Simple Manipulation of a Microwave Dressed-State Ion Qubit

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Many schemes for implementing quantum information processing require that the atomic states used have a nonzero magnetic moment; however, such magnetically sensitive states of an atom are vulnerable to decoherence due to fluctuating magnetic fields. Dressing an atom with external fields is a powerful method of reducing such decoherence [N. Timoney *et al.*, Nature (London) **476**, 185 (2011)]. We introduce an experimentally simpler method of manipulating such a dressed-state qubit, which allows the implementation of general rotations of the qubit, and demonstrate this method using a trapped ytterbium ion.

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A key component of any quantum information processor is a long-lived qubit, isolated from the environment to protect against decoherence [1]. For quantum information processing devices based on trapped ions, dephasing caused by fluctuations of the magnetic field surrounding the ions is a major decoherence source unless a field insensitive clock qubit is used. In many circumstances, however, such clock qubits cannot be used, as they either do not exist, or they are not compatible with the gates which are to be performed. The static magnetic field gradient gate proposed by Mintert *et al.* [2] is a promising gate technology from the point of scaling up an ion based quantum computer. A static magnetic field gradient is present, which produces both individual addressability of ions by a global microwave field [3], and a coupling between the internal and motional states of the ions which can mediate a multi-ion gate. The qubit states are required to have different magnetic moments, meaning a clock qubit cannot be used. Gates using this static field gradient method have been implemented; however, the gate fidelity was limited by such field-induced decoherence [4].

Although the intrinsic coherence time of a qubit may be short, it can be increased by methods such as applying a series of spin-echo pulses to the qubit [5,6]. Recently, Timoney *et al.* [7] proposed and implemented a scheme where two states with opposite magnetic moments are dressed by continuous microwave fields to form a state whose energy has no field dependence; this state is then combined with a third field-independent state to form an effective clock qubit. A method of manipulating the dressed-state qubit was also described, which allows rotation around a specific axis in the $x-y$ plane of the Bloch sphere only. Timoney *et al.* [7] reported a two order of magnitude improvement of coherence time using this dressed-state qubit, which should allow high-fidelity multi-qubit gates based on static field gradients to be performed.

We describe a different method of manipulating the dressed-state qubit which allows direct rotation of the qubit state about any axis in the $x-y$ plane. In addition to allowing more general rotations, this method is simpler to implement experimentally, removing requirements on the setting of the initial relative phases of the driving fields that are needed in the original manipulation method [7]. We demonstrate this new manipulation method using a single $^{171}\mathrm{Yb}^+$ ion.

We start by summarizing the dressed-state qubit and the method of Timoney *et al.* [7], in order to highlight the differences in our method.

The creation and manipulation of the dressed-state qubit is presented here as applied to the ground state of $^{171}\mathrm{Yb}^+$, although the method is applicable to other systems. The hyperfine structure of the $S_{1/2}$ ground state of $^{171}\mathrm{Yb}^+$ is shown in Fig. 1(a) and consists of the $F=0$ level $|0\rangle$, and three levels with $F=1$ ($|-1\rangle$, $|0'\rangle$, and $|+1\rangle$). The degeneracy of the $F=1$ levels is lifted by a static magnetic field. Transitions between $F=0$ and the $F=1$ levels are microwave transitions, and transitions between the $F=1$ levels are radio-frequency (rf) transitions.

By applying continuous microwave excitation to dress the ion, magnetically sensitive states can be stabilized against disturbances caused by magnetic field fluctuations. The three atomic states $|0\rangle$, $|-1\rangle$, and $|+1\rangle$ are dressed by two microwave fields of equal Rabi frequency $\Omega_{\mu\mathrm{w}}$ resonant with the $|0\rangle \leftrightarrow |-1\rangle$ and $|0\rangle \leftrightarrow |+1\rangle$ transitions, resulting in a Hamiltonian $H_{\mu\mathrm{w}} = (\hbar\Omega_{\mu\mathrm{w}}/2)(|+1\rangle\langle 0| + |-1\rangle\langle 0| + \mathrm{H.c.})$, setting the two microwave phases to zero (all Hamiltonians are presented in the interaction picture and after making the rotating wave approximation). The eigenstates of the coupled system are [7]

$$|D\rangle = \frac{1}{\sqrt{2}}(|+1\rangle - |-1\rangle), \tag{1}$$

$$|u\rangle = \frac{1}{2}|+1\rangle + \frac{1}{2}|-1\rangle + \frac{1}{\sqrt{2}}|0\rangle, \tag{2}$$

$$|d\rangle = \frac{1}{2}|+1\rangle + \frac{1}{2}|-1\rangle - \frac{1}{\sqrt{2}}|0\rangle, \tag{3}$$

