Integrating Macroeconomic Variables in Behavioral Models for Interest Rate Risk Measurement in the Banking Book

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Abstract

Recent Basel standards on interest rate risk (IRR) in the banking book requires the consideration of macroeconomic variables for modeling client behaviors while no macroeconomic risk scenarios are prescribed by regulators or are generally agreed in the industry. Since macroeconomic variables and interest rates are correlated, projecting macroeconomic variables for IRR measurement poses a challenge of keeping consistency with regulator-prescribed interest rate scenarios. This paper proposes an approach to integrate macroeconomic variables with interest rate scenarios. The conditional expectation of macroeconomic variables on interest rate variables is used to capture their interdependence. Based on the mathematical properties of conditional expectation we derive its non-parametric estimator. The resulting projections of macroeconomic variables are fully consistent with given interest rate scenarios and are convenient for implementation in practice. An empirical application to Canadian fixed-term deposits is conducted to illustrate the proposed approach.

Key words: interest rate risk, behavioral model, non-parametric, kernel ridge regression, machine learning

1 Introduction

Traditionally for the interest rate risk (IRR hereafter) measurement of banks' banking book, models of client behaviors such as loan prepayment and deposit redemption concern risk scenarios with interest rate variables only. For example,

*Royal Bank of Canada, hezhongfang2004@yahoo.com. I am grateful to the editor for helpful comments on an earlier version of the paper. All remaining errors are my own. The Matlab programs and data used in the paper can be found in its Github site https://github.com/ZhongfangHe/Integrated_IRR_Scenarios. Disclaimer: The analysis and conclusions in this paper are those of the author and do not represent the view of the Royal Bank of Canada.

one risk scenario that is commonly used among banks and is prescribed by regulators for measuring IRR is a parallel shift-up or shift-down of the risk-free yield curve by 200 basis points. Client behaviors are modeled as driven by interest rate variables and possibly other exogenous factors such as seasonality and client characteristics. Given the projections of interest rates under prescribed interest rate scenarios, the change in client behaviors and, consequently, the change in the various balance-sheet and cash-flow items in the banking book are projected over a certain number of future time periods. The resulting changes in bank earning and economic value under the various interest rate scenarios are computed as IRR measures to inform IRR management[1].

Refinement of IRR measurement practice and the strengthened regulation since the 2008 financial crisis, however, ask banks to consider the dimensions of risk scenarios beyond interest rates for modeling client behaviours. The Basel standards "Interest Rate Risk in the Banking Book" of April 2016 explicitly states that "*banks must carefully consider how the exercise of the behavioural optionality will vary not only under the interest rate shock and stress scenarios but also across other dimensions*"(paragraph 46). The macroeconomic variables that are listed in Basel IRR standards (2016) to model client behaviors include stock indices, unemployment rates, GDP, inflation and housing price indices. On the other hand, the risk scenarios that are currently used among banks and are prescribed by regulators to compute IRR measures continue to contain interest rate variables only. For example Basel IRR standards (2016) prescribes six standardized risk scenarios for IRR measurement that include parallel up and down, steepening and flattening, and short rates up and down for interest rates; none of these six risk scenarios concern the macroeconomic variables prescribed to be considered in modeling client behaviors. The lack of prescribed risk scenarios for macroeconomic variables leaves open the question of how to integrate them with the interest rate scenarios to project client behaviors for computing IRR measures.

The key modeling challenge to include macroeconomic variables in IRR measurement arises from the correlation between macroeconomic variables and interest rate movements. Intuitively, consider a macroeconomic variable that is a driver of certain IRR-relevant client behavior but is uncorrelated with interest rates. When we shock the yield curve in the interest rate scenarios, this macroeconomic variable is unaffected given its zero correlation with interest rates and thus its impact on the client behavior would be a constant across the various interest rate scenarios. In other words, no movement in the client behavior across the various interest rate scenarios is associated with this macroeconomic variable. Since the goal of IRR measurement is on the risk generated by interest rate movements, one could simply assume a constant value for this macroeconomic variable, e.g. its historical average, when projecting the client behavior in future time periods for IRR measurement.

