Reconstruction of the Quantum Mechanical State of a Trapped Ion

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(Received 18 May 1995)

It is shown that the full information on the quantum state of the vibrational center-of-mass motion of a trapped ion can be transferred to its electronic dynamics by appropriately irradiating a long-living (e.g., quadrupole) electronic transition by laser light. This allows us to determine the quantum mechanical state of the ion with high quantum efficiency by probing a strong (dipole) transition for the appearance of resonance fluorescence.

PACS numbers: 32.80.Pj, 03.65.Bz, 42.50.Vk

The experimental realization and observation of a single trapped ion [1] opened new possibilities not only for spectroscopy but also for fundamental tests of quantum physics. In the latter context, quantum jumps have been visualized by recording the intermittent resonance fluorescence from a trapped ion [2–4]. Photon antibunching, for the first time observed in resonance fluorescence from an atomic beam [5], has later been demonstrated with a single trapped ion [6,7]. Moreover, a trapped ion in combination with an appropriate observation technique should allow one to detect squeezing in resonance fluorescence [8], an effect that was predicted some years ago [9] but has not been observed yet.

A trapped ion is not only a well defined light source for studying quantum effects of light, it also represents an almost ideal object for fundamental experiments in quantum mechanics. The trap potential may be regarded, to a good approximation, as a quantum mechanical harmonic oscillator [10]. Disturbing dissipation and thermal effects are practically meaningless. In particular, laser sideband cooling allows the preparation of the center-of-mass motion of the ion in the vibrational vacuum state of the trap potential [11]. This is not only of interest for high-resolution spectroscopy, it may also serve as the starting point for the preparation of well defined quantum mechanical states. There exist already some proposals for preparing nonclassical quantum states of the ionic center-of-mass motion [12–15].

Besides the feasibility of studying quantum effects of both the light and the mechanical motion, the trapped ion can be used for dynamical studies of elementary quantum interactions. A vibronic Jaynes-Cummings coupling can be obtained by appropriately irradiating a long-living (e.g., quadrupole) electronic transition of an ion localized within the Lamb-Dicke limit [16]. Beyond the Lamb-Dicke regime, where the spatial extension of the wave function of the vibrational ground state is no longer small compared to the wavelength of the irradiating laser, there even may occur a nonlinear multiquantum Jaynes-Cummings dynamics [17]. Thus interesting features of quantum mechanical dynamics could be experimentally realized.

For studying such fundamental effects of quantum mechanics, one important problem must be solved: To our knowledge until now there existed no method for measuring the full quantum state of the center-of-mass motion of a trapped ion. For a radiation mode the quantum state has been recorded by homodyne detection [18]. Moreover, the quantum mechanical state of a molecular vibration has been derived from nonstationary spectra of the resonance fluorescence [19]. The latter method, however, can hardly be applied for a trapped ion since its vibronic coupling differs significantly from that of a molecule.

In the following we will show that the quantum mechanical state of a trapped ion can be reconstructed from its electronic dynamics, provided the ion is appropriately irradiated by two laser fields. For this purpose let us consider a quadrupole transition of an ion, which is trapped in a harmonic potential within the Lamb-Dicke regime. The first laser is resonant to the well resolved lower vibrational sideband and the second one is tuned to the corresponding upper sideband; see Fig. 1. While the interaction of the ion with the first laser alone can be described by a Jaynes-Cummings Hamiltonian [16], the in-

