Fast and noise-resistant ion-trap quantum computation with inherent dynamical decoupling

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We propose a scheme for realizing quantum logic gates between any pair of ions confined in a linear trap with a pair of laser beams tuned to the carrier. The striking feature of the scheme is that the carrier excitation accompanying the spin-motion coupling does not affect the gate dynamics. As a consequence, the gate not only is much more insensitive to motional heating but also can operate at a higher speed compared to the previous schemes. The other important advantages are that the gate speed does not need to be inversely proportional to the number of ions in the chain, and the accompanying carrier drive results in dynamical decoupling, making the gate performance robust against dephasing noises. We show that for the same error sources the gate infidelity can be decreased by about ten times compared with previous schemes.

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I. INTRODUCTION

One of the most promising candidates for implementation of quantum computation is a system of trapped ions. In such a system, the qubits are represented by long-lived electronic states, whose coupling to the data bus (the vibrational phononic modes) can be controlled by laser fields [1]. Up to now, all of the elementary logic operations required for ion-trap quantum computation have been experimentally achieved [2]. Efforts are being devoted to scaling up the quantum processors and improving the gate fidelity. An important step forward towards scalability has been demonstrated using a multitrap architecture, where ions are shuttled between different regions to perform required gates [3]. It is challenging to realize two-qubit gates with a sufficiently high fidelity so that the imperfection can be rectified by quantum error correction [4,5]. The lowest error threshold for all the fault-tolerant quantum computing models proposed so far is about 10^{-4} [6–8]. For ion-trap quantum computation, there exist different kinds of error sources: unwanted carrier or sideband excitations, unbalanced ac-Stark shifts, fluctuations in the control fields, and an inherent decoherence mechanism, i.e., the spontaneous emission and heating of motional modes. Many ion-trap quantum computational schemes have been suggested to partially suppress these effects [9–15]. The Sørensen-Mølmer gate [9], based on virtual phonon exchange induced by collective interaction of the qubits with the laser fields tuned closely to the sideband transitions, is insensitive to heating. The drawback of this scheme is the gate speed is very low, which is undesirable since other gate error sources may become serious problems during the operation. The speed of this gate can be considerably improved by adequately increasing the laser intensity and detuning at the price of entangling the qubits and the data bus mode during the gate operation [10].

Most of experimental realizations of entangling gates with trapped ions have used collective ion-laser interactions [16–23]. For hyperfine qubits, the highest entanglement fidelity reported so far is about 97% [17,18], with the main sources of gate error being the spontaneous emission and the fluctuations in the detuning and intensity of the Raman laser beams. The decoherence associated with spontaneous emission can be avoided by using magnetic fields and their gradients to coherently manipulate the hyperfine or Zeeman ground states and couple them to motional states [24]. Recently, experimental implementation of this approach has been reported, in which the controlling magnetic fields are generated from oscillating microwave currents in trap electrodes [25]. However, this gate requires a duration much longer than those with laser driving due to the limitation of the available field gradient, and the entangled state between two qubits generated by such a gate operation only has a fidelity of 76%.

The spontaneous photon scattering does not occur during gate operations with optical qubits, which are encoded in one ground state and one long-lived metastable state of ions linked by an optical dipole-forbidden transition. So far, the entangling gate with the highest fidelity (99.3%) was realized with two optical qubits by performing the fast Sørensen-Mølmer gate, which is mediated by the axial center-of-mass vibrational mode [20]. In this scheme, the spin-dependent center-of-mass mode excitation is accompanied by the unwanted carrier and spectator mode excitations. To suppress these detrimental effects the Rabi frequencies of the laser fields need to be much smaller than the vibrational mode frequencies. As the number N of the ions in a linear trap grows, it will be difficult to perform a faithful Sørensen-Mølmer gate operation because the square of the Lamb-Dicke parameter, and hence the gate rate, is proportional to $1/N$. For longer operation times the heating of the phonon modes and fluctuations in the parameters of the control fields will seriously affect the gate fidelity. Another serious problem is the presence of ac-Stark shifts because the presently used compensation technique is not applicable to ions in a thermal state [26].