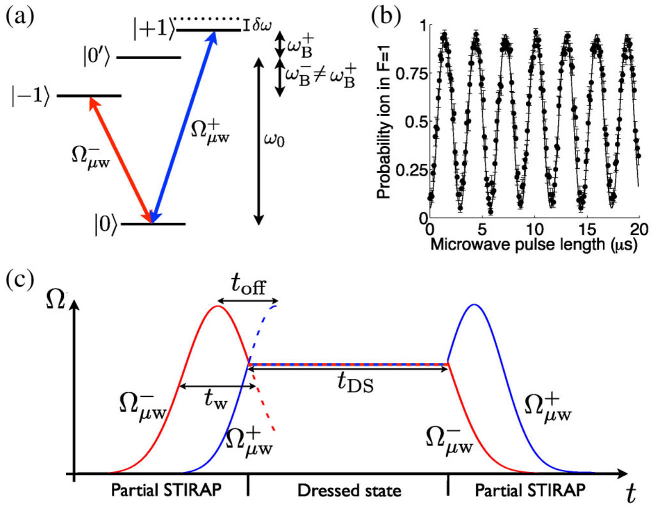

FIG. 1 (color online). (a) The $S_{1/2}$ ground state of the $^{171}Yb^+$ ion consisting of the $F = 0$ state $|0\rangle$ and three $F = 1$ states $|{-1}\rangle$, $|0'\rangle$, and $|{+1}\rangle$ whose degeneracy is lifted by an applied magnetic field. The hyperfine splitting $\omega_0/2\pi$ is 12.6 GHz and $\omega_B^+/2\pi$ is 13.7 MHz for a field of 9.8 G. Due to the second order Zeeman shift $(\omega_B^+ - \omega_B^-)/2\pi = -30$ kHz. Resonant microwave fields can be applied to manipulate or dress the ion, and a radio-frequency field can drive transitions between the $F = 1$ levels. (b) Rabi oscillations between the first-order magnetic field insensitive $|0\rangle$ and $|0'\rangle$ states, with a Rabi frequency of $2\pi \times 342$ kHz. (c) Schematic of the dressed-state pulse sequence. The STIRAP process is "paused" when $\Omega_{\mu w}^+ = \Omega_{\mu w}^-$, dressing the ion. Once dressed-state manipulation is complete the STIRAP process is completed, returning the ion to the bare states. The STIRAP pulses are Gaussian, of width t_w and offset t_{off}.

and the Hamiltonian can be rewritten in this basis as

$$H_{\mu w} = \frac{\hbar\Omega_{\mu w}}{\sqrt{2}}(|u\rangle\langle u| - |d\rangle\langle d|). \quad (4)$$

Without the dressing fields, fluctuations of the magnetic field would cause the $|D\rangle$ superposition to precess to the state $(|{-1}\rangle + |{+1}\rangle)/\sqrt{2} = (|u\rangle + |d\rangle)/\sqrt{2}$; however, the dressing microwaves lift the degeneracy of $|D\rangle$, $|u\rangle$, and $|d\rangle$, so only the part of the fluctuation spectrum around this splitting frequency $\Omega_{\mu w}/\sqrt{2}$ will cause the ion to leave the state $|D\rangle$. Thus, the dressing fields protect the ostensibly field sensitive state $|D\rangle$ from field fluctuations.

The remaining state $|0'\rangle$ does not couple to this dressed subsystem without additional interactions. If the states $|0'\rangle$ and $|D\rangle$ are used as qubit states, then the qubit phase will be unaffected by magnetic field fluctuations (besides those bridging the energy gap).

