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Three-dimensional book data page segmentation and extraction method using Laplace equation

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Abstract. Recently, CT technology (X-ray Computed Tomography) has been widely applied in medical and archaeological aspects because of nondestructive testing it provides. This advantage leads one way to the solution of 3D booklet reconstruction problem, which is helpful to digitize ancient books. Our research explored extracting each page from CT scanned 3D data of the booklet type by establishing and solving one Laplace equation with the principle of electrostatic field analysis among capacitors. Having generated the order of pages, we separated page models.

Keywords: Computer simulation, Numerical methods

1. Introduction

Recently, research cases using CT scanning devices have been published regarding the decoding of old literature that has become fragile and cannot be opened. Recently, methods for digitizing ancient documents buried in the ashes of Mount Vesuvius using phase CT scan data and particle synchrotrons have been proposed. The analytical approach to digital literature is constant, and we are developing an analysis method that assumes the literature is a booklet.

This study developed a method for efficiently extracting page data from 3D booklet images acquired using 3D CT equipment using the Laplace equation and to evaluate its effectiveness. How to correctly extract page information from the 3D images generated by the booklet is our main academic issue. We propose the following hypothesis that if each page surface of a booklet is treated as an isosurface and a scale field is generated appropriately, the page information can be correctly extracted from the three-dimensional image. The booklet is multi-page printed in ink. The main purpose of binding is to display the pages that make up the booklet without flipping the page.

We have already discussed the effects of CT machines on the imaging results of different types of booklets under different voltages and current environments in previous studies. In addition, based on the results of the study, we used the most appropriate inks and printers to make brochures for follow-up research. The book data used in our experiment is as follows: 7 pages of A4 paper printed with the word 'Visualization' are randomly scattered and the volume data of 512x512x276 are obtained from the CT machine.

2. Research methods

In the case of the absorption X-ray CT used in this research, as a matter of fact, if there is not enough difference in the linear absorptance between paper and ink, the difference between the ink area and the paper areas is hardly recognized. To do. In such a case, in order to increase the contrast in the linear absorptance, the wavelength of the X-ray is changed within a possible range, and the wavelength that maximizes the contrast is specified for a given paper and ink pair. In the research “Development of a booklet analysis method using 3D data visualization technology” completed in 2018, we investigated the wavelength-dependent characteristics of the linear absorptance of printing paper and ink distributed in the market, and investigated the combination of printing paper and ink. We have developed a method that can identify the wavelength that maximizes the contrast. In this research, ink containing metal elements is prepared, a booklet composed of printed pages is prepared, and a three-dimensional image with a resolution of 512 x 512 x 276 is created from this. Specifically, the CT to be used this time is inspeXio smxSMX-90CT puls manufactured by Shimadzu Corporation, and the booklet sample is a A4 paper and ink containing 4-50% iron oxide (Canon Cartridge 039 H). In this section, we perform a page extraction experiment based on solving the Laplace equation for this data to verify its feasibility in the research of page segmentation extraction.

2.1. Page extraction

In this section we build an interactive environment for page numbers and 3D coordinates for specifying points within the booklet using WebGL 3D graphics technology. The number of pages and the sufficient number of three-dimensional coordinates of the interior points of the booklet are specified in the interactive environment. The exact solution of the Laplace equation determined in this way is calculated to generate a scalar field, extract the page surface as an isosurface, and map the three-dimensional image to the isosurface.

2.1.1. Construction of an interactive environment

The WebGL (Web Graphics Library) has become one new standard of Web 3D graphics protocol, which could create customized working environments of various projects calling for complex 3D graphic rendering and interaction with users. This tool combines the advantages of JavaScript and OpenGL ES 2.0. We develop scripts to render the scene in HTML5 Canvas with the hardware rendering acceleration by OpenGL. It is beneficial for the promotion of the program, we built on the internet because HTML script doesn't require any extra-plugin beyond the browser itself while the OpenGL is also a cross-platform standard.