In this paper we propose an alternative laser-driving method for implementation of fast and robust phase gates on two optical qubits in an ion crystal. Our scheme uses two laser beams tuned to the carrier instead of a bichromatic field, with detunings close to the sidebands of the center-of-mass vibrational modes. On one hand, the carrier excitation accompanying the spin-boson couplings does not interfere with the conditional dynamics of the two qubits. Due to this dynamical feature the Rabi frequency of the laser beams is not required to be much smaller than the vibrational mode frequencies, and hence the gate speed can be significantly improved. The

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reduction of the Lamb-Dicke parameter due to the increase of N can be compensated by improving the laser intensity, and the gate duration does not need to be proportional to N. On the other hand, the dynamical decoupling arising from the carrier driving greatly improves the robustness of the qubits to dephasing noises. Furthermore, the gate operation is based on virtual excitation of the vibrational modes so that the scheme is insensitive to heating. With these features the gate infidelity can be decreased by about ten times compared with previous schemes for the same decoherence sources.

II. THE FAST AND NOISE-RESISTANT PHASE GATE

We consider N two-level ions confined in a linear trap along the z axis. The quantum information is represented by one metastable state $|\uparrow\rangle$ and one ground state $|\downarrow\rangle$ of the internal degree of freedom. For implementation of the two-qubit gate between ions r and s, the transition $|\uparrow\rangle \leftrightarrow |\downarrow\rangle$ for these two ions is driven by a pair of laser beams in resonance with the electronic transition with the same Rabi frequencies Ω. Under the rotating-wave approximation, the Hamiltonian for the laser-ion coupling is given by (assuming $\hbar = 1$)

$$H = \Omega \sum_{\alpha=r,s} [e^{i(\mathbf{k}_1\cdot\mathbf{r}_\alpha - \phi_1)} + e^{i(\mathbf{k}_2\cdot\mathbf{r}_\alpha - \phi_2)}]|\uparrow\rangle_\alpha\langle\downarrow| + \text{H.c.}, \quad (1)$$

where $\mathbf{k}_j$ and ϕ_j ($j = 1,2$) are the wave vector and phase of the jth laser beam and $\mathbf{r}_\alpha$ is the position operator of ion α relative to its equilibrium position. We here assume that $\mathbf{k}_1$ and $\mathbf{k}_2$ are both in the x-z plane. Then we have $\mathbf{k}_j\cdot\mathbf{r}_\alpha = \sum_{p=1}^{N}[\eta_{p,\alpha}^{z,j}(a_p^\dagger e^{i\omega_{z,p}t} + a_p e^{-i\omega_{z,p}t}) + \eta_{p,\alpha}^{x,j}(b_p^\dagger e^{i\omega_{x,p}t} + b_p e^{-i\omega_{x,p}t})]$, where $a_p^\dagger$ ($b_p^\dagger$) and a_p (b_p) are the creation and annihilation operators for the pth longitudinal (transverse) vibrational mode with frequency $\omega_{z,p}$ ($\omega_{x,p}$) and $\eta_{p,\alpha}^{z,j}$ ($\eta_{p,\alpha}^{x,j}$) is the Lamb-Dicke parameter describing the excursion of the αth ion in the pth longitudinal (transverse) mode associated with the projection of the wave vector $\mathbf{k}_j$ on the z (x) axis. We here have assumed that each laser beam has the same phase on these two ions. This can be achieved by adjusting the wave vectors of the laser beams and/or the ion distance [17,18], so that $|k_{j,z}|d = 2n_j\pi$, where $k_{j,z}$ is the projection of the wave vector for the jth laser beam on the trap axis, d is the distance between the equilibrium positions of these two ions, and n_j is an integer. Since the ions are not homogeneously distributed in a harmonic trap, one should adjust d in every two-qubit gate. This problem can be overcome by confining the ions in an anharmonic linear trap with nearly uniform spacing between ions [27]. In this case, the modification of d for each two-qubit gate is unnecessary. Assume that the Lamb-Dicke parameters are sufficiently small so that the Hamiltonian can be well approximated up to the first order in the Lamb-Dicke parameters,