Timoney *et al.* [7] described a method of manipulating the dressed-state qubit as follows. To first order in the applied magnetic field, the transition frequencies linking $|{-1}\rangle$, $|0'\rangle$, and $|{+1}\rangle$, ω_B^- and ω_B^+, are equal, so a single rf field of Rabi frequency Ω_{rf} and phase ϕ_{rf} will couple all three of the $F = 1$ states. The resultant Hamiltonian $H = H_{\mu w} + H_{rf}$, where the rf terms to add to the microwave terms (4) are

$$H_{rf} = \frac{\hbar\Omega_{rf}}{2}(e^{i\phi_{rf}}|{-1}\rangle\langle 0'| + e^{-i\phi_{rf}}|{+1}\rangle\langle '0| + \text{H.c.}) \quad (5)$$

$$= \frac{\hbar\Omega_{rf}}{2}\Big[\cos\phi_{rf}(|u\rangle + |d\rangle)\langle 0'| - \sqrt{2}i\sin\phi_{rf}|D\rangle\langle 0'| + \text{H.c.}\Big] \quad (6)$$

after rewriting in the dressed-state basis.

The states $|{-1}\rangle$ and $|{+1}\rangle$ were already linked together by the microwave fields, so adding a second linkage between them by applying an rf field results in a looped system. This loop means that the resulting form of the interaction between states is drastically affected by the phase of the rf, the different paths around the loop interfering. This interference means the phase ϕ_{rf} controls the Rabi frequency at which a specific rotation in the Bloch sphere occurs (and also the Rabi frequency at which $|0'\rangle$ is off-resonantly coupled out of the qubit subspace, to the states $|u\rangle$ and $|d\rangle$). Setting the phase ϕ_{rf} to $\pi/2$ produces maximum coupling between the two qubit states and no coupling out of the qubit manifold, resulting in a Hamiltonian $H_{rf} = (\hbar\Omega_{rf}/\sqrt{2})i(|0'\rangle\langle D| - |D\rangle\langle 0'|)$. When implemented experimentally, care must be taken whenever the rf or microwave frequencies are changed that ϕ_{rf} is correctly set to $\pi/2$ to produce the desired rotation.

The rf field produces a σ_y coupling, rotating the qubit about the y axis in the Bloch sphere; however, this is the only coupling obtainable, meaning that arbitrary rotations of the qubit are not possible using this method. In contrast, when a normal two-level system is resonantly driven, the phase of the driving field controls the axis in the $x - y$ plane of the Bloch sphere about which the state rotates (a $\sigma_\phi = \cos\phi\sigma_x + \sin\phi\sigma_y$ coupling from hereon). These more general rotations can then be combined to produce the arbitrary single qubit gates generally required for quantum information processing.

Timoney's method [7] as presented assumes that $\omega_B^+ = \omega_B^-$. For a sufficiently large magnetic field, however, the second order Zeeman shift lifts this degeneracy producing a significant difference between the transition frequencies $\delta\omega = \omega_B^+ - \omega_B^-$. Unless $\Omega_{rf} \gg |\delta\omega|$, an additional rf field is needed, so both transitions can be resonantly addressed. This doubling of the number of rf fields required adds experimental overhead, complicating experiments such as extending the gate to multiple ions [7].

Here, we present a simpler method of performing single qubit rotations making use of $\delta\omega$. It requires only one rf field and does not require the relative phases of the driving fields to be set to specific values. In addition, arbitrary σ_ϕ couplings are obtained with a simple change of the rf phase.

If we have a single rf field, resonant with $|0'\rangle \leftrightarrow |+1\rangle$, then with the condition that the Rabi frequency $\Omega_{\rm rf} \ll |\delta\omega|$ we can ignore directly driven transitions from $|0\rangle$ to $|-1\rangle$ as off resonant, changing the rf part of the Hamiltonian (5) to

$$H_{\rm rf} = \frac{\hbar\Omega_{\rm rf}}{2}(e^{-i\phi_{\rm rf}}|+1\rangle\langle 0'| + \text{H.c.}) \tag{7}$$

$$= \frac{\hbar\Omega'_{\rm rf}}{2}(e^{-i\phi_{\rm rf}}|D\rangle\langle 0'| + \text{H.c.}) + \frac{\hbar\Omega'_{\rm rf}}{2\sqrt{2}}(e^{-i\phi_{\rm rf}}(|u\rangle + |d\rangle)\langle 0'| + \text{H.c.}) \tag{8}$$

in the dressed-state basis where $\Omega'_{\rm rf} = \Omega_{\rm rf}/\sqrt{2}$.