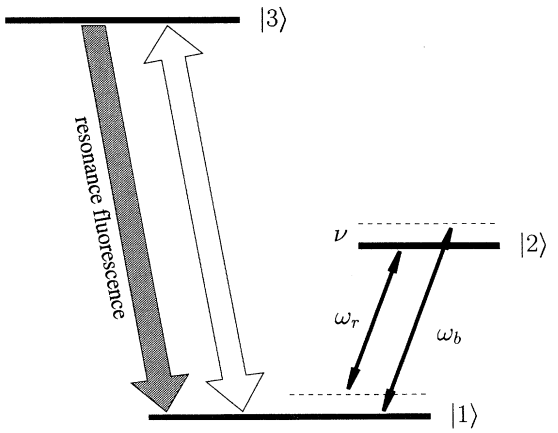

FIG. 1. Scheme of the trapped ion with a weak transition $|1\rangle \leftrightarrow |2\rangle$ and a strong transition $|1\rangle \leftrightarrow |3\rangle$. Two incident lasers of frequencies $\omega_r = \omega_{21} - \nu$ and $\omega_b = \omega_{21} + \nu$ are detuned from the electronic transition by the vibrational frequency ν to the red and blue, respectively. The laser driving the strong transition is used for testing the ground-state occupation probability $\sigma_{11}(t)$ by means of resonance fluorescence.

 0031-9007/95/75(16)/2932(4)$06.00

teraction with the two lasers yields (in the rotating wave approximation with respect to the vibronic transitions) the sum of a Jaynes-Cummings and an anti-Jaynes-Cummings coupling. We consider the particular situation of adjusting the two lasers to have equal intensities. In this case the interaction problem has an exact solution, which is the key for the determination of the quantum state of the ionic center-of-mass motion. The interaction Hamiltonian for the laser-assisted vibronic coupling reads in the interaction picture as

$$\hat{H}_{\text{int}} = \hbar(\Omega^* \hat{A}_{21} + \Omega \hat{A}_{12})\hat{x}_\varphi , \tag{1}$$

$\hat{A}_{ij} = |i\rangle\langle j|$ being the electronic flip operators ($i, j = 1, 2$), $\Omega = \eta\lambda \exp(i\phi)$ the effective Rabi frequency with coupling constant λ and Lamb-Dicke parameter η. The phase $\phi = (\varphi_b + \varphi_r)/2$ contains the phases φ_b and φ_r of the lasers detuned to the blue and red sides of the electronic transition.

The operator $\hat{x}_\varphi$ represents the generalized, phase-dependent center-of-mass position of the ion,

$$\hat{x}_\varphi = \hat{a}\, e^{i\varphi} + \hat{a}^\dagger e^{-i\varphi}, \tag{2}$$

where $\hat{a}$ and $\hat{a}^\dagger$ are the corresponding annihilation and creation operators of vibration quanta. The phase $\varphi = (\varphi_b - \varphi_r)/2$ can be easily changed by altering the phase difference of the two lasers. It simulates the free time evolution of the position operator in the Heisenberg picture. In the interaction picture, however, $\hat{x}_{\varphi=0} = \hat{x}$ and $\hat{x}_{\varphi=-\pi/2} = \hat{p}$ are the scaled position and momentum operators, respectively. For convenience we will denote $\hat{x}_\varphi$ the generalized position operator. Whereas the phase difference φ of the two laser can be well stabilized, the sum ϕ of the laser phases in general undergoes phase diffusion so that lasers of small linewidths are needed. The diffusion effect could be further suppressed by using Raman-type excitations of the weak transitions [20], which for appropriate configurations also leads to the Hamiltonian (1).

In generalized position representation, the Hamiltonian corresponds to a classically driven two-level system with an effective Rabi frequency Ωx_φ, which depends on the generalized position. We exploit this dependence to get information about the mechanical state of the ion from its electronic dynamics. The observed quantity is the occupation probability $\sigma_{11}(t)$ of the electronic ground state, which can be probed using a second, strong (e.g., dipole) transition; see Fig. 1. When the laser on the strong transition is switched on, fluorescence only appears if the ion is in its electronic ground state.

