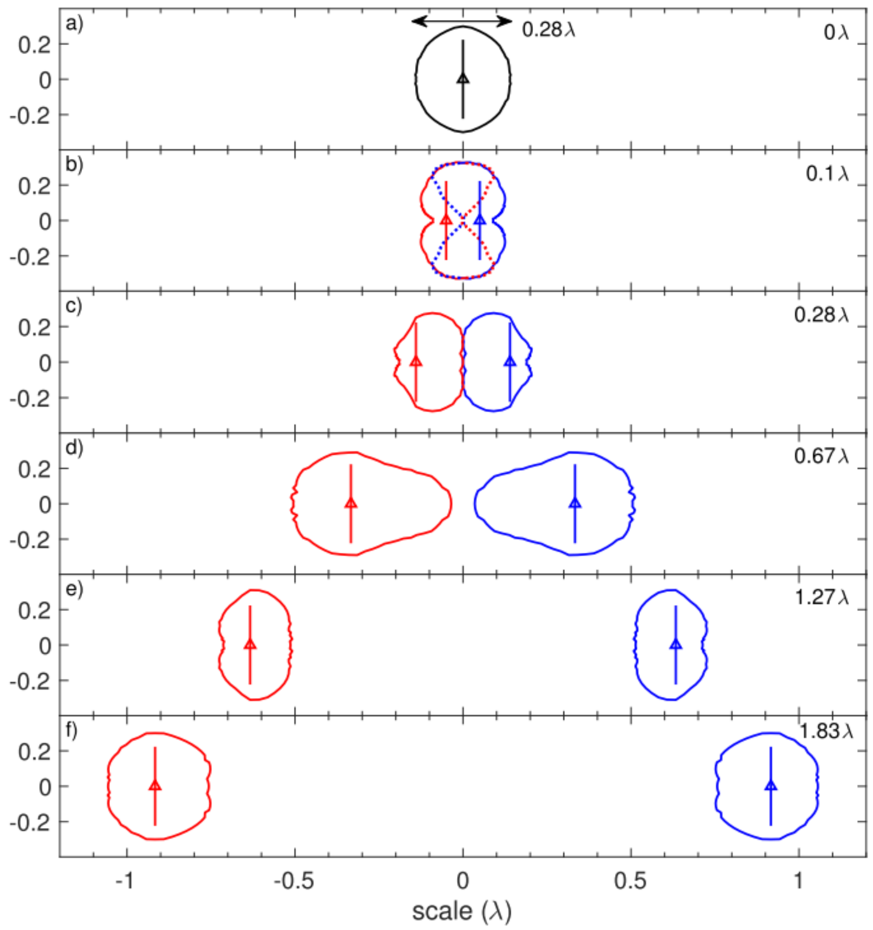
$$\mathrm{R}(\theta, \phi) = \left|\hat{\mathrm{F}}(\theta, \phi) \cdot\ \hat{\mathrm{E}}(\theta, \phi)\right|^2$$

Directivity

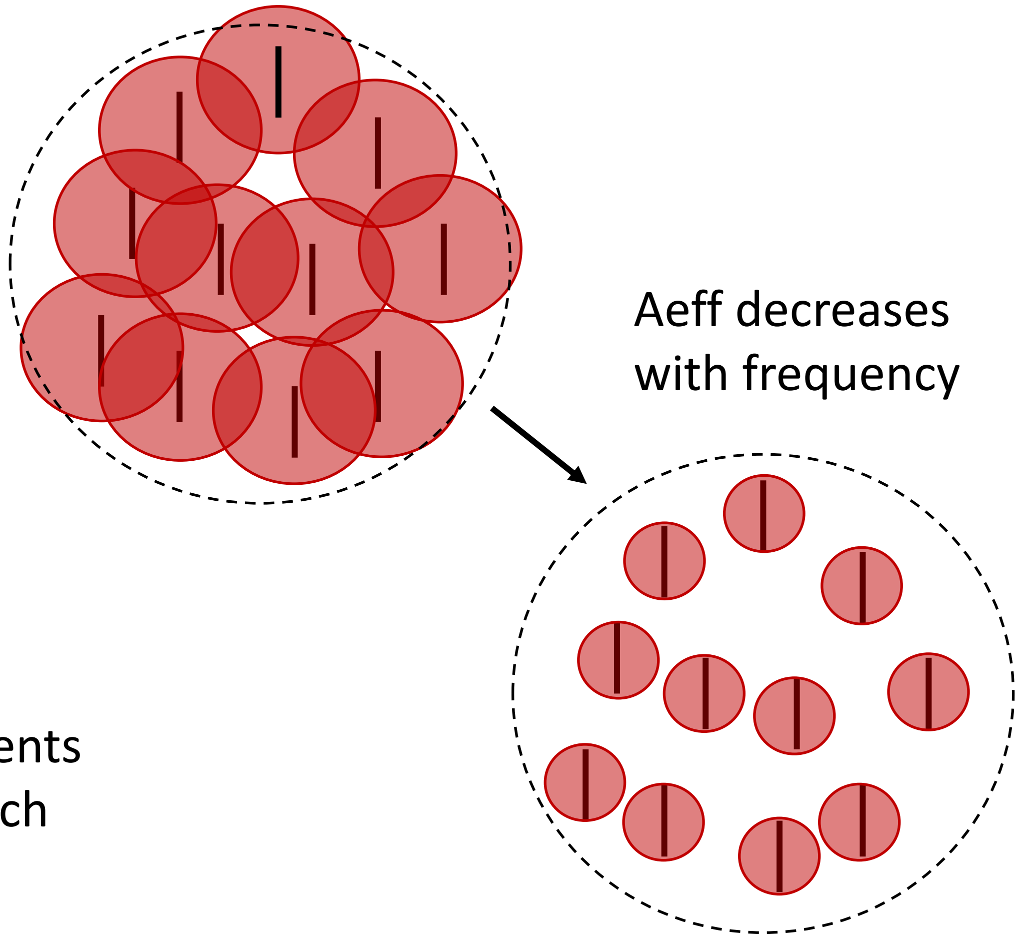
$$\mathrm{D}(\theta, \phi) = 4\pi \frac{|F(\theta, \phi)|^2}{\iint |F(\theta, \phi)|^2 d\Omega}$$



MC impacts the effective area



Aeff decreases when the elements gets close to each other



Aeff decreases with frequency

An Intuitive Way to Understand Mutual Coupling Effects in Antenna Arrays Using the Poynting Streamline Method, IEEE TAP, feb. 2019

Why is it important to radio-astronomy ?

1. System design

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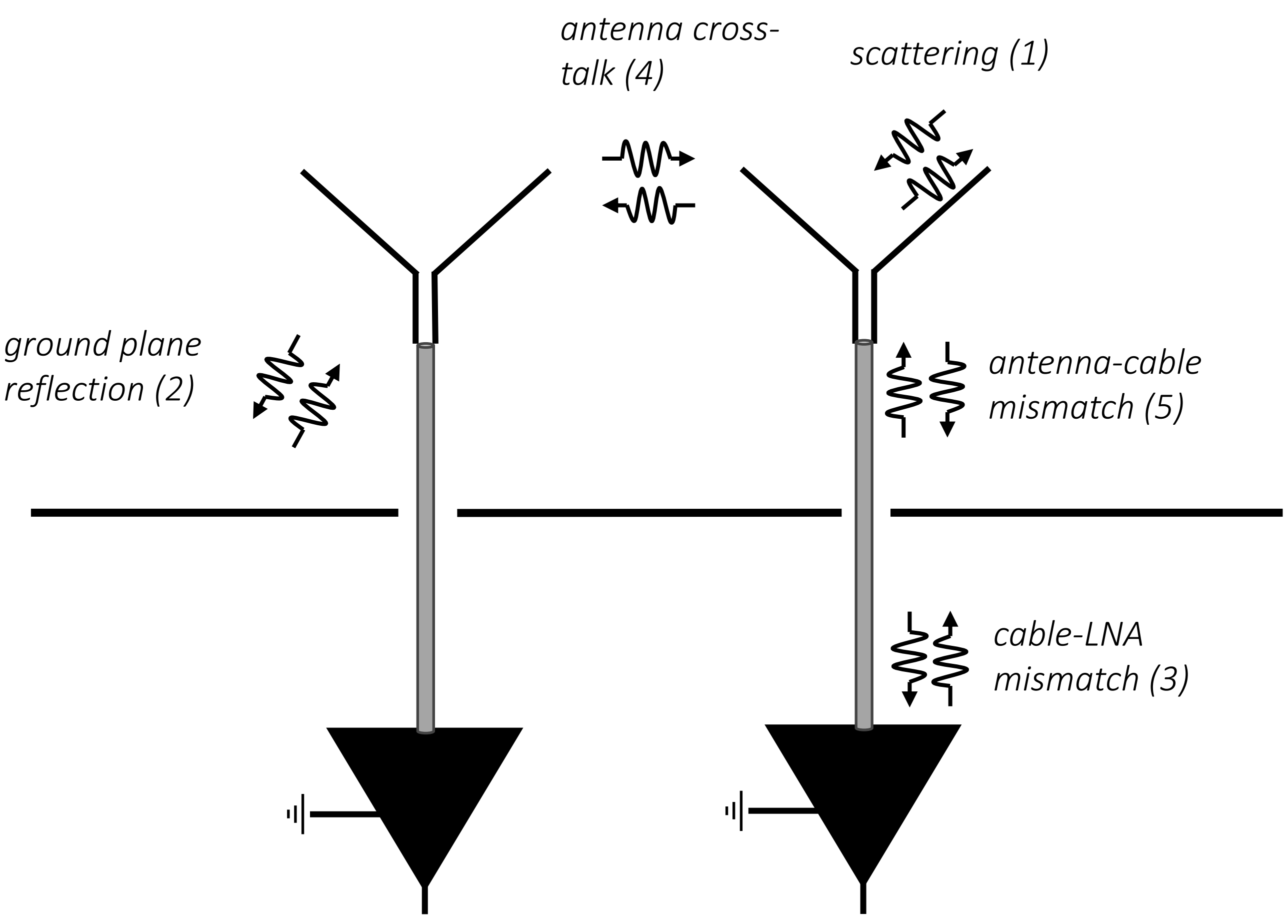
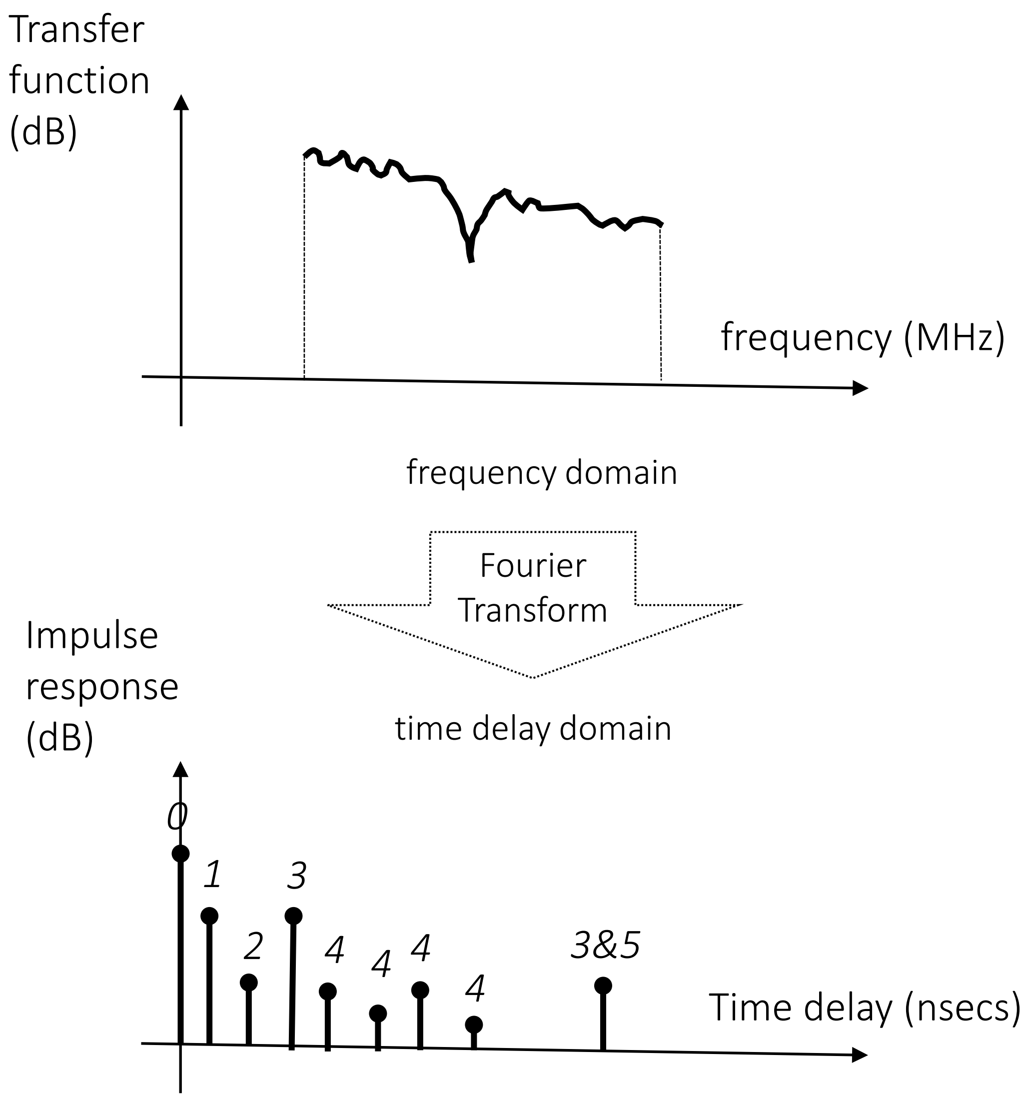
1. System design





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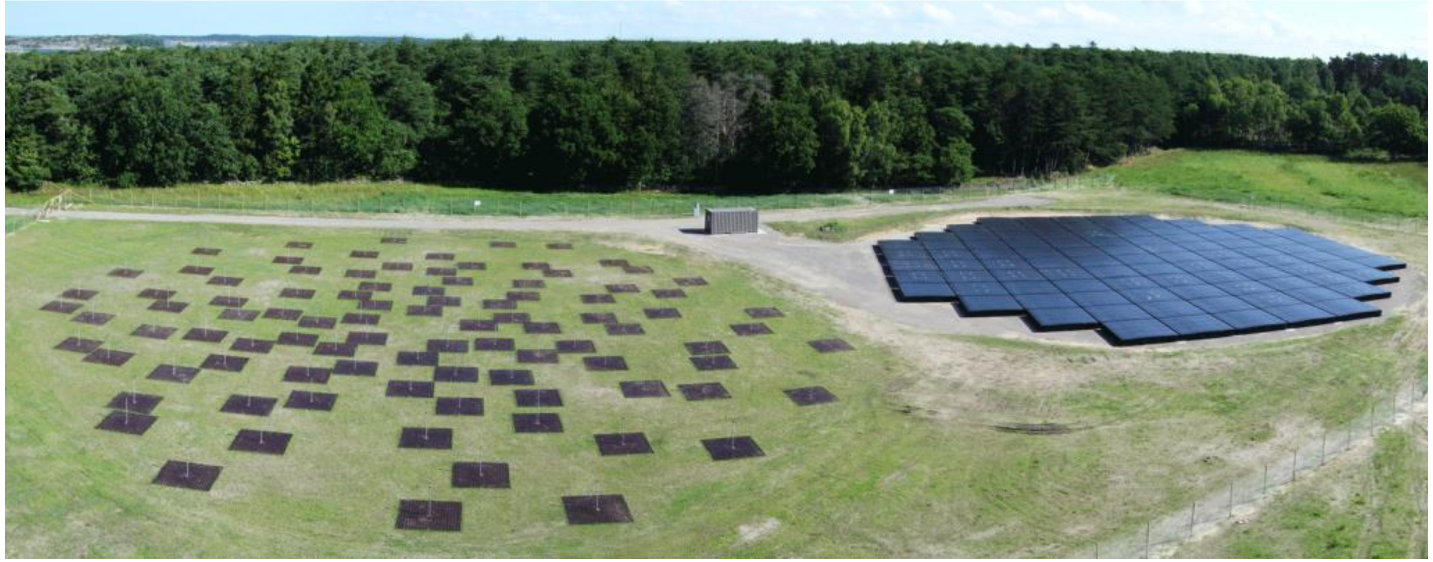
1. System design, LOFAR

