



Modeling and forecasting the volatility of petroleum futures prices



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ABSTRACT

We investigate volatility models and their forecasting abilities for three types of petroleum futures contracts traded on the New York Mercantile Exchange (West Texas Intermediate crude oil, heating oil #2, and unleaded gasoline) and suggest some stylized facts about the volatility of these futures markets, particularly in regard to volatility persistence (or long-memory properties). In this context, we examine the persistence of market returns and volatility simultaneously using the following ARFIMA–GARCH-class models: ARIMA–GARCH, ARFIMA–GARCH, ARFIMA–IGARCH, and ARFIMA–FIGARCH. Although the ARFIMA–FIGARCH model better captures long-memory properties of returns and volatility, the out-of-sample analysis indicates no unique model for all three types of petroleum futures contracts, suggesting that investors should be careful when measuring and forecasting the volatility (risk) of petroleum futures markets.

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1. Introduction

Modeling and forecasting petroleum futures prices and their volatility are of great interest because accurately measuring the volatility of petroleum futures prices is an important component of the price linkage between spot and futures markets (Hammoudeh and Li, 2004; Hammoudeh et al., 2003; Huang et al., 2009; Lin and Tamvakis, 2001; Silvapulle and Moosa, 1999), risk management, such as value-at-risk (Aloui and Mabrouk, 2010; Sadorsky, 2006), jump, or regime switching in energy futures markets (Fong and Kim, 2002; Lee et al., 2010), and option pricing formulas for futures contracts (Wang et al., 2008). Thus, a better understanding of the dynamics of petroleum futures prices and their volatility should be useful to energy researchers, market participants, and policymakers.

It is well known that the volatility or returns of petroleum prices often exhibits long memory or persistence where the market shocks to the volatility or returns decay at a slow rate (Baillie, 1996). Long memory is a particularly interesting feature in that its presence directly conflicts with the validity of the weak-form efficiency of the petroleum market (Tabak and Cajueiro, 2007). Thus, the presence of long memory provides evidence of nonlinear dependence and of a predictable component of returns and volatility of petroleum prices.

There have been many empirical works on long memory in returns and volatilities of petroleum prices. On the one hand, many studies have examined stochastic properties of petroleum price returns by considering various econometric techniques and data frequencies. In particular, some have investigated whether time series of petroleum prices demonstrate long memory properties of returns (Aloui and Mabrouk, 2010; Brunetti and Gilbert, 2000; Elder and Serletis, 2008; Serletis and Andreadis, 2004; Tabak and Cajueiro, 2007; Wang and Liu, 2010). On the other hand, some empirical studies have addressed the modeling and forecasting of long memory volatility in crude oil or petroleum markets using various GARCH-type models (Agnolucci, 2009; Aroui et al., 2012a, 2012b; Kang et al., 2009; Mohammadi and Su, 2010; Sadorsky, 2006; Wei et al., 2010) but their empirical findings have remained with inconclusive results in the petroleum market.

The prior literature considers the presence of long memory in returns and volatility to be irrelevantly appearing phenomena. It is well-known that market shocks have considerable influence on returns and volatility at the same time, and thus they have double long-memory property in the petroleum prices. On the basis of this idea, some empirical studies have considered the relationship between returns and volatility for various economic and financial time series using a joint ARFIMA–FIGARCH model (Aroui et al., 2012a, 2012b; Conrad and Karanasos, 2005a, 2005b; Kang and Yoon, 2007; Kasman et al., 2009). The ARFIMA–FIGARCH model is able to facilitate the analysis of a relationship between returns and volatility for a process exhibiting dual long-memory property.

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The primary objective of this study was to model and forecast price volatility for three types of petroleum futures contracts traded on the New York Mercantile Exchange (NYMEX): West Texas Intermediate (WTI) crude oil, heating oil #2, and unleaded gasoline. This study further extends the works in existing literature by examining long memory in returns and volatilities by using ARFIMA–FIGARCH models that can capture dual long memory property of returns and price volatility for petroleum futures prices, simultaneously. The dual long memory property has been evaluated by four different volatility models: namely, the ARIMA–GARCH, ARFIMA–GARCH, ARFIMA–IGARCH and ARFIMA–FIGARCH models.

This study essentially differs from previous research in regard to two following aspects. First, although many studies have examined long memory in the returns and volatility independently, little work has done dual long memory in the returns and volatility simultaneously. This paper initially explores dual long memory property in the returns and volatility of petroleum futures markets. Second, this study evaluates forecasting performance of the above four models using the out-of-sample analysis based on multiple forecasting horizons (e.g., 1-, 5-, and 20-day-ahead horizons). The multiple forecasting horizons technique provides more elaborate evidence concerning the volatility forecasting ability in the petroleum futures market.

The rest of this paper is organized as follows. Section 2 presents the statistical characteristics of the data. Section 3 discusses the ARFIMA–GARCH-class models and forecast error statistics. Section 4 presents the volatility model estimation and out-of-sample forecasting results, and Section 5 provides conclusions.

2. Data

We investigate the dynamics of futures prices of WTI crude oil, heating oil #2, and unleaded gasoline. In this report, “futures contracts” refer to those contracts with the earliest delivery date (Contract 1). Such futures contracts are traded on NYMEX, and data regarding these contracts are available from the U.S. Energy Information Administration (EIA).

The data used are of daily frequency for the period 3 January 1995 to 31 July 2012; data for the last one and half years are used to evaluate the accuracy of out-of-sample volatility forecasts. The price series were converted into logarithmic percentage return series; that is, $y_t = 100 \times \ln(P_t/P_{t-1})$ for $t = 1, 2, \dots, T$, where y_t indicates returns for each price at time t , P_t is the current price, and P_{t-1} is the price on the previous day. Following Sadorsky (2006), the actual daily volatility (variance) is measured by daily squared returns (y_t^2), as follows:

$$\sigma_t^2 = y_t^2. \quad (1)$$

Fig. 1 shows the dynamics of returns and price volatility for the three types of petroleum futures contracts.

