Integrating Macroeconomic Variables with Interest Rate Scenarios for Interest Rate Risk Measurement in the Banking Book

Zhongfang He*

May 2019

Abstract

Recent Basel standards on interest rate risk (IRR) in the banking book requires the consideration of macroeconomic variables for modeling client behaviors. Since macroeconomic variables and interest rates are correlated, projecting macroeconomic variables for IRR measurement poses a challenge of keeping consistency with interest rate projections from regulator-prescribed interest rate scenarios. This paper proposes an approach to integrate macroeconomic variables with interest rate scenarios. The conditional expectation of macroeconomic variables on interest rate variables is used to capture the dependence between macroeconomic and interest rate variables. Based on the mathematical properties of conditional expectation we derive its non-parametric estimator. The resulting projections of macroeconomic variables are fully consistent with given interest rate scenarios and are convenient for implementation in practice. An empirical application to Canadian fixed-term deposits is conducted to illustrate the proposed approach.

Key words: interest rate risk, non-parametric, kernel ridge regression, machine learning

1 Introduction

Traditionally for the interest rate risk (IRR hereafter) measurement of banks' banking book, models of client behaviors such as loan prepayment and deposit redemption concern risk scenarios with interest rate variables only. For example, one risk scenario that is commonly used among banks and is prescribed by

*Royal Bank of Canada, hezhongfang2004@yahoo.com. The Matlab programs and data used in the paper can be found in this Github site. Disclaimer: The analysis and conclusions in this paper are those of the author and do not represent the view of the Royal Bank of Canada.

regulators for measuring IRR is a parallel shift-up or shift-down of the risk-free yield curve by 200 basis points. Client behaviors are modeled as driven by interest rate variables and possibly other factors such as seasonality and client characteristics. Given the projections of interest rates under prescribed interest rate scenarios, the change in client behaviors and, consequently, the change in the various balance-sheet and cash-flow items in the banking book are projected over a certain number of future time periods. The resulting changes in bank earning and economic value under the various interest rate scenarios are computed as IRR measures to inform IRR management[1].

Refinement of IRR measurement practice and the strengthened regulation since the 2008 financial crisis, however, ask banks to consider the dimensions of risk scenarios beyond interest rates for modeling client behaviours. The Basel standards "Interest Rate Risk in the Banking Book" of April 2016 explicitly states that "*banks must carefully consider how the exercise of the behavioural optionality will vary not only under the interest rate shock and stress scenarios but also across other dimensions*"(paragraph 46). The macroeconomic variables that are listed in Basel IRR standards (2016) to model client behaviors include stock indices, unemployment rates, GDP, inflation and housing price indices. On the other hand, the risk scenarios that are currently used among banks and are prescribed by regulators to compute IRR measures continue to contain interest rate variables only. For example Basel IRR standards (2016) prescribes six standardized risk scenarios for IRR measurement that include parallel up and down, steepening and flattening, and short rates up and down for interest rates; none of these six risk scenarios concern the macroeconomic variables prescribed to be considered in modeling client behaviors. The lack of prescribed risk scenarios for macroeconomic variables leaves open the question of how to integrate them with the interest rate scenarios to project client behaviors for computing IRR measures.

The key modeling challenge to include macroeconomic variables in IRR measurement arises from the correlation between macroeconomic variables and interest rate movements. Intuitively, consider a macroeconomic variable that is a driver of certain IRR-relevant client behavior but is uncorrelated with interest rates. When we shock the yield curve in the interest rate scenarios, this macroeconomic variable is unaffected given its zero correlation with interest rates and thus its impact on the client behavior would be a constant across the various interest rate scenarios. In other words, no movement in the client behavior across the various interest rate scenarios is associated with this macroeconomic variable. Since the goal of IRR measurement is on the risk generated by interest rate movements, one could simply assume a constant value for this macroeconomic variable, e.g. its historical average, when projecting the client behavior in future time periods for IRR measurement.

In reality, the movements of macroeconomic variables and interest rates are generally correlated. Developments in GDP, unemployment rate and inflation

[1] Annex 1 of Basel IRR standards (2016) contains a detailed description of IRR measurement practices in banks.

rate are usually the key ingredients into central banks' monetary policies that determine interest rate levels. Therefore projections of macroeconomic variables for IRR measures need to take into account their correlations with interest rates and to be consistent with interest rate scenarios. For example, what values should the macroeconomic variables be projected in future time periods when the interest rate scenario is a parallel shift up of the current yield curve by 200 basis points? How should the projections of the macroeconomic variables change if the interest rate scenario switches to a steepening shock with lower short rates and higher long rates? IRR managers need to model the correlation between macroeconomic variables and interest rates and ensure their consistency in projections for IRR measurement.