Low-Band Antenna (LBA) 30-80MHz
Dual-polarized inverted V-dipole w. individual ground mesh

54 stations
96 elements per station
5912 signal paths

https://lofar.ie/technology/



M.P. Van Haarlem, LOFAR: The LOw-Frequency ARray, A&A, 556, A2, 2013

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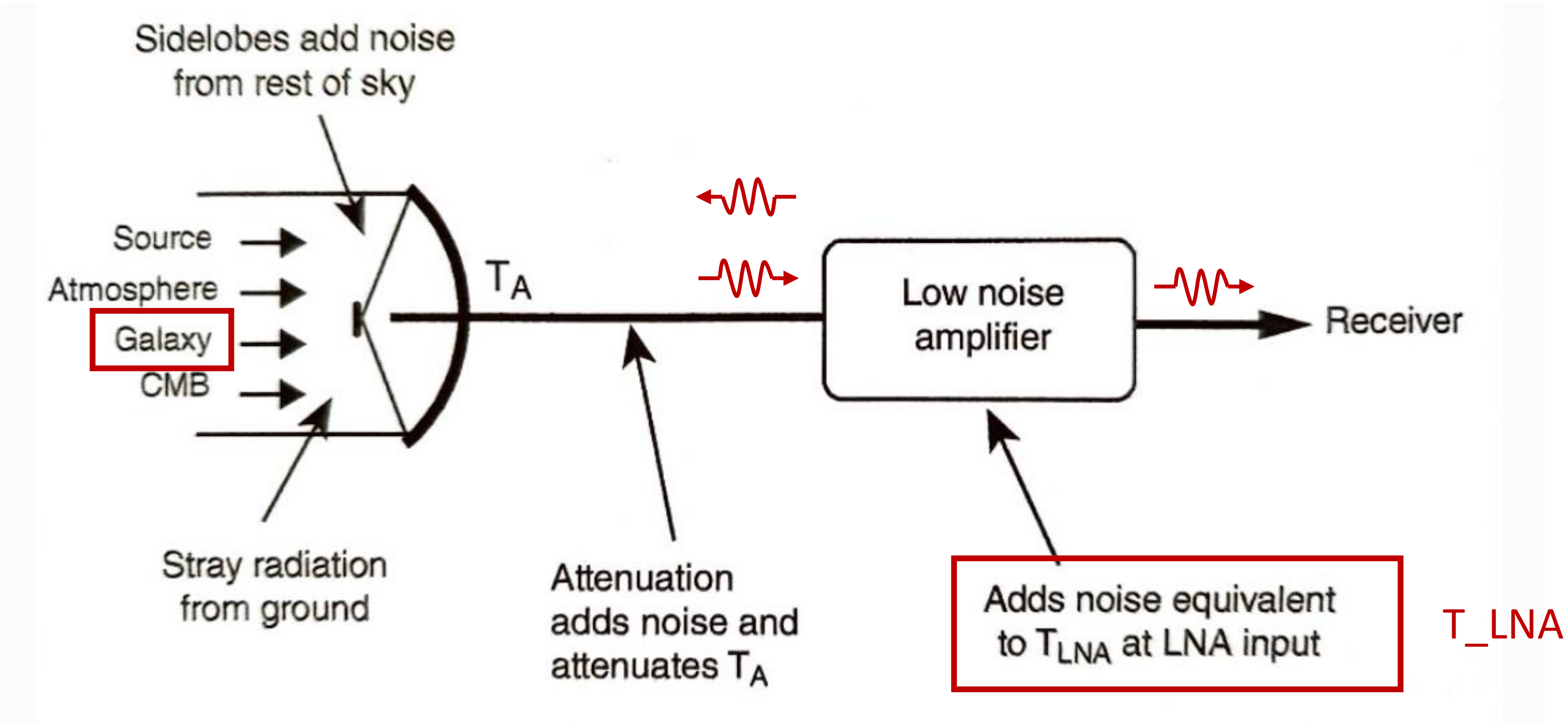
1. System design, LOFAR

T_sky ~ 5000 K @ 50 MHz

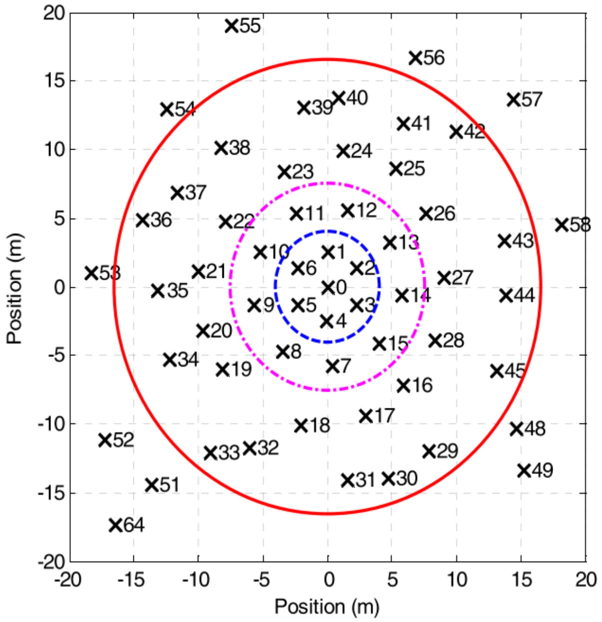
T_sky prop to f^-2.7



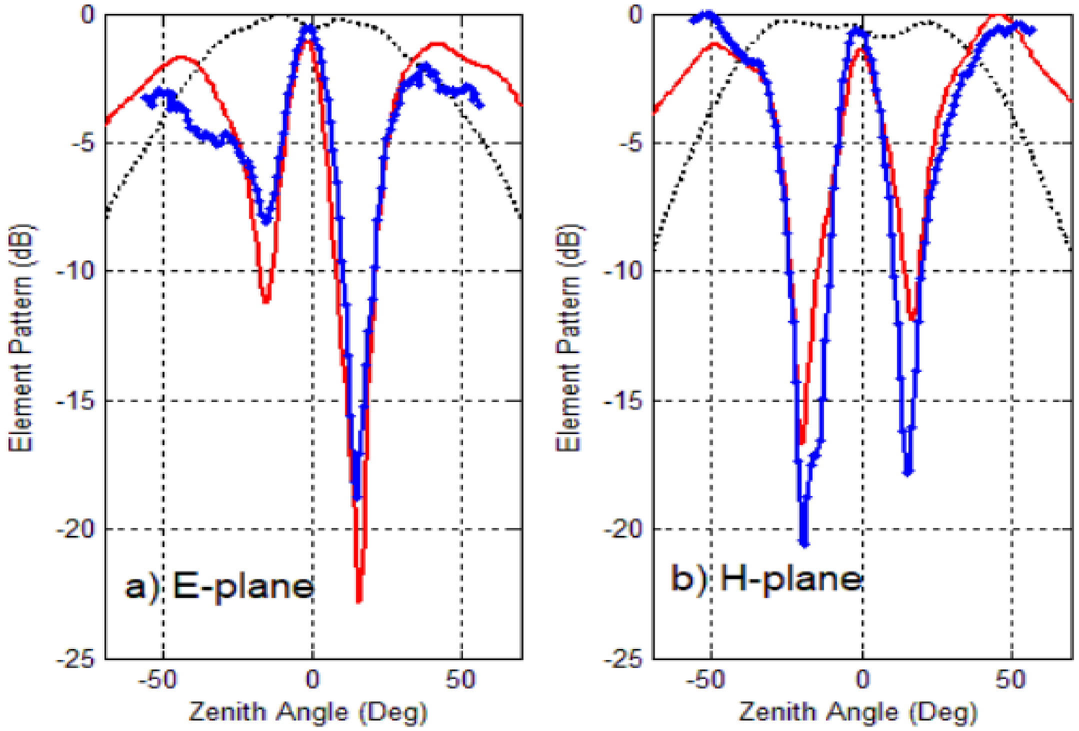
T_LNA ~ 100 K

1. System design, LOFAR

LBA array layout



Cuts of embedded element pattern of central element 0 at 57MHz



G. Virone, et al., *Strong Mutual Coupling Effects on LOFAR: Modeling and In Situ Validation*, TAP 2018

Why is it important to radio-astronomy ?

2. Forward Modelling

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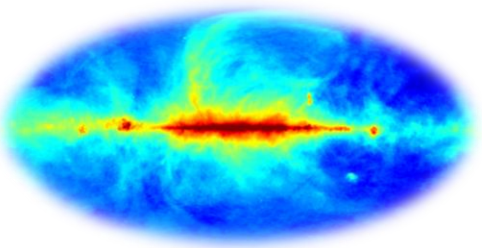
2. Forward Modelling

Cosmology and Astrophysics

Sky models
21-cm EoR models, diffuse emission, point source survey, (GLEAM, HASLAM, …)



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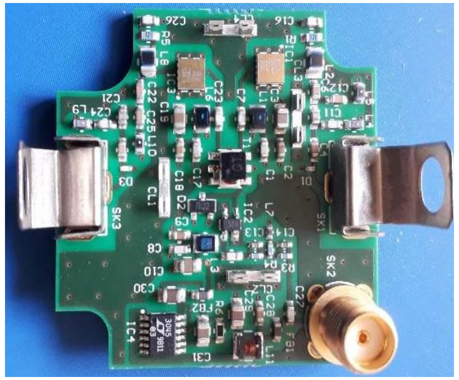
Computational Electromagnetics

Antennas and propagation
antennas, aperture arrays, dishes, ground plane, soil, ionosphere, …

→

Circuit and transmission line theory

Front-end modules
Low-noise amplifiers, coaxial cables, filters, Optical converter, beamforming networks, calibrators, …



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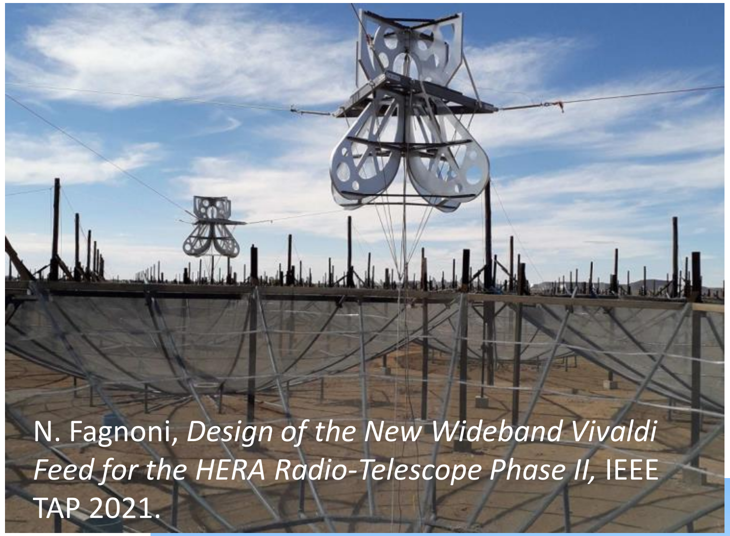
Digital Signal Processing Theory

Back-end modules
FFTs, correlators, power and clock distribution, DACs, network switches, …

2. Forward Modelling, Hydrogen Epoch of Reionization Array (HERA)



N. Fagnoni, *Design of the New Wideband Vivaldi Feed for the HERA Radio-Telescope Phase II*, IEEE TAP 2021.

50-250MHz

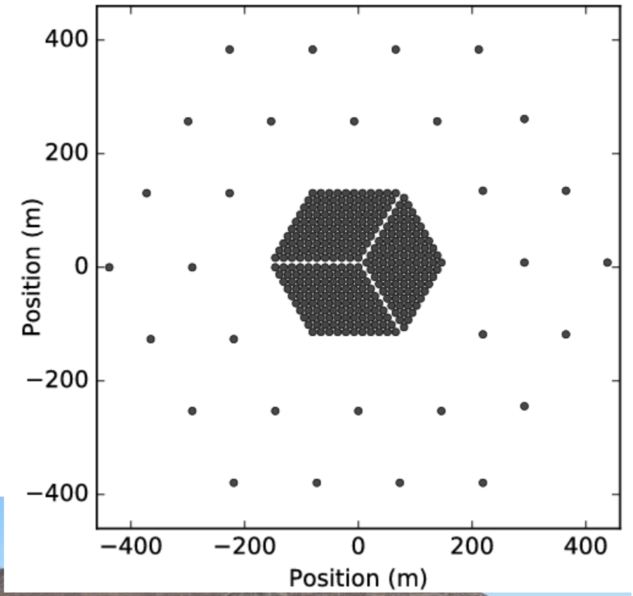
14m dish with Vivaldi antenna feed

hegaxonal lattice with 320 elements

30 outriggers

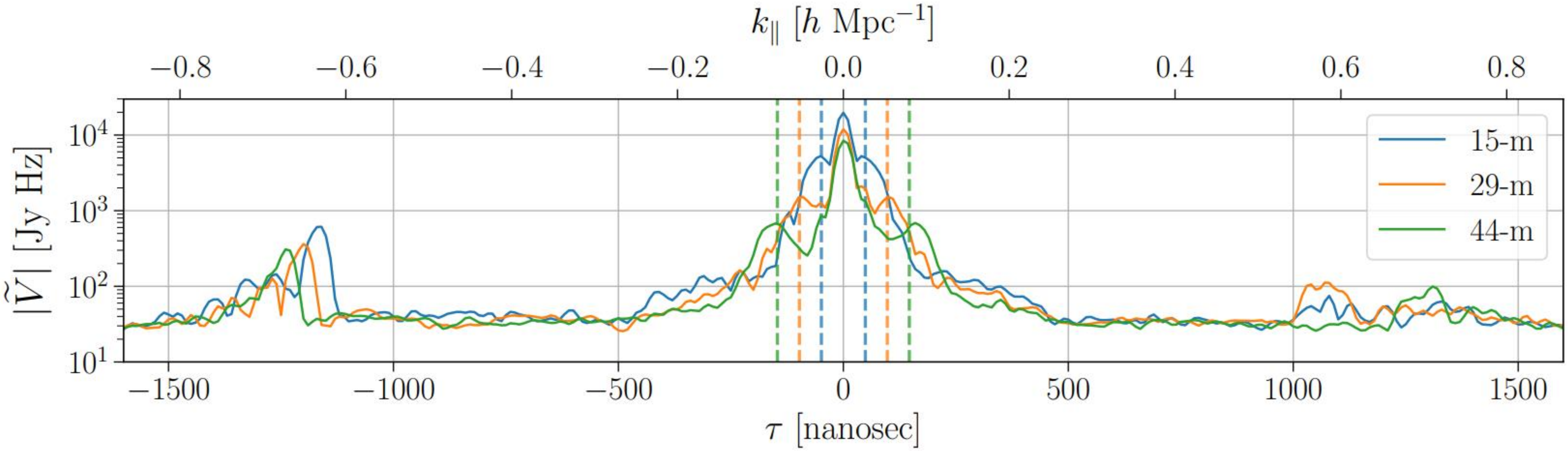
250λ instantaneous uv-coverage at 150 MHz