Table 1 shows the descriptive statistics and the results of the unit root test for both sample returns. As shown in Panel A of Table 1, the mean of these return series is quite small, whereas the corresponding standard deviation of the returns is substantially higher. As indicated by the skewness, kurtosis, and Jarque–Bera results, the returns are not normally distributed. We also examine the null hypothesis of a white-noise process for sample returns using the Box–Pierce test for returns $Q(24)$ and squared returns $Q_s(24)$. Both return and squared return series provide the rejection of the null hypothesis of no serial correlation at the 1% significance level. This evidence indicates significant evidence of serial dependence in the return and squared returns series.

Panel B of Table 1 presents the results of three types of unit root tests for each of the sample returns: augmented Dickey–Fuller (ADF), Phillips–Perron (PP), and Kwiatkowski, Phillips, Schmidt, and Shin (KPSS) tests. These three unit root tests have different null

hypothesis: The null hypothesis of the ADF and PP tests is that a time series contains a unit root, $I(1)$ process, while the KPSS test has the null hypothesis of stationarity, $I(0)$ process. On the one hand, large negative values for the ADF and PP tests for each returns support the rejection of the null hypothesis of a unit root at the 1% significance level. On the other hand, the statistics of the KPSS test indicates that return series are insignificant to reject the null hypothesis of stationarity, implying that they are stationary processes. Thus, we conclude that the petroleum futures return series is a stationary process.

Fig. 2 displays the autocorrelation function (ACF) of daily returns and volatility up to 120 time intervals with two-sided 5% critical values ($\pm 1.96\sqrt{1/T}$). For the returns, most autocorrelations are small, and some significant autocorrelations die out quickly. There seems to be no systemic pattern in the return series of these petroleum futures contracts. However, the autocorrelations for the volatility series are significantly positive and persistence lasts for a substantial number of lags. This indicates that the volatility of petroleum futures contracts exhibits a long-memory process.

3. Model framework

3.1. ARFIMA–FIGARCH model

The ARFIMA model, a well-known parametric method for testing long-memory properties in financial time series, considers the fractionally integrated process $I(d)$ in the conditional mean. The ARFIMA (n, ξ, s) model can be expressed as a generalization of the ARIMA model, as follows:

$$\varepsilon_t = z_t \sigma_t, \quad z_t \sim N(0, 1), \quad (2)$$

$$\Psi(L)(1-L)^\xi(y_t - \mu) = \Theta(L)\varepsilon_t, \quad (3)$$

where y_t is returns for petroleum futures price at time t and ε_t is independently distributed with variance σ_t^2 , L denotes the lag operator, and $\Psi(L) = 1 - \psi_1 L - \psi_2 L^2 - \dots - \psi_n L^n$ and $\Theta(L) = 1 + \theta_1 L + \theta_2 L^2 + \dots + \theta_s L^s$ are, respectively, the autoregressive (AR) and moving-average (MA) polynomials for which all roots lie outside the unit circle. The parameters of the model are ξ, μ, ψ_i , and θ_i .

According to Hosking (1981), if $-0.5 < \xi < 0.5$, then the y_t process is stationary and invertible. For such processes, effects of shocks to ε_t on y_t decay slowly to zero. If $\xi = 0$, then the process is stationary (or short memory), and the effects of shocks to ε_t on y_t decay geometrically. For $\xi = 1$, the process follows a unit root process or ARIMA process. If $0 < \xi < 0.5$, then the process exhibits positive dependence between distant observations, indicating long memory. If $-0.5 < \xi < 0$, then the process exhibits negative dependence between distant observations: that is, anti-persistence.¹

Similar research on volatility has extended the ARFIMA representation of ε_t^2 , the standard GARCH model of Bollerslev (1986) is as follows:

$$\sigma_t^2 = \omega + \alpha(L)\varepsilon_t^2 + \beta(L)\sigma_t^2, \quad (4)$$

where $\omega > 0$, L denotes the lag or backshift operator, and $\alpha(L) \equiv \alpha_1 L + \alpha_2 L^2 + \dots + \alpha_q L^q$, and $\beta(L) \equiv \beta_1 L + \beta_2 L^2 + \dots + \beta_p L^p$. Assuming that $\alpha_i, \beta_i \geq 0$ for all i , the GARCH (p, q) model in Eq. (4) can be rewritten in the form of an ARMA(max{ p, q }, q) model:

$$\varphi(L)\varepsilon_t^2 = \omega + [1 - \beta(L)]v_t, \quad (5)$$

¹ Anti-persistence is a form of long memory characterized by negative autocorrelation that decays very slowly. Peters (1994, p. 61) argued that anti-persistence time series reverses itself more often than a random one would. An anti-persistence process refers to a mean-reverting process.