A straightforward solution would be regressing macroeconomic variables on interest rates by assuming a linear or non-linear functional form such that projections of interest rates can be fed into the estimated model to project the macroeconomic variables. Alternative approaches include modeling the marginal distribution of macroeconomic variables and applying a copula to connect individual models of macroeconomic and interest rate variables. This paper goes one step further to use the conditional expectation of macroeconomic variables on interest rates as a representation of their correlations and motivates the appropriate non-parametric model from the mathematical properties of conditional expectation. Thus macroeconomic variables can be integrated into the interest rate scenarios in a rigorous yet practical way instead of based on *ad hoc* modeling assumptions[2]

The basic idea of the proposed approach is to decompose each macroeconomic variable into the sum of its conditional expectation on interest rates and the residual. By the definition of conditional expectation, the residual is uncorrelated with interest rates and hence its value is immune to the impact of interest rate movements. Such a decomposition ensures that the correlation of each macroeconomic variable with interest rates is fully subsumed in its conditional expectation component. We term this conditional expectation of a macroeconomic variable as its interest-rate-correlated component and the residual as its macro-specific component.

The functional form of the conditional expectation is unknown. To proceed, we show that the conditional expectation of a macroeconomic variable on interest rates minimizes the mean square difference between the macroeconomic variable and any function of interest rates. Therefore the search for the conditional expectation of a macroeconomic variable can be reduced to a function optimization problem with mean square loss. By introducing the assumption of the function space being a reproducing kernel Hilbert one with finite function norm, the solution to the function optimization problem becomes the non-parametric estimator known as the kernel ridge regression in the machine learning literature (Hastie et al. (2009), Murphy (2012)). Thus a non-parametric approach is ap-

[2]Time series copula methods with non-parametric marginal distributions for macroeconomic variables and a non-parametric copula (Patton (2012)) would also require minimal *ad hoc* modeling assumptions but arguably will raise the model maintenance and implementation costs in practice.

plied to estimate the correlation between macroeconomic variables and interest rates that is derived from the mathematical properties of conditional expectation. The final estimate of the conditional expectation is of a linear functional form with a closed-form solution and can be implemented with minimal extra efforts on top of the existing frameworks of IRR measurement in banks.

With the decomposition of macroeconomic variables at hand, it becomes straightforward to project the macroeconomic variable in a given interest rate scenario consistently. Being a function of interest rates, projections of the conditional expectation component of the macroeconomic variable can be computed by using projections of interest rates according to the interest rate scenario. The other part of the macroeconomic variable, namely the macro-specific component, is immune to interest rate movements and hence its projections could be computed as a constant across interest rate scenarios, e.g. its historical average or a chosen tail percentile of its historical distribution. Summing up the projections of these two components produces projections of the macroeconomic variables that are fully consistent with the prescribed interest rate scenario.

Empirical applications of the proposed approach require data on client behavior. As banks' client behavior data are proprietary, we use publicly available aggregate deposit data as the alternative to illustrate the proposed approach. Specifically we apply the proposed approach to project the growth rate of the non-tax-sheltered fixed-term deposits from personal clients in Canadian chartered banks over a 12-month period under the 200-basis-points parallel shift-up and shift-down scenarios of interest rates.

The remainder of the paper is as follows. The details of the decomposition of macroeconomic variables are provided in Section 2. Section 3 shows the empirical illustration of the proposed approach to Canadian fixed-term deposits. Section 4 concludes.

2 The Model

Let y_t be the client behavior variable of interest at time t, e.g. the deposit redemption rate. The variable x_t denotes the set of interest rate variables that are assumed to be correlated with the client behavior y_t and that could include any linear/non-linear transformation of interest rates at time t as well as their lags. In practice, seasonality or other interest-rate-unrelated factors could also be included in x_t to explain the client behavior y_t. With a little abuse of notation, we term x_t as an interest rate variable for expositional convenience when it does not cause confusion. Under a risk scenario with interest rates only, the client behavior model can be written as:

$$y_t = f(x_t) + residual \tag{1}$$

where the function f is known. An example of the function f could be the linear function $f(x_t) = \alpha + x_t'\beta$.