With this technique, we built the working platform to visualize the whole 3D volume data containing the booklet information scanned by the CT machine. That means the we provide one 3D space in accordance with the principle of perspective and set the main object (volume data) in the center of the space and the camera above it at the very beginning. The user could do some basic interactions through simple actions, like rotating the object by dragging with the left button, adjusting the distance from the camera to the object by scrolling the mouse wheel and moving the object by dragging it with the right button. With those actions, the user could clearly find every detail on of the object. To observe the internal structure of the object, we made three slices perpendicular to each other with changeable coordinates. Aiming at highlight the boundary of pages and make it easier to obtain the samples of the feature points on the page, the function to specify the range of gray values that make the points whose gray value is out of that range invisible. By testing, the users could catch a very clear vision on the page boundary in the local image through limit the range, even in some complex structures.

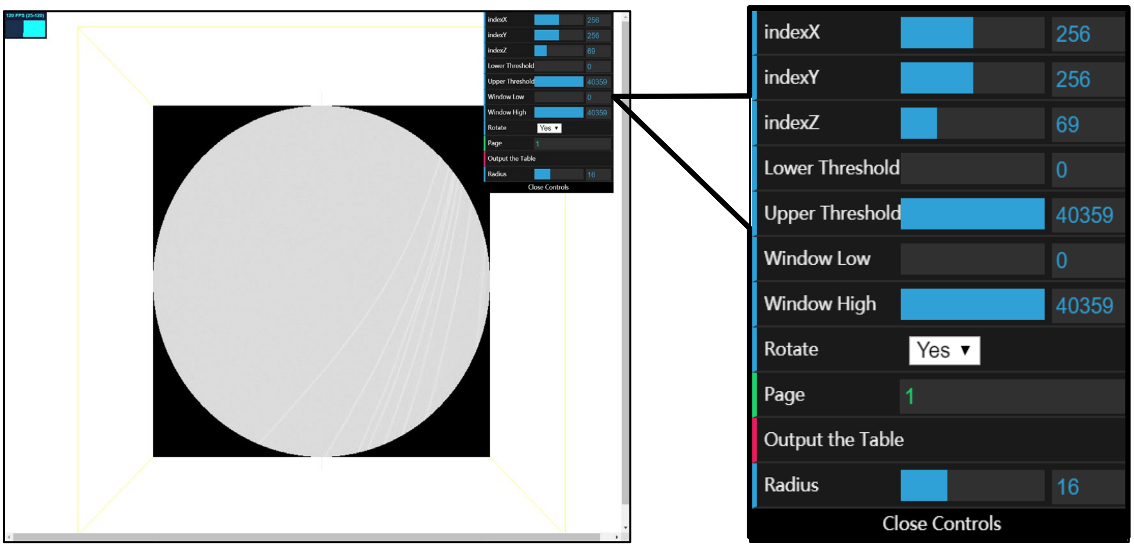


Figure 1: Adjust the perspective on the object (volume data)

In the very first version of our program, the user should select the points on the slice as the point on the actual pages by double click on the points, then the 3D coordinates will be recorded with their page numbers in the csv file, which is also accessible in the console of the browser for users' checking. However, we found this way of sampling is of very low efficiency. After redesign, a sphere following the pointer is created with adjustable radius. Having set appropriate radius shorter than the distance from the selected point on the page to any other points on the next pages or other visible points, all of the points in this sphere should be on the exact page we are searching for, then the information of coordinates about the group of points will be recorded in the csv file when users' double clicking on it. The option of turn on and off the rotating function was also added in the control bars, considering the conflicting between rotating function by single-click and sampling by double-click.