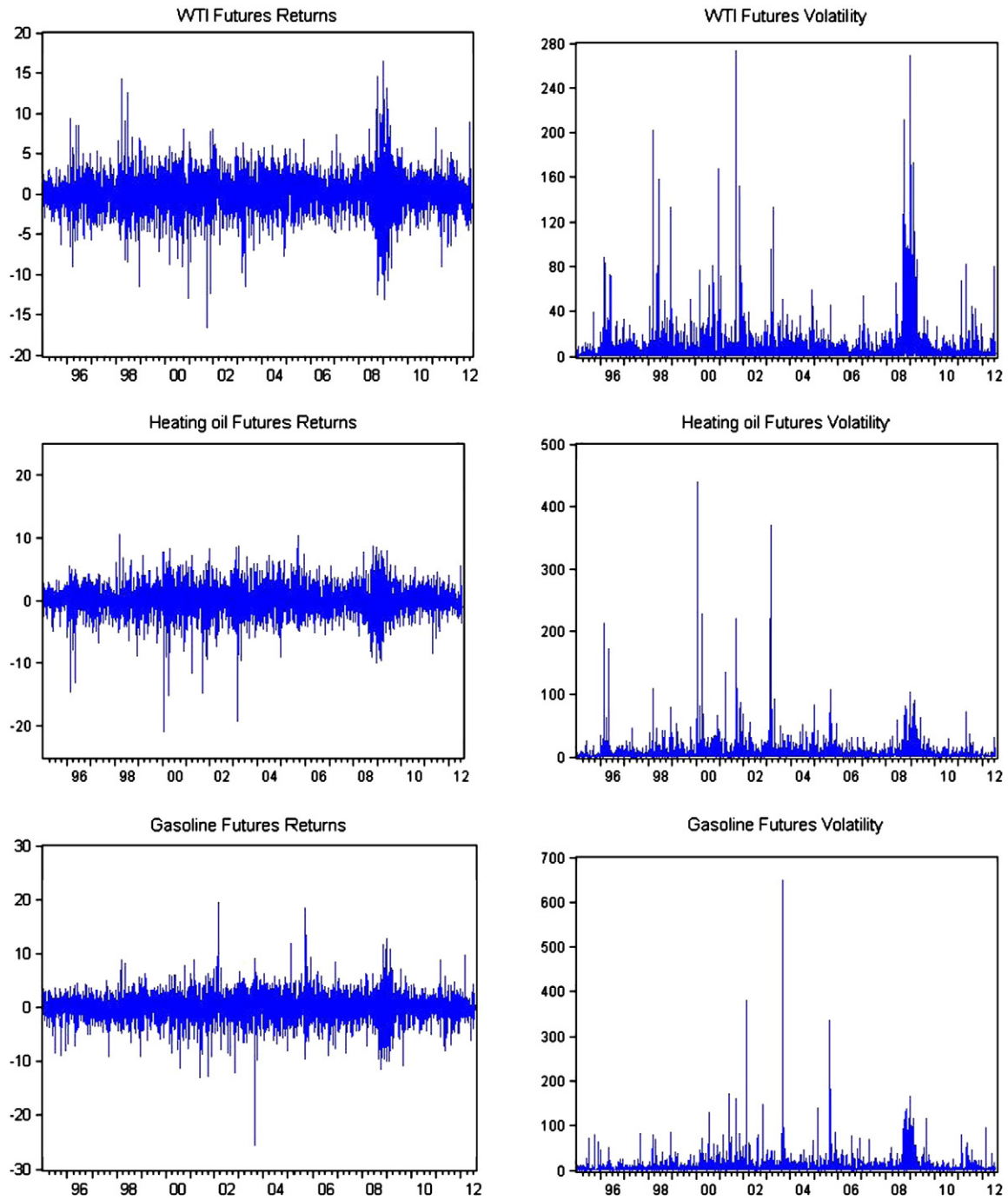


Fig 1. Dynamics of daily returns and price volatility for three types of petroleum futures contracts.

where $v_t \equiv \varepsilon_t^2 - \sigma_t^2$ and $\phi(L) = [1 - \alpha(L) - \beta(L)]$. The $\{v_t\}$ process, which is interpreted as innovations for the conditional variance, has zero mean, serially uncorrelated. Assuming that all the roots of $\phi(L)$ and $[1 - \beta(L)]$ lie outside the unit root circle, the covariance stationary GARCH model is a short memory model because a volatility shock decays at a fast geometric rate. On the other hand, when the autoregressive polynomial $[1 - \alpha(L) - \beta(L)]$ has a unit root, then the GARCH(p, q) process has a unit root in conditional variance. The corresponding IGARCH model of Engle and Bollerslev (1986) is given by

$$\phi(L)(1-L)\varepsilon_t^2 = \omega + [1 - \beta(L)]v_t. \quad (6)$$

However, the IGARCH model cannot allow for modeling long memory in the volatility process, as volatility shocks in the IGARCH model never die out. That is, the IGARCH model is characterized by infinite memory. To overcome this problem, the FIGARCH model of Baillie et al. (1996) can be obtained by replacing the difference operator in Eq. (6) with the fractional differencing operator. The FIGARCH(p, d, q) model is given by

$$\phi(L)(1-L)^d \varepsilon_t^2 = \omega + [1 - \beta(L)]v_t, \quad (7)$$

where $0 \leq d \leq 1$ is the fractional difference parameter. The FIGARCH model provides greater flexibility for modeling the conditional variance,

Table 1
Descriptive statistics and unit root test results for petroleum futures returns.

	WTI futures	Heating oil futures	Gasoline futures
<i>Panel A: Descriptive statistics</i>			
Mean	0.037	0.042	0.039
Std. dev.	2.433	2.367	2.574
Maximum	16.41	10.48	19.48
Minimum	−16.54	−20.95	−25.49
Skewness	−0.285	−0.553	−0.285
Kurtosis	7.084	7.791	8.273
Jarque–Bera	3072***	4289***	5164***
Q(24)	42.89***	41.41***	41.44***
Q _s (24)	2164***	350.25***	405.34***
<i>Panel B: Unit root tests</i>			
ADF	−49.85***	−66.60***	−65.11***
PP	−67.26***	−66.87***	−65.10***
KPSS	0.036	0.051	0.026

Notes: The Jarque–Bera test corresponds to the test statistic for the null hypothesis of normality in the distribution of sample returns. The Ljung–Box statistics, $Q(n)$ and $Q_s(n)$, check for serial correlation of the return series and the squared returns up to the n th order, respectively. MacKinnon's (1991) 1% critical value is −3.435 for the ADF and PP tests. The critical value for the KPSS test is 0.739 at the 1% significance level. *** Indicates rejection of the null hypothesis at the 1% significance level.

because it accommodates the covariance stationary GARCH model when $d=0$, and the IGARCH model when $d=1$, as special cases. For the FIGARCH model in Eq. (7), the persistence of shocks to the conditional variance or the degree of long memory is measured by the fractional differencing parameter d . Thus, the attraction of the FIGARCH model is that, for $0 < d < 1$, it is sufficiently flexible to allow for an intermediate range of persistence.