$$H = \Omega \sum_{\alpha=r,s}\sum_{j=1,2} e^{-i\phi_j}|\uparrow\rangle_\alpha\langle\downarrow| \times \left\{1 + i\sum_{p=1}^{N}\left[\eta_{p,\alpha}^{z,j}(a_p^\dagger e^{i\omega_{z,p}t} + a_p e^{-i\omega_{z,p}t}) + \eta_{p,\alpha}^{x,j}(b_p^\dagger e^{i\omega_{x,p}t} + b_p e^{-i\omega_{x,p}t})\right]\right\} + \text{H.c.} \quad (2)$$

Suppose that $\mathbf{k}_1$ and $\mathbf{k}_2$ have angles θ and $\pi - \theta$ to the trap axis; that is, the wave vectors of the two laser beams along the x axis are identical and those along the z axis are opposite. This implies $\eta_{p,\alpha}^{z,1} = -\eta_{p,\alpha}^{z,2} = \eta_{p,\alpha}^{z} = T_{p,\alpha}^{z}|\mathbf{k}_1|\cos\theta\sqrt{\hbar/(2M\omega_{z,p})}$ and $\eta_{p,\alpha}^{x,1} = \eta_{p,\alpha}^{x,2} = \eta_{p,\alpha}^{x} = T_{p,\alpha}^{x}|\mathbf{k}_1|\sin\theta\sqrt{\hbar/(2M\omega_{x,p})}$, where $T_{p,\alpha}^{z}$ ($T_{p,\alpha}^{x}$) is the normal mode transformation matrix of the αth ion with the pth longitudinal (transverse) mode and M is the mass of a single ion. Then the Hamiltonian can be written as

$$H = 2\Omega e^{-i\phi_+}\sum_{\alpha=r,s}|\uparrow\rangle_\alpha\langle\downarrow| \times \left[\cos\phi_- - \sin\phi_-\sum_{p=1}^{N}\eta_{p,\alpha}^{z}(a_p^\dagger e^{i\omega_{z,p}t} + a_p e^{-i\omega_{z,p}t}) + i\cos\phi_-\eta_{p,\alpha}^{x,j}(b_p^\dagger e^{i\omega_{x,p}t} + b_p e^{-i\omega_{x,p}t})\right] + \text{H.c.}, \quad (3)$$

where $\phi_\pm = (\phi_2 \pm \phi_1)/2$. Here, ϕ_+ corresponds to the phase of the residual carrier drive and decides the relative phase of the state components $|\uparrow\rangle$ and $|\downarrow\rangle$ of the dressed states of the carrier drive. On the other hand, the strength for the carrier excitation and the couplings of the qubits to both the longitudinal and transverse modes depend on ϕ_-.

We here assume that $|\eta_{p,\alpha}^{x,j}|\cos\phi_- \ll |\eta_{p,\alpha}^{z,j}|\sin\phi_-$ so that we can neglect the contributions of the transverse modes. As will be seen in the next section, this condition can be well satisfied. When $\omega_{z,p} \gg 2|\eta_{p,\alpha}^{z}\sqrt{(\bar{n}_{z,p}+1)}\Omega|$, the spin-dependent displacements of all the modes become negligible, and the virtual phonon exchange results in the spin-spin coupling. The effective Hamiltonian for this process is [28]

$$H_e = -4\Omega^2\sum_{p=1}^{N}\frac{\sin^2\phi_-}{\omega_{z,p}}\left[\sum_{\alpha=r,s}\eta_{p,\alpha}^{z}(e^{-i\phi_+}|\uparrow\rangle_\alpha\langle\downarrow| + e^{i\phi_+}|\downarrow\rangle_\alpha\langle\uparrow|)\right]^2. \quad (4)$$