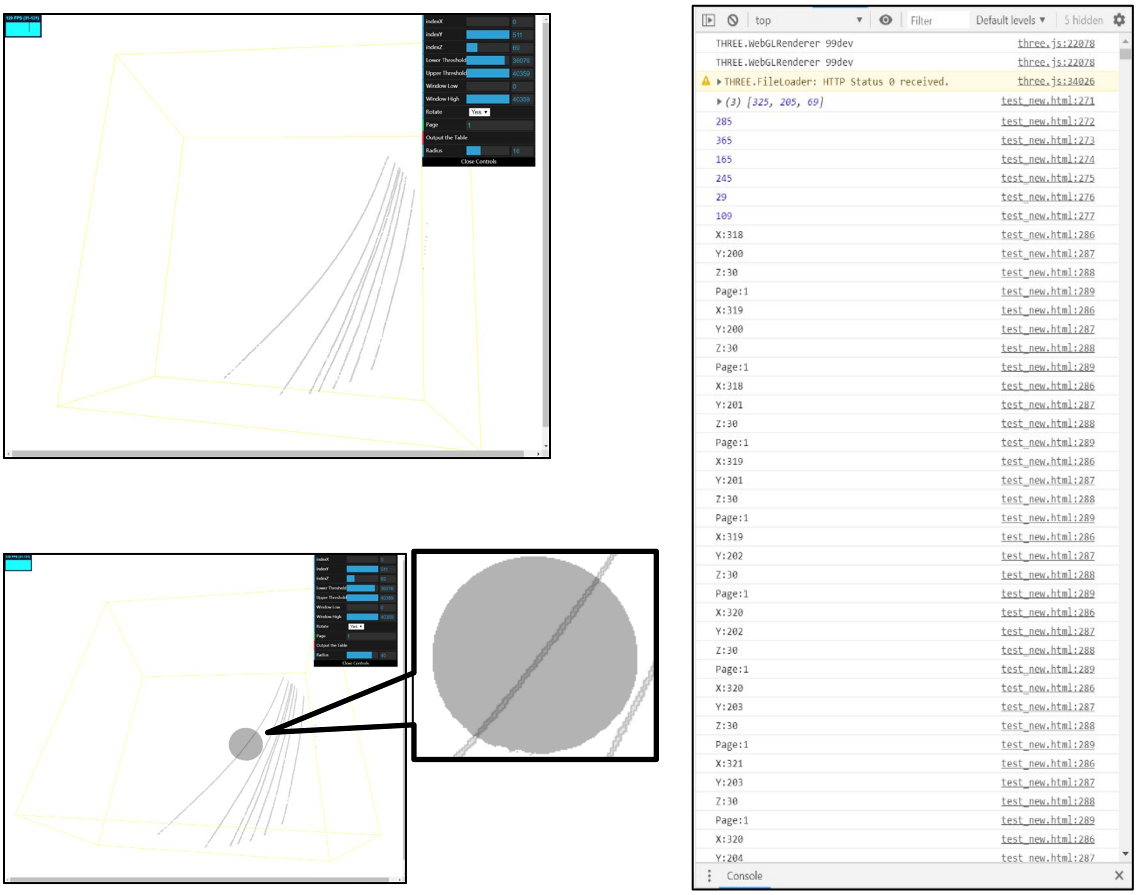


Figure 2: Highlight the boundary of the pages by setting the range of gray value to make the points out of range invisible(Upper left); Set the page number and adjust the radius of the sphere for sampling(Bottom left); The console when double click to sampling (recording the coordinates and page numbers) (right)

2.1.2. Exact solution of the Laplace equation

Considering that the structure of the booklet pages we analyzed has a high degree of similarity to the plate capacitor structure in the electrostatic field problem, we made the hypothesis "page surfaces of the booklet could be extracted by one scalar field describing them as isosurfaces". Therefore, this scalar field could be generated by the Laplace equation, by which we could also solve the electric potential problems inside capacitors. Since the undetermined coefficient of the exact solution of the Laplace equation is determined from the boundary condition, the 3D coordinates with the page number should be specified in advance. That's also the sampling process for which we have designed the GUI above. Above all, an exact solution of Laplace equation determined in this way is calculated to generate a scalar field, the page surface is extracted as an isosurface and a 3D image will be mapped to the isosurface.