The eigenstates of the Hamiltonian (1) are products of electronic and mechanical eigenstates,

$$\hat{H}_{\text{int}} |x_\varphi; \pm\rangle = \pm \hbar|\Omega| x_\varphi |x_\varphi; \pm\rangle , \tag{3}$$

$$|x_\varphi; \pm\rangle = \frac{1}{\sqrt{2}} (|1\rangle \pm e^{-i\phi}|2\rangle)\, |x_\varphi\rangle . \tag{4}$$

They can be used to construct the unitary time evolution operator. Based on this solution of the dynamics, we derive the relation between the occupation probability $\sigma_{11}(t)$ and the initial density operator $\hat{\rho}^{(\text{vib})}(0)$ of the vibrational subsystem. Initially the ion is assumed to be prepared in a decorrelated state of the form

$$\hat{\rho}(0) = \hat{\rho}^{(\text{vib})}(0) \otimes (c_1|1\rangle + c_2|2\rangle)(c_1^*\langle 1| + c_2^*\langle 2|), \tag{5}$$

where c_1 and c_2 are the initial preparation amplitudes of the electronic states of the ion. It is straightforward to derive the ground-state occupation probability as

$$\sigma_{11}(\tau, \varphi) = \frac{1}{2} + \frac{1}{2} \int_{-\infty}^{+\infty} dx\, e^{-ix\tau} \left[(|c_1|^2 - |c_2|^2) p_s(x, \varphi) + \left(c_1^* c_2\, e^{i\phi} - c_1 c_2^*\, e^{-i\phi}\right) p_a(x, \varphi) \right], \tag{6}$$

where $\tau = 2|\Omega|t$ is a dimensionless time. Here we have introduced the symmetrized and antisymmetrized spatial probability densities,

$$p_s(x, \varphi) = \tfrac{1}{2}[p(x, \varphi) + p(-x, \varphi)] \tag{7}$$

and

$$p_a(x, \varphi) = \tfrac{1}{2}[p(x, \varphi) - p(-x, \varphi)], \tag{8}$$

respectively, with $p(x, \varphi) = \langle x_\varphi | \hat{\rho}^{(\text{vib})}(0) | x_\varphi \rangle$ being the probability to find the ion for the chosen phase value φ at the generalized position x. While the first integrand is insensitive to coherences of the initial electronic state of the ion, the second integrand in Eq. (6) is solely due to a coherent electronic preparation.

To reconstruct $p(x, \varphi)$ we have to separate the contributions $p_s(x, \varphi)$ and $p_a(x, \varphi)$ by two measurements of $\sigma_{11}(\tau, \varphi)$, with incoherently and coherently prepared initial electronic states. The latter can be achieved by prepumping the ion with a laser tuned to the pure electronic resonance of the quadrupole transition. An incoherent measurement with an initial preparation in the electronic ground state ($|c_1|^2 = 1, |c_2|^2 = 0$) yields the symmetrized distribution $p_s(x, \varphi)$,

$$\sigma_{11}^{(\text{inc})}(\tau, \varphi) = \frac{1}{2} + \frac{1}{2} \int_{-\infty}^{+\infty} dx\, e^{-ix\tau}\, p_s(x, \varphi) . \tag{9}$$

Performing an additional coherent measurement with the electronic preparation $|c_1|^2 = |c_2|^2 = 1/2$ and $\arg(c_1 c_2^*) = \phi - \pi/2$ allows one to derive the antisymmetrized distribution $p_a(x, \varphi)$,

$$\sigma_{11}^{(\text{coh})}(\tau, \varphi) = \frac{1}{2} + \frac{i}{2} \int_{-\infty}^{+\infty} dx\, e^{-ix\tau}\, p_a(x, \varphi) . \tag{10}$$

Combining both results, we get the characteristic function of the generalized spatial distribution,

$$\Psi(\tau,\varphi) = 2\left[\sigma_{11}^{(\mathrm{inc})}(\tau,\varphi) - \tfrac{1}{2}\right] + 2i\left[\sigma_{11}^{(\mathrm{coh})}(\tau,\varphi) - \tfrac{1}{2}\right]. \tag{11}$$

This function, known for a π interval of the phase φ, provides the full information on the quantum mechanical state of the ionic center-of-mass motion.