The total effective Hamiltonian is $H_{t,e} = H_c + H_e$, where $H_c = 2\Omega\cos\phi_- e^{-i\phi_+}\sum_{\alpha=r,s}|\uparrow\rangle_\alpha\langle\downarrow| + \text{H.c.}$ describes the residual carrier excitation. In terms of the dressed states $|u\rangle = (|\downarrow\rangle - e^{-i\phi_+}|\uparrow\rangle)/\sqrt{2}$ and $|d\rangle = (|\downarrow\rangle + e^{-i\phi_+}|\uparrow\rangle/\sqrt{2}$ produced by the carrier excitation, the effective Hamiltonian can be rewritten as

$$\begin{aligned} H_e &= -4\Omega^2\sum_{p=1}^{N}\frac{\sin^2\phi_-}{\omega_{z,p}}\left[\sum_{\alpha=r,s}\eta_{p,\alpha}^{z}(|u\rangle_\alpha\langle u| - |d\rangle_\alpha\langle d|)\right]^2 \\ &= -\sum_{\alpha=r,s}\lambda_\alpha(|u\rangle_\alpha\langle u| + |d\rangle_\alpha\langle d|) \\ &\quad - \lambda_{r,s}(|u\rangle_r\langle u|\otimes|u\rangle_s\langle u| + |d\rangle_r\langle d|\otimes|d\rangle_s\langle d| \\ &\quad - |u\rangle_r\langle u|\otimes|d\rangle_s\langle d| - |d\rangle_r\langle d|\otimes|u\rangle_s\langle u|), \end{aligned} \quad (5)$$

where $\lambda_\alpha = 4\sum_{p=1}^{N}(\eta_{p,\alpha}^{z}\Omega\sin\phi_-)^2/\omega_{z,p}$ and $\lambda_{r,s} = 8\sum_{p=1}^{N}\eta_{p,1}^{z}\eta_{p,2}^{z}(\Omega\sin\phi_-)^2/\omega_{z,p}$.

Suppose that the qubits are represented by the basis states $|u\rangle$ and $|d\rangle$. Choose the intensity of the laser beams and the interaction time T such that $\lambda_{r,s}T = \pi/4$. Dropping the trivial common phase factor $e^{i(\lambda_r+\lambda_s)T+i\pi/4}$, the evolution of the two-qubit basis states induced by the effective Hamiltonian H_e follows as

$$\begin{aligned} |d\rangle_r\,|d\rangle_s &\longrightarrow |d\rangle_r\,|d\rangle_s\,, \\ |u\rangle_r\,|d\rangle_s &\longrightarrow e^{-i\pi/2}\,|u\rangle_r\,|d\rangle_s\,, \\ |d\rangle_r\,|u\rangle_s &\longrightarrow e^{-i\pi/2}\,|d\rangle_r\,|u\rangle_s\,, \\ |u\rangle_r\,|u\rangle_s &\longrightarrow e^{i\pi}(e^{-i\pi/2}\,|u\rangle_r)(e^{-i\pi/2}\,|u\rangle_s). \end{aligned} \tag{6}$$

This corresponds to a two-qubit π-phase gate plus single-qubit $\pi/2$ -phase shifts on state $|u\rangle$. For the implementation of a given algorithm these additional $\pi/2$-phase shifts can be absorbed into the subsequent single-qubit rotations on the corresponding qubits [17].

We note that the Hamiltonian for the carrier excitation commutes with that for the coupling between the qubits and the longitudinal modes, and thus the carrier drive does not affect the two-qubit conditional phase shift. However, it plays the role of dynamical decoupling, protecting the qubits from dephasing noises. Unlike the previous dynamical decoupling schemes [18,28], here the decoupling is inherent in the gate performance, other than arising from an additional carrier drive not coupled to the motional modes. Furthermore, the Hamiltonian for the coupling between the qubits and the longitudinal modes does not contain any term that induces transitions between different dressed states. Therefore the error due to neglecting such a term in the previous schemes [18,28] is avoided.