The parameters of the ARFIMA–FIGARCH model can be estimated using nonlinear optimization procedures to maximize the logarithm of the Gaussian likelihood function. Under the assumption that the random variable $z_t \sim N(0,1)$, the log-likelihood of the Gaussian or normal distribution (L_{Norm}) can be expressed as

$$L_{Norm} = -\frac{1}{2} \sum_{t=1}^T [\ln(2\pi) + \ln(\sigma_t^2) + z_t^2], \quad (8)$$

where T is the number of observations. The estimation procedure for ARFIMA–FIGARCH-class models requires a minimum number of observations. This minimum number is related to the truncation order of fractional differencing operators $(1-L)^\xi$ and $(1-L)^d$. Following the standard procedure used in previous research, we set the truncation order of infinite $(1-L)^\xi$ and $(1-L)^d$ to 1000 lags as follows:

$$(1-L)^\xi = \sum_{k=0}^{1000} \frac{\Gamma(k-\xi)}{\Gamma(k+1)\Gamma(-\xi)} L^k. \quad (9)$$

3.2. Evaluation of forecasts

We evaluate the out-of-sample forecasting performance of the conditional mean and volatility models using the mean squared error (MSE) and mean absolute error (MAE). The statistics associated with the conditional mean are expressed as:

$$MSE = \frac{1}{T} \sum_{t=1}^T (r_{f,t} - r_{a,t})^2, \quad (10)$$

$$MAE = \frac{1}{T} \sum_{t=1}^T |r_{f,t} - r_{a,t}|, \quad (11)$$

where T denotes the number of forecast data points, $r_{a,t}$ is the actual return and $r_{f,t}$ is its future forecast. Similarly, the forecasting error statistics related to the conditional volatility models are:

$$MSE = \frac{1}{T} \sum_{t=1}^T (\sigma_{f,t}^2 - \sigma_{a,t}^2)^2, \quad (12)$$

$$MAE = \frac{1}{T} \sum_{t=1}^T |\sigma_{f,t}^2 - \sigma_{a,t}^2|, \quad (13)$$

where $\sigma_{f,t}^2$ is the volatility forecast for day t , and $\sigma_{a,t}^2$ is the actual volatility on day t . As a general rule, a smaller forecast error statistic indicates the superior forecasting ability of a given model.

Although the above forecast error statistics are useful for comparing estimated models, they do not allow for statistical analyses of differences in forecast accuracy between two forecasting models. Thus, it is important to determine whether any reduction in forecast errors is statistically significant instead of comparing forecast error statistics between forecasting models. For this reason, Diebold and Mariano (1995) developed a test of forecast accuracy for two sets of forecasts. Having generated n, h -step-ahead forecasts from two different forecasting models, the forecast errors from two competing models $e_{1,t+h}$ and $e_{2,t+h}$, where $t=1,2,\dots,n$ and h is the number of trading days in a given forecast horizon starting on day t , corresponding to 1-, 5-, and 20-day-ahead horizons. $g(e_{1,t+h})$ and $g(e_{2,t+h})$ represent their associated loss functions. The null hypothesis of equal forecast accuracy for two forecasts can be represented as $E(g(e_{1,t+h})) = E(g(e_{2,t+h}))$ or $E[d_{t+h}] = 0$, where $d_{t+h} = g(e_{1,t+h}) - g(e_{2,t+h})$ is the loss differential. Thus, the “equal accuracy” null hypothesis is equivalent to the null hypothesis that the population mean of the loss-differential series is 0 (Diebold and Mariano, 1999, p. 2545).

The mean of differences between forecast errors $d = n^{-1} \sum_{t=1}^n d_{t+h}$ has the approximate asymptotic variance of

$$V(\bar{d}) \approx n^{-1} \left[\gamma_0 + 2 \sum_{k=1}^{k-1} \gamma_k \right], \quad (14)$$

where γ_k is the k th autocovariance of d_{t+h} , which can be estimated as

$$\hat{\gamma} = n^{-1} \sum_{t=k+1}^n (d_t - \bar{d})(d_{t-k} - \bar{d}). \quad (15)$$

Diebold and Mariano's (1995) test statistic for the null hypothesis of equal forecast accuracy is

$$DM = \left[V(\bar{d}) \right]^{-1/2} \bar{d}, \quad (16)$$

where DM has an asymptotic standard normal distribution under the null hypothesis. In this study, the DM test was conducted using the loss differential based on the MSE and MAE of the different forecasting models.

4. Empirical results

4.1. Testing long memory in petroleum futures contracts

To examine the long-memory property, we used three long-memory tests: Lo's (1991) modified R/S analysis and two semi-parametric estimators of the long-memory parameter: the log-periodogram regression