Based on results for the reconstruction of the density matrix of a radiation mode [21], we may relate the electronic dynamics to the vibrational density matrix in the generalized position representation,

$$\langle x + \delta x|\,\hat{\rho}^{(\mathrm{vib})}(0)|x - \delta x\rangle|_{\varphi_0} = \frac{1}{2\pi}\int_{-\infty}^{+\infty} ds\, e^{-ixs}\Psi[\tau(s,\delta x),\varphi(s,\delta x)], \tag{12}$$

simply by Fourier transforming the measured quantity $\Psi(\tau,\varphi)$, using

$$\tau(s,\delta x) = \sqrt{s^2 + \delta x^2}, \tag{13}$$

$$\varphi(s,\delta x) = \varphi_0 + \frac{\pi}{2} - \arg[is - \delta x]. \tag{14}$$

Thus the density matrix of the center-of-mass motion is obtained from the measured occupation probabilities $\sigma_{11}(\tau,\varphi)$ of the electronic ground state. The latter can be expressed according to Eqs. (9) and (10) in terms of $p(x,\varphi)$ [22]. Other representations of the density matrix are readily derived from Eq. (12).

Let us consider some examples for the electronic dynamics, reflecting the quantum states of the mechanical degree of freedom of the ion. The symmetrized spatial distributions are shown in Fig. 2(a) for three different vibrational quantum states, a coherent state, a number state, and a thermal state. The corresponding time evolutions of the ground-state occupation probabilities are given in Fig. 2(b). It is seen that the different features of the spatial distributions are distinguished very well by considering the electronic dynamics.

In the particular case of a coherent state, a further consideration of the phase sensitivity is needed to get the complete information on the quantum state. For this reason we consider the electronic dynamics as a function of the scaled time τ and the phase difference φ of the lasers. The behavior is shown for an incoherent and a coherent initial electronic preparation in Figs. 3(a) and 3(b), respectively. For incoherent vibrational states one gets a constant value of $\sigma_{11}^{(\mathrm{coh})}(\tau) = 1/2$ and a phase-independent behavior of $\sigma_{11}^{(\mathrm{inc})}(\tau)$.

Finally, let us deal with the particular situation where the full information on the vibrational quantum state is contained in the number statistics P_n. This is the case if $\sigma_{11}^{(\mathrm{coh})}(\tau,\varphi)$ is a constant and $\sigma_{11}^{(\mathrm{inc})}(\tau,\varphi)$ is

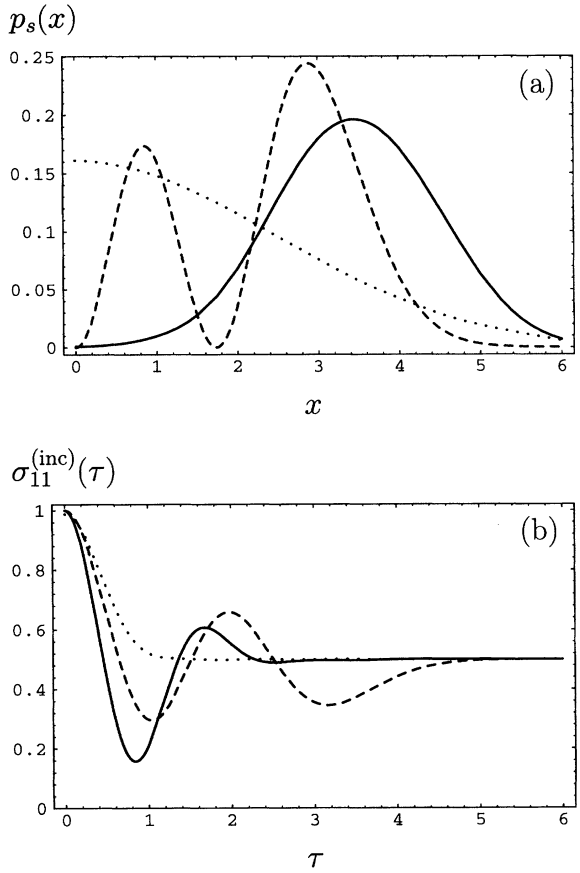

FIG. 2. The distribution $p_s(x,\varphi)$ is given for various initial states of the ionic vibration (a): coherent state $|\alpha\rangle$, $\alpha = \langle n\rangle^{1/2}$, for $\varphi = 0$ (full line); number state (dashed line); thermal state (dotted line). The mean number of vibrational quanta is in all cases $\langle n\rangle = 3$. The spatial distributions of the different states lead to significant differences in the corresponding dynamics of the ground-state occupation probabilities (b).

