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RF quadrupole linear Paul traps are versatile tools in quantum physics experiments. Linear Paul traps with blade-shaped electrodes have the advantages of larger solid angles for fluorescence collection. But with these kinds of traps, the existence of higher-order anharmonic terms of the trap potentials can cause large heating rate for the trapped ions. In this paper, we theoretically investigate the dependence of higher-order terms of trap potentials on the geometry of blade-shaped traps, and offer an optimized design. A modified blade electrodes trap is proposed to further reduce higher-order anharmonic terms while still retaining large fluorescence collection angle. © *2015 AIP Publishing LLC*. [http://dx.doi.org/10.1063/1.4931420]

I. INTRODUCTION

Laser-cooled ions trapped in linear Paul traps can be treated as ideal quantum systems that are free from environmental perturbations. In recent years, ion traps have become powerful tools in experimental investigation of quantum information processing,[1,2] high precision spectroscopy,[3–5] and cavity quantum electrodynamics.[6] The traps are often designed to have enough space for laser cooling, ions state manipulation, and read out.

Using linear Paul traps to control charged particles was first proposed by Prestage *et al.*[7] In an ideal linear Paul trap, the surface of electrodes has hyperbolic shape, and a pure quadrupole electric potential is produced in the radial direction if we neglect the effects of DC electrodes. Under this potential, ion motion is governed by the Mathieu equation. The solution of the Mathieu equation describes ions' motions inside a trap. The motional spectrum contains frequencies of secular motion and micromotion.[8]

For a realistic linear Paul trap, the electrodes cannot be infinitely large to meet the boundary condition of a pure quadrupole potential (the equipotential shape). Electrodes have to be truncated at an acceptable length. In addition, in an ideal linear Paul trap, the electrodes fill up almost all the transverse plane, restricting laser beams' access as well as fluorescence collection access. In order to meet the larger opening requirement for laser cooling and fluorescence collection, the hyperbolic electrodes are replaced with circular shape electrodes, and these electrodes can be truncated at a smaller extension.[9]

In order to provide the required electric potential strength to confine ions with reasonable voltages on the electrodes, the radial distance of the electrodes should be as close as possible. However, when the radial distance of the trap is reduced, it is difficult to properly hold the electrodes during fabrication process, especially when relatively high

assembling precision is required. In addition, when we reduce the radial distance of the trap, the trap is not stable enough with applied high RF voltages. To avoid these problems, blade-shaped electrodes are introduced so that relatively high precision at these small scales could be reached. Blade-shaped electrodes were first used at Innsbruck.[10] Electrostatic simulation of this structure was done to calculate the axial and radial frequency of ions' motion in the trap. In the calculation, the electric potential quadratic term was calculated, but higher-order electric potential terms were not discussed. Since the blade-shaped electrodes may deviate more from the perfect hyperboloid shape, their electric potential will have more higher-order terms contribution. To our knowledge, numerous papers have discussed circular[11–13] and hyperbolic[14,15] electrodes' anharmonic potential terms, but there is no detailed analysis of blade-shaped electrodes anharmonic potential terms.

These anharmonic potentials act on the trapped ions, and give rise to fluctuating forces on the ions. This will heat the ions. The heating rate is used to describe the heating effect, which equals to the average number of quanta of energy gained by the ion secular motion in a given time. For a particle of charge q and mass m trapped in a harmonic potential, the heating rate is generalized to be[16]

$$\frac{dn}{dt} = \frac{q^2}{4m\hbar\omega_m}\left[S_E(\omega_m) + \frac{\omega_m^2}{2\Omega^2}S_E(\Omega\pm\omega_m)\right], \qquad (1)$$

where ω_m is the secular motion frequency of the motion mode under consideration, S_E is the spectral density of electric-field fluctuations in units of $(V/cm)^2 Hz^{-1}$, and Ω is the RF drive frequency of the trap.

Large heating rate has many negative effects. It can drive ions out of an initially prepared motional ground state,[17] and also can lead to decoherence of the quantum superposition states.[18] Additionally, it will potentially limit the achievable temperature of laser cooling, prevent the formation of ion crystals, and limit the trapping lifetime of ions and length of

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quantum logic gate operations, as well as requiring more elaborate cooling and error correction schemes. Therefore, reducing the anharmonic potentials to acceptable values presents a great challenge in ion trap design.

