Advanced Quantum Physics

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Abstract

Abstract of this course

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$$E_n \simeq E_n^{(0)} + \left\langle n^{(0)} \middle| \hat{H}^{(1)} \middle| n^{(0)} \right\rangle$$

$$|n\rangle \simeq \left|n^{(0)}\right\rangle + \sum_{m \neq n} \left|m^{(0)}\right\rangle \frac{\left\langle m^{(0)} \middle| \hat{H}^{(1)} \middle| n^{(0)} \right\rangle}{E_n^{(0)} - E_m^{(0)}}$$

2.3 Second-order Perturbation Theory

$$E_n^{(2)} = \sum_{m \neq n} \frac{\left|\left\langle m^{(0)} \middle| \hat{H}^{(1)} \middle| n^{(0)} \right\rangle\right|^2}{E_n^{(0)} - E_m^{(0)}}$$

Example: Infinite square well with central bump

... **Example: Infinite square well in an electric field**

... **Example: Harmonic Oscillator + Linear perturbation**

... **Example: Van der Waals Interaction**

...

2.4 Degenerate Perturbation Theory

Example: Perturbed 2D infinite square well

...

2.5 Variation Method

Example: Hydrogen atom ground state energy

...

2.5.1 Rayleigh-Ritz Method

Example: Hydrogen atom with finite proton mass

...

3 Wigner-Eckart theorem

Let $\hat{K}$ be a scalar operator with respect to an angular momentum operator $\hat{\boldsymbol{J}}$:

$$[\hat{\boldsymbol{J}}, \hat{K}] = 0$$

Then the Wigner-Eckart theorem states that matrix elements of $\hat{K}$ taken between total angular momentum eigenstates $|\alpha'' j'' m''\rangle$ and $|\alpha' j' m'\rangle$ must be of the form

$$\left\langle \alpha'' j'' m'' | \hat{K} | \alpha' j' m' \right\rangle = C\left(\alpha'' \alpha'; j'\right) \delta_{j'' j'} \delta_{m'' m'}$$

where $C(\alpha''\alpha'; j')$ is a complex constant known as the reduced matrix element which is independent of the quantum numbers m'' and m'. The quantities α'' and α' collectively label all other quantum numbers needed to uniquely identify the angular momentum eigenstates involved.

3.1 Consequence of Wigner-Eckart theorem (Scalar Hamiltonian)

Since $\hat{H}$ and $\hat{\boldsymbol{J}}$ commute, they possess a simultaneous set of eigenstates $|\alpha jm\rangle$, where " α " represents all other quantum numbers needed to uniquely identify a particular energy eigenstate of $\hat{H}$. From the Wigner-Eckart theorem, the matrix elements of $\hat{H}$ between these eigenstates must be of the form

$$\left\langle \alpha'' j'' m'' | \hat{H} | \alpha' j' m' \right\rangle = \left\langle \alpha'' j'' \| \hat{H} \| \alpha' j' \right\rangle \delta_{j''j'} \delta_{m''m'}$$

where the reduced matrix element $\left\langle \alpha'' j'' \| \hat{H} \| \alpha' j' \right\rangle$ is a constant, independent of the quantum number m. In particular, the expectation values of $\hat{H}$ are

$$\langle \alpha jm | \hat{H} | \alpha jm \rangle = \langle \alpha j \| \hat{H} \| \alpha j \rangle$$

The Wigner-Eckart theorem for scalar operators, Equation, to be written in the form

$$\left\langle \alpha'' j'' m'' | \hat{K} | \alpha' j' m' \right\rangle = \left\langle \alpha'' j'' || \hat{K} \| \alpha' j' \right\rangle \langle 00; j'm' \mid j''m'' \rangle .$$

The reason for writing the Wigner-Eckart theorem in this way will become clear once we have also considered the equivalent result for vector operators.

3.2 consequence of Wigner-Eckart theorem (Vector Operators)

Then the Wigner-Eckart theorem states that the matrix elements of $\hat{\boldsymbol{V}}$ between eigenstates $|\alpha' j' m'\rangle$ and $|\alpha'' j'' m''\rangle$ of $\hat{\boldsymbol{J}}$ must be of the form

$$\left\langle \alpha'' j'' m'' \left| \hat{V}_m \right| \alpha' j' m' \right\rangle = \left\langle \alpha'' j'' || \hat{\boldsymbol{V}} \| \alpha' j' \right\rangle \langle 1m; j'm' \mid j''m'' \rangle ,$$

where the final factor, $\langle 1m; j'm' \mid j''m'' \rangle$, is a Clebsch-Gordan coefficient which can be obtained by considering the angular momentum combination $j'' = 1 \otimes j' = j', j' \pm 1$

4 Electromagnetism

Definition cyclotron frequency:

$$\omega_c = \frac{qB}{m}$$

This is the frequency with which particles moving transverse to a magnetic filed B undergo circular orbit.
ω_c should be really close to the spin precesssion frequency ω_s.

