

Fast and robust two-qubit gates for scalable ion trap quantum computing

J. J. García-Ripoll,^{1,*} P. Zoller,² and J. I. Cirac¹

¹*Max-Planck-Institut für Quantenoptik, Hans-Kopfermann-Str. 1, Garching, D-85748, Germany.*

²*Innsbruck Universität, Technikerstr. 24, 6020 Innsbruck, Austria.*

(Dated: January 6, 2018)

We propose a new concept for a two-qubit gate operating on a pair of trapped ions based on laser coherent control techniques. The gate is insensitive to the temperature of the ions, works also outside the Lamb-Dicke regime, requires no individual addressing by lasers, and can be orders of magnitude faster than the trap period.

Trapped ions constitute one of the most promising systems to implement scalable quantum computation. [1] In an ion trap quantum computer qubits are stored in long-lived internal atomic states. A universal set of single and two qubit gates is obtained by manipulating the internal states with lasers, and entangling the ions via the motional states [2]. During the last years a remarkable experimental progress in building an ion trap quantum computer has allowed to realize two-qubit gates [3, 4, 5, 6] and also to prepare entangled states [7, 8, 9]. The ultimate challenge is now the development of scalable ion trap quantum computing. Scalability is based on storing a set of ions, and moving ions independently, in particular to bring together *pairs of ions* to perform a two-qubit gate [10, 11]. Basic steps towards this goal have already been demonstrated experimentally [12].

An important question to be addressed is to identify the current limitations of the two-qubit gates with trapped ions (given the fact that one-qubit gates are significantly simpler with those systems). The ideal scheme should: (i) be independent of temperature (so that one does not need to cool the ions to their ground state after they are moved to or from their storage area); (ii) require no addressability (to allow the ions to be as close as possible during the gate so as to strengthen their interaction), and (iii) be fast (in order to minimize the effects of decoherence during the gate, and to speed up the computation). This last property has been identified [1] as a key limitation: in essentially all schemes suggested so far [2, 13, 14, 15, 16, 17] one has to resolve spectroscopically the motional sidebands of the ions with the exciting laser, which limits the laser intensity and therefore the gate time.

The two-qubit gate between pairs of ions analyzed below solves the problem of speed by using mechanical effects instead of spectral methods to couple the motion and internal states of the ions. In this way the new limits on the time of the quantum gate are those of laser control, which can be orders of magnitudes faster than the present limits dictated by trap design. Thus, the method proposed here is a significant step forward towards fast and efficient scalable quantum computations with trapped ions.

Below we will first study the dynamics of a pair of

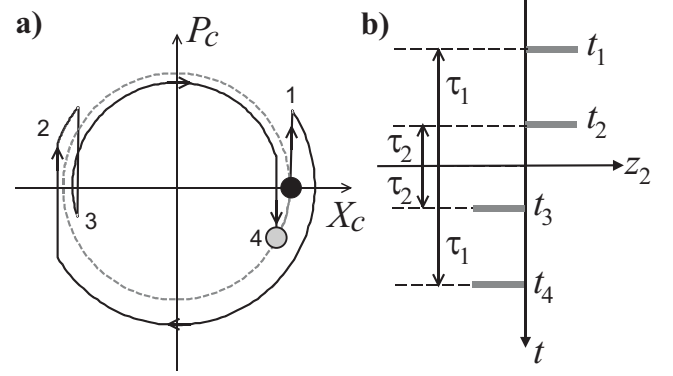


FIG. 1: a) Trajectory in phase space of the center-of-mass state of the ion (X_c, P_c) (where $(X_c + iP_c)/\sqrt{2} = \langle a \rangle$) during the 2-qubit gate (solid line), connecting the initial state (black filled circle) to the final state (grey filled circle) at the gate time T . The time evolution consists of a sequence of kicks (vertical displacements), which are interspersed with free harmonic oscillator evolution (motion along the arcs). A pulse sequence satisfying the commensurability condition (3) guarantees that the final phase space point is restored to the one corresponding to a free harmonic evolution (dashed circle). The particular pulse sequence plotted corresponds to a four pulse sequence given in the text (Protocol I). Figure b) shows how the laser pulses (bars) distribute in time for this scheme.

trapped ions under the influence of short laser pulses with varying directions. We will prove that there exist certain laser pulse sequences which perform a phase gate on the two qubits, while leaving the motional state unchanged. We illustrate this with two protocols for laser pulses: (i) a sequence of four pulses which gives a gate time of $T = 1.08/\nu$ with ν the trap frequency, and (ii) a protocol which allows us to perform a gate in a time $T \sim N_P^{-2/3}/\nu$ where N_P is the number of laser pulses. Finally, we will complement our study of the gate dynamics with an analysis of possible errors, which includes fluctuations of the intensity or the duration of the pulses, and temperature. The gate will be shown to be extremely robust to these perturbations.