To add macroeconomic variables z_t, the model of the client behavior changes to:

$$y_t = g(x_t, z_t) + residual \tag{2}$$

where the function g could be, say, $g(x_t, z_t) = \alpha + x_t'\beta + z_t'\gamma$. Parameters in the function g can be estimated by using historical data on the client behavior y_t, interest rates x_t and macroeconomic variables z_t, $t = 1, 2, ..., T$.

A prescribed interest rate scenario specifies the shocked yield curve at time T that leads to the projection of future interest rates $\{x_{T+j}\}_{j=1}^{H}$ through forward rate calculation or term structure model of interest rates. To project the future client behavior $\{y_{T+j}\}_{j=1}^{H}$, ones needs to compute the projections of macroeconomic variables $\{z_{T+j}\}_{j=1}^{H}$ that are consistent with the projected interest rates $\{x_{T+j}\}_{j=1}^{H}$ in the scenario. To do that, let $E(z_t|x_t) = \int z_t p(z_t|x_t)dz_t$ be the expectation of the macroeconomic variables z_t conditional on the interest rates x_t. Denote the residual $u_t = z_t - E(z_t|x_t)$. It follows that the macroeconomic variables are decomposed as $z_t = E(z_t|x_t) + u_t$ and that $E(u_t|x_t) = 0$. The latter follows from the law of iterated expectation $E(u_t|x_t) = E(z_t - E(z_t|x_t)|x_t) = 0$. It is straightforward to see that:

$$\begin{aligned} cov(x_t, u_t) &= E(x_t u_t) - E(x_t)E(u_t) \\ &= E(x_t u_t) \quad (E(u_t) = E(z_t - E(z_t|x_t)) = 0) \\ &= E(E(x_t u_t|x_t)) \\ &= E(x_t E(u_t|x_t) \\ &= 0 \end{aligned} \tag{3}$$

Given that the component u_t is uncorrelated with the interest rate variables x_t, we term u_t as the "macro-specific" component of the macroeconomic variables z_t. The conditional expectation $E(z_t|x_t)$ is by its definition a function of the interest rates x_t and hence is termed the "interest-rate-correlated" component of the macroeconomic variables z_t. It can be seen that the correlations of the macroeconomic variables with interest rates are fully subsumed in their interest-rate-correlated components:

$$\begin{aligned} cov(x_t, z_t) &= cov(x_t, E(z_t|x_t) + u_t) \\ &= cov(x_t, E(z_t|x_t)) + cov(x_t, u_t) \\ &= cov(x_t, E(z_t|x_t)) \quad (cov(x_t, u_t) = 0 \text{ from Equation 3}) \end{aligned} \tag{4}$$

With the decomposition $z_t = E(z_t|x_t) + u_t$ at hand, it is straightforward to project the macroeconomic variables $\{z_{T+j}\}_{j=1}^{H}$ in future time periods consistent with interest rate projections. Inserting the interest rate projections $\{x_{T+j}\}_{j=1}^{H}$ into the function of each macroeconomic variable's interest-rate-correlated component $E(z_t|x_t)$ produces the projections $\{E(z_{T+j}|x_{T+j}\}_{j=1}^{H}$. The macro-specific component u_t of a macroeconomic variable is uncorrelated with interest rates x_t and therefore can be specified as a constant across the various interest rate scenarios. As $E(u_t) = 0$ by its definition, a natural candidate of the projections is $u_{T+j} = 0$ for $j = 1, 2, ..., H$. If macro-specific stress is desired in IRR measurement, a tail percentile of the historical data $\{u_t\}_{t=1}^{T}$ could be used instead.

2.1 Estimating the Conditional Expectation

To perform the decomposition of the macroeconomic variables $z_t = E(z_t|x_t) + u_t$, ones needs to estimate the functional form of the conditional expectation $E(z_t|x_t)$. A linear function of x_t could be a convenient approximation of the conditional expectation $E(z_t|x_t)$ but would be subject to unforeseen approximation errors. A non-parametric method is more appropriate to estimate the conditional expectation $E(z_t|x_t)$.