4.1 Aharanov-Bohm Effect

4.2 Gauge Invariance

4.2.1 Couloumb Gauge

4.2.2 Symmetric Gauge

4.3 Orbital Magnetic moment

In Hamiltonian, the $L \cdot B$ term can be written as: $\hat{H} = -\hat{\mu}_L \cdot B$ *Definition Orbital magnetic moment operator:*

$$-\hat{\mu}_L = \frac{q}{2m}\hat{L}\gamma_L$$

... *Definition Gyromagnetic ratio, γ_L:*

$$\gamma_L = \frac{q}{2m}$$

... For an electron (q=-e), the orbital magnetic moment operator is

4.4 Magnetic Moments

4.4.1 Electron

4.4.2 Muon

4.4.3 p, n, nuclei

4.5 Spin

4.5.1 Particle magnetic moment: spin-half

4.5.2 Spin Precession

4.5.3 Spin-half

4.5.4 Energy Eigenstates

4.5.5 Wave-function Evolution

4.6 Stern-Gerlach

4.7 Landau Levels

4.7.1 Landau Gauge

Example: 2D Electron Gas

5 Real Hydrogen Atom

5.1 Relativistic Corrections

5.2 Fine Structure

5.3 Hyperfine Structure

6 Symmetries

6.1 Symmetry Transformation

6.1.1 Time translation

Take way: The time-dependent Shrodinger equation is a consequence of the invariance under time transformation

6.2 The Wigner-Eckart Theorem(selection rule)

Wigner-Eckart Theorem: $\langle\alpha''j''m''|\,\hat{K}\,|\alpha'j'm'\rangle = \langle\alpha''j''|\,|\hat{K}|\,|\alpha'j'\rangle$
A particular case is that $\langle\alpha jm|\,\hat{K}\,|\alpha jm\rangle = \langle\alpha j|\,|\hat{K}|\,|\alpha j\rangle$

i.e. the expectation values of a scalr opertator are independent of m and are given by the appropriate reduced matrix element of K

6.3 Combining magnetic moment

7 Identical Particles

Identical particles are indistinguishable.

1. The normalized two-particle wavefunction $\psi(x_1, x_2)$, which gives the probability $|\psi(x_1, x_2)|^2 dx_1 dx_2$ of finding simultaneously one particle in the interval x_1 to $x_1 + dx_1$ and another between x_2 to $x_2 + dx_2$,
2. only makes sense if $|\psi(x_1, x_2)|^2 = |\psi(x_2, x_1)|^2$, since we can't know which of the two indistinguishable particles we are finding where.
3. It follows from this that the wavefunction can exhibit two (and, generically, only two) possible symmetries under exchange: $\psi(x_1, x_2) = \psi(x_2, x_1)$ or $\psi(x_1, x_2) = -\psi(x_2, x_1)$.[2]
4. f two identical particles have a symmetric wavefunction in some state, particles of that type always have symmetric wavefunctions, and are called bosons.Similarly, particles having antisymmetric wavefunctions are called fermions.
5. We could achieve the necessary antisymmetrization for particles 1 and 2 by subtracting the same product wavefunction with the particles 1 and 2 interchanged, i.e. $\psi_a(1)\psi_b(2)\psi_c(3) \mapsto (\psi_a(1)\psi_b(2) - \psi_a(2)\psi_b(1))\,\psi_c(3)$, ignoring the overall normalization for now. Such a sum over permutations is precisely the definition of the determinant. So, with the appropriate normalization factor:
$$\psi_{abc}(1,2,3) = \frac{1}{\sqrt{3!}} \begin{vmatrix} \psi_a(1) & \psi_b(1) & \psi_c(1) \\ \psi_a(2) & \psi_b(2) & \psi_c(2) \\ \psi_a(3) & \psi_b(3) & \psi_c(3) \end{vmatrix}$$

7.1 Space and spin

7.2 Spin and statistics(fermions and bosons)

7.3 Exchange forces

7.4 The Helium atom

8 Multi-electron atoms

8.1 Periodic table

8.2 LS coupling(Hund's rule)

8.3 jj coupling

9 Zeeman effect/Stark effect/Molecules: H_2^+ and H_2

10 Time-dependent perturbation

11 Fermi Golden Rule