Online Appendix

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This is the online appendix of our submitted manuscript "Feasibility in Multistage Robust Dispatch with Renewables: A Recursive Characterization and Scalable Approximation". Reviewers/researchers are requested to read the full text of both the manuscript and this appendix for better understanding.

In a backward recursion, given a polyhedron $\mathcal{F}_t$ and a polyhedral uncertainty set Ξ_t, we need to compute $\mathcal{F}_{t-1}$ where

$$\mathcal{F}_{t-1} = \left\{ x_{t-1} \middle| \begin{array}{c} \forall \xi_t \in \Xi_t, \exists (x_t, y_t): x_t \in \mathcal{F}_t \\ Ax_t + By_t + Cx_{t-1} \leq b + D\xi_t \end{array} \right\}$$

The submitted manuscript has provided the solution when the uncertainty set Ξ_t is low-dimensional, whose vertices can be enumerated. This appendix focuses on the case where vertex enumeration is intractable.

To this end, define

$$\mathcal{F}_{t-1}(\xi_t) = \left\{ x_{t-1} \middle| \begin{array}{c} \exists (x_t, y_t): x_t \in \mathcal{F}_t \\ Ax_t + By_t + Cx_{t-1} \leq b + D\xi_t \end{array} \right\} \quad (1)$$

Obviously, we have

$$\mathcal{F}_{t-1} = \bigcap_{\forall \xi_t \in \Xi_t} \mathcal{F}_{t-1}(\xi_t) \quad (2)$$

$\mathcal{F}_{t-1}(\xi_t)$ is the orthogonal projection of $\mathcal{Q}_{t-1}(\xi_t)$ where

$$\mathcal{Q}_{t-1}(\xi_t) = \left\{ (x_{t-1}, x_t, y_t) \middle| \begin{array}{c} x_t \in \mathcal{F}_t \\ Ax_t + By_t + Cx_{t-1} \leq b + D\xi_t \end{array} \right\} \quad (3)$$

By denoting $\mathcal{F}_{t-1}(\xi_t) = \mathrm{Proj}_{x_{t-1}}[\mathcal{Q}_{t-1}(\xi_t)]$, condition (2) is equivalent to

$$\mathcal{F}_{t-1} = \bigcap_{\forall \xi_t \in \Xi_t} \mathrm{Proj}_{x_{t-1}}[\mathcal{Q}_{t-1}(\xi_t)] \quad (4)$$

Due to the difficulty in computing large-scale polyhedral projection, the idea is to approximate $\mathcal{F}_{t-1}$ from within by an affine transformation $\Lambda\mathbb{B} + \mu$. Polyhedron $\mathbb{B}$ is a template, diagonal matrix Λ is for stretch, and column vector μ is for translation. Please refer to the submitted manuscript for details.

To find the maximal inner approximation of $\mathcal{F}_{t-1}$, we resort to the following optimization problem:

$$\begin{aligned} &\max_{\Lambda \geq 0, \mu} \det(\Lambda) \\ &\mathrm{s.t.}\ \Lambda\mathbb{B} + \mu \subseteq \mathcal{F}_{t-1} = \bigcap_{\forall \xi_t \in \Xi_t} \mathrm{Proj}_{x_{t-1}}[\mathcal{Q}_{t-1}(\xi_t)] \end{aligned} \quad (5)$$

Problem (5) is equivalent to

$$
\begin{aligned}
&\max_{\Lambda \geq 0, \mu} \det(\Lambda) \\
&\quad \text{s.t.} \forall \xi_t \in \Xi_t : \Lambda \mathbb{B} + \mu \subseteq \mathrm{Proj}_{x_{t-1}}[\mathcal{Q}_{t-1}(\xi_t)]
\end{aligned} \tag{6}
$$

By **Claim 1** in the submitted manuscript, problem (6) is further equivalent to

$$
\begin{aligned}
&\max_{\Lambda \geq 0, \mu} \det(\Lambda) \\
&\text{s.t.} \forall \xi_t \in \Xi_t, \forall u \in \mathbb{B}, \exists \hat{u} : \begin{bmatrix} u \\ \hat{u} \end{bmatrix} \in \Phi \mathcal{Q}_{t-1}(\xi_t) + \eta
\end{aligned} \tag{7}
$$

where Φ is a diagonal matrix built with Λ, and η is a vector that depends on μ. Please refer to the submitted manuscript for details.

Apparently, problem (7) is a two-stage robust optimization and shares the same mathematical structure with the problem (25) in the submitted manuscript. Hence, the proposed solution procedure in **Algorithm 1,** which consists of a master problem and a sub-problem, is applicable to problem (7).

In conclusion, this online appendix reveals that when the vertices of uncertainty set Ξ_t cannot be enumerated, the proposed method still works with minor amendments to the model.