In this work, we calculate the anharmonic trapping potential terms of blade-shaped electrodes linear Paul traps. Influence of geometry dimension of blade-shaped electrodes on the anharmonic potential terms is theoretically studied. Fluorescence collection efficiency is also considered in the trap design. Based on the calculation results, an optimized trap design is given. The results demonstrate a blade-shaped electrodes linear Paul trap with low anharmonic terms and favorable fluorescence collection angle. In addition, we propose a novel design of blade to further reduce the anharmonic terms while keeping the large fluorescence collection angle.

II. TRAP DESIGN

The initial design of Paul trap uses hyperbolic quadrupole electrodes.[19] Inside the trap where no charge exists, electrical potential distribution obeys the Laplace equation $\nabla^2\Phi = 0$. Therefore, the potential is determined by the voltages on the electrodes surfaces which function as boundary conditions for the potential distribution. If the inner zone of trap is not well shielded by the electrodes, the external environment can also affect the potential distribution. However, this is usually not considered as a problem due to its small effect. Assuming the electrodes are long enough, electrical potential at the transverse plane near the trap center would be uniform.

In order to realize larger opening for laser cooling and fluorescence collection, in many applications, the hyperbolic electrodes are replaced with circular shape electrodes, which are easier to fabricate and mount with high precision. The potential distribution of circular electrodes in the transverse plane near the trap center can be expressed as an infinite series[11]

$$\Phi(r, \theta) = (U_0 + V_0 \cos \omega t) \sum_n C_n \left(\frac{r}{r_0}\right)^n \cos n\theta, \quad (2)$$

where U_0 is the applied DC voltage, V_0 is the peak voltage of the applied RF drive, r_0 is the distance from the center of the trap to the electrodes. When distortions of the quadrupole potential at the traps' center plane are discussed, the potential is usually given by the multipole expansion of the potential $\Phi(x, y)$:[12]

$$\Phi(x, y) = \sum_{n=0} \frac{K_n \phi_n}{r_0^n} = A_0 + \sum_{n=2} A_n \phi_n, \quad (3)$$

where K_n is the amplitude of a multipole ϕ_n with $2n$ poles. With the exception of the first term, the symmetry of electrodes' geometry and boundary conditions requires that only $n = 2(2m + 1)$ ($m = 0, 1, 2, \ldots$) terms exist. The first few terms of $\Phi(x, y)$ are

$$\Phi_0 = A_0,$$
$$\Phi_2 = A_2(x^2 - y^2),$$
$$\Phi_6 = A_6(x^6 - 15x^4y^2 + 15x^2y^4 - y^6),$$
$$\Phi_{10} = A_{10}(x^{10} - 45x^8y^2 + 210x^6y^4 - 210x^4y^6 + 45x^2y^8 - y^{10}),$$
$$\Phi_{14} = A_{14}(x^{14} - 91x^{12}y^2 + 1001x^{10}y^4 - 3003x^8y^6 + 3003x^6y^8 - 1001x^4y^{10} + 91x^2y^{12} - y^{14}).$$

Here, the first term is quadrupole offset potential. The second term is the quadrupole term, which is called A_2 term since it corresponds to $n = 2$. The third term is called A_6 term which corresponds to $n = 6$, and it is the second largest potential terms in the central trap region. In order to obtain the best approximation to a pure quadrupole potential, the A_6 term should be minimized in the trap design. It has been found that for circular electrodes, the A_6 term vanishes when $r_e/r_0 = \eta = 1.14511$,[20] where r_e is the radius of the circular electrodes. Definition of r_0 and r_e for the circular electrodes trap is shown in Fig. 1(a).

The circular electrodes' distance to trap center r_0 greatly affects the trap's required RF drive power. If we only consider regular RF power that can be realized in a typical experimental setup, a reasonable value of r_0 would be less than 1 mm. Under this situation, it is difficult to hold the electrodes in the fabrication process, especially when a relatively high precision assembling is required. In our case, the precision should be about $20\,\mu$m. Trap metal electrodes fabricated using the wire electrical discharging machining can reach a precision of $10\,\mu$m. The trap holders machined using the computer numerical controlled milling machine can reach precision of $10–20\,\mu$m. In previous experiments, we assembled a trap with $200\,\mu$m displacement. In that case, the micromotion is very hard to compensate since the needed compensation voltage is more than 2500 V. For another trap with $70\,\mu$m displacement, the compensation voltage is decreased to 1200 V.