We consider two ions in a one-dimensional harmonic trap, interacting with a laser beam on resonance. The

Hamiltonian describing this situation[2] can be written as $H = H_0 + H_1$, where $H_0 = \nu_c a^\dagger a + \nu_r b^\dagger b$ describes the motion in the trap and

$$H_1 = \frac{\Omega(t)}{2} \sigma_1^+ e^{i\eta_c(a^\dagger + a) + \frac{1}{2}\eta_r(b^\dagger + b)} + \frac{\Omega(t)}{2} \sigma_2^+ e^{i\eta_c(a^\dagger + a) - \frac{1}{2}\eta_r(b^\dagger + b)} + h.c. \quad (1)$$

Here, $\nu_c = \nu$ and $\nu_r = \sqrt{3}\nu_c$ are the frequencies of the center of mass and stretching mode, respectively; a and b are the corresponding annihilation operators, and $\eta_c = \eta/\sqrt{2}$ and $\eta_r = \eta\sqrt{4/3}$ are proportional to the Lamb-Dicke parameter, η . Note that the Rabi frequency Ω is the same for both ions, since *we have not assumed individual addressing*. Also notice that replacing η with $(-\eta)$ is equivalent to reversing the direction of the laser beam.

In the following we will consider two different kind of processes: (i) Free evolution, where the laser is switched off ($\Omega = 0$) for a certain time; (ii) Sequences of pairs of very fast laser pulses, each of them coming from opposite sides. If we denote by δt the duration of a pulse and by Ω the corresponding Rabi frequency, we are interested in the limit $\delta t \rightarrow 0$ with $\Omega \delta t = \pi$. Processes (i) and (ii) will be alternated (See Fig. 1a): at time t_1 a sequence of z_1 pulses is applied, followed by free evolution until at time t_2 another sequence of z_2 pulses is applied followed by free evolution and so on. The z_k are integer numbers, whose sign indicates the direction of the laser pulses.

For a pulse sequence, consisting of kicks interspersed with free harmonic time evolution (Fig. 1), we write the evolution operator as $\mathcal{U} = \mathcal{U}_c \mathcal{U}_r$, where $\mathcal{U}_{c,r} = \prod_{k=1}^N U_{c,r}(\Delta t_k, z_k)$ has contributions of the center-of-mass and relative motions,

$$U_c(t_k, z_k) = e^{-i2z_k\eta_c(a+a^\dagger)(\sigma_1^z + \sigma_2^z)} e^{-i\nu_c \Delta t_k a^\dagger a},$$

$$U_r(t_k, z_k) = e^{-iz_k\eta_r(b+b^\dagger)(\sigma_1^z - \sigma_2^z)} e^{-i\nu_r \Delta t_k b^\dagger b}.$$

The integers z_k indicate the direction of the initial pulse in the sequence of pairs of very fast laser pulses, each of them coming from opposite sites.

In order to fully characterize \mathcal{U} , we only have to investigate its action on states of the form $|i\rangle_1 |j\rangle_2 |\alpha\rangle_c |\beta\rangle_r$, where $i, j = 0, 1$ denote the computational basis, and $|\alpha\rangle$ and $|\beta\rangle$ are coherent states. This task can be easily carried out once we know the action of $\mathcal{U} = \prod_{k=1}^N U(\phi_k, p_k)$ on an arbitrary coherent state $|\alpha\rangle$, where

$$U(\phi_k, p_k) = e^{-ip(a+a^\dagger)} e^{-i\phi_k a^\dagger a}.$$

We obtain $\mathcal{U}|\alpha\rangle = e^{i\xi} |\tilde{\alpha}\rangle$, where

$$\tilde{\alpha} = \alpha e^{-i\theta_N} - i \sum_{k=1}^N p_k e^{i(\theta_k - \theta_N)},$$

$$\xi = - \sum_{m=2}^N \sum_{k=1}^{m-1} p_m p_k \sin(\theta_k - \theta_m) - \Re \left[\alpha \sum_{k=0}^N p_k e^{-i\theta_m} \right],$$