If $\Omega_{\rm rf} \ll \Omega_{\mu\rm w}$, then transitions from $|0'\rangle$ to $|d\rangle$ and $|u\rangle$ are suppressed by the energy gap, and we are left with a resonant interaction between $|0'\rangle$ and $|D\rangle$, with Rabi frequency $\Omega'_{\rm rf}$. However, the rf field no longer links $|-1\rangle$ to $|+1\rangle$, so there is no loop and $\phi_{\rm rf}$ can be freely chosen without causing interference effects. Changes to $\phi_{\rm rf}$ produce arbitrary σ_ϕ couplings as occurs in a driven two-level system. Thus, as long as $\Omega_{\rm rf} \ll |\delta\omega|, \Omega_{\mu\rm w}$, a single rf field resonant with $|0'\rangle \leftrightarrow |+1\rangle$ is able to manipulate the dressed-state qubit and, by changing the phase of the rf, perform arbitrary σ_ϕ rotations. Detuning the rf field would allow general qubit rotations about any axis in the Bloch sphere.

As in Timoney *et al.*, the addition of a static magnetic field gradient would allow the rf field to drive transitions which change the motional state of the ion, by detuning the rf frequency by a motional trap frequency [2,7]. Multi-ion entangling gates, such as the Mølmer-Sørensen gate [8], would then be possible, requiring two rf frequencies per ion, four in total for a two-qubit gate. This compares favorably with the method of Timoney *et al.* which would require eight rf frequencies in total to drive a two-qubit Mølmer-Sørensen gate once the second order Zeeman shift is accounted for.

We demonstrate this manipulation method of the dressed state qubit using a single trapped $^{171}\mathrm{Yb}^+$ ion [9]. Preparation of the ion's initial state and measurement of its final state are performed on the bare ion without the presence of the dressing fields. We will briefly describe these steps and the method used to switch between the bare and dressed states, before describing the dressed-state ion manipulation.

To initialize the ion 369 nm light resonant with $S_{1/2}$, $F = 1 \rightarrow P_{1/2}$, $F = 1$ (along with 935 nm light to depopulate $D_{3/2}$) is used to optically pump into the $F = 0$ state $|0\rangle$.

The state of the ion is measured using a fluorescence technique to distinguish between the ion being in the $F = 0$ state $|0\rangle$ or one of the $F = 1$ states $|-1\rangle$, $|0'\rangle$, and $|+1\rangle$. The $S_{1/2}$, $F = 1 \rightarrow P_{1/2}$, $F = 0$ cycling transition is driven (and the ion repumped from $D_{3/2}$). If the ion is in one of the $F = 1$ states, photons will be scattered and detected using a photomultiplier tube while an ion in the $F = 0$ state will be dark. The number of photons detected is then used to decide if the ion is fluorescing. The efficiency of this technique is limited by off-resonant excitation which can cause transitions between the fluorescing $F = 1$ states and the nonfluorescing $F = 0$ state, the detection efficiency, and the dark count rate [10]. With our setup, we currently achieve a detection fidelity of ≈ 0.93. This could be improved significantly by increasing the collection efficiency of our imaging optics and reducing the dark count rate; further improvements are also available if the arrival times of collected photons are taken into account in determining the state [10,11].

Figure 1(b) shows a typical set of Rabi oscillations with a Rabi frequency of $2\pi \times 342$ kHz obtained by driving the magnetic field insensitive $|0'\rangle \leftrightarrow |0\rangle$ microwave transition at 12.6 GHz via a microwave horn 4 cm from the ion. In all data shown here, each repeat of the experimental sequence was started at the same phase of the AC line cycle. The coherence of the clock qubit $|0\rangle$-$|0'\rangle$ was measured to be $\sim$1.6 s by a Ramsey split pulse method, with a single spin-echo π pulse used in the middle of the Ramsey sequences to remove the effect of any slow drifts in the qubit and microwave frequencies. The $|0\rangle$-$|+1\rangle$ qubit dephases much more quickly, with a coherence time $\sim$200 μs, illustrating the way that magnetic field fluctuations dominate the dephasing of field sensitive states.