We derive our non-parametric estimator from the mathematical properties of the conditional expectation $E(z_t|x_t)$. Let $m(x_t)$ be any function of the interest rates x_t. It follows that:

$$\begin{aligned} E((z_t - m(x_t))^2) &= E((z_t - E(z_t|x_t) + E(z_t|x_t) - m(x_t))^2) \\ &= E(u_t^2) + E((E(z_t|x_t) - m(x_t))^2) + 2E(u_t(E(z_t|x_t) - m(x_t))) \\ &= E(u_t^2) + E((E(z_t|x_t) - m(x_t))^2) \\ &\geq E(u_t^2) \end{aligned} \tag{5}$$

where $E(u_t(E(z_t|x_t) - m(x_t))) = 0$ by applying $E(u_t|x_t) = 0$ and the law of iterated expectation. Equation 5 shows that the conditional expectation $E(z_t|x_t)$ is the function that minimizes its mean square difference from the macroeconomic variables z_t. Thus the estimation problem for the conditional expectation can be framed as:

$$\min_{m} \quad E((z_t - m(x_t))^2) \tag{6}$$

Equation 6 is an infinite-dimensional optimization problem and can not be directly solved. To find a solution, we make two additional assumptions:

1. The function space to search for the solution m is a reproducing kernel Hilbert space (RKHS) $\mathcal{H}_K$ spanned by a kernel function K.

2. The norm of the function $||m||^2_{\mathcal{H}_K}$ is below some finite level A.

RKHS is a general function space and covers a wide variety of possible functions (Wahba (2002)). The assumption of RKHS with finite norm is standard in the non-parametric statistics and machine learning literature as it provide a general and unified context for solving function estimation problems.

Given these two assumptions, the problem of finding the conditional expectation becomes:

$$\begin{aligned} \min_{m \in \mathcal{H}_K} \quad & E((z_t - m(x_t))^2) \\ s.t. \quad & ||m||^2_{\mathcal{H}_K} \leq A \end{aligned} \tag{7}$$

The constrained optimization of Equation 7 is equivalent to a regularization problem by applying the Lagrange Multiplier method (Kloft et al. (2009)):

$$\min_{m \in \mathcal{H}_K} \quad E((z_t - m(x_t))^2) + \lambda ||m||^2_{\mathcal{H}_K} \tag{8}$$

where λ is the Lagrange multiplier for the constraint on the function norm $||m||^2_{\mathcal{H}_K}$. Viewed as a penalty term, the constraint on the function norm $||m||^2_{\mathcal{H}_K}$ serves to reduce in-sample overfit and improves the generality of the estimate. The optimization problem of Equation 8 is known as the kernel ridge regression in the machine learning literature and has the finite-dimensional solution:

$$m(x_t) = \sum_{j=1}^{T} \phi_j K(x_t, x_j) \tag{9}$$

where $\phi = [\phi_1, ..., \phi_T]' = (K + \lambda I_T)^{-1} z^*$ and $z^* = [z^*_1, ..., z^*_T]'$. The matrix K is T-by-T with the (i,j)-th element being $K(x_i, x_j)$, i, j = 1, 2, ..., T. The details of the kernel ridge regression and the penalized function optimization in general can be found in Wahba (1990), Girosi et al. (1995), Evgeniou et al. (2000) and Hastie et al. (2009).

Common choices of the kernel function K includes the polynomial function $K(x, y) = (1+x'y)^d$ and the Gaussian function $K(x, y) = exp(-v(x-y)'(x-y))$ where d and v are hyper-parameters to be determined by the research. In the estimation, the penalty weight λ along with any additional hyper-parameters in the kernel function need to be determined. A time-series cross-validation procedure could be applied to determine such free parameters (Bergmeir et al. (2018)). To proceed, we first create a grid of possible values of the free parameters. For a given value in the grid, we use the first Q data in our data sample to estimate the function m, denoted as $m_Q(\cdot)$, and compute the squared 1-step-ahead forecast error $(z_{Q+1} - m_Q(x_{Q+1}))^2$. Then we increase one more data and use the first $Q + 1$ data to produce the estimate $m_{Q+1}(\cdot)$ and the squared 1-step-ahead forecast error $(z_{Q+2} - m_{Q+1}(x_{Q+2}))^2$. This procedure is repeated sequentially for the first Q, Q+1, ..., T-1 data and results in a sequence of squared 1-step-ahead forecast errors $\{(z_{j+1} - m_j(x_{j+1}))^2\}_{j=Q}^{T-1}$. The root mean squared error (RMSE) of forecasts $\sqrt{\frac{1}{T-Q}\sum_{j=Q}^{T-1}(z_{j+1} - m_j(x_{j+1}))^2}$ is recorded. The grid value that leads to the smallest RMSE of forecasts is chosen as the optimal value for the free parameters.