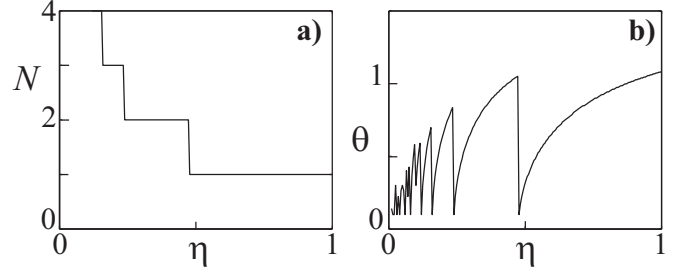


FIG. 2: (a) Number of pairs of pulses, and (b) relative angle of the two laser beams required to produce a phase gate using the first exact scheme developed in the paper.

with $\theta_k = \sum_{m=1}^k \phi_m$.

The crucial point is to realize that if $\sum_{k=1}^N p_k e^{i\theta_k} = 0$ the motional state $|\alpha\rangle$ after the evolution is the same as if there was only free evolution [Fig. 1a], and a global phase ξ appears which does not depend on the motional state. Translating this result to the operators $\mathcal{U}_c|\alpha\rangle$ and $\mathcal{U}_r|\beta\rangle$, we obtain the conditions

$$C_c \equiv \sum_{k=1}^N z_k e^{-i\nu t_k} = 0, \quad (3a)$$

$$C_r \equiv \sum_{k=1}^N z_k e^{-i\sqrt{3}\nu t_k} = 0. \quad (3b)$$

If these commensurability conditions are satisfied, the motional state of the ion will not depend on the qubits and the evolution operator will be given by

$$\mathcal{U}(\Theta) = e^{i\Theta \sigma_1^z \sigma_2^z} e^{-i\nu_c T a^\dagger a} e^{-i\nu_r T b^\dagger b}. \quad (4)$$

The value T is the total time required by the gate and

$$\Theta = 4\eta^2 \sum_{m=2}^N \sum_{k=1}^{m-1} z_k z_m \left[\frac{\sin[\sqrt{3}\nu \Delta t_{km}]}{\sqrt{3}} - \sin(\nu \Delta t_{km}) \right], \quad (5)$$

where $\Delta t_{km} = t_k - t_m$. Therefore, if Eqs. (3) are fulfilled, and $\Theta = \pi/4$ we will have a controlled-phase gate (which is equivalent to a controlled-NOT gate up to local operations) which is *completely independent of the initial motional state*, i.e. there are no temperature requirements.

It is straightforward to show that for any value of the time T it is always possible to find a sequence of laser pulses which implements the gate, and therefore the gate operation can be, in principle, arbitrarily fast. The search for a sequence of pulses may be done numerically, or even semianalytically. In the following we give two simple protocols which are not optimized in order to reduce the number of pulses.

The first protocol (Protocol I) requires the least number of pulses to produce the gate in a fixed time $T \simeq 1.08(2\pi/\nu)$. The recipe is illustrated in Fig. 1, which

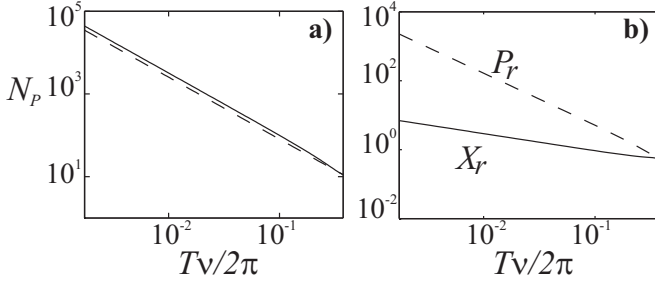


FIG. 3: (a) Log-log plot of the number of pairs of pulses required to produce a phase gate using protocol II, as a function of the duration of the gate, T , for a realistic value[4] of the Lamb-Dicke parameter, $\eta = 0.178$. We plot both the exact result (solid line) and a rough estimate $N_p = 40(\nu T/2\pi)^{-3/2}$ (dashed line) based on perturbative calculations. (b) Maximum relative displacement, X_r (solid), and maximum momentum acquired, P_r (dashed line), for scheme II. These quantities are adimensionalized versions of the real observables, $X_r = \max[\langle x_r(t) \rangle / a_0]$, and $P_r = \max[\langle p_r(t) \rangle a_0 / \hbar]$.

provides the phase space plots for the evolution of the motional state. The sequence of pulses is defined as $(z_n/N, t_n) = \{(\gamma, -\tau_1), (1, -\tau_2), (-1, \tau_2), (-\gamma, \tau_1)\}$. Here $0 < \gamma = \cos(\theta) < 1.0$ is a real number, which may be introduced by tilting both lasers a small angle θ with respect to the axis of the trap, so that no transverse motion is excited. It is always possible to find a solution to Eq. (3) with $\tau_1 \simeq 0.538(4)(2\pi/\nu) > \tau_2 > 0$. The results for the performance of the gate are summarized in Fig. 2. As shown in Fig. 2(a), for realistic values of the Lamb-Dicke parameter [4] we only need to apply the sequence of pulses one or two times to implement a phase gate.