A blade-shaped electrodes' construction with high precision at these small scales is a good solution for trap fabrication. The basic idea is to connect each rod electrode with a blade together, and the blade has relatively large area and is easy to be supported on the trap holder. In practice, blade-shaped electrodes are designed to be an oblique body with rounded edge. The farther part of the blade is used for supporting the critical edge, and to be mounted in the trap holder.

Just like in the design of circular electrodes linear Paul traps, in the design of blade-shaped electrodes linear traps, it
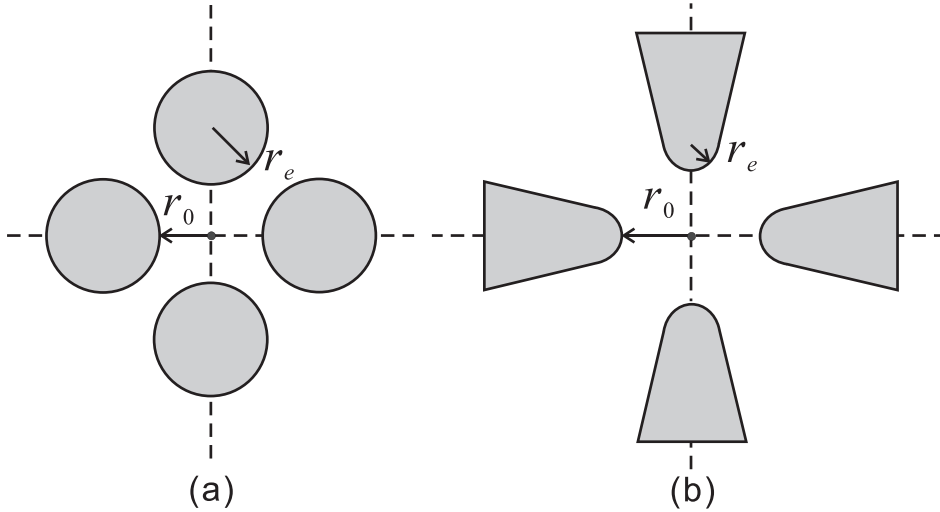
FIG. 1. Definition of r_0 and r_e in cross section of circular electrodes trap (a) and blade electrodes trap (b).

is necessary to consider two parameters, distance of the blades' inner surface to the trap center r_0, and the radius of curvature of the blades r_e. Definition of r_0 and r_e for the blade-shaped electrodes linear traps is shown in Fig. 1(b). In the following part, the two parameters are set as variable parameters to find a configuration with the smallest anharmonic potential terms.

First, we considered the effect of the distance of the two opposite blades $2r_0$. If r_0 is too large, the secular frequency would be low since

$$\omega_{r_0} \propto \frac{qV_0}{m\Omega r_0^2}. \tag{4}$$

However, in many experiments, higher secular frequency is preferred so that strong binding limit and smaller Lamb-Dicke parameter are easier to be achieved. Higher secular frequency will also facilitate faster ions motional ground state cooling. Therefore, a large r_0 is not appropriate. On the other hand, down-sizing the trap will cause serious heating problems according to earlier experiments.[16] We choose r_0

to be a moderate value of 0.8 mm, and then we will try to find the best r_e for the ion traps.

Since A_2 term is the needed quadrupole term to drive and confine ions, we are interested in ratio of A_6, A_{10}, A_{14} to A_2. Since they have different units, we choose to compare their normalized counterparts, namely, K_2, K_6, K_{10}, K_{14}. The relationship between them is $A_n = K_n/r_0^n$, where the normalization factor r_0 is chosen to be 0.8 mm. Apparently with this normalization, all K_n terms have the same unit.