The magnitude of the magnetic field at the ion, B, was determined to be 9.80(1) G by measuring the frequencies of the three microwave transitions. From these measurements, we find $\delta\omega = -2\pi \times 29(1)$ kHz. This is consistent with the theoretical value of $\delta\omega = -[2g_J\mu_B B/(2I+1)]^2/2\hbar^2\omega_0 = -2\pi \times 0.31\ \mathrm{kHz/G^2} \times B^2$ [12].

In order to dress the ion we require two microwave dressing fields. To generate these, we begin with two different low frequency signals (0–30 MHz) which are then individually amplitude modulated before being combined and then shifted into the microwave domain by mixing with a 12.6 GHz source before being amplified and sent to the microwave horn. The amplitude of intermodulation frequencies generated by mixing are minimized by limiting the power of the low frequency signals. Overall, this results in reduced microwave Rabi frequencies compared with driving the amplifier directly from the microwave source.

An interrupted stimulated Raman adiabatic passage (STIRAP) process [13] is used to controllably dress and undress the ion. The ion is prepared in $|+1\rangle$ (using a microwave π pulse from $|0\rangle$) and a STIRAP sequence started by adiabatically modulating the Rabi frequencies $\Omega^+_{\mu\rm w}$ and $\Omega^-_{\mu\rm w}$ of the two microwave fields, as though to transfer the ion to $|-1\rangle$. At the point at which $\Omega^+_{\mu\rm w} = \Omega^-_{\mu\rm w}$ the ion is in $|D\rangle$, and the Rabi frequencies are then held

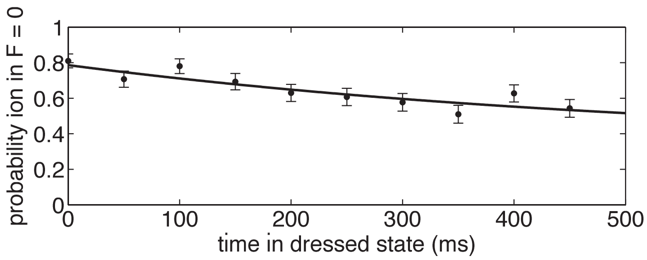

FIG. 2. Decay of the $|D\rangle$ state. The ion is held in the $|D\rangle$ state for a variable length of time. After the STIRAP is completed, a π pulse swaps the population in $|-1\rangle$ and $|0\rangle$ before readout. The lifetime of the dressed-state $|D\rangle$ is 550 ms, for microwave Rabi frequencies during t_{DS} of $2\pi \times 16$ kHz. The peak microwave Rabi frequency during the STIRAP was $2\pi \times 25$ kHz, and the pulses were characterized by $t_{\mathrm{w}} = 450\ \mu$s and $t_{\mathrm{off}} = 356\ \mu$s.

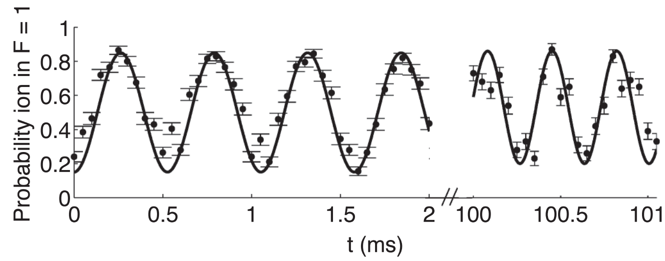

FIG. 3. Rabi oscillations between the $|D\rangle$ and $|0'\rangle$ states. Based on the short time behavior, the Rabi frequency $\Omega'_{\mathrm{rf}} = 2\pi \times 1.9$ kHz. The frequency of oscillation appears changed for pulse lengths over 100 ms due to slow fluctuations of experimental parameters; a steady increase in the rf Rabi frequency of only 0.5% over the 4.5 minutes required to take the data is sufficient to create the observed apparent frequency of oscillation.

constant, "pausing" the STIRAP process for a time t_{DS}, during which manipulation of the dressed-state qubit can take place. Once the qubit is to be measured, the STIRAP process is resumed to transfer the population in state $|D\rangle$ to $|-1\rangle$ before a second π pulse transfers it to $|0\rangle$. A schematic of this is shown in Fig. 1(c).