The second protocol (Protocol II) performs the gate in an arbitrarily short time T . The pulses are now distributed according to $(z_n/N, t_n) = \{(-2, -\tau_1), (3, -\tau_2), (-2, -\tau_3), (2, \tau_3), (-3, \tau_2), (2, \tau_1)\}$. The whole process takes a time $T = 2\tau_1$ and requires $N_p = \sum |z_n| = 14N$ pairs of pulses. As Fig. 2 shows, the number of pulses increases with decreasing time as $N_p \propto T^{-3/2}$.

In order to study the main potential limitations of our scheme, we define the error of the gate E in terms of the gate fidelity [18] as $E = 1 - \text{Tr}_{\text{mot}} \{Q_{\text{mot}} \rho_{\text{mot}} Q_{\text{mot}}^\dagger\}$. Here Tr_{mot} and Tr_{int} denote traces over motional and internal degrees of freedom, and $Q_{\text{mot}} = \text{Tr}_{\text{int}} \{U(\pi/4) U_{\text{real}}^\dagger\}$, depends on U_{real} , the gate performed in the presence of imperfections.

We now turn to a discussion of the possible sources of errors. A limiting factor for the gate is the anharmonicities of the restoring forces. The more pulses we apply, the larger the relative displacement of the ions, as Fig. 3(b) shows. When the ions become too close to each other, the increasing intensity of the Coulomb force can lead to a breakdown of the harmonic approximation which is implicit in Eq. (1). In order to analyze this ef-

fect, we have made a perturbation analysis for $\nu T \ll 1$ and found that such an anharmonicity causes an error $E \simeq |0.4a_0/d|^2/(2\pi\nu T)$, where a_0 is the ground state size of the external potential and d is the ion separation in equilibrium. For typical parameters and imposing an error $E \simeq 10^{-4}$ we obtain $\nu T \simeq 10^{-3}$. A similar analysis could be applied to study anharmonicities of the trap itself.

In addition, we have studied the influence of errors in the laser pulses of our scheme. Up to now, our analytical calculations assumed that the intensity of the laser is very large during each pulse, and that therefore one may neglect the influence of the trap during this process. To validate this assumption we have simulated numerically a system of two ions with only one vibrational mode. We have used the exact sequences developed above to produce the phase gate using only eight laser pulses. In Fig. 4(a) we plot the error of the gate as a function of the duration of the laser pulse, $\tau = \pi/2\Omega$. The longer the pulse, the more important the effect of the trap, and the larger the error. But even for relatively long pulses, we obtain a fidelity which is comparable to the results obtained in current setups [4, 5, 6]. We have also studied the influence of noise in the intensity of the laser pulses, or, what is equivalent, random errors in its duration. The larger the amplitude of the error the lower the fidelity of the gate, as Fig. 4(b) shows.

As mentioned before, the scheme is insensitive to temperature. If the commensurability condition (3) is not perfectly satisfied due to, for example, errors in timing of laser pulses, or misalignment of the lasers, then the corresponding contribution to the gate error is

$$E = (C_1^4 + C_2^4 + C_1 C_2 - 6)/8$$

with

$$C_1 = \exp[-(1/2 + k_b T / \hbar \nu_c) |2\eta_c C_c|^2] \quad (6)$$

$$C_2 = \exp[-(1/2 + k_b T / \hbar \nu_r) |\eta_r C_r|^2], \quad (7)$$

which is a smooth function of temperature T .

Finally, we would like to make some remarks regarding the experimental implementation of this scheme. First, it is not necessary to kick the atoms using pairs of counter-propagating laser beams. The same effect (i.e. a change of sign in η) may also be achieved in current experiments by reverting the internal state of both ions simultaneously. One then only needs a laser beam aligned with the trap to kick the atoms, and another laser orthogonal to the axis of the trap to produce the NOT-gate.