In reality, the movements of macroeconomic variables and interest rates are

[1] Annex 1 of Basel IRR standards (2016) contains a detailed description of IRR measurement practices in banks.

generally correlated. Developments in GDP, unemployment rate and inflation rate are usually the key ingredients into central banks' monetary policies that determine interest rate levels. Therefore projections of macroeconomic variables for IRR measures need to take into account their correlations with interest rates and to be consistent with interest rate scenarios. For example, what values should the macroeconomic variables be projected in future time periods when the interest rate scenario is a parallel shift up of the current yield curve by 200 basis points? How should the projections of the macroeconomic variables change if the interest rate scenario switches to a steepening shock with lower short rates and higher long rates? IRR managers need to model the correlation between macroeconomic variables and interest rates and ensure their consistency in projections for IRR measurement.

A straightforward solution would be regressing macroeconomic variables on interest rates by assuming a linear or non-linear functional form such that projections of interest rates can be plugged into the estimated function to project the macroeconomic variables. Alternative approaches include modeling the joint distribution of interest rates and macroeconomic variables via methods such as vector autoregression (Stock and Watson (2001)) or copula (Patton (2012)). The downside of these parametric approaches, however, is that the resulting correlation structure between interest rates and macroeconomic variables may be *ad hoc* and restrictive. The assumed functional forms in the parametric approaches could be subject to unforeseen approximation errors to the relation of macroeconomic variables with interest rates. This paper goes one step further to use the conditional expectation of macroeconomic variables on interest rates as a representation of their correlations and motivates the appropriate non-parametric model from the mathematical properties of conditional expectation. Thus macroeconomic variables can be integrated into the interest rate scenarios in a rigorous yet practical way instead of based on *ad hoc* modeling assumptions[2]

The basic idea of the proposed approach is to decompose each macroeconomic variable into the sum of its conditional expectation on interest rates and the residual. By the definition of conditional expectation, the residual is uncorrelated with interest rates and hence its value is immune to the impact of interest rate movements. Such a decomposition ensures that the correlation of each macroeconomic variable with interest rates is fully subsumed in its conditional expectation component. We term this conditional expectation of a macroeconomic variable as its interest-rate-correlated component and the residual as its macro-specific component.

The functional form of the conditional expectation is unknown. To proceed, we show that the conditional expectation of a macroeconomic variable on interest rates minimizes the mean square difference between the macroeconomic vari-

[2]Copula methods with non-parametric marginal distributions for macroeconomic variables and a non-parametric copula linking interest rates and macroeconomic variables would also require minimal *ad hoc* modeling assumptions but faces the challenge of the curse of dimensionality (Kauermann et al. (2013)) and arguably will raise the model maintenance and implementation costs in practice.

able and any function of interest rates. Therefore the search for the conditional expectation of a macroeconomic variable can be reduced to a function optimization problem with mean square loss. By introducing the assumption of the function space being a reproducing kernel Hilbert one with finite function norm, the solution to the function optimization problem becomes the non-parametric estimator known as the kernel ridge regression in the machine learning literature (Hastie et al. (2009), Murphy (2012)). Thus a non-parametric approach is applied to estimate the correlation between macroeconomic variables and interest rates that is derived from the mathematical properties of conditional expectation. The final estimate of the conditional expectation is of a linear functional form with a closed-form solution and can be implemented with minimal extra efforts on top of the existing frameworks of IRR measurement in banks.

With the decomposition of macroeconomic variables at hand, it becomes straightforward to project the macroeconomic variable in a given interest rate scenario consistently. Being a function of interest rates, projections of the interest-rate-correlated component of the macroeconomic variable can be computed by using projections of interest rates according to the interest rate scenario. The other part of the macroeconomic variable, namely the macro-specific component, is immune to interest rate movements and hence its projections could be computed as a constant across interest rate scenarios, e.g. its historical average or a chosen tail percentile of its historical distribution. Summing up the projections of these two components produces projections of the macroeconomic variables that are fully consistent with the prescribed interest rate scenario.