The set of interest rate variables x_t is huge theoretically, containing all functions of observable interest rates and their lags. To implement the kernel ridge regression in practice, it is necessary to condense the set x_t to a manageable level. Since principal component analysis of interest rates has revealed that a small number of principal components of interest rates (specifically the first three components) can explain a large part of their variations and thus be useful representations of the information contained in interest rates (Litterman and Scheinkman (1991)), we recommend using the first three principal components and a fixed number of their lags to approximate the set x_t in estimation in practice.

3 Empirical Illustration

Data of a bank's client behavior is proprietary and is not publicly available. In this paper, we resort to aggregate banking data for an empirical illustration of the proposed approach. Specifically we consider the monthly log growth rate of non-tax-sheltered Canadian-dollar fixed-term deposit balance in Canadian chartered banks from personal clients in the monthly sample from January 2000 to January 2019. The data of the deposit balance is obtained from Bank of Canada website. The interest rate variables we postulate to be correlated with the deposit growth rate is the monthly change in 12-month Canadian Dollar Offered Rate (CDOR hereafter)[3] and their lags up to 3 months. Following Basel IRR standards (2016), we consider five additional explanatory variables of macroeconomic factors: Canadian real GDP growth rate, Canadian unemployment rate, 12-month Canadian inflation rate, 12-month average log return of the SP&TSX stock index (stock market return hereafter), and 12-month average log return of the Canadian new housing price index (housing price change hereafter).

3.1 Regression of Deposit Growth Rate

An OLS regression of the deposit growth rate on the CDOR rate change and its lags yields an adjusted-R-square of 5%, while adding the five macroeconomic variables raises the adjusted-R-square to 32%. The regression results are shown in Table 3.1. Lags of the CDOR rate change are statistically significant, suggesting that higher interest rates tend to be associated with subsequent fixed-term deposit growth. The statistically significant coefficients on the unemployment rate and the housing price change suggest that fixed-term deposit growth is procyclical, growing in economic booms and falling in economic recessions. The coefficient on the stock market return is statistically significant and is negative, suggesting a substitution effect of stock investment on fixed-term deposit.

3.2 Decomposition of Macroeconomic Variables

We collect the monthly data on CDOR rates of 1-, 2-, 3-, 6- and 12-months as well as Canadian dollar swap rates of 2- to 12-, 15-, 20- and 30-years from January 2000 to January 2019. The first 3 principal components of the interest rates are able to capture over 99.9% of the interest rates' variations and thus are used to summarize the information in the interest rates. We use both the contemporaneous value and up to 3 lags of the first 3 principal components to represent the information in interest rates available at each month.

We focus on the decomposition of the three macroeconomic variables that are statistically significant in the regression of deposit growth rate: unemployment rate, stock market return and housing price change. A cubic polynomial kernel is used in this exercise to reduce the number of free hyper-parameters. To estimate the conditional expectation of each macroeconomic variable on the

[3]The CDOR rates are the main short-term benchmark rates in Canadian financial market. Details of the CDOR rates can be found in McRae and Auger (2018).

Table 1: Regressions of Deposit Growth Rate

	Regression: interest rate variables only	Regression: including macroeconomic variables
Constant	0.27	3.88
2-month lag of CDOR change		0.41
3-month lag of CDOR change	1.05	0.70
Unemployment rate		-0.53
Stock market return		-0.14
Housing price change		0.67
Adjusted R square	0.05	0.32

Note: The dependent variable is the monthly log growth rate of non-tax-sheltered Canadian-dollar fixed-term deposit balance in Canadian chartered banks from personal clients in the monthly sample from January 2000 to January 2019. The regressor "CDOR change" is the monthly change of 12-month CDOR rate. The regressor "unemployment rate" is the Canadian unemployment rate. The regressor "stock market return" is the 12-month average log return of the SP&TSX stock index. The regressor "housing price change" is the 12-month average log return of the Canadian new housing price index. The standard errors of the regression coefficients are computed by the Newey-West method with 4 lags. Only the regressors with coefficients that are statistically significant at the 5% level are retained in the final forms of the regressions.