The second and more important remark is that it is possible to avoid errors in the laser pulses by using more sophisticated kicking methods. One possibility consists in using STIRAP [19, 20]. Only one of the qubit states would be connected by two on-resonance laser beams to a third atomic state, $|e'\rangle$. In the first part of the kicking process, the Rabi frequencies of the lasers Ω_a

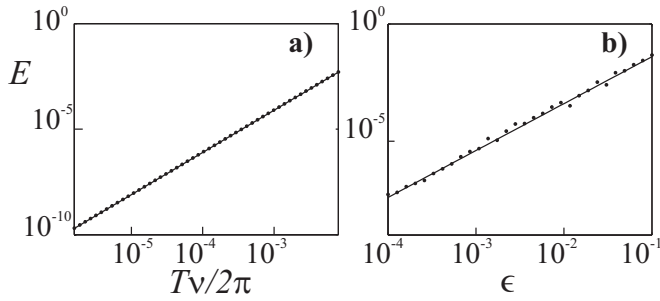


FIG. 4: For the sequences of four kicks designed in the paper, using [4] $\eta = 0.178$ and a duration of the gate $T = \mathcal{O}(1/\nu)$, we have computed the dependence of the gate error with respect to the characteristics of the laser pulses. In (a) the error is plotted versus the duration of the pulse, τ , rescaled using the period of the trap. It perfectly fits the estimate $E = 2(\tau\nu/2\pi)^2$. In (b) the mean error of the gate is plotted against the amplitude of random errors in the duration of the laser pulses. Our errors are modeled setting $\tau_k = \pi/(2\Omega)(1 + \epsilon r_k)$, with random numbers r_k uniformly distributed in $[-1/2, 1/2]$. In solid lines we show a visual aid which follows the formula $E = 4\epsilon^2$.

and Ω_b are adiabatically switched on an off respectively. The momenta of both laser beams should be different, so that as we slowly proceed from $\Omega_a/\Omega_b \simeq 0$ to the opposite regime $\Omega_b/\Omega_a \simeq 0$, the ions in the state $|1\rangle$ are completely transferred to the new dark state $|e'\rangle$ and get a kick $|1\rangle \rightarrow e^{i(\vec{k}_a - \vec{k}_b)\vec{x}} |e'\rangle$. Next we must change the sense of the laser beams ($\vec{k}_{a,b} \rightarrow -\vec{k}_{a,b}$), and perform the adiabatic transfer from $\Omega_b/\Omega_a \simeq 0$ to $\Omega_a/\Omega_b \simeq 0$. The total transformation will be $|0\rangle \rightarrow |0\rangle$, $|1\rangle \rightarrow e^{2i(\vec{k}_a - \vec{k}_b)\vec{x}} |1\rangle$. The advantages of this method are: (i) the system remains all the time in a dark state, avoiding spontaneous emission; (ii) the process is insensitive to fluctuations of the intensity; (iii) the duration of the pulse need not be precisely adjusted, and (iv) the intensity of the laser need not be the same for both ions.

Summing up, in this work we have developed a new concept of two-qubit quantum gate for trapped ions, in which the trap frequency poses no longer a limitation on the speed of the gate. Rather than performing sideband transitions which weakly couple the internal and the motional states of the atoms, we suggest to push the atoms resonantly during very short times and along different directions. The limitations in that case come from: (i) the anharmonicities of the restoring force that the ions experience when pushed far away from each other, and (ii) the ability to control the laser pulses. The first limitation still allows to perform the gates in a time which is three orders of magnitude smaller than the one imposed by the trap frequency. The second one can be overcome by using adiabatic passage techniques which make these

laser excitations tolerant against laser imperfections. In any case, the rapid experimental progress in laser control with very short pulses indicates that it may be possible soon to perform quantum gates with a very high speed. In addition, our scheme is independent of the temperature, requires no addressability, and works beyond the Lamb-Dicke regime.

J.I.C. thanks D. Leibfried for discussions and NIST for the hospitality during his stay in Boulder. P.Z. thanks the Innsbruck ion trap group for discussions. Part of this work was supported by the EU IST project RESQ, the project TOPQIP, the DFG (Schwerpunktprogramm Quanteninformationsverarbeitung) and the Kompetenznetzwerk Quanteninformationsverarbeitung der Bayerischen Staatsregierung. Research at the University of Innsbruck is supported by the Austrian Science Foundation, EU Networks and the Institute for Quantum Information.

* Electronic address: Juan.Ripoll@mpq.mpg.de

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