1. Let $f(x) = e^x$

$(e^x)' = e^x$ 이므로 $f^{(n)}(x) = e^x$.

$$\therefore f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n$$

$$= \sum_{n=0}^{\infty} \frac{x^n}{n!}.$$

2. Let $\begin{cases} x - y = r \\ y - x = s \end{cases} \Rightarrow w \begin{cases} r \begin{cases} x \\ y \end{cases} \\ s \begin{cases} x \\ y \end{cases} \end{cases}$.

$$\therefore \frac{\partial w}{\partial x} = \frac{\partial w}{\partial r}\overset{1}{\frac{\partial r}{\partial x}} + \frac{\partial w}{\partial s}\overset{-1}{\frac{\partial s}{\partial x}}$$

$$= \frac{\partial w}{\partial r} - \frac{\partial w}{\partial s} \qquad \Rightarrow \frac{\partial w}{\partial x} + \frac{\partial w}{\partial y} = 0.$$

$$\frac{\partial w}{\partial y} = \frac{\partial w}{\partial r}\overset{-1}{\frac{\partial r}{\partial y}} + \frac{\partial w}{\partial s}\overset{1}{\frac{\partial s}{\partial y}}$$

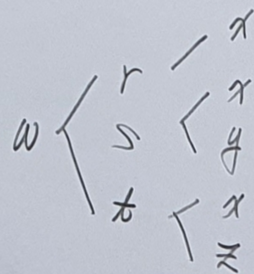
$$= -\frac{\partial w}{\partial r} + \frac{\partial w}{\partial s}$$

3. $$E[e^{uX}] = \int_{-\infty}^{\infty} e^{ux} \cdot f(x)\,dx = \frac{1}{\sqrt{2\pi t}} \int_{-\infty}^{\infty} e^{ux} \cdot e^{-\frac{x^2}{2t}}\,dx$$

$$= \frac{1}{\sqrt{2\pi t}} \int_{-\infty}^{\infty} e^{-\frac{(x-tu)^2}{2t} + \frac{1}{2}u^2 t}\,dx$$

$$= e^{\frac{1}{2}u^2 t} \cdot \underbrace{\frac{1}{\sqrt{2\pi t}} \int_{-\infty}^{\infty} e^{-\frac{(x-tu)^2}{2t}}\,dx}_{1}$$

$$= e^{\frac{1}{2}u^2 t}.$$

4. Let $\begin{cases} x^3 = r \\ \dfrac{y}{x} = s \\ \dfrac{z}{x} = t \end{cases}$ $\Rightarrow$ 

$\therefore \dfrac{\partial w}{\partial x} = \dfrac{\partial w}{\partial r}\cdot\dfrac{dr}{dx} + \dfrac{\partial w}{\partial s}\cdot\dfrac{\partial s}{\partial x} + \dfrac{\partial w}{\partial t}\cdot\dfrac{\partial t}{\partial x}$

일변수

$= 3x^2\dfrac{\partial w}{\partial r} - \dfrac{y}{x^2}\dfrac{\partial w}{\partial s} - \dfrac{z}{x^2}\dfrac{\partial w}{\partial t}$

$\therefore \dfrac{\partial w}{\partial y} = \dfrac{\partial w}{\partial s}\dfrac{\partial s}{\partial y}$

$= \dfrac{1}{x}\dfrac{\partial w}{\partial s}$

$\therefore \dfrac{\partial w}{\partial z} = \dfrac{\partial w}{\partial t}\cdot\dfrac{\partial t}{\partial z}$

$= \dfrac{1}{x}\dfrac{\partial w}{\partial t}$

$\Rightarrow$ $\therefore x\cdot\dfrac{\partial w}{\partial x} + y\cdot\dfrac{\partial w}{\partial y} + z\cdot\dfrac{\partial w}{\partial z}$

$= 3x^3\dfrac{\partial w}{\partial r} - \cancel{\dfrac{y}{x}\dfrac{\partial w}{\partial s}} - \cancel{\dfrac{z}{x}\dfrac{\partial w}{\partial t}} + \cancel{\dfrac{y}{x}\dfrac{\partial w}{\partial s}} + \cancel{\dfrac{z}{x}\dfrac{\partial w}{\partial t}}$

$= 3r\dfrac{\partial w}{\partial r}$

$= 3rf(s,t)$ $\quad (\because w = rf(s,t) \Rightarrow \dfrac{\partial w}{\partial r} = f(s,t))$

$= 3w.$