9 deg field of view



Picture from Josh Dillon, HERA construction photo log, https://reionization.org/

2. Forward Modelling, HERA



N. S. Kerns et al., *"Mitigating internal instrument coupling for 21cm cosmology II: a method demonstrated with the Hydrogen Epoch of Reionization Array"*, AJ, 2020

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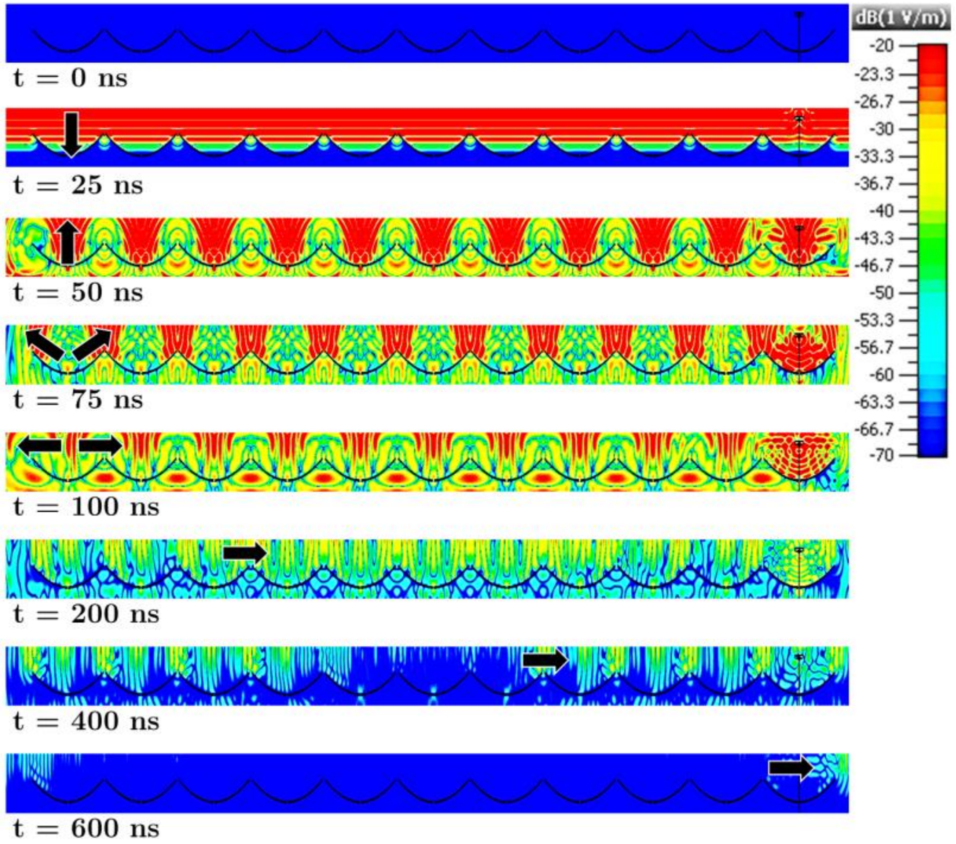
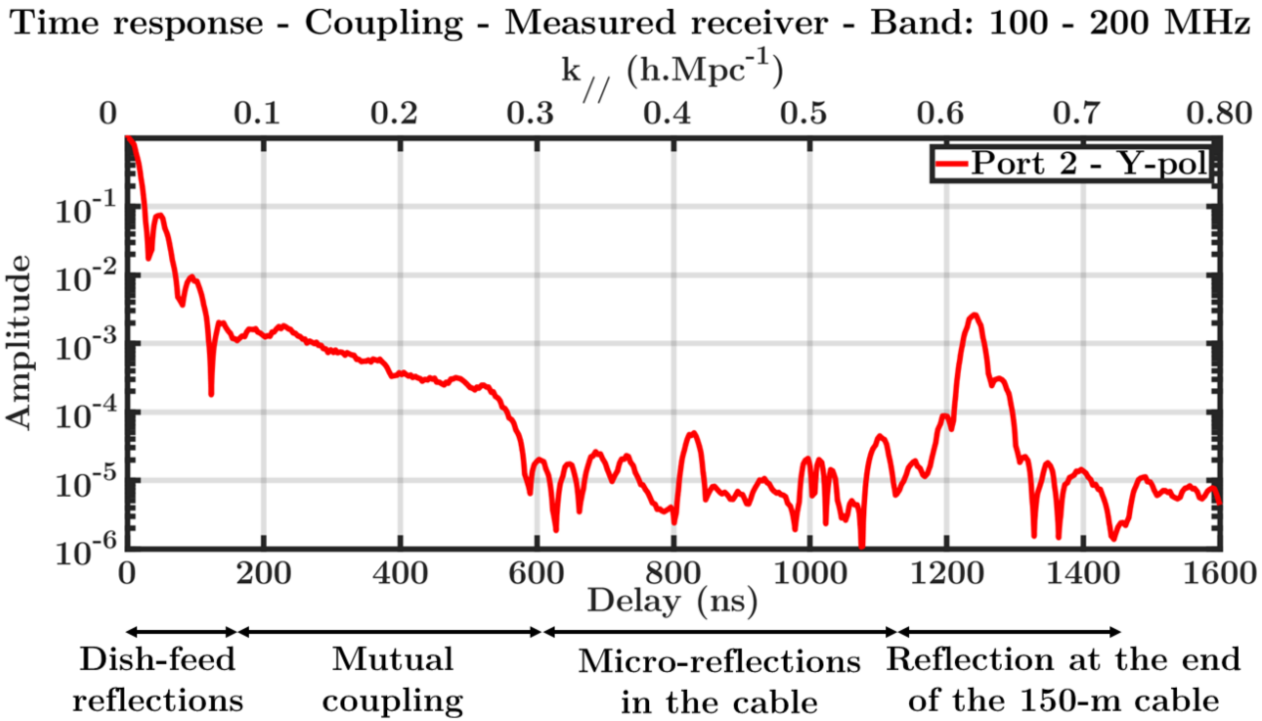
2. Forward Modelling, HERA





N. Fagnoni et al., “Understanding the HERA Phase I receiver system with simulations and its impact on the detectability of the EoR delay power spectrum”, *MNRAS*, 2021

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Why is it becoming important to radio-astronomy ?

3. Station calibration

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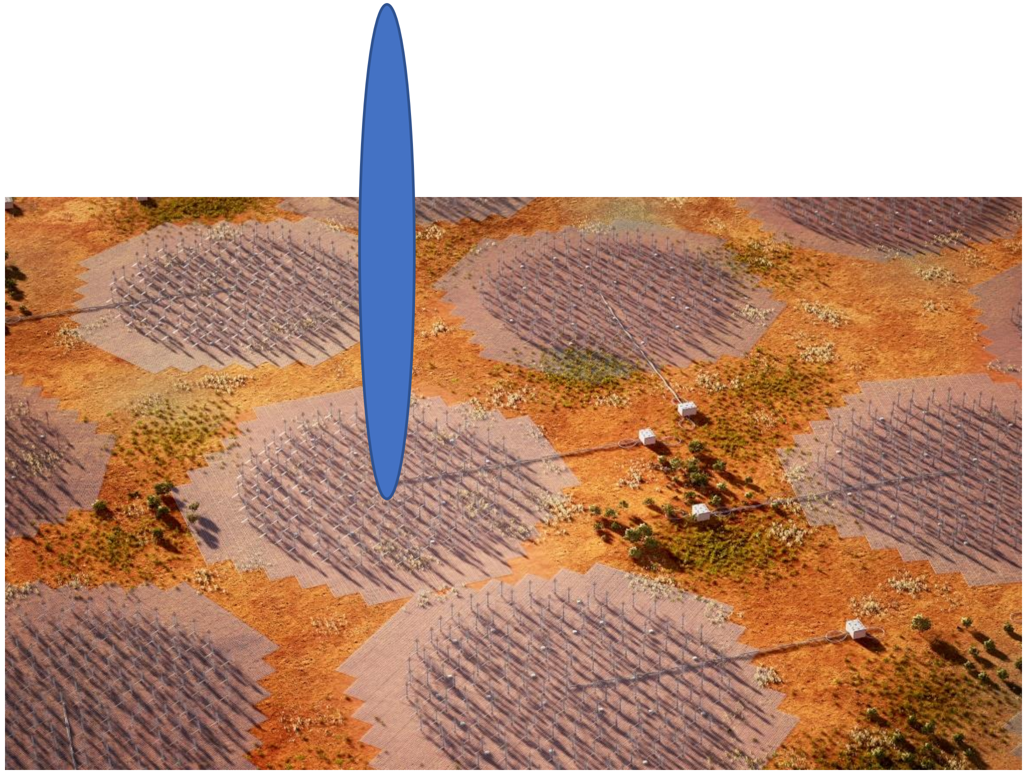
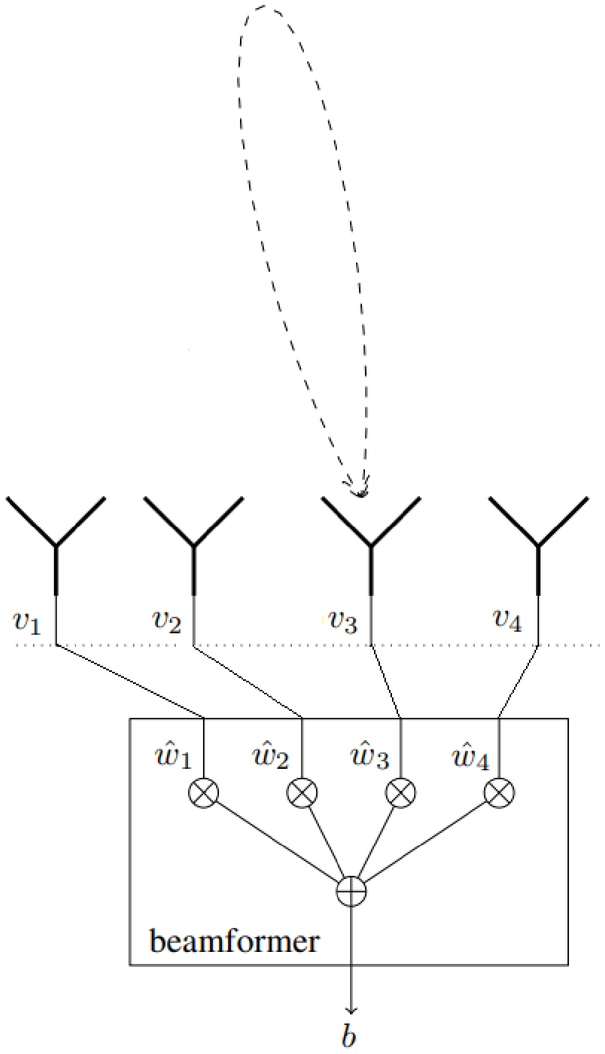
Station beam





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Station beam

The voltage at the output of the beamformer is given by

$$b = \sum_{i=1}^{N_{ant}} \hat{w}_i v_i$$

$\hat{w}_i$ is the beamforming weight applied to antenna « i »

The voltage received by antenna "i" is

$$v_i = c_i \vec{\mathbf{F}}_i(\theta, \phi) \, . \, \vec{\mathbf{E}}(\theta, \phi)$$

So, the station beam voltage is simply

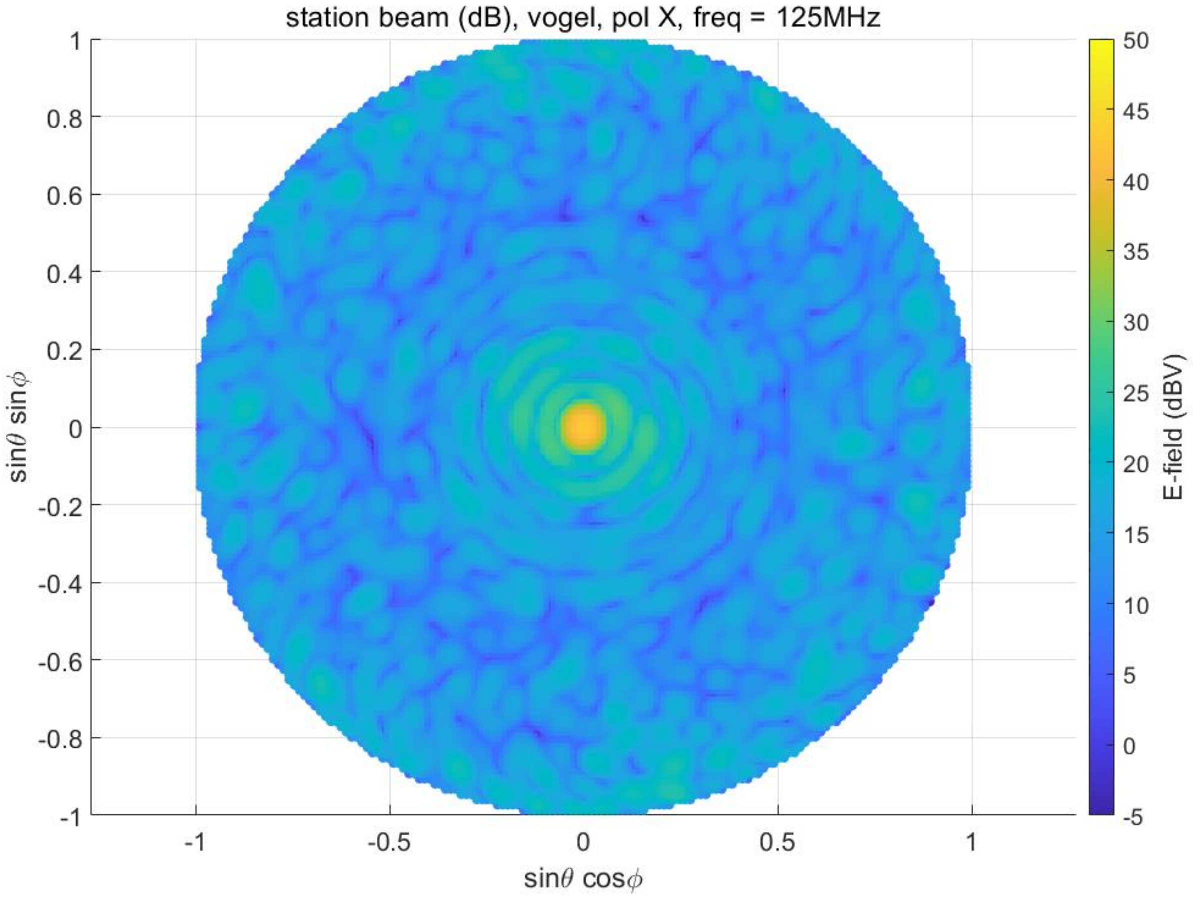
$$b = \vec{\mathbf{P}}(\theta, \phi) \, . \, \vec{\mathbf{E}}(\theta, \phi)$$