phase independent, which is expected to be of practical relevance, for example, in laser cooling. From Eq. (12) we may derive

$$P_n = 4\int_0^{\infty} d\tau\,\tau\, e^{-\tau^2/2} L_n(\tau^2)\left[\sigma_{11}^{(\mathrm{inc})}(\tau) - \tfrac{1}{2}\right]. \tag{15}$$

Provided this distribution is a thermal one, the temperature T of the ionic center-of-mass motion is related to the exponential decay time τ_e of the quantity $\sigma_{11}^{(\mathrm{inc})}(\tau) - 1/2$ by

$$T = \frac{\hbar\nu}{k_B}\left[\ln\left(\frac{2+\tau_e^2}{2-\tau_e^2}\right)\right]^{-1}, \tag{16}$$

where ν is the vibrational frequency of the ion.

In conclusion, we have shown that the full information on the quantum mechanical state of a trapped ion can be

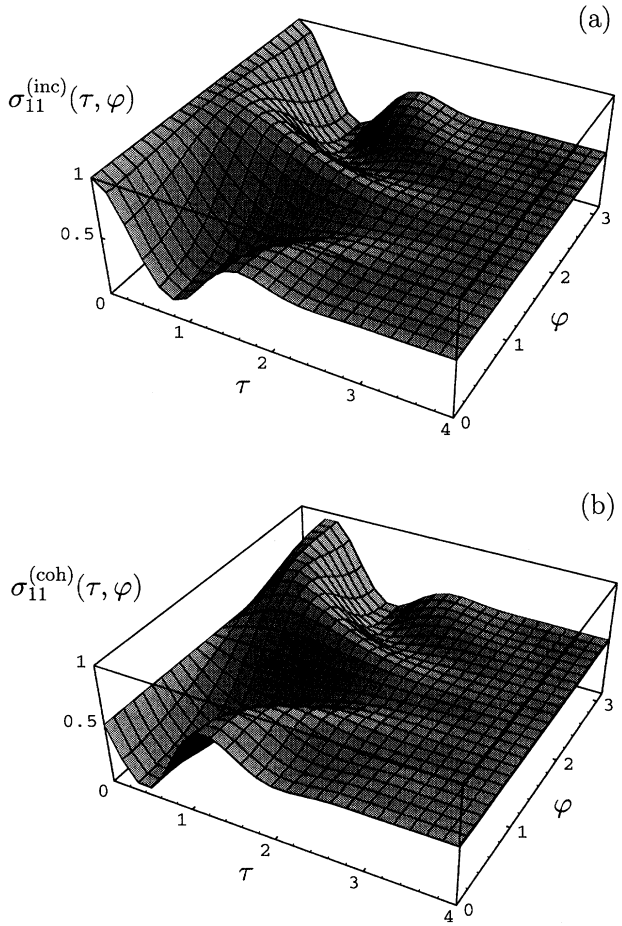

FIG. 3. The phase-sensitive properties are shown for the ground-state occupation probabilities, where $\sigma_{11}^{(inc)}(\tau,\varphi)$ is measured with an initially incoherent electronic preparation (a) and $\sigma_{11}^{(coh)}(\tau,\varphi)$ is recorded with a coherent initial preparation of the electronic subsystem (b). The ionic vibration is prepared in the same coherent state as in Fig. 2.

obtained from the dynamics of the ground state of a long-living electronic transition. To achieve this, the transition is irradiated by two lasers of the same intensity, tuned to the well resolved upper and lower vibrational sidebands. The occupation probability of the electronic ground state is readily observed by switching on a laser, which pumps a second, strong transition. When the ion is in its ground state, this leads to the onset of resonance fluorescence. The ground-state occupations measured in this manner allow us to reconstruct the density matrix of the center-of-mass motion with high quantum efficiency.

This research was supported by the Deutsche Forschungsgemeinschaft.

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