Compared to the fast Sørensen-Mølmer gate [10], the carrier excitation does not affect the two-qubit conditional dynamics, and thus the restriction $\Omega \ll \omega_{z,1}$ is removed. Furthermore, the motional excitation is highly suppressed during the gate operation. Another important advantage of our scheme is that the accompanying carrier driving suppresses the dephasing errors due to the resulting dynamical decoupling. We have assumed that the relative phase of the two laser beams is the same on these two ions (the present beam geometry guarantees that the sum of the phases of the two laser beams is the same on different ions). If that is not the case, the coefficient $\sin^2\phi_-$ in the qubit-qubit coupling $\lambda_{r,s}$ should be replaced by $\sin(\phi_{r,1}/2)\sin(\phi_{r,2}/2)$, where $\phi_{r,1}$ and $\phi_{r,2}$ are the relative phases of the lasers on these two ions. The same gate operation can be implemented within a time $T = 30\pi/\omega_{z,1}$ if the condition $2\eta_1^z\Omega\sqrt{\sin(\phi_{r,1}/2)\sin(\phi_{r,2}/2)} = 0.079\,06\omega_{z,1}$ is satisfied. These features offer the possibility for implementation of faithful quantum gates in a large crystal. We note this gate is also fundamentally different from the light-shift-induced gate [11]: (i) Here the light-shift term commutes with the spin-motion coupling term, and hence the conditional phase shift has no dependence upon light shift. (ii) Since the effective Hamiltonian does not involve the motional-state-dependent part, it is unnecessary to invert the phase of the laser at a high frequency to cancel the unbalanced Stark shifts even for an ion chain with $N > 2$. (iii) The time required for the gate performance is considerably reduced.

III. GATE ERROR ANALYSIS

For the case with $N = 2$ we have $\omega_{z,2} = 3^{1/2}\omega_{z,1}$ and $\eta_{2,1}^z = -\eta_{2,2}^z = \eta_2^z = 3^{-1/4}\eta_1^z$, where η_1^z is Lamb-Dicke parameter for the longitudinal center-of-mass mode with the frequency $\omega_{z,1}$. Then the time required to produce the controlled π-phase gate is $T = 3\pi\omega_{z,1}/[64(\eta_1^z\Omega\sin\phi_-)^2]$. If the qubits are initially in the state $|\downarrow\rangle_1|\downarrow\rangle_2$, they evolve to the maximally entangled state $\frac{1}{\sqrt{2}}(|\downarrow\rangle_1|\downarrow\rangle_2 + ie^{-2i\phi_+}|\uparrow\rangle_1|\uparrow\rangle_2)$ after the gate operation. To obtain a stable entangled state one should lock the phases of the laser beams through an additional reference laser. For $2\eta_1^z\Omega\sin\phi_- = 0.1\omega_{z,1}$, the time required for the generation of this state is $T = 18.75\pi/\omega_{z,1}$. To further suppress the entanglement between the motional modes and the qubits, we here take $T = 30\pi/\omega_{z,1}$, which corresponds to $2\eta_1^z\Omega\sin\phi_- = 0.079\,06\omega_{z,1}$. For $\eta_1^z = 0.05$ and $\phi_- = 0.35\pi$, the required Rabi frequency is $\Omega = 0.8873\omega_{z,1}$.