We used the Computer Simulation Technology (CST) software to calculate electrical potential distribution of the trap with a r_e. Then we least-squares fit the ratios of K_6, K_{10}, K_{14} to K_2. Then the simulation was repeated under different r_e. These values as a function of rounding diameter $2r_e$ are plotted in Fig. 2. It can be seen that when the rounding diameter is less than 1.8 mm, larger rounding diameter produces smaller higher order potential terms. However, larger rounding diameter may block the path for fluorescence collection. Figure 3 shows the fluorescence collection solid angle as a function of rounding diameter. At 1.8 mm, the fluorescence collection solid angle is only 0.16 Sr. Since in most trap
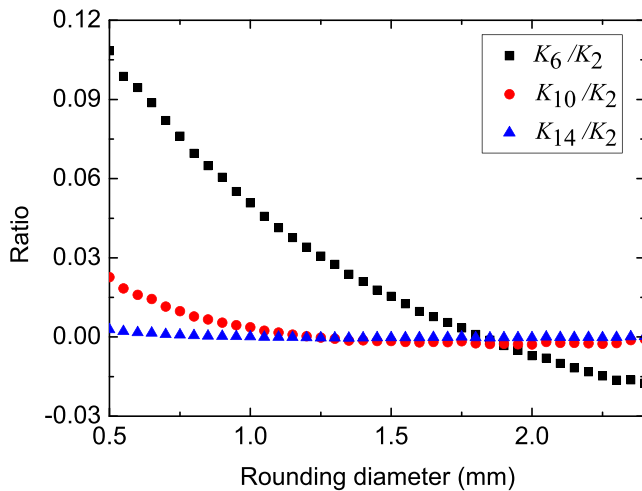

FIG. 2. Ratio of anharmonic terms K_6, K_{10}, K_{14} to K_2 as a function of rounding diameter $2r_e$ for the blade electrodes trap. K_n is defined in Eq. (3). In the simulation, r_0 is set to 0.8 mm.
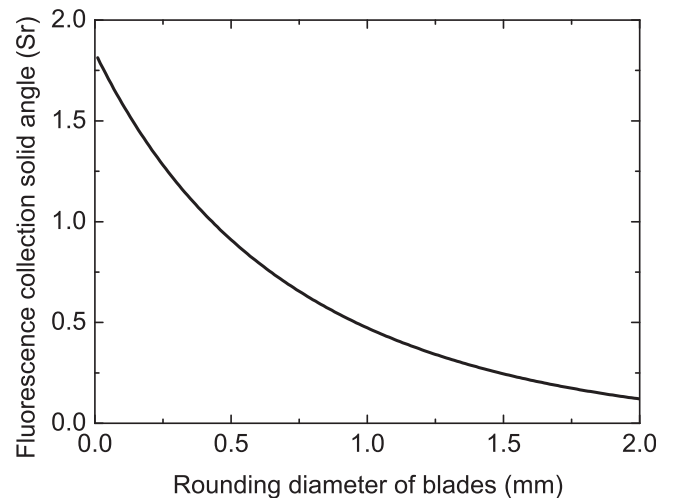

FIG. 3. Fluorescence collection solid angle of blade electrodes traps as a function of rounding diameter $2r_e$.

applications a larger fluorescence collection solid angle is preferred, a smaller rounding diameter is required. To balance these two requirements, radius of the electrodes edge is chosen to be $r_e = 0.3$ mm. This design enables a rather large fluorescence collection solid angle of about 0.80 Sr, which is four times more than that of circular electrodes at its optimum design of $r_e/r_0 = \eta = 1.14511$. Based on a trap in our lab with the collection solid angle of about 0.80 Sr, a count rate of about 250 kHz is observed from a single $^{25}Mg^+$ ion at saturation intensity, and a signal to noise ratio of 100 is reached. In this case, the corresponding anharmonic electrical potential terms are still reasonably small ($K_6/K_2 = 0.1$).

III. A NOVEL DESIGN OF BLADE ELECTRODES

If the higher order anharmonic terms need to be further reduced while keeping the trap's fluorescence collection solid angle, a geometry compensation method for blade-shaped electrodes can be implemented. To explain the mechanism of this compensation method, equipotential contours for hyperbolic electrodes, blade-like electrodes, circular electrodes, and the novel electrodes are shown in Fig. 4. The equipotential contours of pure quadrupole term are plotted as the white dashed lines. In Fig. 4(a), it can be seen that for the hyperbolic electrodes, the equipotential contours of quadrupole term well match the equipotential lines of real electrodes. For circular electrodes in Fig. 4(b), the ratio $r_e/r_0 = \eta = 1.14511$, so in the center region, the equipotential contours still match quadrupole lines well, but they deviate