The Array Pattern (AP) is defined as

$$\vec{\mathbf{P}}(\theta, \phi) = \sum_{i=1}^{N_{ant}} \hat{w}_i c_i \vec{\mathbf{F}}_i(\theta, \phi) e^{-jk(\sin\theta\cos\phi x_i + \sin\theta\cos\phi y_i)}$$

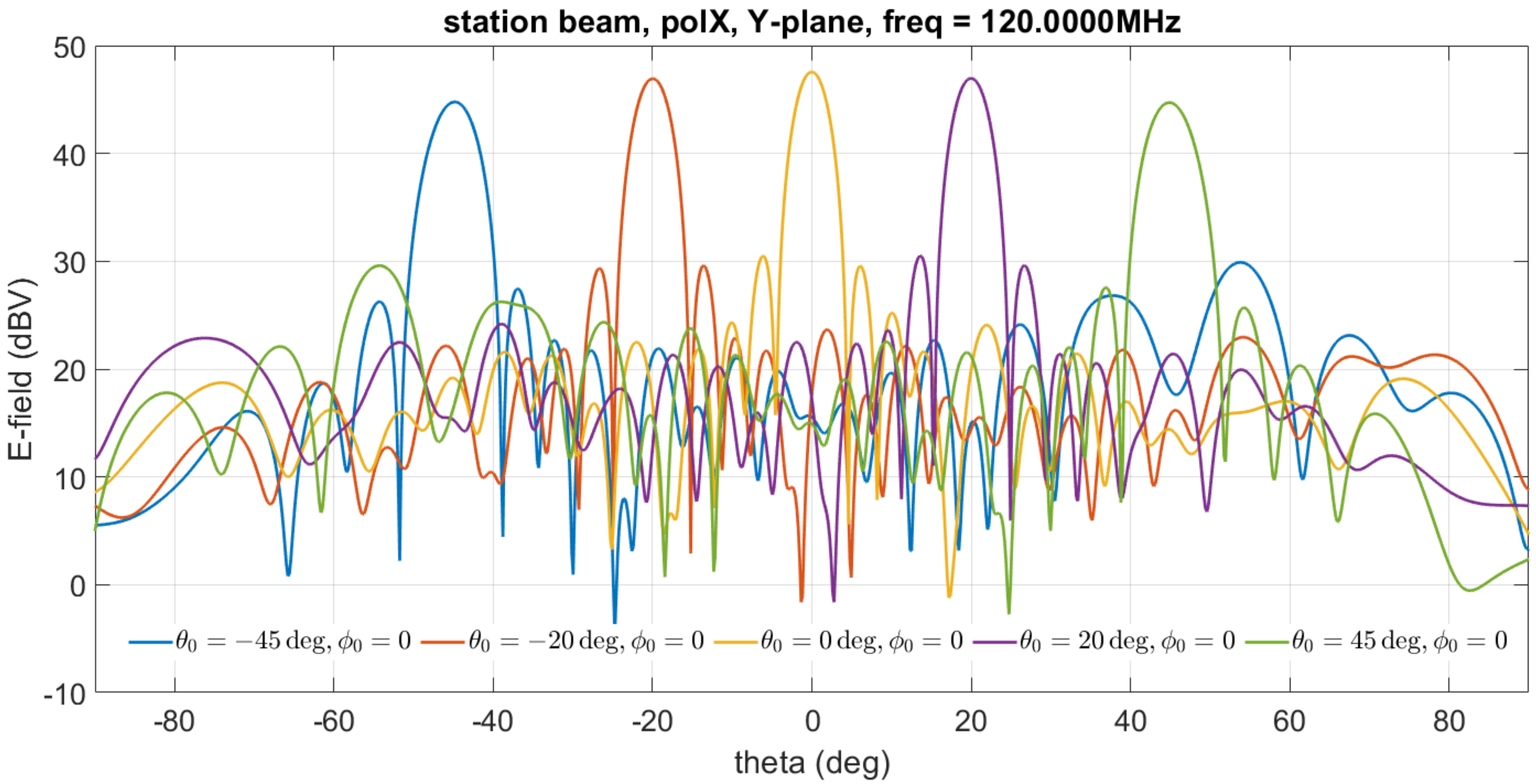
x_i, y_i are the position of antenna « i »

Array Pattern example



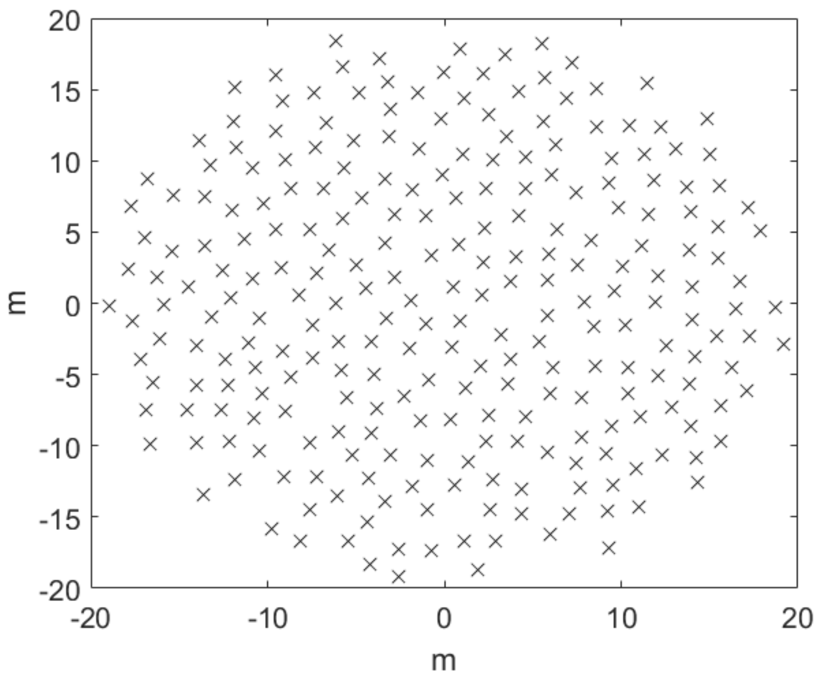
Beam scanning

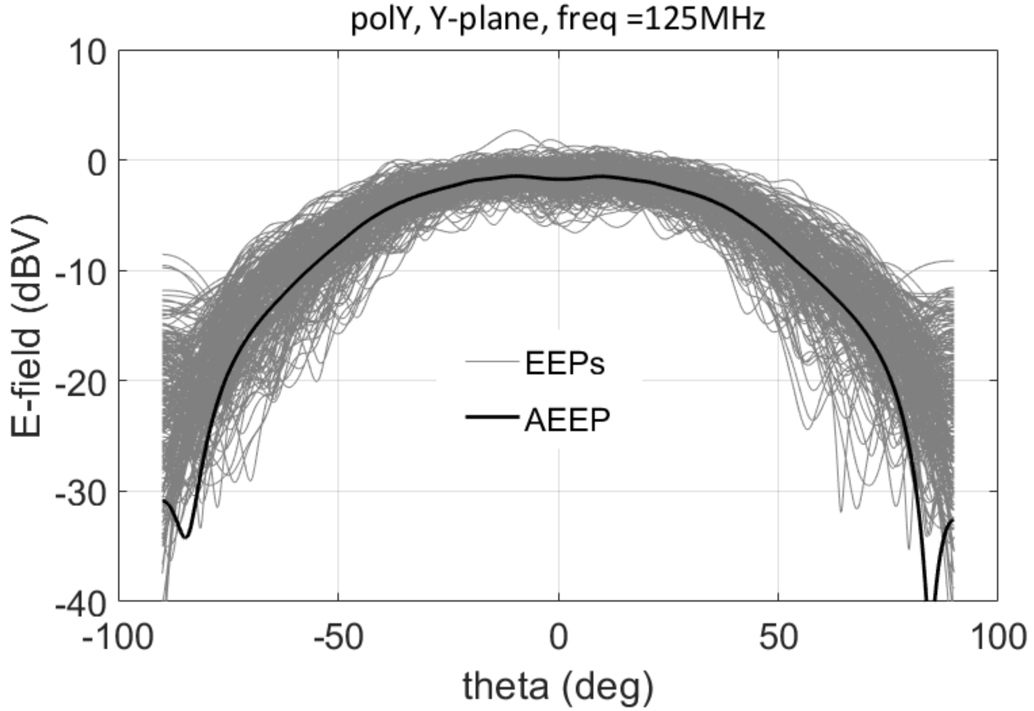
$$\hat{w}_i = e^{jk(\sin\theta_0\cos\phi_0 x_i + \sin\theta_0\sin\phi_0 y_i)}$$



Embedded Element Patterns

$\vec{\mathbf{F}}_i(\theta, \phi)$ is called the *Embedded Element Patterns* (EEP) of antenna "i" and is computed by considering the antenna "i" as active and all the other antennas as passively terminated.





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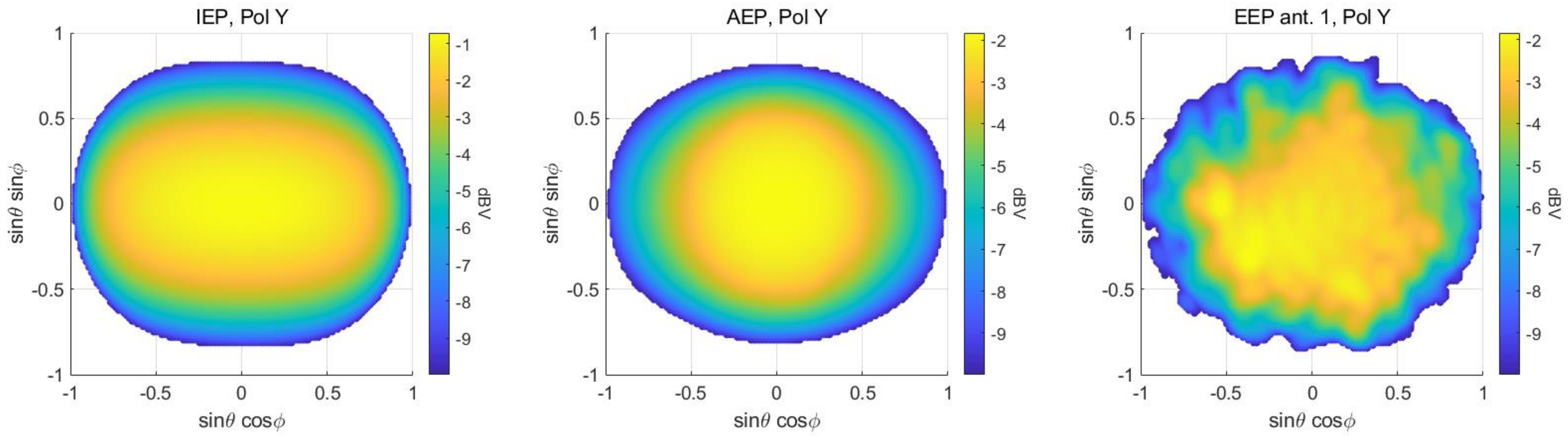
SKA-low, IEP vs AEP vs EEP

Approx 1: Isolated Element Pattern (IEP)

MC is *negligible*, $\vec{\mathbf{F}}_i(\theta,\phi) \approx \vec{\mathbf{F}}_{iso}(\theta,\phi)$

Approx 2: Average Element Pattern (AEP)

MC *averages out*, $\vec{\mathbf{F}}_{av}(\theta,\phi) \approx \frac{1}{\mathrm{N_{ant}}} \sum_{i=1}^{N_{ant}} c_i \vec{\mathbf{F}}_i(\theta,\phi)$



Sunflower layout, 100 MHz

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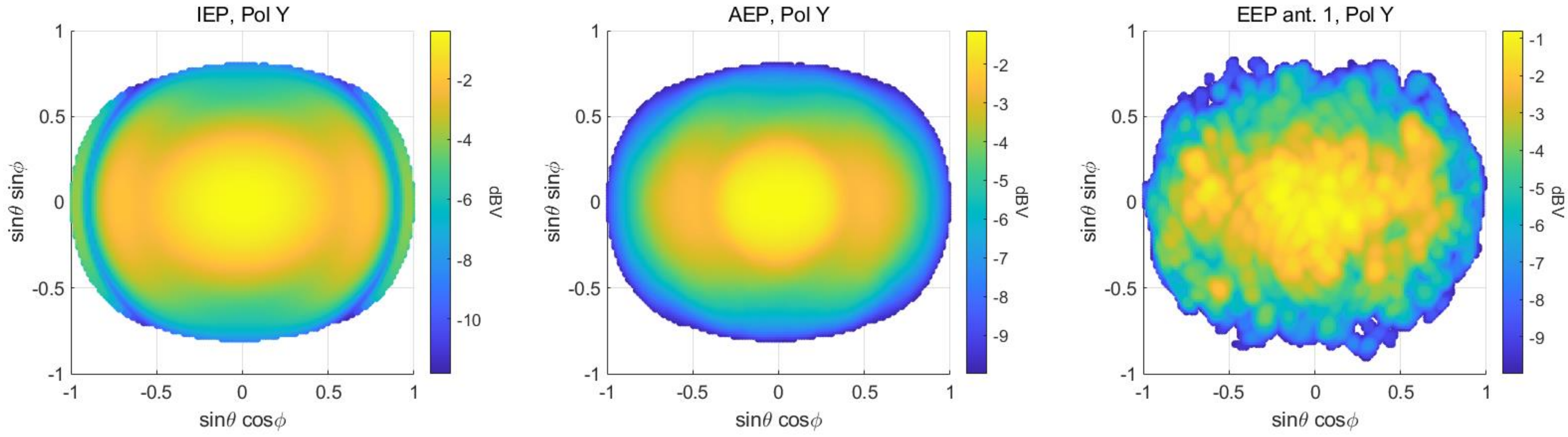
SKA-low, IEP vs AEP vs EEP

Approx 1: Isolated Element Pattern (IEP)

MC is *negligible*, $\vec{\mathbf{F}}_i(\theta,\phi) \approx \vec{\mathbf{F}}_{iso}(\theta,\phi)$

Approx 2: Average Element Pattern (AEP)

MC *averages out*, $\vec{\mathbf{F}}_{av}(\theta,\phi) \approx \frac{1}{\mathrm{N_{ant}}}\sum_{i=1}^{N_{ant}} c_i \vec{\mathbf{F}}_i(\theta,\phi)$



sunflower layout, 200 MHz

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SKA-low, EEP ripples

Radiation patterns $\vec{\mathbf{F}}$ are obtained from current distribution $\vec{\mathbf{J}}$

$\hat{\mathrm{k}} = \sin\theta\cos\phi\,\hat{x} + \sin\theta\sin\phi\,\hat{y} + \cos\theta\hat{z}$ is a unit direction over the sky

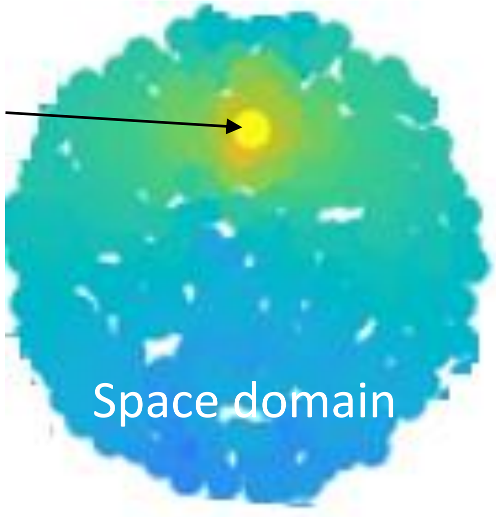
$$\vec{\mathbf{F}}(\hat{\mathbf{k}}) = -\frac{j\eta}{2\lambda}\iint\left(\vec{\mathbf{J}}(\vec{\mathbf{r}}) - (\vec{\mathbf{J}}(\vec{\mathbf{r}})\cdot\hat{\mathbf{k}})\hat{\mathbf{k}}\right)e^{jk\hat{\mathbf{k}}\cdot\mathrm{r}}\,dS$$

S is the surface of every antennas. $\vec{\mathbf{r}}$ is a position of a point on the surface S

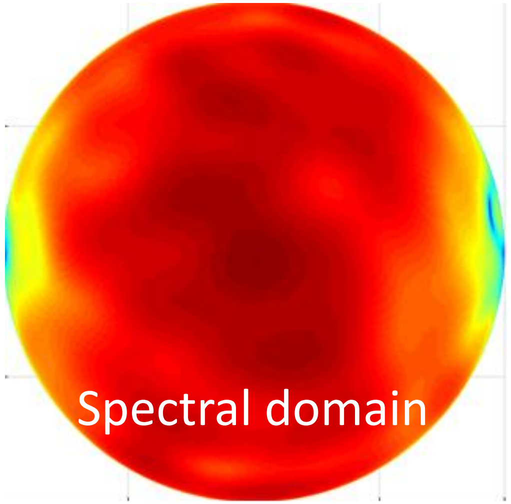
Currents $\vec{\mathbf{J}}$ induced on every antenna in the array when one element is active !