For the Laplace equation, it could be shown as follows in 3D space of the Cartesian coordinate system:

$$\nabla^2 \psi = \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} = 0 \tag{2.1}$$

Assuming

$$\psi(x, y, z) = X(x)Y(y)Z(z) \tag{2.2}$$

Thus

$$\nabla^2 \psi = \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} \tag{2.3}$$

$$\frac{\partial^2 \psi}{\partial x^2}\frac{1}{X(x)} + \frac{\partial^2 \psi}{\partial y^2}\frac{1}{Y(y)} + \frac{\partial^2 \psi}{\partial z^2}\frac{1}{Z(z)} = 0 \tag{2.4}$$

Then we could find there are three constants λ_1, λ_2, λ_3 to meet:

$$\begin{cases} \dfrac{\partial^2 \psi}{\partial x^2}\dfrac{1}{X(x)} = -\lambda_1 \\ \dfrac{\partial^2 \psi}{\partial y^2}\dfrac{1}{Y(y)} = -\lambda_2 \\ \dfrac{\partial^2 \psi}{\partial z^2}\dfrac{1}{Z(z)} = -\lambda_3 \\ -\lambda_1 - \lambda_2 - \lambda_3 = 0 \end{cases} \tag{2.5}$$

For the Laplace equation above as a linear partial differential equation, the principle of linear superposition is satisfied.

In our experiments, considering the surface should be developable surface, we determined the solution form as

$$\psi = \left(A_1 e^{\sqrt{-\lambda_1}x} + B_1 e^{-\sqrt{-\lambda_1}x} + C_1 x + D_1\right) * \left(A_2 e^{\sqrt{-\lambda_2}y} + B_2 e^{-\sqrt{-\lambda_2}y} + C_2 y + D_2\right) * \left(A_3 cos\sqrt{\lambda_3}z + C_3 z + D_3\right) \tag{2.6}$$

In the experiment, we used the previous UI to collect coordinate points, and used the above formula to estimate the parameters of the scalar field.

2.1.3. Extraction of isosurface

According to the obtained three-dimensional coordinates, we can obtain a series of parameters by solving the Laplace equation. This allows us to calculate the scalar of each coordinate point in a three-dimensional space of the same size as the volume data, and generates a scalar field to simulate the distribution of pages in the volume data.
As Figure 3, when collecting the coordinates of the internal points, each page is given a different mark, so that in the generated scalar field, the scalar values of these coordinate points are also equal to the mark value (page number) of the page.

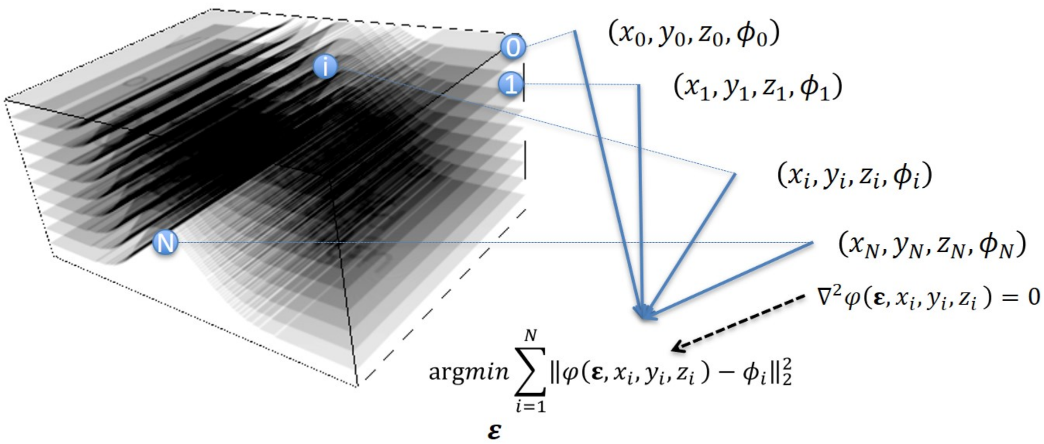


Fig 3: Using the Laplace equation, generating a scalar field with each page surface of the booklet as an isosurface

By generating such a scalar field, we can extract the pages of corresponding values (number of pages) by extracting the isosurface of different values. Finally, we corresponded the isosurface coordinates with the data in the volume data, and extracted each page in the volume data according to the value of the isosurface.

