Notes

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Contents

Abstract

Abstract of this course

1 Spacetime Curvature

Definition Riemann Curvature Tensor:

$$R^d_{abc} = \nabla_a \nabla_b v_c - \nabla_b \nabla_a v_c$$

The reason for doing so is that covariant derivative of dual vector field is not commutative

1.1 Subsection 1

2 Gravitation field equations

2.1 Energy Momentum Tensor

Definition Energy Momentum Tensor:

$$T^{\mu\nu}(x) = \rho_0(x) u^\mu(x) u^\nu(x)$$

T^{00}: energy density
T^{i0}: i-th component of 3-momentum density (times c)
T^{ij}: flux of i-component of 3-momentum in j-direction

2.1.1 Properties of Energy-momentum tensor

- Always symmetric

2.2 Energy-momentum Tensor in Ideal Fluid

Consider Ideal Fluid:
We can find a local inertial frame so that $T^{i0} = 0$; and the spatial components are isotropic: $T^{ij} \propto \delta^{ij}$. Hence, in **instantaneous rest frame**:

$$T^{\mu\nu} = \mathrm{diag}(\rho c^2, p, p, p)$$

where ρc^2 is the rest frame energy density and p is isotropic pressure. While in general:

$$T^{\mu\nu} = (\rho + \frac{p}{c^2}) u^\mu u^\nu - p g^{\mu\nu} \tag{1}$$

Equation 1 is the energy momentum tensor, valid in any coordinate system.

2.2.1 Continuity Equation

The energy-momentum tensor satisfies continuity equation:

$$\nabla_\mu T^{\mu\nu} = 0$$

2.3 Einstein Field Equation

Definition Einstein Field Equation:

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\kappa T_{\mu\nu}$$

Constant proportionality and negative sign on the right is required for consistency with the weak field limit

3 The Schwarzchild Solution

Adopting a passive view point: change the coordinate system without changing the functional form of the fields on our coordinates.