Empirical applications of the proposed approach require data on client behavior. As banks' client behavior data are proprietary, we use publicly available aggregate deposit data as the alternative to illustrate the proposed approach. Specifically we apply the proposed approach to project the growth rate of the non-tax-sheltered fixed-term deposits from personal clients in Canadian chartered banks over a 12-month period under the 200-basis-points parallel shift-up and shift-down scenarios of interest rates. A detailed step-by-step illustration of implementing the proposed method is provided in this paper.

1.1 Related Literature

Our paper fits into the literature of behavioral models for banking risk. One strand of the literature concerns modeling the prepayment behavior of residential mortgages. Coriazzi and Signani (2019) and Baccaglini (2019) are examples of prepayment models that are used in the industry for IRR measurement. A common feature of these prepayment models is that the impact of macroeconomic factors on clients' prepayment behavior is not considered. Sirignano et al. (2018) applies a deep neural network to study the mortgage risk in the U.S. and relates the prepayment rate of mortgages to macroeconomic variables such as unemployment rate. However the macroeconomic variables in Sirignano et al. (2018) are treated as exogenous inputs to their model. Chernov et al. (2018) finds that two latent factors in their model play an important role in capturing the prepayment behavior in the U.S. mortgage-backed securities and that these two latent factors are correlated with macroeconomic variables such as unemployment rate and housing price. The latent factors are modeled as mean-reverting square-root processes in Chernov et al. (2018) but the macroeconomic variables are not directly modeled to explain the prepayment behavior.

Another important strand of behavioral models is on measuring the risk of non-maturity deposits (NMD), *i.e.* deposits that have no contractual maturity and can be withdrawn by clients on demand. Traditional models of NMD volume mainly focus on interest rate factors as the explanatory variables (e.g. see Rienzo (2019) for an overview of the NMD models commonly used in the industry). Recent NMD models begin to incorporate macroeconomic factors. Frascarelli and Pagliaccia (2019) provides a model of NMD volume that takes into account economic variables such as equity market return but treats these economic variables as exogenous model inputs. Castagna and Manenti (2013), inspired by Nystrom (2008), studies a model of NMD volume as a time-varying stochastic fraction of the clients' income which in turn is modeled as a deterministic time trend. Similarly, O'Brien (2000) considers a regression model of NMD volume that includes the national income as an explanatory variable but models the national income as a deterministic time trend.

The focus of our approach differs from the existing behavioral models. We focus on how to consistently integrate the macroeconomic variables in behavioral models for IRR measurement under prescribed interest rate risk scenarios. The integration method can be applied to any behavioral model that incorporates economic variables as explanatory variables, regardless of the specific functional form of the behavioral model. Rather than treating macroeconomic variables as deterministic time trends or restricting them as linearly correlated with interest rate variables, we model the relation of macroeconomic variables with interest rates in a flexible non-parametric way. Thus the system of IRR measurement does not need to develop a separate set of risk scenarios specifically for macroeconomic variables but instead could use prescribed interest rate scenarios to generate consistent projections of future values of macroeconomic variables and IRR measures. As we are aware, this paper is the first in the literature to tackle the integration of macroeconomic variables in behavioral models for IRR measurement in practice.

The remainder of the paper is structured as follows. The details of the decomposition of macroeconomic variables are provided in Section 2. Section 3 shows the empirical illustration of the proposed approach to Canadian fixed-term deposits. Section 4 concludes.

2 The Model

Let y_t be the client behavior variable of interest at time t, e.g. the deposit redemption rate. The variable x_t denotes the set of interest rate variables that are assumed to be correlated with the client behavior y_t and that could include any linear/non-linear transformation of interest rates at time t as well as their lags. In practice, interest-rate-unrelated factors such as seasonality could also be included in x_t to explain the client behavior y_t. With a little abuse of notation, we term x_t as an interest rate variable for expositional convenience when it does not cause confusion. Under a risk scenario with interest rates only, the client behavior model can be written as:

$$y_t = f(x_t) + residual \tag{1}$$

where the function f is known. An example of the function f could be the linear function $f(x_t) = \alpha + x_t'\beta$.