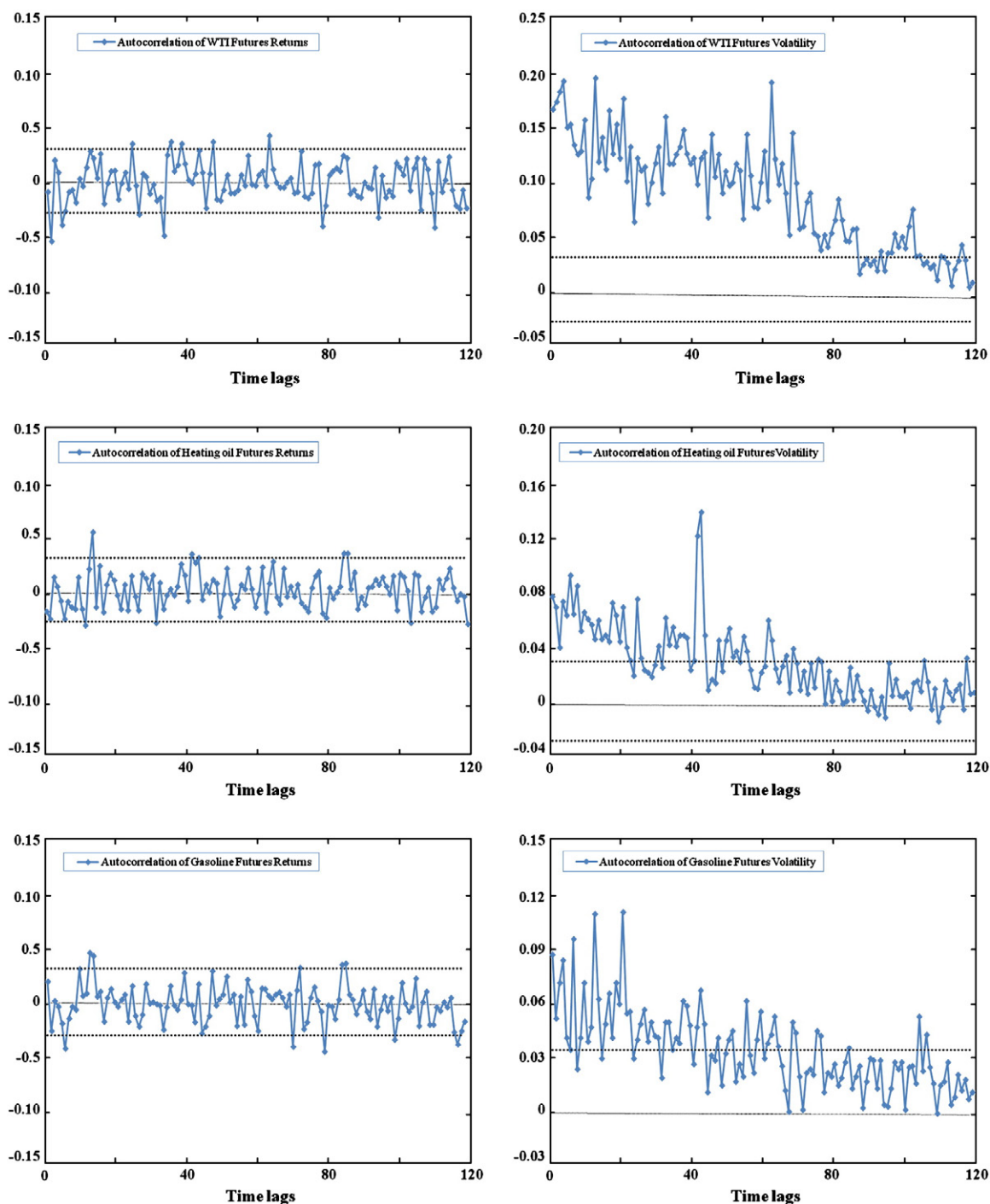


Fig 2. Autocorrelation of daily returns and price volatility for three types of petroleum futures contracts.

(GPH) of Geweke and Porter-Hudak (1983) and the Gaussian semi-parametric (GSP) of Robinson and Henry (1999).² Panel A of Table 2 provides the results of Lo's R/S test for daily returns and volatility. For returns, the value of the modified R/S statistic supports the null hypothesis of short memory, while the volatility displays strong evidence of persistence.

² The choice of these alternative tests is justified by the fact that several authors have questioned the relevance of Lo's (1991) modified R/S. Practically, Lo's modified R/S analysis has a strong preference for accepting the null hypothesis of no long-range dependence, regardless of whether long memory is present in a time series (Hiemstra and Jones, 1997; Teverovsky et al., 1999).

However, Panels B and C of Table 2 show that both the semi-parametric test (GPH and GSP tests) results reject the null hypothesis of short memory in returns and volatility of sample prices. Note that the results of GPH and GSP tests of long memory properties are sensitive to the size of the bandwidth.³ As a result, the evidence of long memory in returns is inconclusive by these different long-memory tests, while

³ The GPH test was implemented with different bandwidths: $m = T^{0.5}$, $m = T^{0.6}$, $m = T^{0.8}$. The GSP test statistic was also estimated with diverse bandwidths: $m = T/4$, $m = T/16$, $m = T/64$.

Table 2
Results of long-memory tests: Lo's R/S test, the GPH test, and the GSP test.

	WTI futures	Heating oil futures	Gasoline futures
<i>Panel A: Lo's R/S test</i>			
Returns	1.075	1.111	0.900
Volatility	4.016***	3.654***	4.175***
<i>Panel B: GPH test</i>			
Returns			
$m = T^{0.5}$	0.052 [0.552]	0.162 [0.064]	−0.046 [0.600]
$m = T^{0.6}$	0.068 [0.218]	0.091 [0.097]	−0.020 [0.716]
$m = T^{0.8}$	−0.039 [0.087]	−0.051 [0.028]	−0.069 [0.002]
Volatility			
$m = T^{0.5}$	0.734 [0.000]	0.476 [0.000]	0.475 [0.000]
$m = T^{0.6}$	0.591 [0.000]	0.397 [0.000]	0.421 [0.000]
$m = T^{0.8}$	0.287 [0.000]	0.225 [0.000]	0.187 [0.000]
<i>Panel C: GSP test</i>			
Returns			
$m = T/4$	−0.037 [0.012]	−0.024 [0.115]	−0.031 [0.042]
$m = T/16$	0.014 [0.629]	−0.004 [0.894]	0.035 [0.250]
$m = T/64$	−0.001 [0.874]	0.066 [0.271]	−0.148 [0.015]
Volatility			
$m = T/4$	0.308 [0.015]	0.219 [0.000]	0.216 [0.000]
$m = T/16$	0.432 [0.030]	0.351 [0.000]	0.384 [0.000]
$m = T/64$	0.710 [0.000]	0.482 [0.000]	0.470 [0.000]

Notes: The critical value of Lo's modified R/S analysis is 2.098 at the 1% significance level. m denotes the bandwidth for the GPH and the GSP tests. *, **, and *** indicate significance levels of 10%, 5%, and 1%, respectively.

the volatilities of crude oil, heating oil, and gasoline seem to be well fitted by a fractionally integrated process. From this point, our research evolved with the ARFIMA–FIGARCH model to identify the long-memory property in returns and volatility in the three energy markets.

4.2. Estimation results using the ARFIMA–FIGARCH model

We estimated the ARFIMA–FIGARCH model to capture possible long-memory properties in both the mean and conditional variance. We also evaluated the performance of the ARFIMA–GARCH, ARFIMA–IGARCH, and ARFIMA–FIGARCH models in terms of their ability to capture long-memory properties of returns and volatility simultaneously. In addition, we consider a simple ARIMA (1,1,1)–GARCH (1,1) model to compare the performance of volatility forecasting ability with the sophisticated above models.

Table 3 shows the estimation results obtained using these models.⁴ In the mean equation, an ARFIMA (0, ξ , 1) model is the best representation of both WTI and heating oil returns for a long-memory process, whereas an ARFIMA (1, ξ , 0) is the best representation of the long-memory process for the gasoline returns. This suggests that gasoline prices have more pronounced short-run dynamics relative to those of WTI and heating oil prices, which are affected by market shocks. Generally, the results confirm the ability of the ARFIMA–FIGARCH model to capture the dynamics of returns for the three types of petroleum futures contracts. For example, the estimated values of the parameter ξ are negative and statistically different from zero, providing evidence of negative dependence (or anti-persistence) among the returns. This result is consistent with the findings of Elder and Serletis (2008), who examined prices of crude oil futures contracts using GPH and wavelet OLS estimators.