As shown in Figure 4, we sample the coordinates of 3,515 points to calculate the Laplace equation, and reduce the total error (the square of the difference between the value of each point and the number of pages) to 1400. We visualize the resulting scalar field and fit each page by generating isosurface.

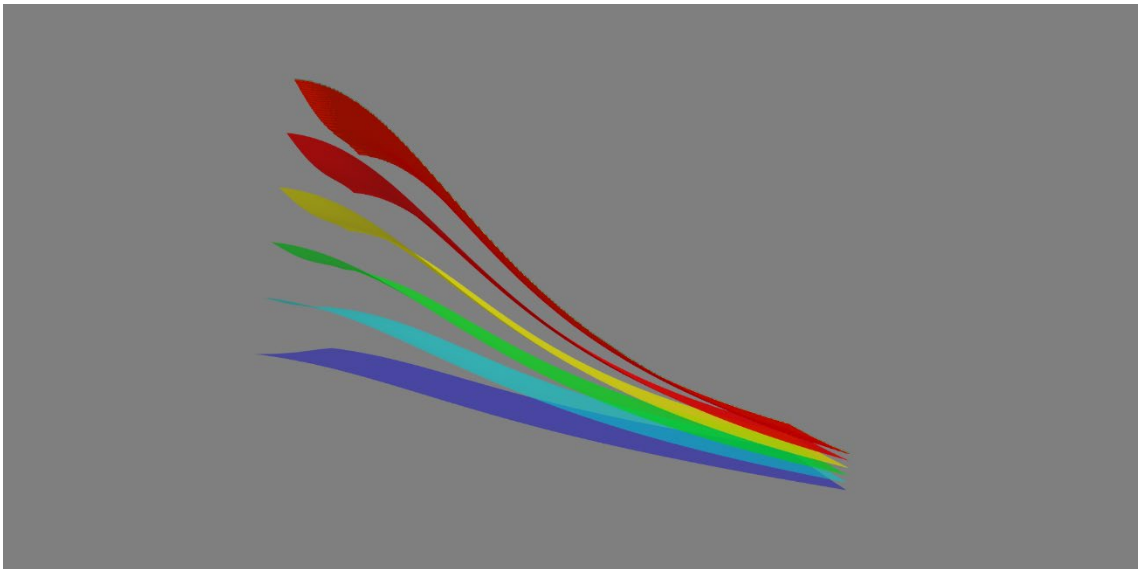


Figure 4: Visualization results of scalar field isosurface

By visualizing the results, we can conclude that if the error of the scalar field obtained by the parameters of the Laplace equation is small enough, it can fit each page in the volume data. We calculated by the coordinate values of different numbers of points, and obtained various scalar field parameters. We searched for the optimal solution among these parameters to extract the corresponding pages in subsequent experiments. Based on the results, it is easy to extract each page from the volume data through the program and evaluate it in subsequent experiments to determine the feasibility of the research method.

2.1.4. Criterion for the number of internal points

Regarding the appropriateness of the scalar field constructed using the interactive environment, a three-dimensional image is generated by using the exact solution of the Laplace equation for which the undetermined coefficient is predetermined, and the difference from the coefficient determined through this interactive environment is calculated. Formalization, the closer the value (error) is to zero, the more appropriate the scalar field is generated. Using this result, it is possible to determine a criterion for the number of internal points to be specified that are necessary for generating the scalar field.

We experimented with different numbers of internal points extracted from the same individual data. By determining the parameters of the exact solution of the Laplace equation, the error in the scalar field is minimized as much as possible. The data in the above experimental process show that the larger the number of internal points, the easier it is to fit the curved shape of the page, but at the same time, it will also cause the parameters of the Laplace equation that cannot perfectly match the volume data. As shown in Figure 5, when using the coordinate data of 881 internal points to calculate and reduce the error to 716, although the overall brochure shape is consistent with the real situation, the details are warped, which makes it difficult to extract the pages. Will cause the loss of data, which will greatly reduce the experimental results.