To add macroeconomic variables z_t, the model of the client behavior changes to:

$$y_t = g(x_t, z_t) + residual \tag{2}$$

where the function g could be, say, $g(x_t, z_t) = \alpha + x_t'\beta + z_t'\gamma$. Parameters in the function g can be estimated by using historical data on the client behavior y_t, interest rates x_t and macroeconomic variables z_t, t = 1, 2, ..., T.

A prescribed interest rate scenario specifies the shocked yield curve at time T that leads to the projection of future interest rates $\{x_{T+j}\}_{j=1}^{H}$ through forward rate calculation or term structure model of interest rates. To project the future client behavior $\{y_{T+j}\}_{j=1}^{H}$, ones needs to compute the projections of macroeconomic variables $\{z_{T+j}\}_{j=1}^{H}$ that are consistent with the projected interest rates $\{x_{T+j}\}_{j=1}^{H}$ in the scenario. To do that, let $E(z_t|x_t) = \int z_t p(z_t|x_t) dz_t$ be the expectation of the macroeconomic variables z_t conditional on the interest rates x_t. Denote the residual $u_t = z_t - E(z_t|x_t)$. It follows that the macroeconomic variables are decomposed as $z_t = E(z_t|x_t) + u_t$ and that $E(u_t|x_t) = 0$. The latter follows from the law of iterated expectation $E(u_t|x_t) = E(z_t - E(z_t|x_t)|x_t) = 0$. It is straightforward to see that:

$$\begin{aligned}
cov(x_t, u_t) &= E(x_t u_t) - E(x_t)E(u_t) \\
&= E(x_t u_t) \quad (E(u_t) = E(z_t - E(z_t|x_t)) = 0) \\
&= E(E(x_t u_t|x_t)) \\
&= E(x_t E(u_t|x_t) \\
&= 0
\end{aligned} \tag{3}$$

Given that the component u_t is uncorrelated with the interest rate variables x_t, we term u_t as the "macro-specific" component of the macroeconomic variables z_t. The conditional expectation $E(z_t|x_t)$ is by its definition a function of the interest rates x_t and hence is termed the "interest-rate-correlated" component of the macroeconomic variables z_t. It can be seen that the correlations of the macroeconomic variables with interest rates are fully subsumed in their interest-rate-correlated components:

$$\begin{aligned}
cov(x_t, z_t) &= cov(x_t, E(z_t|x_t) + u_t) \\
&= cov(x_t, E(z_t|x_t)) + cov(x_t, u_t) \\
&= cov(x_t, E(z_t|x_t)) \quad (cov(x_t, u_t) = 0 \text{ from Equation 3})
\end{aligned} \tag{4}$$

With the decomposition $z_t = E(z_t|x_t) + u_t$ at hand, it is straightforward to project the macroeconomic variables $\{z_{T+j}\}_{j=1}^{H}$ in future time periods consistent with interest rate projections. Inserting the interest rate projections $\{x_{T+j}\}_{j=1}^{H}$ into the function of each macroeconomic variable's interest-rate-correlated component $E(z_t|x_t)$ produces the projections $\{E(z_{T+j}|x_{T+j}\}_{j=1}^{H}$. The macro-specific component u_t of a macroeconomic variable is uncorrelated with interest rates x_t and therefore can be specified as a constant across the various interest rate scenarios. As $E(u_t) = 0$ by its definition, a natural candidate of the projections is $u_{T+j} = 0$ for $j = 1, 2, ..., H$. If macro-specific stress is desired in IRR measurement, a tail percentile of the historical data $\{u_t\}_{t=1}^{T}$ could be used instead.

2.1 Estimating the Conditional Expectation

To perform the decomposition of the macroeconomic variables $z_t = E(z_t|x_t) + u_t$, ones needs to estimate the functional form of the conditional expectation $E(z_t|x_t)$. A linear function of x_t could be a convenient approximation of the conditional expectation $E(z_t|x_t)$ but would be subject to unforeseen approximation errors. A non-parametric method is more appropriate to estimate the conditional expectation $E(z_t|x_t)$. In the following, the discussion will treat the macroeconomic variables z_t as a scalar variable but it should be understood that the approach applies to each variable in the vector z_t.

