

Notes

Xinyu Zhong
Wolfson College

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Abstract

Abstract of this course

1 Classical Fields

2 Symmetries and conservation laws

2.1 Noether's theorem

Noether's theorem: there is a **conserved current** associated with every continuous symmetry of the Lagrangian

2.2 Symmetries and conserved currents

2.3 Global phase symmetry

Consider the Klein-Gordon Lagrangian density for a complex field:

$$L_{KG} = \partial_\mu \psi^* \partial^\mu \psi - m^2 \psi^* \psi$$

We can then find the conserved Noether current, as well as the conserved charge

2.4 Local phase (gauge) symmetry

We now allow the phase change ϵ to be dependent on the space-time coordinates x^μ . We realise that electromagnetic fields/ covariant derivative is an essential requirement for a complex field to remain invariant under local phase transformation.

2.5 Electromagnetic interaction

Expanding the Klein-Gordon equation:

$$\begin{aligned} L_{KG} &= (D_\mu \psi)^* (D^\mu \psi) - m^2 \psi^* \psi \\ &= \partial_\mu \psi^* \partial^\mu \psi - m^2 \psi^* \psi + ie A_\mu [(\partial_\mu \psi)^* \psi - \psi (\partial_\mu \psi)] + e^2 A_\mu A^\mu \psi^* \psi \end{aligned}$$

We see that the third term being the interaction term $e A_\mu J^\mu$, where J^μ is the free-field current. The

2.6 Stress-energy tensor, angular momentum tensor

2.7 Quantum fields