We now analyze the effects of different noises on the gate fidelity. The fluctuation $\delta\Omega$ in the Rabi frequency, which mainly comes from lower-frequency laser-intensity noise and thermal phonons of the radial modes [20], leads to a gate infidelity $\varepsilon_1 \simeq (\pi/2)^2(\delta\Omega/\Omega)^2$. Taking $\delta\Omega/\Omega = 1.4\times10^{-2}$ yields $\varepsilon_1 \simeq 4.8\times10^{-4}$. The frequency noise results in the deviation $\delta\omega_l$ of the laser frequency from the atomic transition frequency, causing dephasing. Due to the dynamical decoupling arising from the carrier driving, the error ε_2 due to laser-frequency noise is reduced to the order of $[\delta\omega_l/(4\Omega\cos\phi_-)]^2$. With $\delta\omega_l = 1.3\times10^{-4}\omega_{z,1}$ [20], this error is $\varepsilon_2 \sim 10^{-8}$. The relative phase fluctuation $\delta(\phi_2-\phi_1)$ of the laser fields causes an error in the spin-motion coupling, resulting in an infidelity $\varepsilon_3 \simeq (\pi/2)^2(\delta\phi_-\cot\phi_-)^2$. Setting $\delta(\phi_2-\phi_1) = 0.01$, we obtain $\varepsilon_3 \simeq 1.6\times10^{-5}$. The error caused by the total phase fluctuation of the laser beams is $\varepsilon_4 = (\delta\phi_+)^2 = 10^{-4}$ for $\delta(\phi_2+\phi_1) = 0.02$.

The effective decoherence rates due to the heating of the two longitudinal modes are about $\Gamma_{1,e} \simeq 2(4\eta_1^z\Omega\sin\phi_-/\omega_{z,1})^2 d\bar{n}_{z,1}/dt$ and $\Gamma_{2,e} \simeq 2(4\eta_2^z\Omega\sin\phi_-/\omega_{z,2})^2 d\bar{n}_{z,2}/dt$. For $d\bar{n}_{\xi,p}/dt \simeq 2.5\times10^{-6}\omega_{z,1}$ $(\xi = z,x)$ [20,26], the error due to heating is $\varepsilon_5 \sim (\Gamma_{1,e}+\Gamma_{2,e})T \simeq (1.79\pi/\omega_{z,1})d\bar{n}_{z,1}/dt \simeq 1.4\times10^{-5}$. This is more than one order of magnitude lower than the heating-induced error in the Sørensen-Mølmer gate [10], which was estimated to be $[\pi/(\omega_{z,1}-\delta)]d\bar{n}_{z,1}/dt$, with the sideband detuning $(\omega_{z,1}-\delta)$ being much smaller than $\omega_{z,1}$.

Now we analyze the error arising from the interactions between the qubits and the transverse modes that have been discarded. On one hand, these off-resonant interactions result in unequal energy shifts for the dressed states, with the difference $\delta_d = \sum_{p=1}^2(2\eta_{p,1}^x\Omega\cos\phi_-)^2(1/\omega_{x,p}^- - 1/\omega_{x,p}^+)$, where $\omega_{x,p}^\pm = \omega_{x,p} \pm 4\Omega\cos\phi_-$. Setting $\omega_{x,1} = 3.252\omega_{z,1}$ [20] and $\theta = 0.1\pi$, we have $\delta_d = 4.66\times10^{-5}\omega_{z,1}$. The resulting error is $\varepsilon_6 = (\delta_d T)^2 = 1.9\times10^{-5}$. On the other hand, these interactions induce the effective coupling between $|u\rangle_1|d\rangle_2$ and $|u\rangle_2|d\rangle_1$ with the coupling strength $\mu = \sum_{p=1}^2(2\Omega\cos\phi_-)^2\eta_{p,1}^x\eta_{p,2}^x(1/\omega_{x,p}^- + 1/\omega_{x,p}^+)$. The corresponding error is $\varepsilon_7 = (\mu T/2)^2 \simeq 8.7\times10^{-8}$. The error resulting from the off-resonant two-phonon excitations is proportional to $(\eta_{p,1}^z\cos\phi_-)^8$, which is negligible compared to other errors.

