1. Product Rule

↳ $d(XY) = Y\,dX + X\,dY + dX\,dY$ 증명.

Hint. 2-dimensional Itô-Doeblin formula.

2. $d(tW(t))$ 를 이용하여

$\int_0^t (t-s)\,dW(s) = \int_0^t W(s)\,ds$ 임을 보이고

$\int_0^t W(s)\,ds$ 의 분포과 분산을 구하시오.

Hint. Martingale 와 Itô Isometry

3. $dX(t) = -aX(t)\,dt + dW(t)$

$X(t) = ?$

Hint. $d(e^{at} \cdot X(t))$ 이용 생각.

4. $X(t) = ?$

(1) $dX(t) = dt + 2\sqrt{X(t)}\,dW(t)$

(2) $dX(t) = \frac{1}{3}X(t)^{\frac{1}{3}}\,dt + X(t)^{\frac{2}{3}}\,dW(t)$

(3) $dX(t) = \frac{1}{2}a^2m\,X(t)^{2m-1}\,dt + aX(t)^m\,dW(t)$

where m, a : constant 와 $m \neq 1$

Hint. Stratonovich Integral.

5.

$\int_0^t W(s)^2 \circ dW(s) = \frac{1}{3}W(t)^3$ 증명.

Hint. $d(W(t)^3) = ?$

Stratonovich integral 의 정의

6.

(1) $d(t^2 e^{W(t)})$

(2) $d(e^{W(t)^2})$

Hint. Itô-Doeblin formula.