The estimates of the long-memory parameter d are positive and significant at the 1% level, indicating rejection of $d=0$ (GARCH

model) and $d=1$ (IGARCH model). This suggests that the volatility of petroleum futures returns has long-memory properties. Previous studies have reported similar findings (Aloui and Mabrouk, 2010; Brunetti and Gilbert, 2000; Kang et al., 2009; Sadorsky, 2006).

In Table 4, we present the accuracy of model specifications using several diagnostic tests: three residual tests and three model selection criteria. To check the residual test, we applied the Box–Pierce test, $Q(24)$ for up to 24th-order serial correlation in the residuals, Engle's (1982) LM ARCH (10) test for the presence of ARCH effects in residuals up to lag 10, and the RBD (10) test for conditional heteroscedasticity in residuals up to lags 10.⁵ Additionally, the Akaike information criterion (AIC), the Shibata criterion (SC), and the Hannan–Quinn criterion (HQ) were used to choose the best specification model among the given models in Table 3.

As presented in Table 4, the results of $Q(24)$ and the ARCH (10) show no serial correlation and no remaining ARCH effect. The insignificance of RBD (10) statistics indicates that the ARFIMA–FIGARCH model is suitable for depicting heteroscedasticity exhibited in the petroleum futures markets, indicating that there is no statistically significant evidence of misspecification in the ARFIMA–FIGARCH model. Additionally, the lowest values of three model selection criteria (AIC, SC and HQ) indicate that the ARFIMA–FIGARCH model best captures the long-memory dynamics of both returns and price volatility simultaneously for petroleum futures contracts.

4.3. Out-of-sample forecast results

Tables 5 and 6 report out-of-sample forecasting performance of the conditional mean and volatility models using the error statistics (MSE and MAE), respectively. We evaluated 398 out-of-sample forecasts between 3 January 2011 and 31 July 2012 and assessed the accuracy of these forecasts. The out-of-sample forecast analysis considers 1, 5, and 20 forecast horizons, corresponding to 1-day, 1-week, and 1-month trading periods, respectively.

Table 5 shows that both MSE and MAE statistics have no support for the only one model in the conditional mean of three futures contracts. The WTI futures returns prefer the ARIMA–GARCH model at the 1-day forecasting horizon, but the ARFIMA–FIGARCH model is superior to others at the 20-day forecasting horizon. The heating oil futures returns choose the ARFIMA–FIGARCH model whereas the gasoline futures returns attain the ARFIMA–IGARCH model at the 1-day horizon, the ARIMA–GARCH model at the 5-day horizon and the ARFIMA–FIGARCH model at the 20-day horizon.

Table 6 presents the calculated values of the out-of-sample volatility forecast error statistics. Now evidence is mixed. In WTI futures, the simple ARIMA–GARCH model provides the lowest MSE and MAE values and shows a superior ability to forecast volatility for all three forecast horizons. However, in both heating oil and gasoline futures, the ARFIMA–FIGARCH model is more suitable than the other models (i.e., the ARIMA–GARCH, ARFIMA–IGARCH and ARFIMA–FIGARCH models). Thus, the results of the forecasting errors indicate that none of the models assessed provides the best fit for all of the three series considered.

Additionally, Table 7 reports the estimated values of DM test statistic for out-of-sample performance of conditional volatility models using differentials from both MSE and MAE . We tested the null hypothesis of no difference in forecast accuracy between the conditional models in three petroleum futures. In WTI futures, the DM test statistic rejects the null hypothesis that the forecasting errors of the ARIMA–FIGARCH model are the same as either the ARFIMA–GARCH or ARFIMA–IGARCH or ARFIMA–FIGARCH model. Thus, the ARIMA–FIGARCH model outperforms other models (ARFIMA–GARCH, ARFIMA–IGARCH and ARFIMA–FIGARCH models). However, the values of the DM test in both heating

⁴ To determine the orders n and s of the ARFIMA (n, d, s) specification in Eq. (3), we estimates all the possible combinations for the ARFIMA (n, d, s) part with maximum $n=0, 1, 2$ and $s=0, 1, 2$, based on the Akaike information criterion (AIC). The minimum value of AIC selected the best lag ARFIMA model: A MA (1) specification was retained for WTI and heating oil futures returns, whereas an AR (1) specification was chosen for unleaded gasoline futures returns. Upon requirement, we do present the results of order selection.

⁵ Tse (2002) developed residual-based diagnostics (RBD) for conditional heteroscedasticity to test the null hypothesis of a correct model specification.

Table 3
Estimation results of volatility models.