The phase shifts of the dressed states induced by the carrier driving can be canceled by the procedure analogous to spin echoes. If at time $T/2$ one shifts the phases of both laser beams by π, the carrier-excitation-induced phase shifts accumulated during the two halves of the evolution cancel each other. Recently, this technique has been applied in the demonstration of the dressed-state phase gate with hyperfine qubits [18]. The results reported in Ref. [18] show that the error due to the carrier drive infidelity mainly arises from the fluctuations of the Rabi frequency on the time scale of the gate duration. When the fluctuations are slow, which is the case in the experiments with optical qubits [20], the spin-echo technique effectively cancels the corresponding errors. Note that due to the laser phase shift the center-of-mass mode does not disentangle from the qubits after the gate operation. We can insert a free evolution of time $\tau = \pi/\omega_{z,1}$ in the middle of the gate operation to cancel this entanglement. Then the error due to the entanglement between the qubits and the stretch mode results in an error $\varepsilon_8 \simeq |4\eta_2^z\Omega\sin\phi_-(e^{i\omega_2 T}-1)/\omega_{z,2}|^2/2 \simeq 5.8\times 10^{-7}$.

Due to the relative amplitude fluctuation between the two lasers, the carrier drive Hamiltonian contains a term that couples the dressed states $|u\rangle$ and $|d\rangle$. If we set the Rabi frequencies for these two lasers to be $\Omega_{1,2} = \Omega(1\mp\xi/2)$, the coupling strength between $|u\rangle$ and $|d\rangle$ is $g = \xi\Omega\sin\phi_-$. Owing to the dynamical decoupling, this coupling is far off-resonant and results in energy shifts for the dressed states. The spin-echo technique cancels the error arising from these energy shifts by reversing the effective detuning (the light shift) during the second half of the evolution. On the other hand, the error associated with the off-resonant dressed-state transition is $\varepsilon_9 \sim 2(g/4\Omega\cos\phi_-)^2$. Taking $\xi = 0.01$, we have $\varepsilon_8 \sim 4.8\times 10^{-5}$.

For optical qubits, the ac-Stark shift mainly arises from the far-off-resonant dipole transition, which is proportional to the square of the carrier Rabi frequency. We roughly model it as $\delta_{ac} = (2\Omega\cos\phi_-)^2/\chi$ and estimate χ to be approximately $40\omega_{z,1}$ with the parameters reported in [20,26]. This energy shift can be compensated by shifting the laser frequency accordingly. Due to the fluctuations of Ω and ϕ_- the residual ac-Stark shift is $2\delta_{ac}(\delta\Omega/\Omega + \delta\phi_-\tan\phi_-)$. In the presence of the dynamical decoupling this results in the far-off-resonant coupling between the two dressed states, causing an error $\varepsilon_{10} \sim 2[(\delta\Omega/\Omega)^2 + (\delta\phi_-\tan\phi_-)^2](\delta_{ac}/4\Omega\cos\phi_-)^2 \simeq 6\times 10^{-8}$. With all the sources of decoherence being included, the gate error is about 6.8×10^{-4}, one order of magnitude smaller than the best result achieved so far [20].

We note that for a two-ion crystal the speed of the controlled π-phase gate can be further considerably increased by sandwiching a free evolution time $\tau \simeq 1.6836/\omega_{z,2}$ in between two pulses with the same duration $2\pi/\omega_{z,1}$ and Rabi frequency $0.1165\omega_{z,1}/\eta_1^z$. The phases of the lasers in the second pulse are shifted by π with respect to those in the first one. In this case the total time required to complete the gate operation is only $T \simeq 4.309\pi/\omega_{z,1}$.

Since the carrier excitation does not interfere with the gate dynamics, the reduction of the Lamb-Dicke parameters can be compensated by increasing the Rabi frequency so that the gate duration does not need to be proportional to the number of the qubits N. We here take an example. For $N = 10$, the controlled π-phase gate between the two ions at different ends of the linear crystal can be completed in a time $T = 30\pi/\omega_{z,1}$ with the choice $2\eta_1^z\Omega = 0.0942\omega$, and the coupling between these two qubits is mainly mediated by the first five longitudinal modes, which are numbered in increasing order according to their frequencies [29]. With a free evolution of time $\pi/\omega_{z,1}$ being sandwiched in between two laser-ion interactions as discussed above, the error due to the motional-internal state entanglement is mainly contributed by the third and fifth longitudinal modes. For $\bar{n}_{z,5} = 0$ and $\bar{n}_{\xi,p} = 3$ ($\xi \neq z$ or $p \neq 5$), the corresponding error is about 4.0×10^{-3} (cooling a single mode to the ground state can be achieved even for an ion crystal containing 14 qubits [22]).