EEP $\vec{\mathbf{F}}$ is a bandlimited function!
(in theory …)

Active element



Fourier transform



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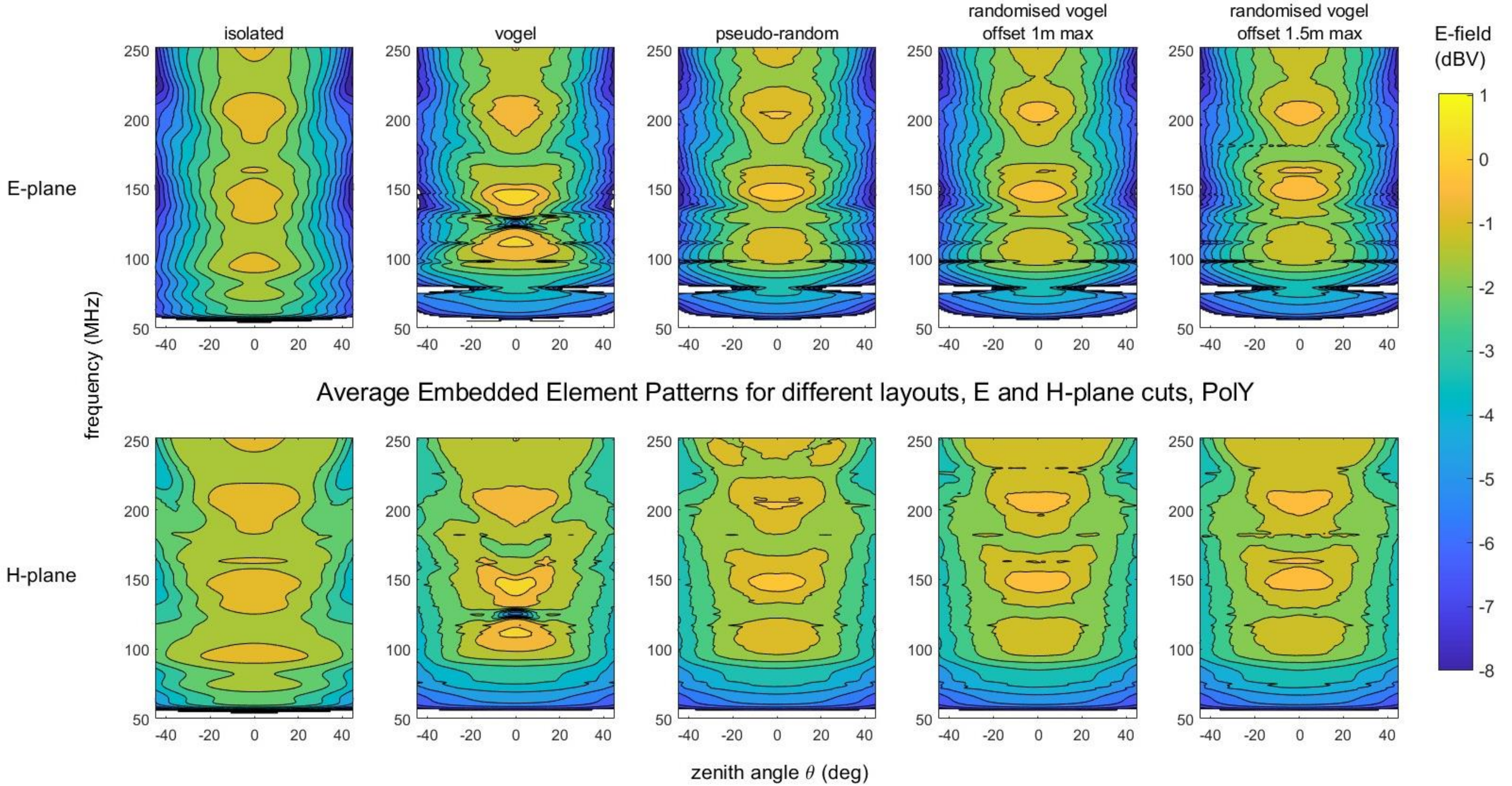
Average Element Pattern (AEP) vs freq



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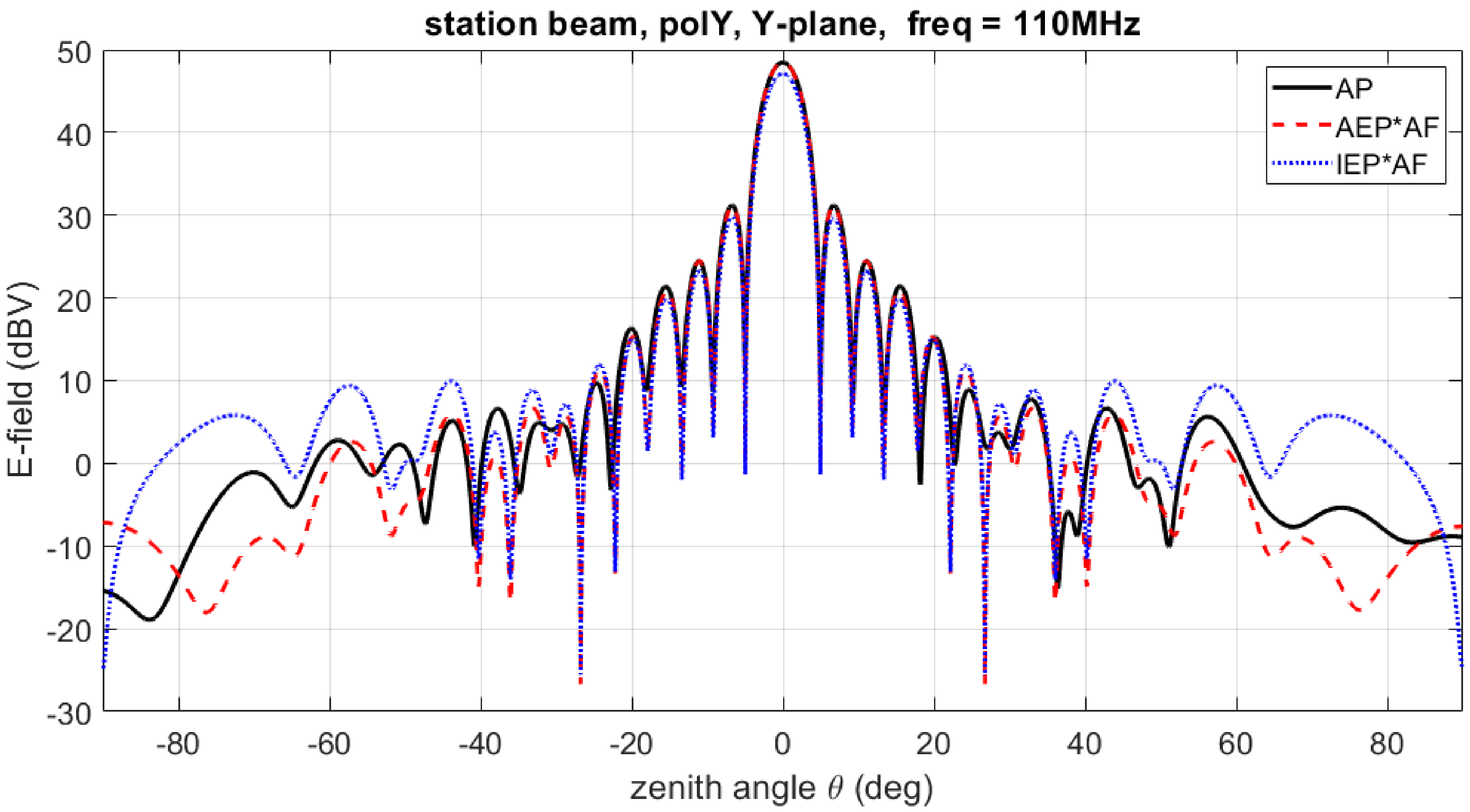
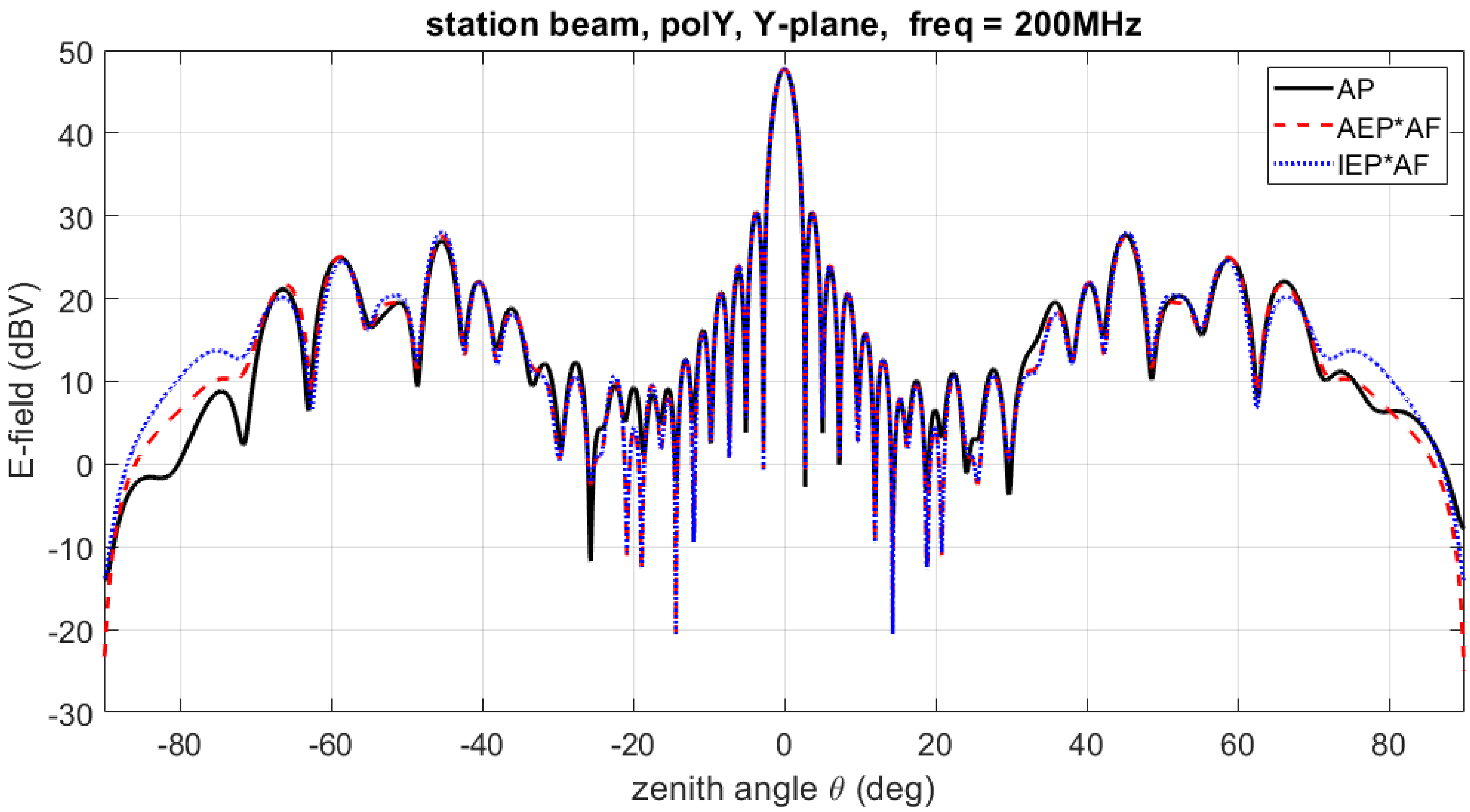
SKA-low, IEP vs AEP vs EEP