principal components, the penalty weight λ in the optimization of Equation 8 is selected by grid search through time-series cross validation. We use a trial and error process to determine the grid of penalty weights. We first randomly draw a number of possible penalty weights from a uniform distribution between 0.01 and 5,000 to see the general shape of the RMSEs and identify the trough segment of the RMSE curve. We then create a finer grid in the trough segment of the RMSE curve to pin down the optimal penalty weight while reducing the number of grid points in non-trough segment to reduce computation burden. Figure 3.2 plots the final grid of the penalty weight λ against the corresponding RMSEs of one-step-ahead out-of-sample forecasts. All the RMSE curves exhibit a U shape. The resulting optimal penalty weight λ is 2,100 for unemployment rate, 1,100 for stock market return, and 620 for housing price change.

The resulting in-sample decomposition of the three macroeconomic variables is shown in Figure 3.2. While the interest-rate-correlated component captures most of the time-series variations of unemployment rate and, to a less extent, of housing price change, stock market return appears to be mainly affected by its macro-specific component.

3.3 Projection of Deposit Growth Rate

We consider two interest rate scenarios: parallel shift up and down of the yield curve at January 2019 by 200 basis points (the zero lower bound of interest rates is preserved in the shift down scenario). The deposit growth rates are projected

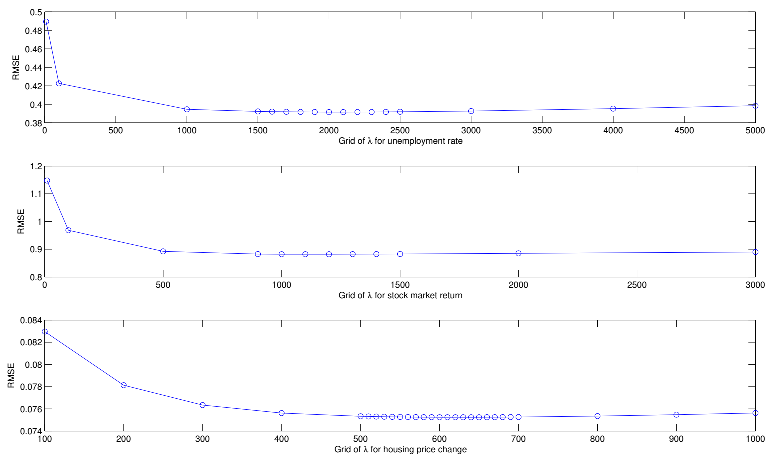

Figure 1: Selecting Penalty Weights in Kernel Ridge Regressions for Decomposing Macroeconomic Variables

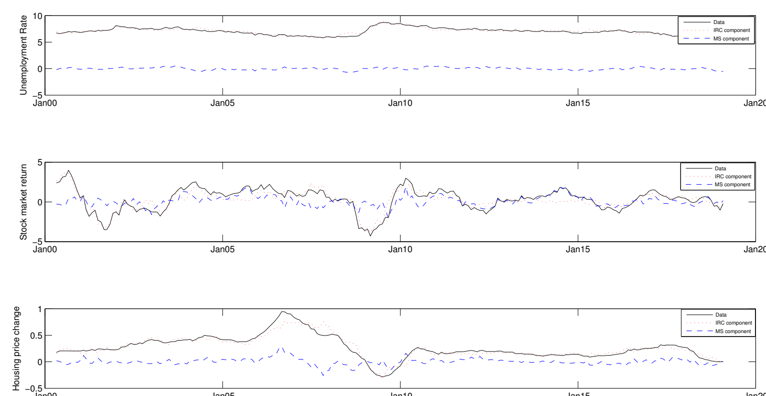

Figure 2: In-Sample Decomposition of Macroeconomic Variables (the abbreviation "IRC" refers to the interest-rate-correlated component while "MS" refers to the macro-specific component)

for each of the subsequent 12 months from February 2019 to January 2020 under these two interest rate scenarios. To integrate macroeconomic variables into the interest rate scenario, we need to further specify the value of the macro-specific components of the macroeconomic variables in the 12 projection months. In this exercise we set the projections of the macro-specific components of the macroeconomic variables to be at their mean values of zero.