We derive our non-parametric estimator from the mathematical properties of the conditional expectation $E(z_t|x_t)$. Let $m(x_t)$ be any function of the interest rates x_t. It follows that:

$$\begin{aligned} E((z_t - m(x_t))^2) &= E((z_t - E(z_t|x_t) + E(z_t|x_t) - m(x_t))^2) \\ &= E(u_t^2) + E((E(z_t|x_t) - m(x_t))^2) + 2E(u_t(E(z_t|x_t) - m(x_t))) \\ &= E(u_t^2) + E((E(z_t|x_t) - m(x_t))^2) \\ &\geq E(u_t^2) \end{aligned} \tag{5}$$

where $E(u_t(E(z_t|x_t) - m(x_t))) = 0$ by applying $E(u_t|x_t) = 0$ and the law of iterated expectation. Equation 5 shows that the conditional expectation $E(z_t|x_t)$ is the function that minimizes its mean square difference from the macroeconomic variables z_t. Thus the estimation problem for the conditional expectation can be framed as:

$$\min_{m} \quad E((z_t - m(x_t))^2) \tag{6}$$

Equation 6 is an infinite-dimensional optimization problem and can not be directly solved. To find a solution, we make two additional assumptions:

1. The function space to search for the solution m is a reproducing kernel Hilbert space (RKHS) $\mathcal{H}_K$ spanned by a kernel function K.

2. The norm of the function $||m||^2_{\mathcal{H}_K}$ is below some finite level A.

RKHS is a general function space and covers a wide variety of possible functions (Wahba (2002)). The assumption of RKHS with finite norm is standard in the non-parametric statistics and machine learning literature as it provide a general and unified context for solving function estimation problems.

Given these two assumptions, the problem of finding the conditional expectation becomes:

$$\begin{aligned} \min_{m\in\mathcal{H}_K} \quad & E((z_t - m(x_t))^2) \\ s.t. \quad & ||m||^2_{\mathcal{H}_K} \le A \end{aligned} \tag{7}$$

The constrained optimization of Equation 7 is equivalent to a regularization problem by applying the Lagrange Multiplier method (Kloft et al. (2009)):

$$\min_{m\in\mathcal{H}_K} \quad E((z_t - m(x_t))^2) + \lambda ||m||^2_{\mathcal{H}_K} \tag{8}$$

where λ is the Lagrange multiplier for the constraint on the function norm $||m||^2_{\mathcal{H}_K}$. Viewed as a penalty term, the constraint on the function norm $||m||^2_{\mathcal{H}_K}$ serves to reduce in-sample overfit and improves the generality of the estimate. The optimization problem of Equation 8 is known as the kernel ridge regression in the machine learning literature and has the finite-dimensional solution:

$$m(x_t) = \sum_{j=1}^{T} \phi_j K(x_t, x_j) \tag{9}$$

where $\phi = [\phi_1, ..., \phi_T]' = (K + \lambda I_T)^{-1} z$ and $z = [z_1, ..., z_T]'$. The matrix K is T-by-T with the (i,j)-th element being $K(x_i, x_j)$, $i, j = 1, 2, ..., T$. The details of the kernel ridge regression and the penalized function optimization in general can be found in Wahba (1990), Girosi et al. (1995), Evgeniou et al. (2000) and Hastie et al. (2009).