from quadrupole lines when they are far from the trap center (close to electrodes). However, for this trap geometry, the fluorescence collection angle is not favorable. For the blade-shaped electrodes in Fig. 4(c), the ratio r_e/r_0 is 0.37, which gives a large collection solid angle of 0.80 Sr. The deviation of equipotential contours from quadrupole lines is more serious. Cause of this deviation is obvious from the comparison between Figs. 4(b) and 4(c). The blade-shaped electrodes are too sharp. For each fixed voltage, the real equipotential exceeds the quadrupole's at the radial direction. Based on above observation, we propose a geometry compensation method to refine the non-hyperbolic quadrupole traps. Inspiration of this method comes from harmonic tuning of ancient eastern musical instruments *Xiao* and *Shakuhachi*. A cutting in the electrodes' front draws back the equipotential lines radially as a compensation. The modified electrodes together with the equipotential contour are shown in Fig. 4(d) for these modified-blade electrodes. Compared with Fig. 4(c), it can be seen that the real equipotential agrees quadrupole potential very well in the area near the trap center even though the potential near electrodes deviates seriously.

In order to tune the electrodes' potential terms, the geometry of the blade should be carefully designed. We defined two characteristic parameters for modified-blade electrodes. They are cutting and filleting, as shown in Fig. 5. Simulation results show that the A_6 term can be made zero by changing the characteristic parameters. In simulation, the cutting is chosen from 0.2 mm to 1.5 mm. Note that the
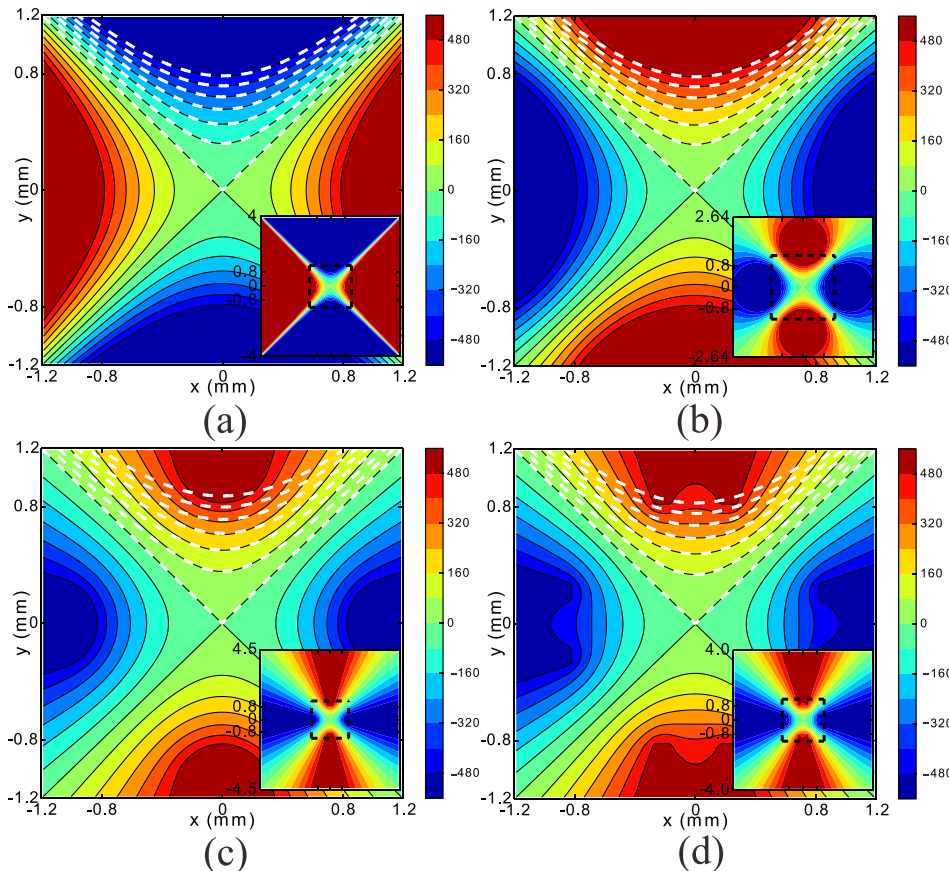