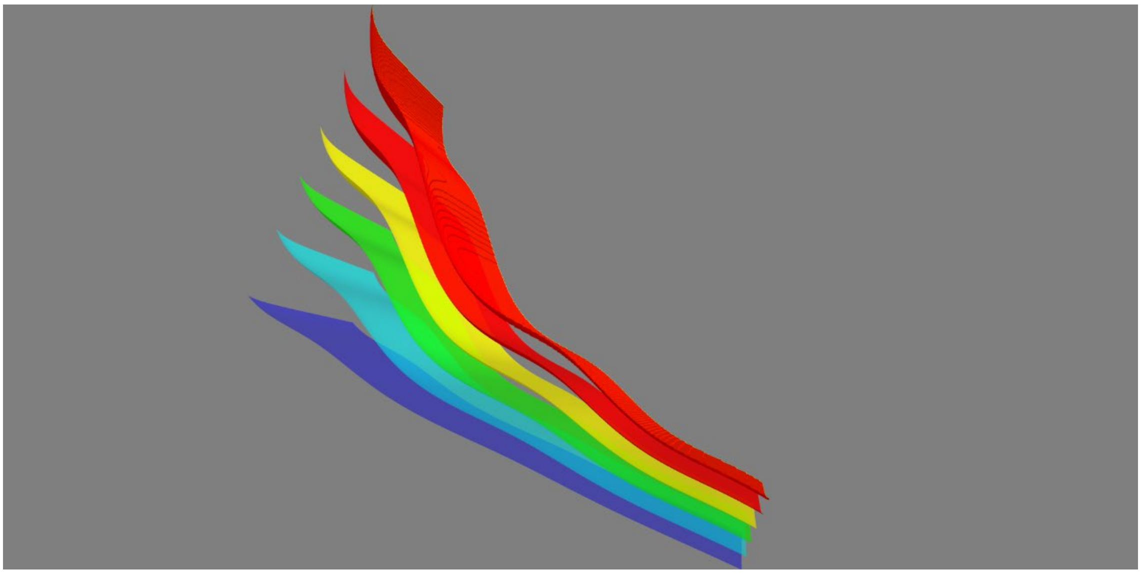


Figure 5: Isosurface visualization results when the number of points is insufficient

The conclusion is that when sampling points, it is necessary to collect as uniformly as possible. Simultaneously, to ensure the fitting effect, it is necessary to collect the feature points of the page (sudden distortion, flicking). Based on this, we need to collect more points while ensuring the validity of the Laplace equation parameters.

2.2. Result

According to the experiments, we successfully obtained the coordinate range of each page in the book data through the calculated scalar field. Based on these results, we can easily extract each page of the book data from the volume data and visualize it.

2.2.1. The calculated field

The scalar field to show the page numbers was generated by the means we mentioned above. The result was as follows. Theoretically, in this harmony field fitting the distributed points on pages we sampled, each point whose ψ equals one natural number stands for one physical point on the real page whose number is that ψ. However, most of those points own the coordinates which is not integers. This cause a lot of losses on the information on pages. More seriously, there are still errors in sampling and calculation that lead to coordinate deviations on the calculated page from the real one.