Common choices of the kernel function K includes the polynomial function $K(x, y) = (1+x'y)^d$ and the Gaussian function $K(x, y) = exp(-v(x-y)'(x-y))$ where d and v are hyper-parameters to be determined by the researcher. In the estimation, the penalty weight λ along with any additional hyper-parameters in the kernel function need to be determined. A time-series cross-validation procedure could be applied to determine such free parameters (Bergmeir et al. (2018)). To proceed, we first create a grid of possible values of the free parameters. For a given value in the grid, we use the first Q data in our data sample to estimate the function m, denoted as $m_Q(\cdot)$, and compute the squared 1-step-ahead forecast error $(z_{Q+1} - m_Q(x_{Q+1}))^2$. Then we increase one more data and use the first $Q + 1$ data to produce the estimate $m_{Q+1}(\cdot)$ and the squared 1-step-ahead forecast error $(z_{Q+2} - m_{Q+1}(x_{Q+2}))^2$. This procedure is repeated sequentially for the first Q, Q+1, ..., T-1 data and results in a sequence of squared 1-step-ahead forecast errors $\{(z_{j+1} - m_j(x_{j+1}))^2\}_{j=Q}^{T-1}$. The root mean squared error (RMSE) of forecasts $\sqrt{\frac{1}{T-Q}\sum_{j=Q}^{T-1}(z_{j+1} - m_j(x_{j+1}))^2}$

is recorded. The grid value that leads to the smallest RMSE of forecasts is chosen as the optimal value for the free parameters.

The set of interest rate variables x_t is huge theoretically, containing all functions of observable interest rates and their lags. To implement the kernel ridge regression in practice, it is necessary to condense the set x_t to a manageable level. Since principal component analysis of interest rates has revealed that a small number of principal components of interest rates (specifically the first three components) can explain a large part of their variations and thus be useful representations of the information contained in interest rates (Litterman and Scheinkman (1991)), we recommend using the first three principal components and a fixed number of their lags to approximate the set x_t in estimation in practice.

3 Empirical Illustration

Data of a bank's client behavior is proprietary and is not publicly available. In this paper, we use aggregate banking data for an empirical illustration of the proposed approach. Specifically we consider the monthly log growth rate of non-tax-sheltered Canadian-dollar fixed-term deposit balance in Canadian chartered banks from personal clients in the monthly sample from January 2000 to January 2019. The data of the deposit balance is obtained from Bank of Canada website. The interest rate variables we postulate to be correlated with the deposit growth rate is the monthly change in 12-month Canadian Dollar Offered Rate (CDOR hereafter)[3] and their lags up to 3 months. Following Basel IRR standards (2016), we consider five additional explanatory variables of macroeconomic factors: Canadian real GDP growth rate, Canadian unemployment rate, 12-month Canadian inflation rate, 12-month average log return of the SP&TSX stock index (stock market return hereafter), and 12-month average log return of the Canadian new housing price index (housing price change hereafter).

3.1 Regression of Deposit Growth Rate

An OLS regression of the deposit growth rate on the CDOR rate change and its lags yields an adjusted-R-square of 5%, while adding the five macroeconomic variables raises the adjusted-R-square to 32%. The regression results are shown in Table 3.1. Lags of the CDOR rate change are statistically significant, suggesting that higher interest rates tend to be associated with subsequent fixed-term deposit growth. The statistically significant coefficients on the unemployment rate and the housing price change suggest that fixed-term deposit growth is procyclical, growing in economic booms and falling in economic recessions. The coefficient on the stock market return is statistically significant and is negative, suggesting a substitution effect of stock investment on fixed-term deposit.

[3]The CDOR rates are the main short-term benchmark rates in Canadian financial market. Details of the CDOR rates can be found in McRae and Auger (2018).

Table 1: Regressions of Deposit Growth Rate

	Regression: interest rate variables only	Regression: including macroeconomic variables
Constant	0.27	3.88
2-month lag of CDOR change		0.41
3-month lag of CDOR change	1.05	0.70
Unemployment rate		-0.53
Stock market return		-0.14
Housing price change		0.67
Adjusted R square	0.05	0.32

Note: The dependent variable is the monthly log growth rate of non-tax-sheltered Canadian-dollar fixed-term deposit balance in Canadian chartered banks from personal clients in the monthly sample from January 2000 to January 2019. The regressor "CDOR change" is the monthly change of 12-month CDOR rate. The regressor "unemployment rate" is the Canadian unemployment rate. The regressor "stock market return" is the 12-month average log return of the SP&TSX stock index. The regressor "housing price change" is the 12-month average log return of the Canadian new housing price index. The standard errors of the regression coefficients are computed by the Newey-West method with 4 lags. Only the regressors with coefficients that are statistically significant at the 5% level are retained in the final forms of the regressions.