SKA-low, IEP vs AEP vs EEP



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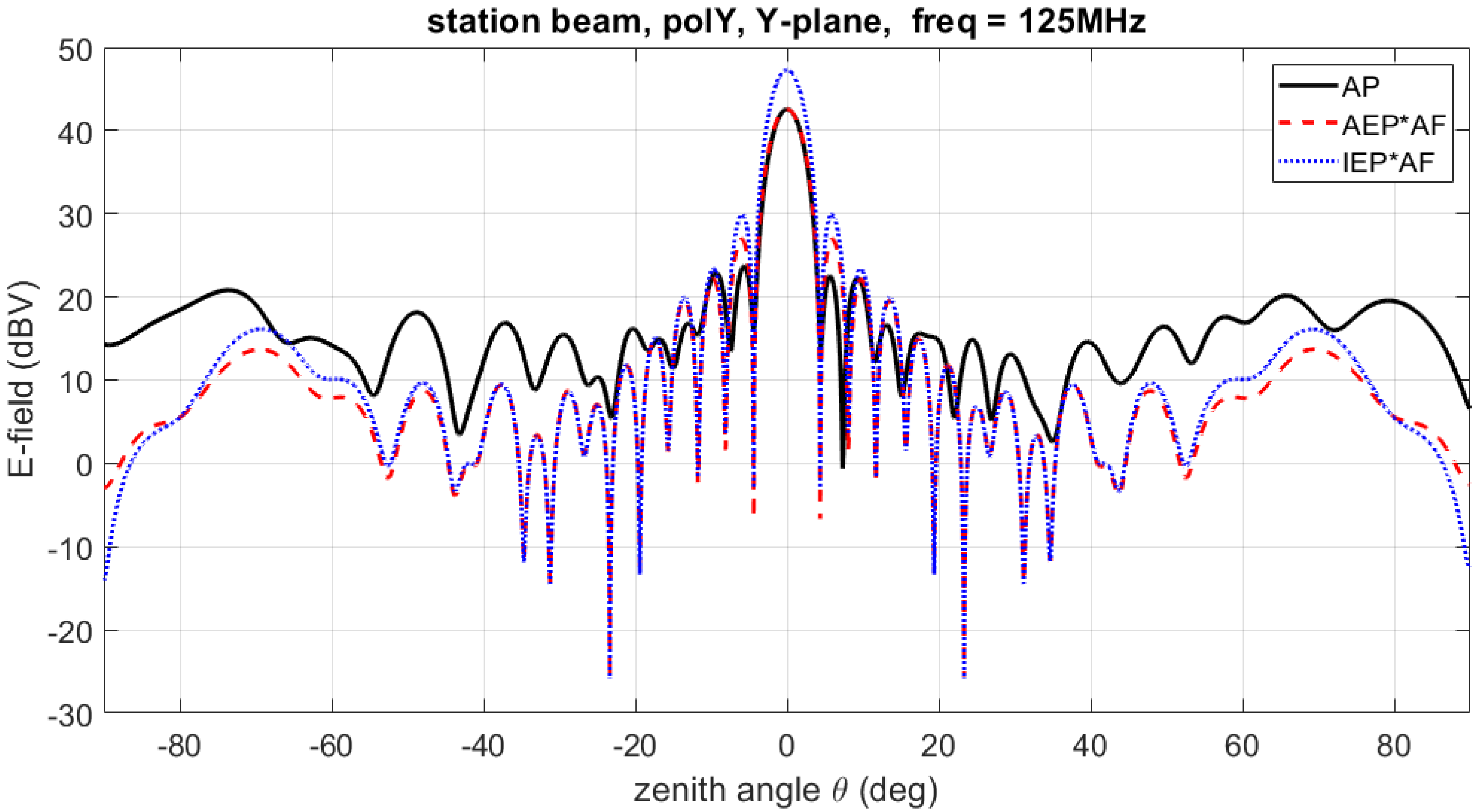
SKA-low, IEP vs AEP vs EEP



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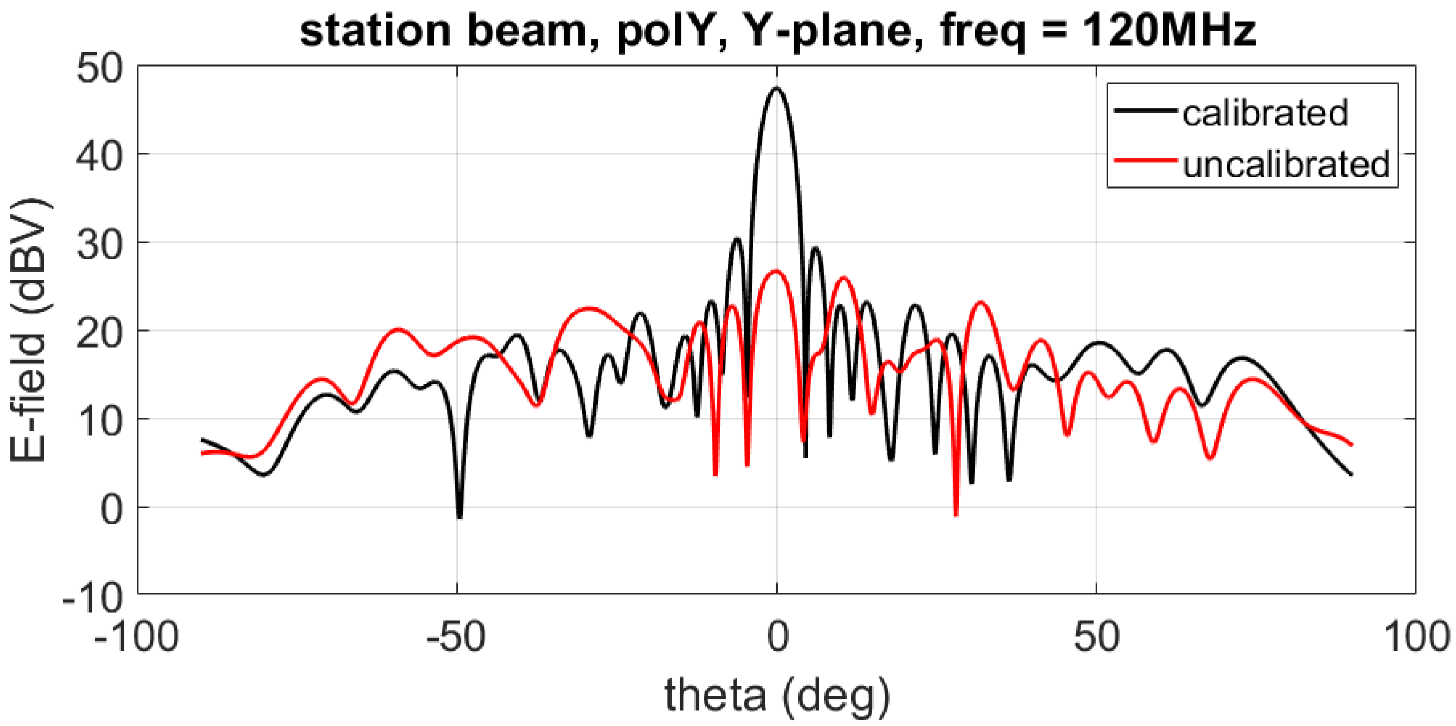
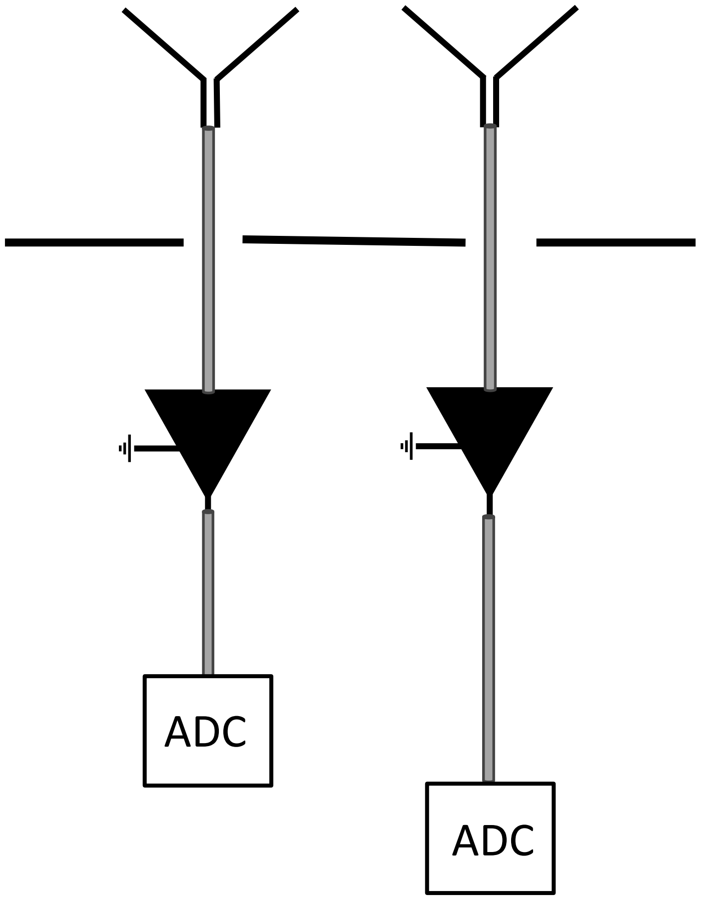
Without calibration





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Calibration Problem

The voltage just at the input of the digital converter is given by

$$v_i = G_i \vec{\mathbf{F}}_i(\theta, \phi) \,.\, \vec{\mathbf{E}}(\theta, \phi)$$

G_i is an unknown complex-valued direction-independent gain accounting for the propagation in the analog chain of each antenna

The intra-station visibilities are modelled by

$$\mathrm{V_{ij}} = <\mathrm{v_i(t) v_j^\star(t)}> = \mathrm{G_i G_j^\star} \iint \left(\vec{\mathbf{F}}_\mathrm{i}(\theta, \phi) \,.\, \vec{\mathbf{F}}_\mathrm{j}^\star(\theta, \phi) \right) \mathrm{T_{sky}}(\theta, \phi)\; \mathrm{e}^{-\mathrm{j}k(\sin\theta\cos\phi\, \mathrm{u_{ij}} + \sin\theta\cos\phi\, \mathrm{v_{ij}})} \sin\theta \, d\theta d\phi$$

$\mathrm{V_{ij}}$ is measured

$\mathrm{G_i G_j^\star}$ are the unknowns gains

$\vec{\mathbf{F}}_\mathrm{i}(\theta, \phi)$ are known beam models

$\mathrm{T_{sky}}$ is known from low resolution sky map

$u_{ij} = x_i - x_j$,
$v_{ij} = y_i - y_j$
are baseline coordinates

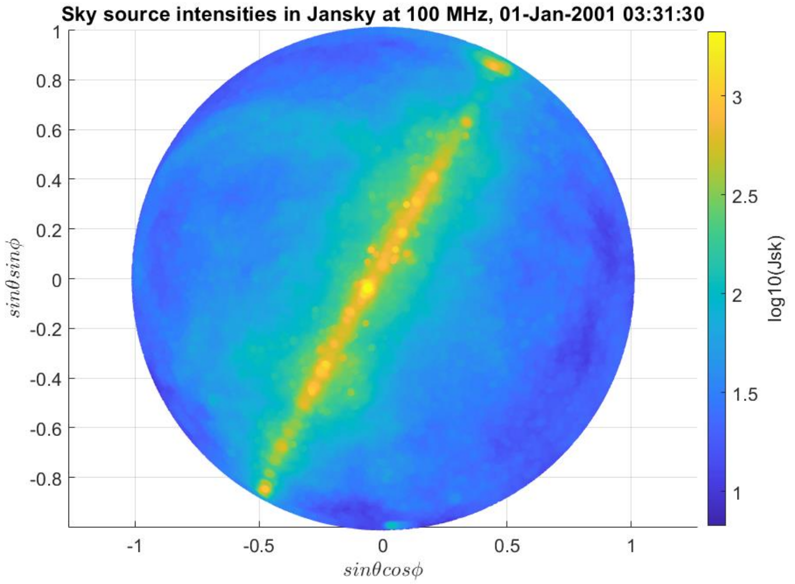
Calibration Problem

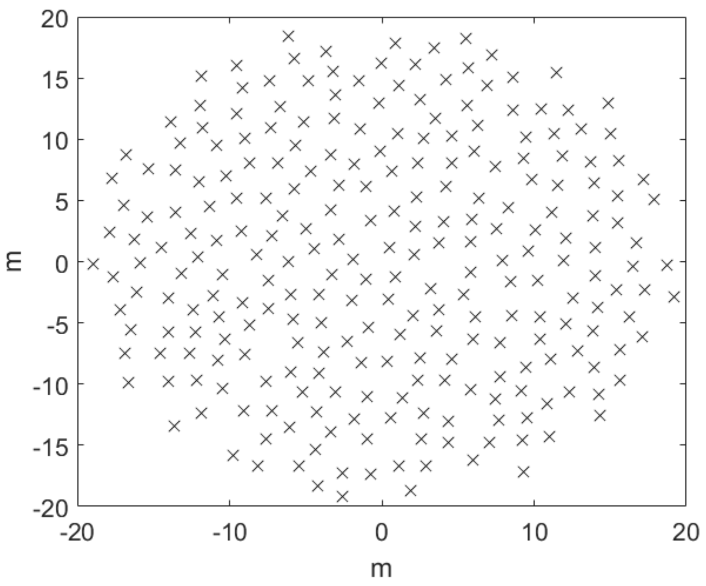
$\mathbf{V} = \mathbf{G}\,\mathbf{I}\,\mathbf{G}^{\star}$

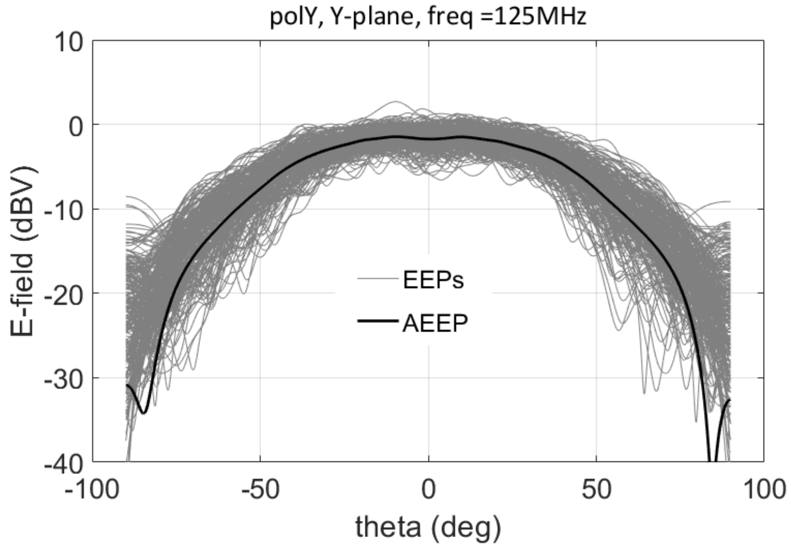
$\mathrm{N_{ant}}$ x $\mathrm{N_{ant}}$ matrix with measured visibilities

$\mathrm{N_{ant}}$ x $\mathrm{N_{ant}}$ diagonal matrix with gain unknowns

$\mathrm{N_{ant}}$ x $\mathrm{N_{ant}}$ matrix with modelled cross-correlations







S. Salvini and S.J. Wijnholds, “Fast gain calibration in radio astronomy using alternating direction implicit methods: Analysis and applications,” AA, 2014.