Series	WTI futures				Heating oil futures				Gasoline futures			
Model	ARIMA-GARCH	ARFIMA-GARCH	ARFIMA-IGARCH	ARFIMA-FIGARCH	ARIMA-GARCH	ARFIMA-GARCH	ARFIMA-IGARCH	ARFIMA-FIGARCH	ARIMA-GARCH	ARFIMA-GARCH	ARFIMA-IGARCH	ARFIMA-FIGARCH
μ	0.068 (0.037)*	0.069 (0.021)**	0.067 (0.021)**	0.073 (0.022)**	0.052 (0.035)	0.060 (0.025)**	0.061 (0.025)**	0.042 (0.031)	0.049 (0.041)	0.051 (0.027)*	0.043 (0.027)	0.055 (0.027)**
ψ_i	−0.521 (0.225)**	–	–	–	0.846 (0.098)**	–	–	–	−0.554 (0.260)**	0.078 (0.031)**	0.081 (0.030)**	0.077 (0.032)**
ξ	1.000	−0.072 (0.022)**	−0.073 (0.021)**	−0.065 (0.023)**	1.000	−0.046 (0.023)**	−0.047 (0.022)**	−0.055 (0.028)**	1.000	−0.054 (0.024)**	−0.057 (0.023)**	−0.054 (0.024)**
θ_1	0.536 (0.223)**	0.078 (0.027)**	0.078 (0.026)**	0.072 (0.027)**	−0.870 (0.092)**	0.039 (0.027)	0.040 (0.026)	0.049 (0.034)	0.576 (0.253)**	–	–	–
ω	0.088 (0.042)**	0.078 (0.033)**	0.030 (0.014)**	0.231 (0.100)**	0.111 (0.054)**	0.072 (0.029)**	0.032 (0.014)**	0.275 (0.177)	0.188 (0.100)*	0.137 (0.083)*	0.040 (0.045)	0.318 (0.126)**
α_1	0.059 (0.016)**	0.057 (0.014)**	0.060 (0.013)**	–	0.071 (0.019)**	0.061 (0.013)**	0.065 (0.013)**	–	0.060 (0.027)**	0.056 (0.027)**	0.058 (0.037)	–
β_1	0.926 (0.020)**	0.930 (0.017)**	1–0.060	0.568 (0.094)**	0.912 (0.023)**	0.927 (0.015)**	1–0.065	0.550 (0.185)**	0.914 (0.037)**	0.924 (0.035)**	1–0.058	0.634 (0.085)**
φ	–	–	–	0.247 (0.080)**	–	–	–	0.225 (0.132)*	–	–	–	0.380 (0.100)**
d	–	–	–	0.401 (0.072)**	–	–	–	0.409 (0.130)**	–	–	–	0.347 (0.126)**

Notes: Standard errors are in parentheses below the corresponding parameter estimates. **, and * indicate rejection of the null hypothesis at the 5%, and 10% significance levels, respectively.

oil and gasoline futures contracts reject the null hypothesis that the forecasting errors of the ARFIMA-FIGARCH model are the same as either the ARFIMA-GARCH or ARFIMA-IGARCH or ARFIMA-FIGARCH model, indicating better performance by the ARFIMA-FIGARCH model than the other models. As a result, the WTI futures prices choose the simple ARIMA-GARCH model, whereas the heating oil and gasoline futures prefer the dual long memory ARFIMA-FIGARCH model.

In contrast to Kang et al. (2009), who suggested that the fractionally integrated model provided the best fit for the volatility of WTI spot prices, the results of the present study indicate that shocks to the volatility of WTI futures returns dissipate exponentially, pointing to the GARCH model. These findings have important implications for measuring value-at-risk estimations, determining optimal hedging ratios, and pricing derivatives in petroleum futures markets. For example, (1) an appropriate volatility model provides accuracy for capital reserve requirements in quantifying value-at-risk estimations (Aloui and Mabrouk, 2010; Fan et al., 2008), (2) accurate conditional variance from the volatility model is used for calculating hedging ratios and enhancing hedging effectiveness in the price change regression (Wilson et al., 1996; Zanotti et al., 2010), and (3) an accurate long-memory volatility model is an important input in measuring option

pricing in the Black-Scholes model (Bollerslev and Mikkelsen, 1996; Taylor, 2000).

5. Conclusions

In this study, we sought to identify a good model for forecasting volatility and examined some stylized facts about the volatility (particularly in regard to long memory or persistence) of three types of petroleum futures contracts. For this, we calculated the out-of-sample forecasts of the volatility and evaluated the performance of the ARIMA-GARCH, ARFIMA-GARCH, ARFIMA-IGARCH, and ARFIMA-FIGARCH models in terms of their ability to capture long-memory properties of returns and volatility simultaneously.

The estimation results suggest that the ARFIMA-FIGARCH model can better capture long-memory features than can other models (the ARIMA-GARCH, ARFIMA-GARCH and ARFIMA-IGARCH models), indicating that returns and volatility for the three types of petroleum futures contracts have dual long-memory properties. The presence of long-memory properties casts doubt on the weak-form efficiency of petroleum futures markets.

Table 4
Diagnostic tests of volatility models.

Series	WTI futures				Heating oil futures				Gasoline futures			
Model	ARIMA-GARCH	ARFIMA-GARCH	ARFIMA-IGARCH	ARFIMA-FIGARCH	ARIMA-GARCH	ARFIMA-GARCH	ARFIMA-IGARCH	ARFIMA-FIGARCH	ARIMA-GARCH	ARFIMA-GARCH	ARFIMA-IGARCH	ARFIMA-FIGARCH
Q(24)	19.33 [0.624]	13.14 [0.948]	12.97 [0.952]	14.05 [0.925]	20.37 [0.559]	16.52 [0.832]	15.31 [0.882]	16.23 [0.844]	26.78 [0.219]	20.03 [0.639]	19.30 [0.683]	16.17 [0.847]
Q _s (24)	34.80 [0.040]	33.99 [0.050]	34.19 [0.047]	32.88 [0.063]	20.40 [0.557]	22.64 [0.422]	25.90 [0.255]	21.03 [0.518]	28.27 [0.167]	26.98 [0.211]	26.00 [0.251]	22.09 [0.454]
ARCH (10)	1.445 [0.153]	1.399 [0.173]	1.411 [0.168]	0.987 [0.451]	0.714 [0.711]	0.834 [0.594]	0.824 [0.604]	0.599 [0.815]	1.414 [0.167]	1.405 [0.171]	1.318 [0.214]	1.186 [0.294]
RBD (10)	14.82 [0.138]	9.336 [0.500]	13.38 [0.202]	9.873 [0.451]	7.837 [0.644]	6.665 [0.756]	9.645 [0.472]	5.511 [0.854]	14.28 [0.161]	10.85 [0.369]	11.77 [0.300]	9.583 [0.477]
AIC	4.512751	4.473758	4.473502	4.470926	4.509664	4.474311	4.474992	4.473313	4.728666	4.677373	4.681301	4.673728
SC	4.512746	4.484752	4.481354	4.480349	4.509656	4.485305	4.482845	4.482736	4.728661	4.686798	4.689155	4.684724
HQ	4.516417	4.477655	4.476285	4.474266	4.514101	4.478207	4.477775	4.476653	4.732332	4.680713	4.684084	4.677625
ln(L)	−8141.77	−8960.64	−8962.13	−8955.97	−6560.07	−8961.76	−8965.12	−8960.76	−8531.61	−9367.46	−9376.33	−9359.15

Notes: Q(24) and Q_s(24) are Box-Pierce statistics for return series and squared return series, respectively, for up to 24th-order serial correlation. ARCH (10) is Engle's (1982) ARCH LM test to check the presence of ARCH effects in residuals up to lag 10. RBD (10) is the residual-based diagnostic for conditional heteroscedasticity, using 10 lags. ln(L) is the maximized Gaussian log-likelihood value. Numbers in brackets are p-values.