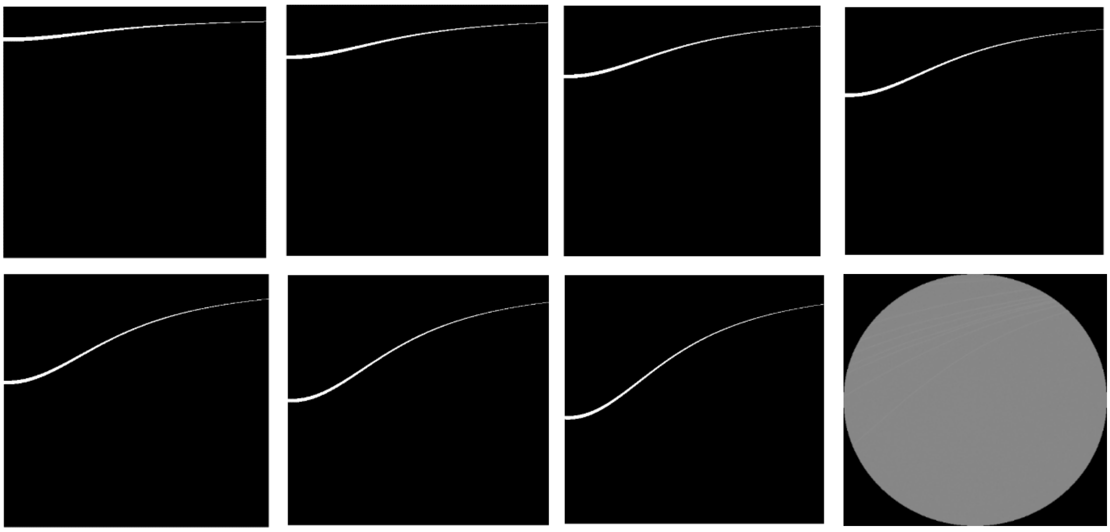


Figure 6: extracted pages 1-7

In the actual situation, we found that, because the parameters that generate the scalar field are not unique, this will cause the scalar field to have some errors with the real volume data. This result will cause us to specify the number of pages we want to extract, not a certain number, but a range. For example, we extracted the fourth page in the experiment. However, in the scalar field, the value of the coordinates representing the data of the fourth page will fluctuate between 3.4 and 4.1 due to the error. This direct result of this error is that the extracted page will be thicker than the original page, and when the page gap is small, the data of other pages will also be included. However, we believe that this problem can be solved by reducing the error of the Laplace equation parameters.

2.2.2. The tolerance of page extracting

For dealing with the problem above, we must set one tolerance of errors to make sure to keep the valid data from discarding. This means that the extracted volume data of one page should not be one single layer with the exact ψ, but one region with sufficient thickness contains those valid data, which calls for one range $[\psi-\delta_1, \psi+\delta_2]$ when extracting page data instead of ψ.

To show this model with tolerance, we do one experiment on page 4 as shown in Figure

7.

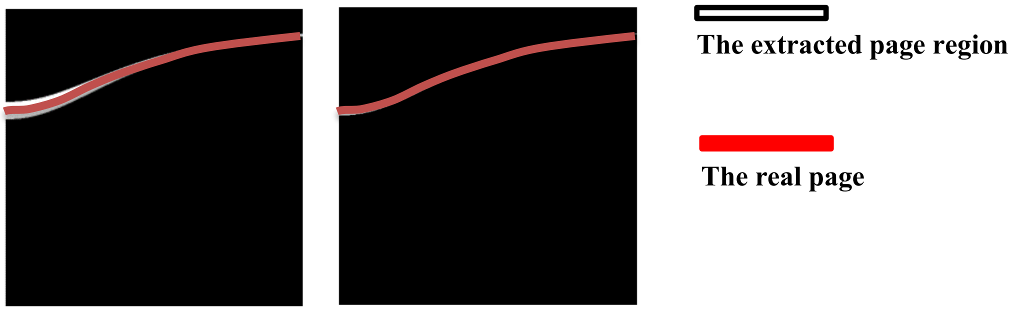


Figure 7: extracted volume data of page 4

As shown in the result, the right range would lose many valid data, but when the range was widened into the left one, it owned a good ability to contain the entire page data.

2.2.3. The visualization of pages

The output of the extracted pages above is actually series of coordinates of points. Then the problem comes that how can we judge the result. For reflecting the main information of those data, we made the projection from the volume data onto one plane. More detailly speaking, we do the compression through the thickness direction, using the average of gray values, and increase the contrast of the image. The images generated from the 7 pages of our sample are shown in Figure 8.