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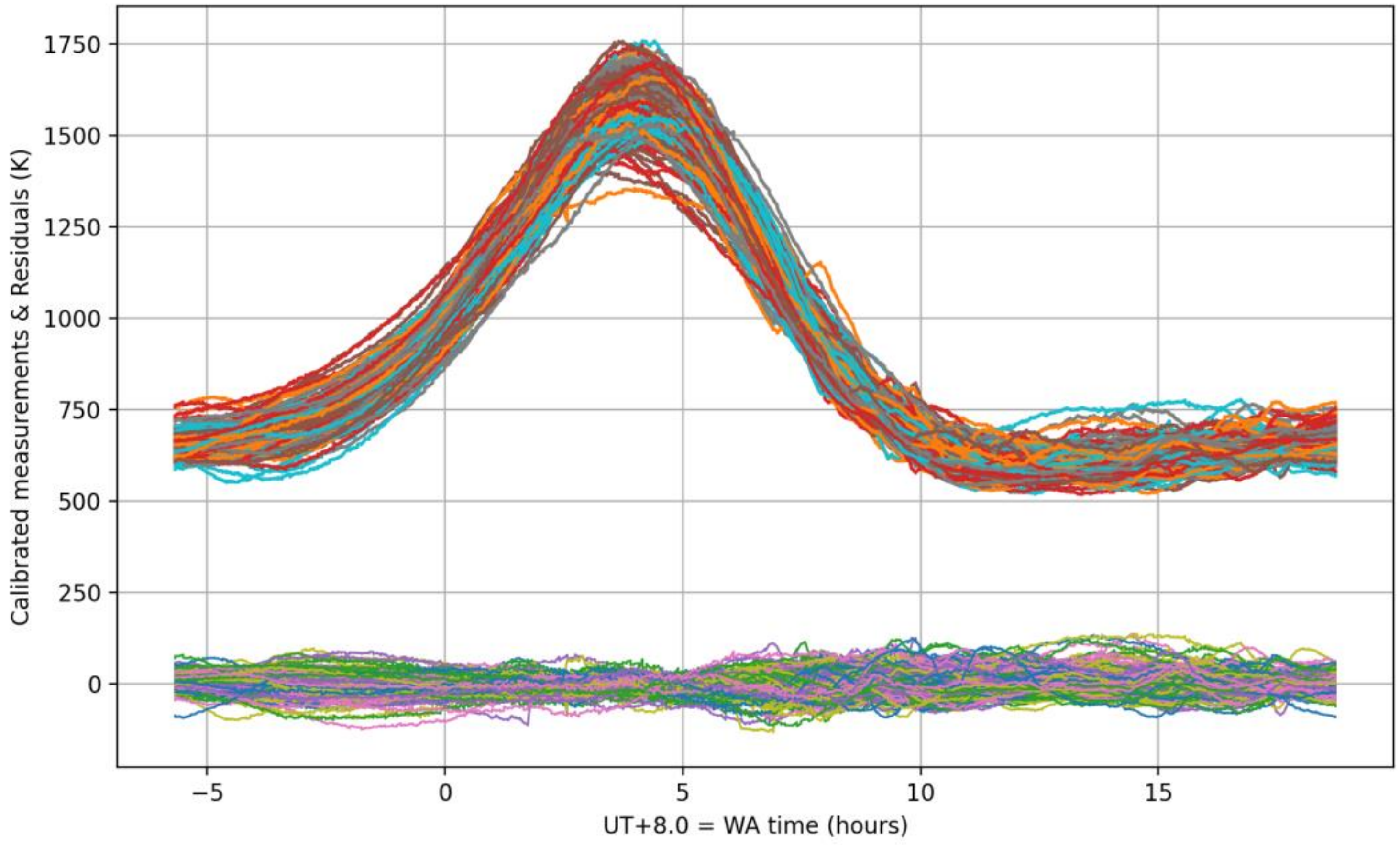
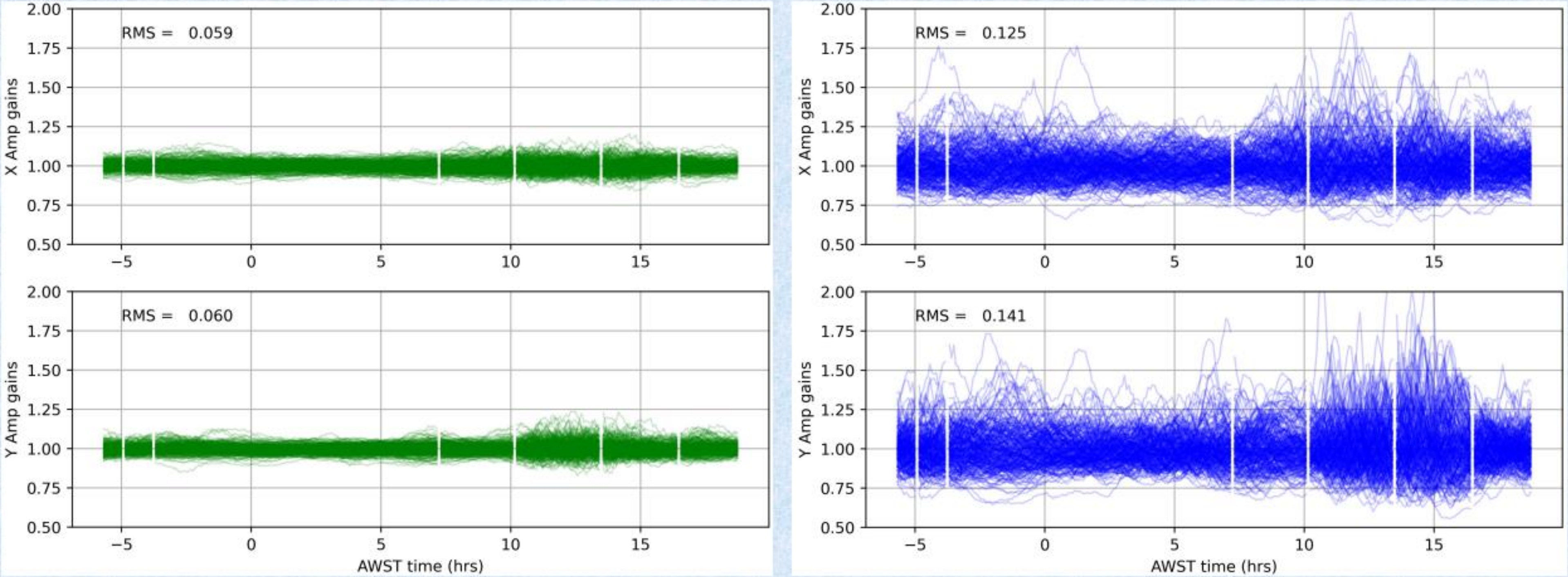
3. Calibration, SKA-low, AAVS2



R. Subrahmanyan CSIRO-CASS, Curtin-CIRA March 2021, *AAVS2 antenna total powers versus time at 110 MHz*

3. Calibration, SKA-low, AAVS2

Comparison of amplitude solutions when predictions use EEPs versus using average beam pattern:



R. Subrahmanyan CSIRO-CASS, Curtin-CIRA March 2021, *AAVS2 antenna total powers versus time at 110 MHz*

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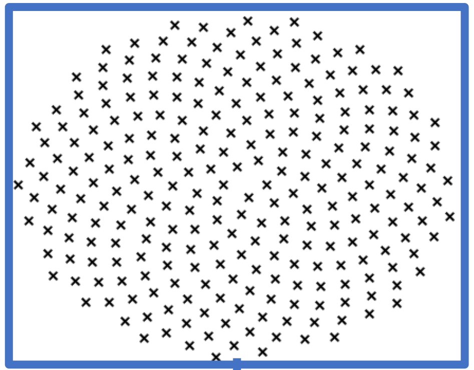
1. System design, SKA-low

16 SKALA2, Cambridge

256 SKALA4.1, Australia



SKALA4.1, new layout ?

AAVS0 (2012) → AAVS1 (2016) → AAVS2 (2019) → SKA64 (2021) → AAVS3 (2024?)

256 SKALA2, Australia

64 SKALA4, Cambridge

39

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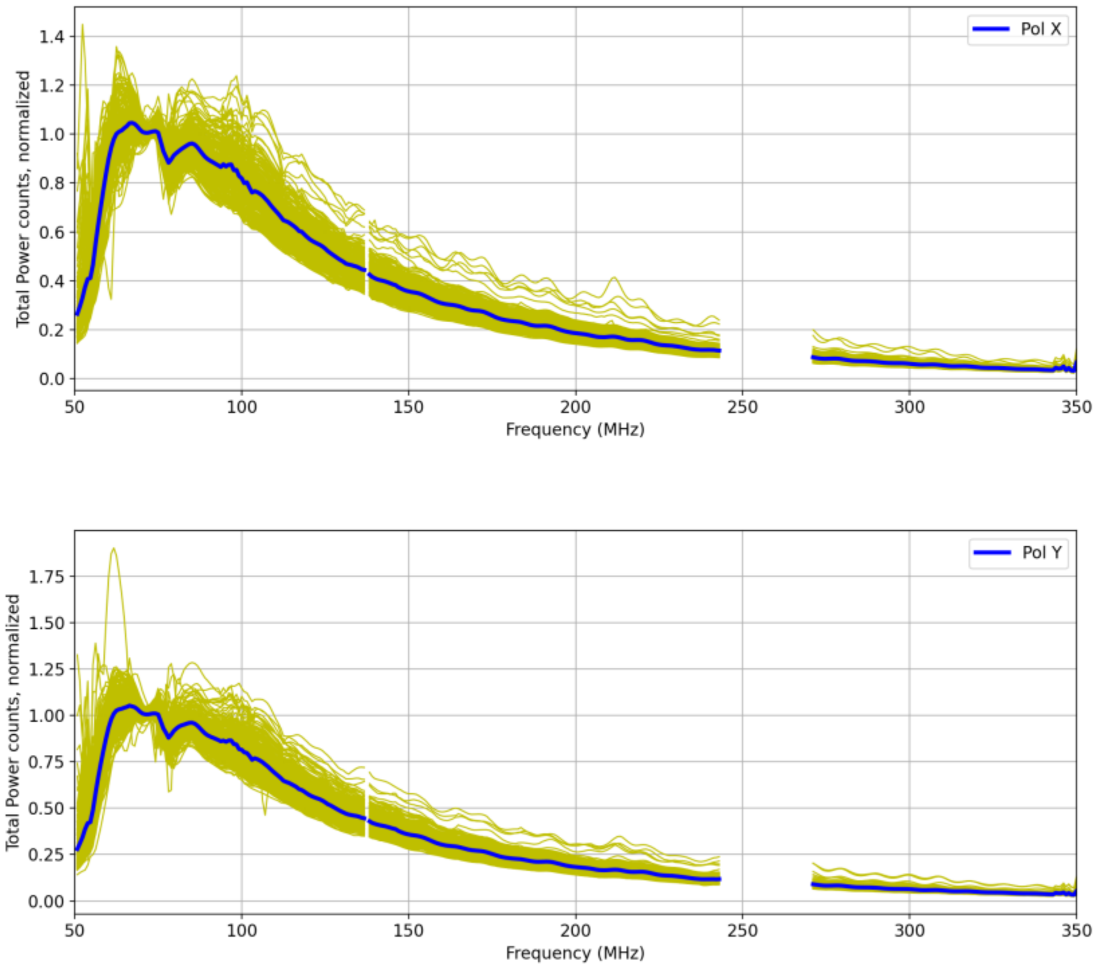
1. System design , SKA-low

$$P_r = \frac{1}{2} k_B B \iint T_{sky}(\Omega)\, A_{eff}(\Omega)\, d\Omega$$

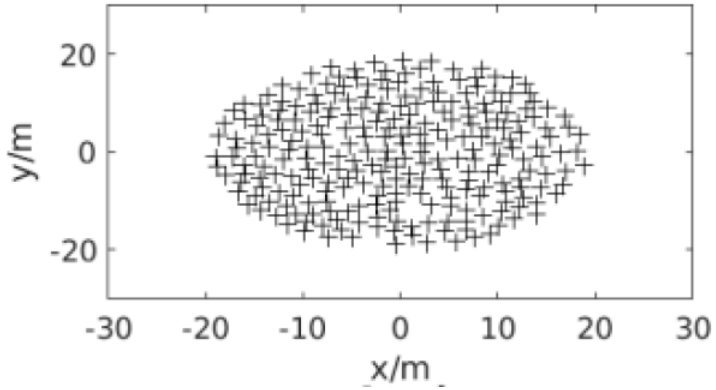
Band-pass measurements, AAVS2, April 2021

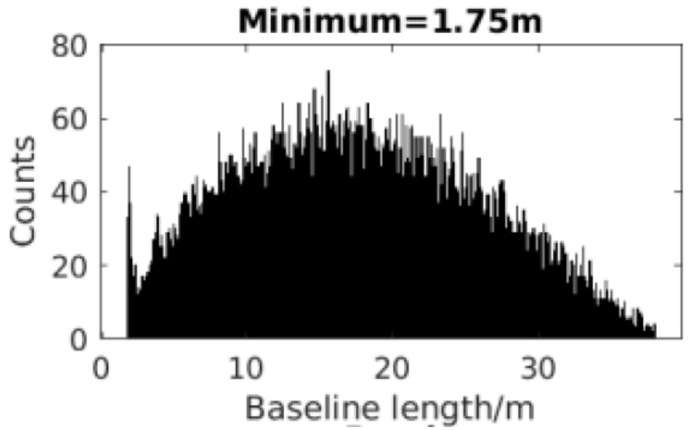


Ravi Subrahmanyan CSIRO-CASS, Curtin-CIRA April 2021, *Analysis of total power spectra from AAVS2 antennas*