Table 5

Accuracy of out-of-sample forecasts for the conditional mean models.

Horizon		1-day horizon		5-day horizon		20-day horizon	
Series	Models	MSE	MAE	MSE	MAE	MSE	MAE
WTI futures	ARFIMA–FIGARCH	0.333	0.576	3.193	1.528	4.710	1.836
	ARFIMA–IGARCH	0.313	0.559	3.254	1.532	4.713	1.833
	ARFIMA–GARCH	0.318	0.564	3.237	1.531	4.712	1.834
	ARIMA–GARCH	0.279	0.528	2.983	1.555	5.524	2.016
Heating oil futures	ARFIMA–FIGARCH	1.337	1.156	1.789	1.177	6.747	2.308
	ARFIMA–IGARCH	1.342	1.158	1.789	1.177	6.753	2.309
	ARFIMA–GARCH	1.339	1.157	1.789	1.177	6.752	2.309
	ARIMA–GARCH	1.433	1.197	2.776	1.212	9.823	2.761
Gasoline futures	ARFIMA–FIGARCH	0.155	0.394	3.176	1.576	5.588	1.952
	ARFIMA–IGARCH	0.145	0.381	3.215	1.580	5.576	1.947
	ARFIMA–GARCH	0.156	0.395	3.178	1.576	5.587	1.951
	ARIMA–GARCH	0.245	0.495	2.972	1.551	5.597	1.961

Notes: MSE is the mean squared error; MAE is the mean absolute error. Lowest values for the statistics are denoted in bold face.

However, the out-of-sample analyses suggest that none of the volatility models is adequate for all three petroleum futures series. The WTI futures prices choose the simple ARIMA–GARCH model, whereas the heating oil and gasoline futures prefer the ARFIMA–FIGARCH model. This suggests that investors should be careful when measuring volatility (risk) in petroleum futures markets. The findings of this study should be useful in facilitating accurate value-at-risk management, developing futures pricing models, and determining optimal hedge ratios with respect to petroleum markets.

A number of avenues could be followed to extend this research. First, it would be interesting to consider high-frequency data in measuring the long-memory property in energy markets. Second, our long-memory result would be sensitive to the presence of structural breaks in energy markets. Thus, it would be worthwhile to include Markov switching-type volatility models in capturing regime shifts and comparing the forecasting ability with the long-memory volatility models. Third, it emerged that the volatility asymmetric feature occurs in crude oil markets (Mohammadi and Su, 2010). Our future research attempts to model and forecast the volatility asymmetry in petroleum futures prices. Finally, it would be interesting to check the relevance of different return distributions to enhance the forecasting ability of volatility models.

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Table 6

Accuracy of out-of-sample forecasts for the conditional volatility models.

Horizon		1-day horizon		5-day horizon		20-day horizon	
Series	Models	MSE	MAE	MSE	MAE	MSE	MAE
WTI futures	ARFIMA–FIGARCH	131.2	9.828	132.6	9.845	133.9	9.863
	ARFIMA–IGARCH	237.1	13.86	243.4	14.08	250.6	14.34
	ARFIMA–GARCH	85.35	6.195	87.01	6.203	88.95	6.217
	ARIMA–GARCH	85.04	6.151	86.76	6.166	88.80	6.183
Heating oil futures	ARFIMA–FIGARCH	35.86	4.230	37.00	4.317	38.24	4.401
	ARFIMA–IGARCH	79.05	7.226	81.75	7.432	89.91	7.656
	ARFIMA–GARCH	40.62	4.793	41.62	4.902	43.40	5.013
	ARIMA–GARCH	45.61	5.460	46.89	5.548	48.25	5.635
Gasoline futures	ARFIMA–FIGARCH	89.12	5.908	92.14	6.043	95.22	6.154
	ARFIMA–IGARCH	153.0	9.602	158.5	9.889	164.2	10.16
	ARFIMA–GARCH	92.75	6.427	95.91	6.581	99.13	6.712
	ARIMA–GARCH	95.42	6.828	98.23	6.931	101.4	10.15

Notes: See Table 5.

Table 7

Diebold–Mariano (DM) test statistic for forecast accuracy of conditional variance.

		1-day horizon	5-day horizon	20-day horizon	1-day horizon	5-day horizon	20-day horizon
		Mean Squared Error (MSE)			Mean Absolute Error (MAE)		
WTI futures	ARFIMA–FIGARCH	–	–	–	–	–	–
	ARFIMA–IGARCH	20.05**	15.21**	16.23**	8.37**	6.58**	9.58**
	ARFIMA–GARCH	–	–	–	–	–	–
	ARIMA–GARCH	20.14**	9.44**	6.32**	12.10**	6.38**	4.53**
Heating oil futures	ARFIMA–FIGARCH	–	–	–	–	–	–
	ARFIMA–IGARCH	9.97**	4.73**	3.34**	3.08**	2.24**	2.32**
	ARFIMA–GARCH	–	–	–	–	–	–
	ARIMA–GARCH	–	–	–	–	–	–
Gasoline futures	ARFIMA–FIGARCH	–	–	–	–	–	–
	ARFIMA–IGARCH	12.51**	5.03**	3.20**	15.22**	5.78**	3.59**
	ARFIMA–GARCH	–	–	–	–	–	–
	ARIMA–GARCH	9.63**	6.01**	4.65**	16.59**	9.27**	6.39**
Gasoline futures	ARFIMA–FIGARCH	–	–	–	–	–	–
	ARFIMA–IGARCH	8.10**	6.75**	9.22**	19.57**	19.96**	39.23**
	ARFIMA–GARCH	–	–	–	–	–	–
	ARIMA–GARCH	–	–	–	–	–	–

** Indicates that the null hypothesis of the DM test is rejected at the 5% significance level.

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