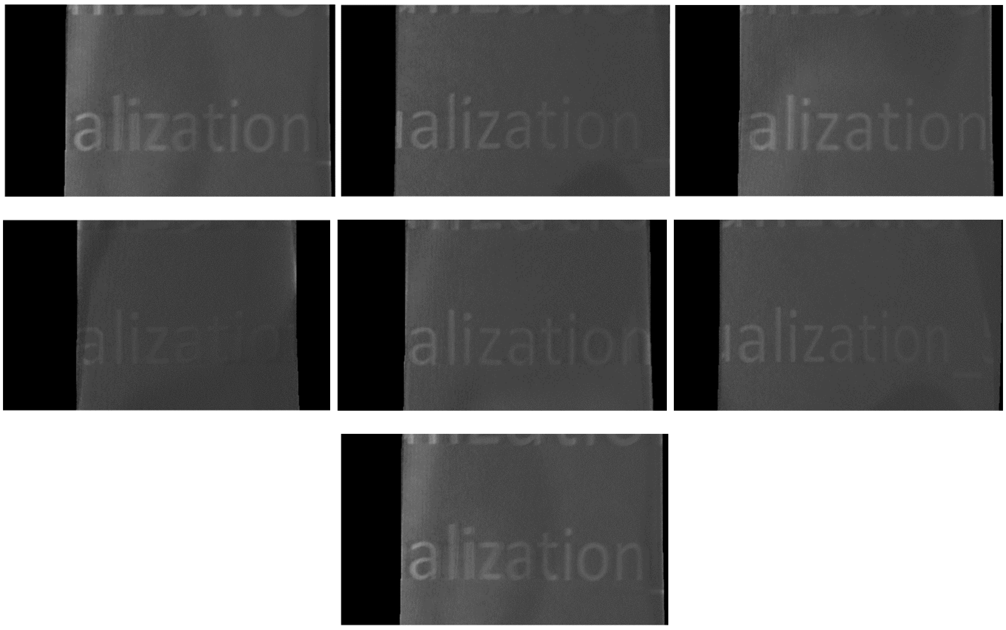
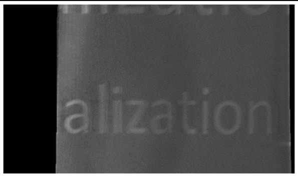
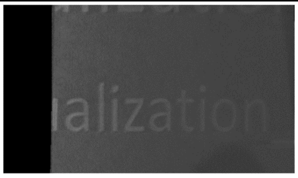
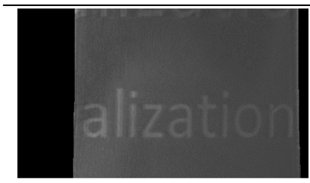
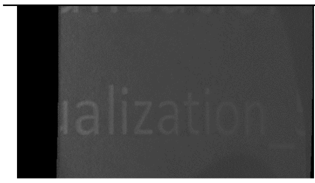


Figure 8: the projection of the 7 pages on plane

We found that the results are identifiable as simple text, and they could be recognized by the OCR system from Google with the correct contents.

3. Conclusion

The output so far is acceptable in the OCR system of Google. The recognition results with the original page image are shown below:

Output of our system	Original page image	Output of OCR
		alization
		alization
		alization
		alization
		alization
		ialization
		alization

As the form shows, all of the internal texts could be recognized correctly, except the incomplete letter “u” misrecognized into “I”. After all, the result is of high practicality.

We set up the spatial coordinates corresponding to the points on the page in the volume data collected in the UI environment to solve the Laplace equation in the electrostatic field, and fit each page in the booklet through the obtained scalar field to propose Page separation model for book data. During the experiment, we also optimized the UI for the difficulties in sampling the 3D model. According to the experimental results, the influence of the number of sampling points in the research method on the results is analyzed, and a mature page separation model is obtained. In the future, we will collect more data to optimize the page separation model, and build a system to make the page separation process more concise and accurate.

4. References

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