FIG. 4. The equipotential contours for (a) hyperbolic electrodes, (b) circular electrodes, (c) blade electrodes, and (d) blade-modified electrodes. The equipotential contours of pure quadrupole are plotted as the white dashed lines. For (a) and (b), the equipotential contour matches the quadrupole lines well but the fluorescence collection angle is very small. For (c), the real equipotential exceeds the quadrupole's at the direction of blade to electrodes center. For (d), the equipotential contour matches the quadrupole potential lines very well again in the region near the trap center. Insets show fuller view of the potential distribution.
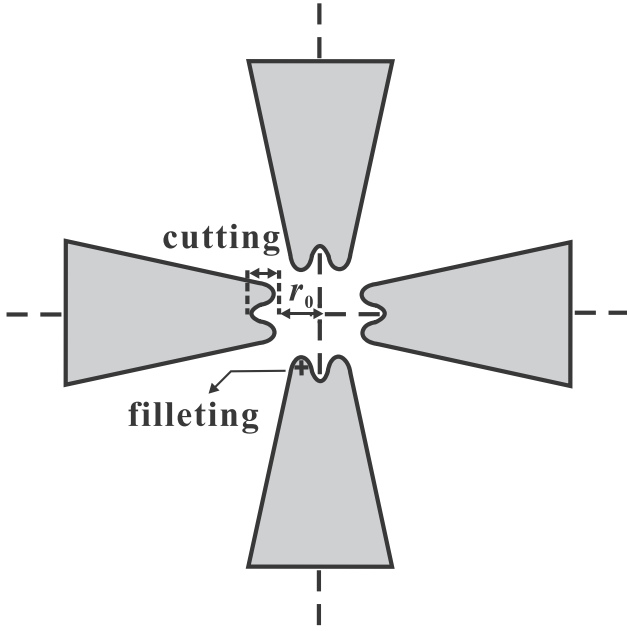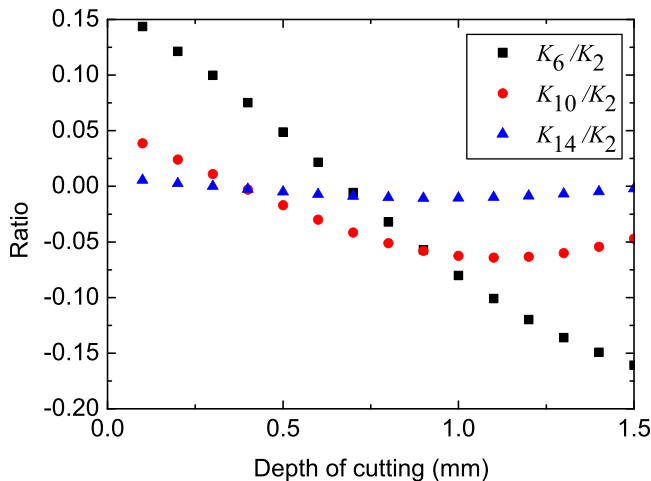
FIG. 5. Cross section of the modified-blade electrodes.

bottom of the cutting should not be too sharp, or it may affect the vacuum pumping ability. Some typical results are shown in Fig. 6. In Fig. 6, r_0 is chosen to be 0.8 mm, this is the same as discussed before in Section II, and filleting is 0.1 mm. The figure shows that the ratio of anharmonic terms K_6, K_{10}, K_{14} to K_2 as a function of cutting depth. It can be seen that when the cutting depth is adjusted appropriately, we can tune the A_6 term to be zero, which is the most important operating parameter for ion traps. In comparison with blade-electrodes trap design and modified-blade electrodes trap design, under the same fluorescence collection angle, the modified-blade electrodes A_6 terms will be much smaller, and consequently contributing less to the heating rate.

In real applications, due to machining tolerance, there is uncertainty for the amount of filleting. In Fig. 6, where the filleting is 0.1 mm, if the machining uncertainty is 10 μm, the calculated ratio of K_6/K_2 has an uncertainty of 0.001. For comparison, the ratio of K_6/K_2 has an uncertainty of 0.002 if the cutting uncertainty is also 10 μm. From our experience, a machining tolerance of 20 μm is acceptable.

IV. CONCLUSION

In conclusion, we study the effect of trap geometry on anharmonic potential terms for blade-shaped electrodes linear Paul traps. The simulation results show how the trap design can be optimized to reduce anharmonic terms while meeting requirement of large fluorescence collection angle. Furthermore, a modified blade model is proposed to null the A_6 term while keeping a large fluorescence collection solid angle. The study will be useful in ion-trap-related precision measurement experiments.

FIG. 6. Ratio of anharmonic terms K_6, K_{10}, K_{14} to K_2 as a function of depth of cutting for the modified-blade electrodes traps.

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