3.2 Decomposition of Macroeconomic Variables

We collect the monthly data on CDOR rates of 1-, 2-, 3-, 6- and 12-months as well as Canadian dollar swap rates of 2- to 12-, 15-, 20- and 30-years from January 2000 to January 2019. The first 3 principal components of the interest rates are able to capture over 99.9% of the interest rates' variations and thus are used to summarize the information in the interest rates. We use both the contemporaneous value and up to 3 lags of the first 3 principal components to represent the information in interest rates available at each month.

We focus on the decomposition of the three macroeconomic variables that are statistically significant in the regression of deposit growth rate: unemployment rate, stock market return and housing price change. A cubic polynomial kernel is used in this exercise to reduce the number of free hyper-parameters. To estimate the conditional expectation of each macroeconomic variable on the principal components, the penalty weight λ in the optimization of Equation 8 is selected by grid search through time-series cross validation. We use a trial and error process to determine the grid of penalty weights. We first randomly draw a number of possible penalty weights from a uniform distribution between 0.01 and 5,000 to see the general shape of the RMSEs and identify the trough segment of the RMSE curve. We then create a finer grid in the trough segment of the RMSE curve to pin down the optimal penalty weight while reducing the number of grid points in non-trough segment to reduce computation burden. Figure 1 plots the final grid of the penalty weight λ against the corresponding

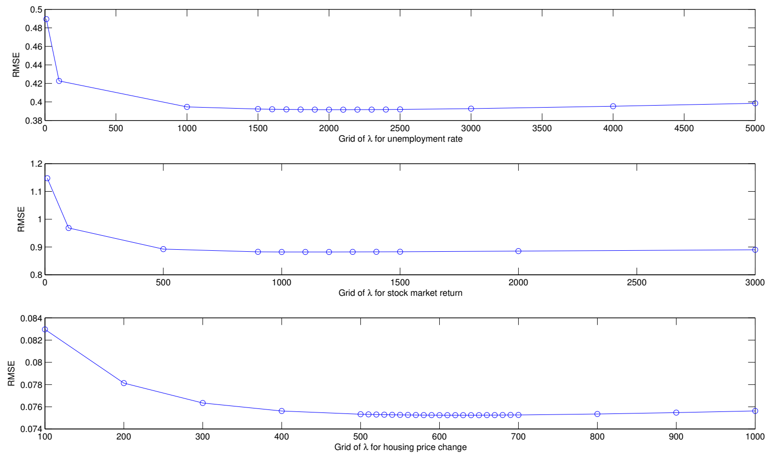

Figure 1: Selecting Penalty Weights in Kernel Ridge Regressions for Decomposing Macroeconomic Variables

RMSEs of one-step-ahead out-of-sample forecasts. All the RMSE curves exhibit a U shape. The resulting optimal penalty weight λ is 2,100 for unemployment rate, 1,100 for stock market return, and 620 for housing price change.

The resulting in-sample decomposition of the three macroeconomic variables is shown in Figure 2. While the interest-rate-correlated component captures most of the time-series variations of unemployment rate and, to a less extent, of housing price change, stock market return appears to be mainly affected by its macro-specific component.

3.3 Projection of Deposit Growth Rate

We consider two interest rate scenarios: parallel shift up and down of the yield curve at January 2019 by 200 basis points (the zero lower bound of interest rates is preserved in the shift down scenario). The deposit growth rates are projected for each of the subsequent 12 months from February 2019 to January 2020 under these two interest rate scenarios. To integrate macroeconomic variables into the interest rate scenario, we need to further specify the value of the macro-specific components of the macroeconomic variables in the 12 projection months. In this exercise we set the projections of the macro-specific components of the macroeconomic variables to be at their mean values of zero.