The efficiency of the STIRAP process depends on the process being adiabatic. For a peak microwave Rabi frequency of $2\pi \times 25$ kHz and using Gaussian amplitude envelopes, taking detection fidelity into account a maximum transfer efficiency of ~91% was obtained for a t_{w} in the range 250–500 μs; higher peak Rabi frequencies should improve this transfer efficiency. All data presented here have $t_{\mathrm{w}} = 450\ \mu$s and $t_{\mathrm{off}} = 356\ \mu$s.

A measure of the robustness of the dressed-state qubit is the lifetime of the dressed-state $|D\rangle$. This can be determined by measuring the population of the $|D\rangle$ state as the pause time t_{DS} is varied, as shown in Fig. 2. A fitted exponential gives a lifetime of $|D\rangle$ of 550 ms, with microwave Rabi frequencies $\Omega_{\mu\mathrm{w}} = 2\pi \times 16$ kHz dressing the ion.

Since the lifetime of the $|D\rangle$ state is limited by fluctuations in the magnetic field at a frequency of $\Omega_{\mu\mathrm{w}}/\sqrt{2}$ and magnetic field fluctuations typically fall off rapidly with frequency [14,15], increases in the microwave Rabi frequency should extend this $|D\rangle$ state lifetime. An order of magnitude increase in microwave Rabi frequency should be relatively easily attainable by changing the frequency generation setup feeding the microwave horn and even larger improvements could be made by trapping ions on a surface trap incorporating microwave waveguides [16]. Increases in $\Omega_{\mu\mathrm{w}}$ would also allow Ω_{rf} to be increased, reducing gate times.

To coherently manipulate our dressed qubit, an rf field tuned to resonance with $|0'\rangle \leftrightarrow |+1\rangle$ is used. Figure 3 shows Rabi oscillations driven between the $|D\rangle$ and $|0'\rangle$ states, for both short and long pulses of the rf field, with a Rabi frequency $\Omega'_{\mathrm{rf}} = 2\pi \times 1.9$ kHz. Even at long flopping times, there is little dephasing of the qubit. The nonunity contrast is due to a combination of imperfect bare-state discrimination and imperfect transfer between bare and dressed states. The coherence time of the dressed state qubit exceeds 500 ms, implying the lifetime of the $|D\rangle$ state is the dominant source of decoherence.

To demonstrate the ability to perform arbitrary rotations, we perform a Ramsey split pulse experiment. By detuning the rf field from resonance by $\delta_{\mathrm{rf}} \ll \Omega'_{\mathrm{rf}}$ the $\pi/2$ nature of the pulses are unaffected; however, the ion and the rf develop a relative phase proportional to the time between Ramsey pulses, changing the rotation axis in the Bloch sphere about which the second $\pi/2$ pulse operates and producing a Ramsey fringe as shown in Fig. 4.

Microwave dressing the magnetically sensitive levels of an atom allows the construction of a dressed-state qubit that is robust against decoherence due to magnetic field fluctuations. We have described and implemented a simple single-qubit gate allowing arbitrary σ_ϕ couplings on such a dressed-state qubit. It is experimentally simple to implement, requiring only a single rf field to be applied to the ion, and does not require knowledge of the phase of the rf relative to the microwaves. Adding a magnetic field

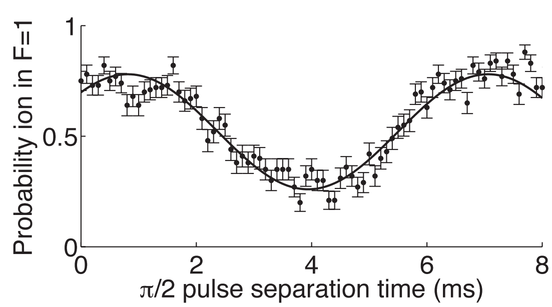

FIG. 4. Ramsey fringe within the dressed-state qubit. Two $\pi/2$ pulses (between the $|D\rangle$ and $|0'\rangle$ states), detuned from resonance, are separated by a variable time. During the separation time the ion and rf develop a phase difference, causing the second $\pi/2$ rotation to be performed about a different axis in the $x-y$ plane of the Bloch sphere. From the fringe period, a detuning $\delta_{\mathrm{rf}} = 2\pi \times 160$ Hz is inferred.

gradient at the center of the trap will allow us to explore the implementation of laser-free Mølmer-Sørensen gates.

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