

# Advanced Quantum Physics Notes

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## 1 Revision

## 2 Perturbation Theory

### 2.1 Time-Independent Perturbation Theory

### 2.2 First-order Perturbation Theory

### 2.3 Second-order Perturbation Theory

Example: Infinite square well with central bump

...

Example: Infinite square well in an electric field

...

Example: Harmonic Oscillator + Linear perturbation

...

Example: Van der Waals Interaction

...

### 2.4 Degenerate Perturbation Theory

Example: Perturbed 2D infinite square well

...

### 2.5 Variation Method

Example: Hydrogen atom ground state energy

...

#### 2.5.1 Rayleigh-Ritz Method

Example: Hydrogen atom with finite proton mass

...

## 3 Electromagnetism

*Definition cyclotron frequency:*

$$\omega_c = \frac{qB}{m}$$

This is the frequency with which particles moving transverse to a magnetic field  $B$  undergo circular orbit.

$\omega_c$  should be really close to the spin precession frequency  $\omega_s$ .

### 3.1 Aharanov-Bohm Effect

### 3.2 Gauge Invariance

#### 3.2.1 Coulomb Gauge

#### 3.2.2 Symmetric Gauge

### 3.3 Orbital Magnetic moment

In Hamiltonian, the  $L \cdot B$  term can be written as:  $\hat{H} = -\hat{\mu}_L \cdot B$  *Definition Orbital magnetic moment operator:*

$$-\hat{\mu}_L = \frac{q}{2m} \hat{L} \gamma_L$$

...

*Definition Gyromagnetic ratio,  $\gamma_L$ :*

$$\gamma_L = \frac{q}{2m}$$

...

For an electron ( $q=-e$ ), the orbital magnetic moment operator is

### 3.4 Magnetic Moments

#### 3.4.1 Electron

#### 3.4.2 Muon

#### 3.4.3 p, n, nuclei

### 3.5 Spin

#### 3.5.1 Particle magnetic moment: spin-half

#### 3.5.2 Spin Precession

#### 3.5.3 Spin-half

#### 3.5.4 Energy Eigenstates

#### 3.5.5 Wave-function Evolution

### 3.6 Stern-Gerlach

### 3.7 Landau Levels

#### 3.7.1 Landau Gauge

Example: 2D Electron Gas

## 4 Real Hydrogen Atom

### 4.1 Relativistic Corrections

### 4.2 Fine Structure

### 4.3 Hyperfine Structure

## 5 Symmetries

### 5.1 Symmetry Transformation

#### 5.1.1 Time translation

Take way: The time-dependent Shrodinger equation is a consequence of the invariance under time transformation

### 5.2 The Wigner-Eckart Theorem(selection rule)

**Wigner-Eckart Theorem:**  $\langle \alpha'' j'' m'' | \hat{K} | \alpha' j' m' \rangle = \langle \alpha'' j'' | \hat{K} | \alpha' j' \rangle$

A particular case is that  $\langle \alpha j m | \hat{K} | \alpha j m \rangle = \langle \alpha j | \hat{K} | \alpha j \rangle$

i.e. the expectation values of a scalar operator are independent of m and are given by the appropriate reduced matrix element of K

### 5.3 Combining magnetic moment

## 6 Identical Particles

### 6.1 Spin and statistics(fermions and bosons)

### 6.2 Exchange forces

### 6.3 The Helium atom

## 7 Multi-electron atoms

### 7.1 Periodic table

### 7.2 LS coupling(Hund's rule)

### 7.3 jj coupling

## 8 Zeeman effect/Stark effect/Molecules: $H_2^+$ and $H_2$