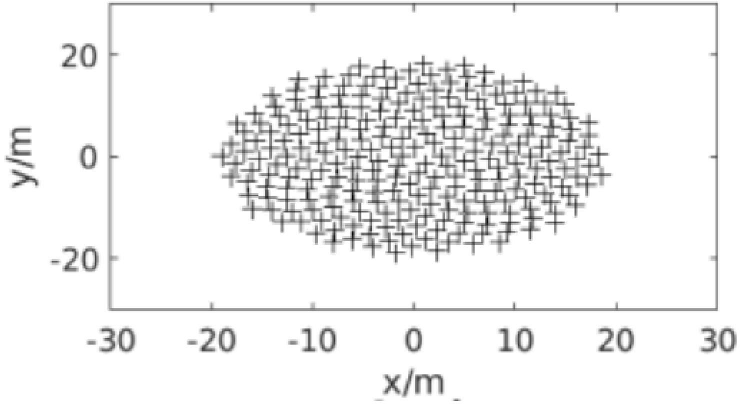
1. System design, SKA-low

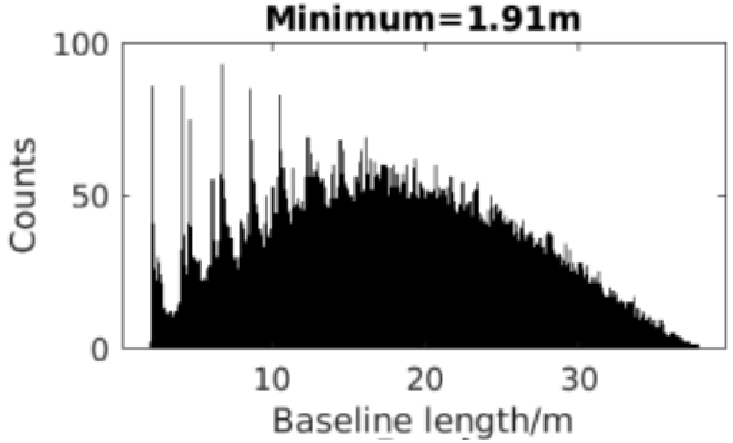
Pseudo Random





Vogel





D. Anstey, et al., Mitigating zenith blindness in

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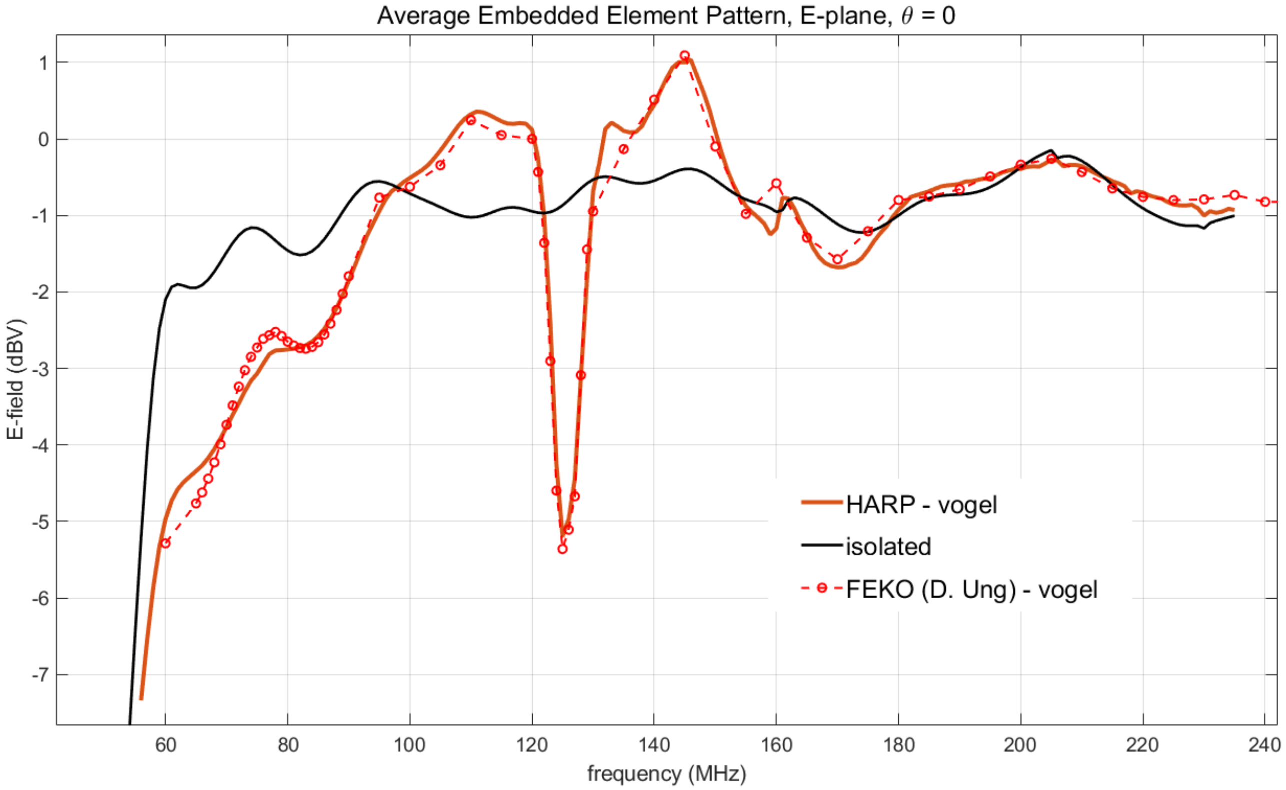
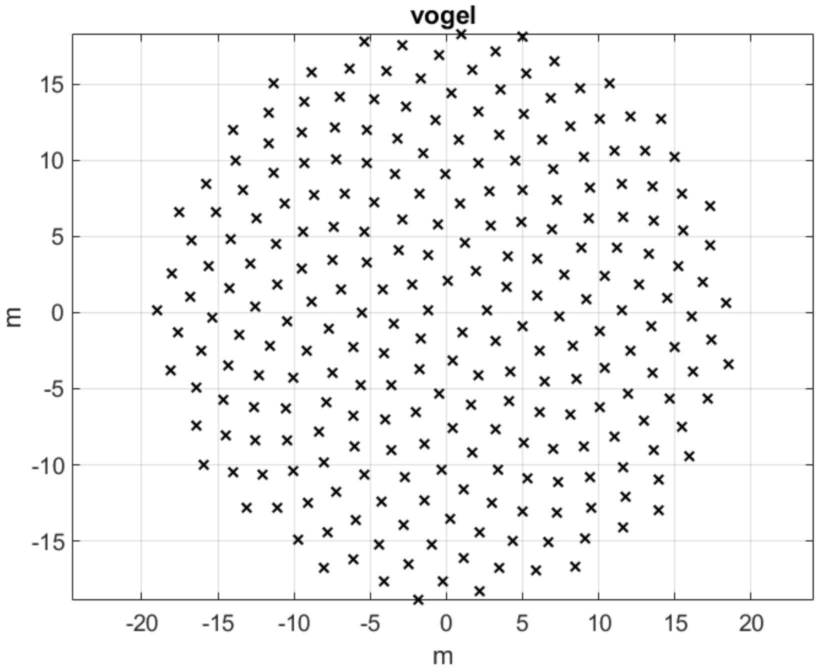
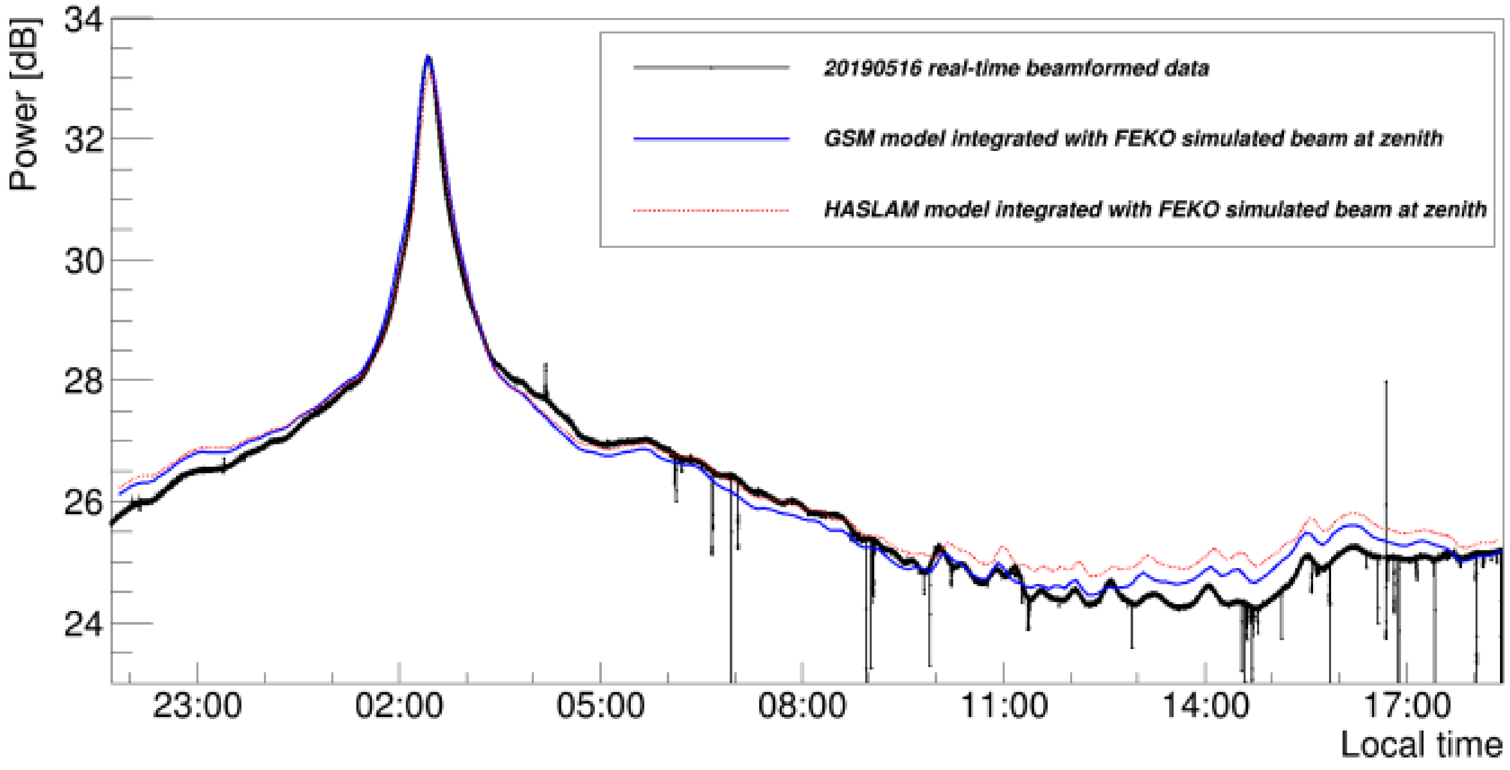
1. System design, SKA-low





3. Calibration, SKA-low, AAVS1



P. Benthem, et al., *The Aperture Array Verification System 1: System overview and early commissioning results*, A&A, 2021.

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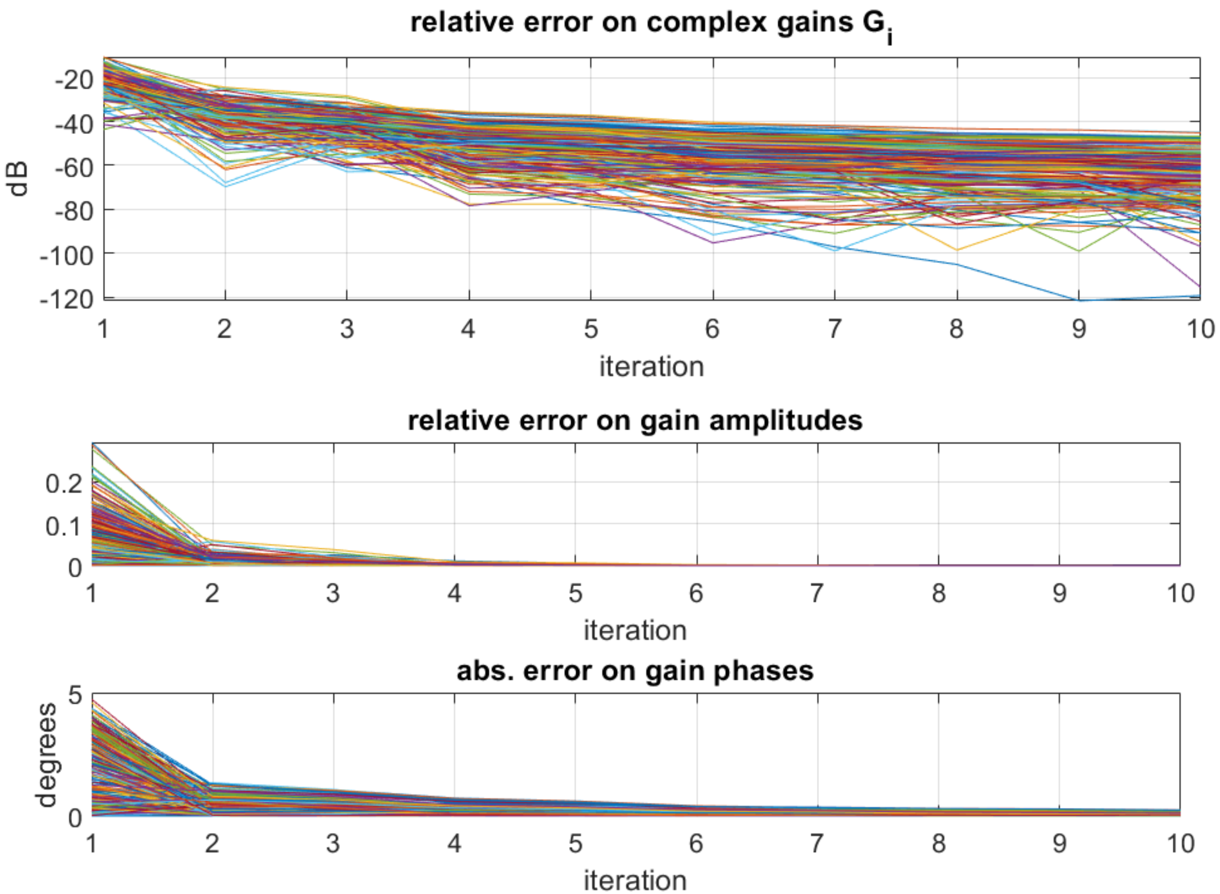
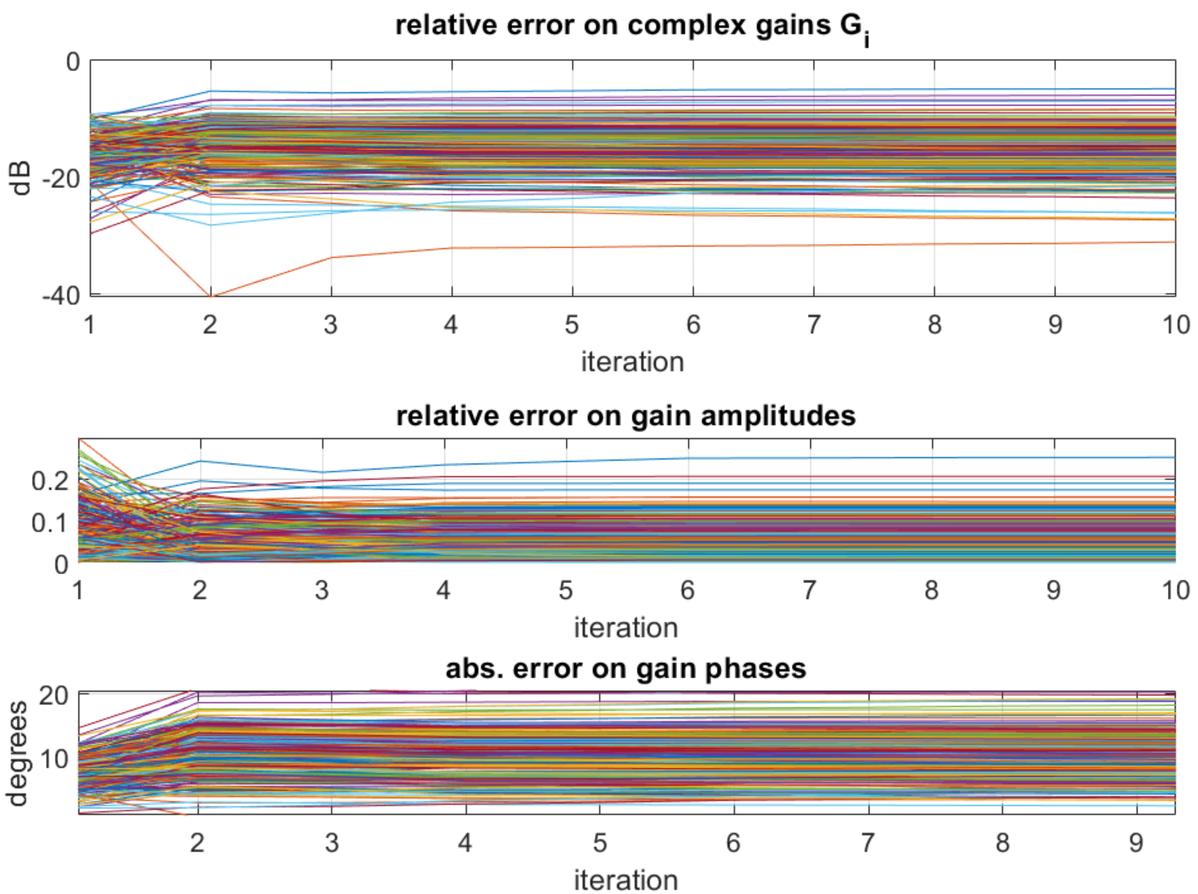
Calibration iterative solution

With all the EEPs



With the AEP



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Calibration solution, example