Forward rates are used in this exercise as projections of interest rates. We first linearly interpolate the unobserved points of the shifted yield curve at January 2019 to compute the forward curves for each of the subsequent 12 months. Projections of the variables involving the monthly change of 12-month CDOR rate and the principal components are calculated based on the forward

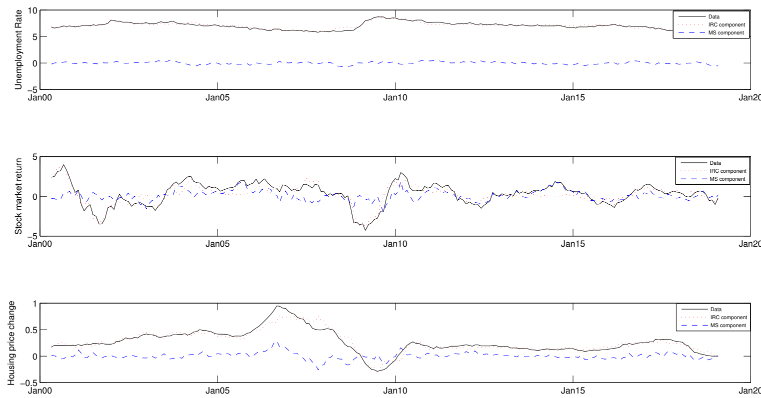

Figure 2: In-Sample Decomposition of Macroeconomic Variables (the abbreviation "IRC" refers to the interest-rate-correlated component while "MS" refers to the macro-specific component)

curves in the 12 months after January 2019. The projected principal components are then fed into the estimated kernel ridge regression functions to project the interest-rate-correlated component of the macroeconomic variables in the 12 months after January 2019. Combining the projections of the interest-rate-correlated and the macro-specific components produces the projections of the macroeconomic variables that are consistent with the scenario of shifted yield curve in January 2019.

The projections of deposit growth rate are computed by inserting projections of the macroeconomic variables and the monthly change of 12-month CDOR rate into the regressions of Table 3.1 and are plotted in Figure 3 along with the historical deposit growth rate. The projected deposit growth is generally higher in the parallel shift-up scenario than in the parallel shift-down one, consistent with the procyclicality of the deposit growth rate seen in its regression estimates.

4 Conclusion

This paper proposes a non-parametric approach to decompose a macroeconomic variable into an interest-rate-correlated component and a macro-specific component. Projections of interest rates per prescribed interest rate scenarios can be fed into the estimated decomposition to project the interest-rate-correlated components of macroeconomic variables. The proposed approach ensures that the projections of interest rates and macroeconomic variables are consistent with each other under the IRR measurement framework and thus integrate

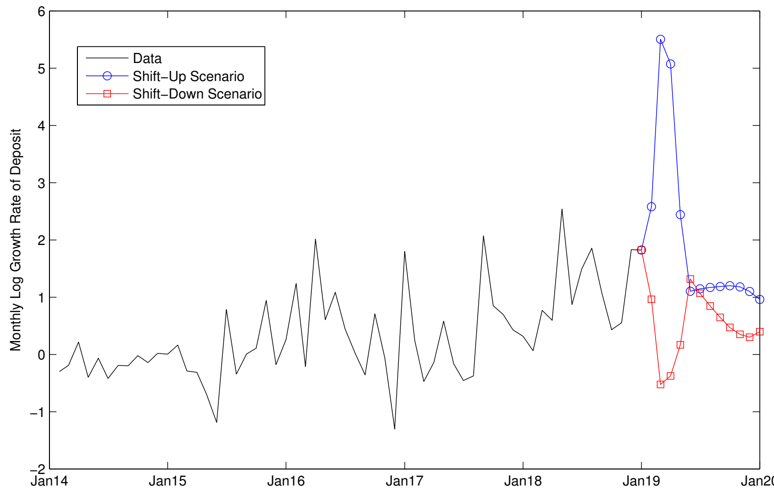

Figure 3: Projections of Deposit Growth Rate

macroeconomic variables into interest rate scenarios for modeling client behaviors. Implementation cost of the proposed approach in banks' IRR measurement systems is minimal as the final estimate is of a linear functional form and has a closed-form solution.

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