Forward rates are used in this exercise as projections of interest rates. We first linearly interpolate the unobserved points of the shifted yield curve at January 2019 to compute the forward curves for each of the subsequent 12 months. Projections of the variables involving the monthly change of 12-month CDOR rate and the principal components are calculated based on the forward curves in the 12 months after January 2019. The projected principal components are then fed into the estimated kernel ridge regression functions to project the interest-rate-correlated component of the macroeconomic variables in the 12 months after January 2019. Combining the projections of the interest-rate-correlated and the macro-specific components produces the projections of the macroeconomic variables that are consistent with the scenario of shifted yield curve in January 2019.

The projections of deposit growth rate are computed by inserting projections of the macroeconomic variables and the monthly change of 12-month CDOR rate into the regressions of Table 3.1 and are plotted in Figure 3.3 along with the historical deposit growth rate. The projected deposit growth is generally higher in the parallel shift-up scenario than in the parallel shift-down one, consistent with the procyclicality of the deposit growth rate seen in its regression estimates.

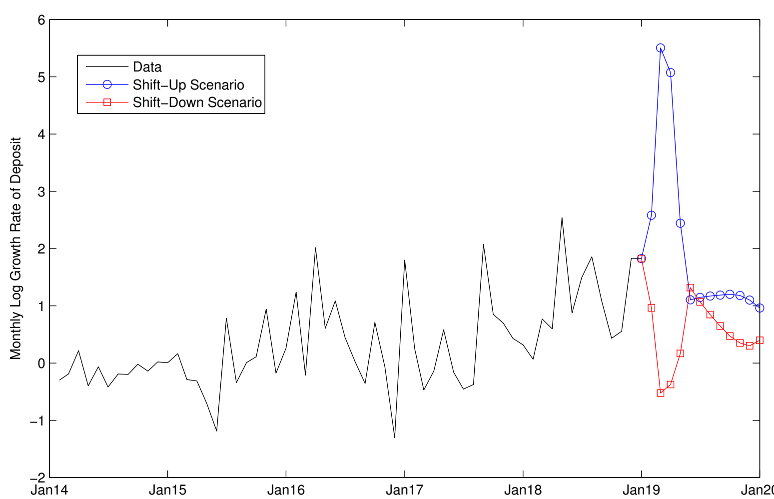

Figure 3: Projections of Deposit Growth Rate

4 Conclusion

This paper proposes a non-parametric approach to decompose a macroeconomic variable into an interest-rate-correlated component and a macro-specific component. Projections of interest rates per prescribed interest rate scenarios can be fed into the estimated decomposition to project the interest-rate-correlated components of macroeconomic variables. The proposed approach ensures that the projections of interest rates and macroeconomic variables are consistent with each other under the IRR measurement framework and thus integrate macroeconomic variables into interest rate scenarios for modeling client behaviors. Implementation cost of the proposed approach in banks' IRR measurement systems is minimal as the final estimate is of a linear functional form and has a closed-form solution.

References

C. Bergmeir, R. Hyndman, and B. Koo. A note on the validity of cross-validation for evaluating autoregressive time series prediction. *Computational Statistics and Data Analysis*, 120:70–83, 2018.

T. Evgeniou, M. Pontil, and T. Poggio. Regularization networks and support vector machines. *Advances in Computational Mathematics*, 13:1–50, 2000.

F. Girosi, M. Jones, and T. Poggio. Regularization theory and neural network architectures. *Neural Computation*, 7:219–269, 1995.

T. Hastie, R. Tibshirani, and J. Friedman. *The Elements of Statistical Learning*. Springer, 2009.

M. Kloft, U. Brefeld, P. Laskov, K. Muller, A. Zien, and S. Sonnenburg. Efficient and accurate lp-norm multiple kernel learning. *Advances in Neural Information Processing Systems*, 22, 2009.

R. Litterman and J. Scheinkman. Common factors affecting bond returns. *Journal of Fixed Income*, 1:54–61, 1991.

K. McRae and D. Auger. A primer on the canadian bankers' acceptance market. *Bank of Canada Staff Discussion Paper*, 2018.

K. Murphy. *Machine Learning: A Probabilistic Perspective*. The MIT Press, 2012.

A. Patton. A review of copula models for economic time series. *Journal of Multivariate Analysis*, 110:4–18, 2012.

G. Wahba. *Spline Models for Observational Data*. SIAM Philadelphia, 1990.

G. Wahba. Soft and hard classification by reproducing kernel hilbert space methods. *Proceedings of the National Academy of Sciences of the United States of America*, 99:16524–16530, 2002.