Comments on "Time-dependent shrinkage of time-varying parameter models", submitted to *Econometric Reviews*

Summary This is an interesting paper that makes several contributions in the context of time-varying parameter (TVP) models which became an indispensable and valuable tool for modelling economic time series in the past decade. The contribution of the paper is two-fold:

- The author introduces a new hierarchical prior for variance selection in TVP models. This *Gamma Horse prior* provides an important and useful extension to the existing literature and is shown to exhibit excellent shrinkage properties both for simulated and real data.

- To improve MCMC estimation, a new strategy is proposed for applying boosting via ASIS to TVP models, by defining new conditional SA-AA pairs to boost the global shrinkage parameter ν_j for each TV coefficient as well as the local shrinkage parameter $d_{j,t}$.

The paper is well motivated and well-written and, in my opinion, would make a fine contribution to *Econometric Reviews*. Please find below specific comments how to improve this interesting paper further.

Specific comments

1. In the paper, the author uses the rather uncommon representation of the horseshoe prior as a scale mixture involving the inverted beta distribution $IB(0.5, 0.5)$ (which is also known as the beta prime distribution). However, for the $a = b$, the $IB(a, a)$-distribution is equal to the $\mathrm{F}(2a, 2a)$-distribution. Hence, the representation introduced in this paper is equivalent to the representation of the horseshoe prior introduced in Cadonna et al. (2020), where the local shrinkage parameter follows the $\mathrm{F}(1, 1) = IB(0.5, 0.5)$-distribution.

2. Establishing this relationship could be helpful with regard to studying the theoretical properties of the suggested prior in Appendix A. Following Carvalho et al. (2010) it is claimed that the marginal density of the horseshoe prior is not available in closed form. However, this is no longer the case, since Cadonna et al. (2020) provided a closed form for this density involving Kummer's U-function. This paper also studies the mathematical behaviour of the density both in the spike and in the tails and these results very likely are useful also for the present paper.

3. Regarding the priors for heteroscedastic variances based on the SV model, the author suggests a hierarchical prior for the volatility of volatility parameter σ_y based on the F $(1, 1) = IB(0.5, 0.5)$-distribution which I find an interesting extension. Most details of this extension are provided in the Appendix B, but it would be helpful to have a more detailed discussion of this extension in the main part of the paper.

4. Regrettably, the author never follows up on this extension in the applied part of the paper. The simulation study in Section 4 is based on homoscedastic errors. The SV model is applied to the inflation data in Section 5. However, as far as I can see, no details are provided what the effect of introducing a hierarchical prior for σ_y actually is. The author should comment on this (and provide details in a Webappendix, such as the estimated s_t^2 and posterior distributions of σ_y under the two priors).

5. Regarding the inflation data, I wonder what the estimated TV coefficients reported for the GHS in Figure 7 look like for the other three priors. The author should comment on that and provide these additional results in a web-appendix.

6. Currently, the data sample runs till Q4 2020. Given the recent drastic increase in inflation, it might be worthwhile to include more recent data. Most importantly, this could be a situation where the specification of a sensible prior for the TV coefficients matters even more than for the current data set.

Minor comments

1. p.6, l.9: $\log \sigma_1^2 \Rightarrow \log \sigma_0^2$?

2. p.13, footnote 9: Frühwirth-Schnatter (1994) is another pioneering paper for sampling the latent states in linear Gaussian state space models.

References

Cadonna, A., S. Frühwirth-Schnatter, and P. Knaus (2020). Triple the gamma – A unifying shrinkage prior for variance and variable selection in sparse state space and TVP models. *Econometrics 8*, 20.

Carvalho, C. M., N. G. Polson, and J. G. Scott (2010). The horseshoe estimator for sparse signals. *Biometrika 97*, 465–480.

Frühwirth-Schnatter, S. (1994). Data augmentation and dynamic linear models. *Journal of Time Series Analysis 15*, 183–202.