The error arising from the thermal-phonon-induced reduction of the Rabi frequency depends upon the Lamb-Dicke parameters. We here choose $\omega_{x,1} = 10\omega_{z,1}$ and suppose that the center-of-mass trap frequency for the longitudinal direction is the same as in the two-ion crystal. Then the Lamb-Dicke parameters for the longitudinal and transverse center-of-mass modes are $\eta_1^z \simeq 0.022\,36$ and $\eta_p^x \simeq 0.0071$, respectively. With this choice the error due to the thermal fluctuation of the Rabi frequency is [15]

$$\left(\frac{\pi}{4}\right)^2\left[\sum_{p=1}^{10}4\left(\eta_p^z\right)^4\left(\bar{n}_{z,p}^2+\bar{n}_{z,p}\right)+\sum_{q=1}^{10}4\left(\eta_q^x\right)^4\left(\bar{n}_{x,q}^2+\bar{n}_{x,q}\right)\right]$$
$$+\left(\frac{\pi}{4}\right)^2\left[\sum_{p=1}^{10}\left(\eta_p^z\right)^2(2\bar{n}_{z,p}+1)+\sum_{q=1}^{10}\left(\eta_q^x\right)^2(2\bar{n}_{x,q}+1)\right]^2$$
$$\simeq 2.3\times 10^{-4}.$$

For $d\bar{n}_{\xi,p}/dt \simeq 2.5\times 10^{-6}\omega_{z,1}$, the error induced by heating is about 3.0×10^{-5}. With all the decoherence sources being included, the gate infidelity is estimated to be approximately 4.8×10^{-3}.

As has been shown, the entanglement of the qubits with the third and fifth longitudinal modes is the main source of the gate error. If a free evolution $\tau = 3.5\pi/\omega_{z,3}$ is sandwiched in between two laser pulses with the same duration $\tau = 30\pi/\omega_{z,1}$ but opposite phases, then the qubit-motion entanglement can be considerably suppressed. In this case the required Rabi frequency is $\Omega = 0.0942\omega_{z,1}/(2\sqrt{2}\eta_1^z\sin\phi_-)$. For $\bar{n}_{\xi,p} = 3$, the errors due to the residual qubit-motion entanglement and the thermal fluctuation of the Rabi frequency are 1.45×10^{-3} and 2.4×10^{-4}, respectively. Then the total error is reduced to about 2.4×10^{-3}.

It should be noted that for a given duration the gate between these two ions requires the highest laser intensity and thus has the lowest fidelity. For example, for the given ion-laser interaction time $60\pi/\omega_{z,1}$ the Rabi frequency required for implementing the gate between the ion at one end and its nearest neighbor is reduced to $\Omega = 0.0523\omega_{z,1}/(2\eta_1^z\sin\phi_-)$. Then the gate infidelity is about 2.0×10^{-3}.

IV. SUMMARY

In conclusion, we have proposed a scheme for implementing controlled quantum phase gates for two ion qubits with two laser beams resonant with the dipole-forbidden transition. In our scheme, the carrier-driving Hamiltonian commutes with the spin-motion coupling Hamiltonian used for the gate performance. This allows the laser intensity and hence the gate rate to be significantly improved, and therefore the gate rate does not need to be inversely proportional to the number of qubits in the ion array. Meanwhile, the effect of heating is suppressed since all the vibrational modes are only virtually excited. Another important feature is that the accompanying carrier driving has the benefit of reducing the sensitivity of the qubits to various dephasing noises. The idea can be directly applied to the hyperfine qubits by using two pairs of Raman beams tuned to the carrier.

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