Online Appendix: CCG Algorithm for MRUC-PAF

Zhongjie Guo, Wei Wei *Senior Member, IEEE*

This is the online appendix of our manuscript entitled *Partially Affine Policy for Multistage Robust Unit Commitment*. This appendix presents the customized column-and-constraint generation (CCG) algorithm for the multistage robust unit commitment under partially affine policy (MRUC-PAF).

We rewrite MRUC-PAF below:

$$\begin{aligned}\min_{\widehat{x}\in\widehat{\mathcal{X}}} & \left\{ e^\top x + \sum_{t=1}^{T} f^\top q_t \right. \\ & \left. + \max_{\xi\in\Xi} \min_{z\in\widehat{\mathcal{F}}(\widehat{x},\xi)} \left\{ \sum_{t=1}^{T} \left[f^\top Q_t \xi_{[t]} + h^\top z_t \right] \right\} \right\}\end{aligned} \tag{1}$$

where $\widehat{x} = \{x, (Q_t)_{t=1}^T, (q_t)_{t=1}^T\}^\top$, $z = [z_1; \cdots; z_T]$, $\xi = [\xi_1; \cdots; \xi_T]$. The uncertainty set is

$$\Xi = \{\xi | G\xi \le g\} = \Xi_1 \times \cdots \times \Xi_T$$

The feasible set of real-time dispatch is

$$\widehat{\mathcal{F}}(\widehat{x}, \xi) = \{z | Nz \le l + J\widehat{x} + L\xi + U\widehat{x}V\xi\}$$

where matrices N, J, L, U, V, and l are constant coefficients depending on matrices A_t, B_t, C_t, D_t, E_t, and b_t. Please see the main manuscript for more details of the notations.

MRUC-PAF (1) is a special two-stage adaptive robust optimization problems; the product terms of $\widehat{x}$ and ξ appear in the objective function and constraints of the second stage.

I. CCG Algorithm

The CCG algorithm handles problem (1) by solving a master problem and two subproblems iteratively. The master problem update $\widehat{x}$, given which the subproblems identify non-trivial scenarios from the uncertainty set regarding optimality and feasibility.

A. Master problem

The master problem finds the optimal $\widehat{x}$ considering all the identified scenarios so far, i.e.

$$\begin{aligned}\min_{\widehat{x}\in\widehat{\mathcal{X}},\eta,z^k} & \left\{ e^\top x + \sum_{t=1}^{T} f^\top q_t + \eta \right\} \\ \text{s.t. } & \eta \ge \sum_{t=1}^{T} \left[f^\top Q_t \xi^k_{[t]} + h^\top z^k_t \right], \forall \xi^k \in \Pi \\ & Nz^k \le l + J\widehat{x} + L\xi^k + U\widehat{x}V\xi^k, \forall \xi^k \in \Pi\end{aligned} \tag{2}$$

The optimal value is a lower bound of MRUC-PAF since this master problem enumerates the scenarios in Π, a subset of Ξ.

B. Feasibility subproblem

Given $\widehat{x}$, the feasibility subproblem checks whether there exists at least one feasible z for any $\xi \in \Xi$; if not, the scenario that causes the most serious infeasibility is identified. The feasibility subproblem is

$$\begin{aligned}\max_{\xi\in\Xi} \min_{z,s} \ & \mathbf{1}^\top s \\ \text{s.t. } & Nz - \boldsymbol{I}^\top s \le l + J\widehat{x} + L\xi + U\widehat{x}V\xi \\ & s \ge 0\end{aligned} \tag{3}$$

where s is a slack vector, $\mathbf{1}$ is an all-one vector, and $\boldsymbol{I}$ is an identity matrix. If the optimal value of feasibility subproblem is zero, the current $\widehat{x}$ is robustly feasible; else, identify ξ at the optimal solution.

C. Optimality subproblem

Given $\widehat{x}$, the optimality subproblem is

$$\begin{aligned}\max_{\xi\in\Xi} \min_{z} \ & \sum_{t=1}^{T} \left[f^\top Q_t \xi_{[t]} + h^\top z_t \right] \\ \text{s.t. } & Nz \le l + J\widehat{x} + L\xi + U\widehat{x}V\xi\end{aligned} \tag{4}$$

The optimality subproblem identifies the scenario that causes the worst cost; the optimal value will build the upper bound.

D. CCG procedure

The flowchart of the customized CCG procedure is given in Algorithm 1.

Algorithm 1 CCG algorithm for MRUC-PAF

Step 0: Set the tolerance ε, lower bound $LB = -\infty$, upper bound $UB = +\infty$, iteration count $I = 1$, and arbitrary set $\Pi = \{\xi^0\}$
Step 1: Solve the master problem (2); record the optimal solution $\widehat{x}^* = \{x^*, (Q_t^*)_{t=1}^T, (q_t^*)_{t=1}^T\}^\top$ and optimal value v^*. Update the lower bound $LB = v^*$.
Step 2: Given $\widehat{x}^*$, solve the feasibility subproblems (3); record the optimal solution ξ^* and optimal value w^*. If $w^* > 0$, update $\Pi \leftarrow \Pi \cup \{\xi^*\}$, $I \leftarrow I + 1$, and go to step 1; if $w^* = 0$, proceed.
Step 3: Given x^*, solve the optimality subproblem (4); record the optimal solution ξ^* and optimal value r^*. Update the upper bound $UB = e^\top x^* + \sum_{t=1}^T f^\top q_t^* + r^*$. If $UB - LB \le \varepsilon$, terminate; else, update $\Pi \leftarrow \Pi \cup \{\xi^*\}$, $I \leftarrow I + 1$, and go to step 1.

II. Solving Subproblems

In Algorithm 1, the master problem (2) is a mixed-integer linear program (MILP) and can be sovled by commercial solvers. However, both the feasibility and optimality subproblems are linear max-min programs. Since (3) and (4) share the same structure, we take (3) as an example to introduce the solution method. Rewrite (3) as follows:

$$\begin{aligned}\max_{\xi\in\Xi}\min_{z,s}\ & \mathbf{1}^\top s \\ \text{s.t. } & Nz-\boldsymbol{I}^\top s \le l+J\widehat{x}+L\xi+U\widehat{x}V\xi:\lambda \\ & s\ge 0\end{aligned}$$

where λ is the vector of dual variables. Given ξ, the inner minimization problem becomes a linear program. Hence, we can dualize the inner problem, i.e.

$$\begin{aligned}\max_{\xi\in\Xi,\lambda}\ & \lambda^\top\left(l+J\widehat{x}+L\xi+U\widehat{x}V\xi\right) \\ \text{s.t. } & N^\top\lambda=0,-1\le\lambda\le 0\end{aligned} \tag{5}$$

The difficulty in solving problem (5) is the bilinear term $\lambda^\top\left(L\xi+U\widehat{x}V\xi\right)$. Since the feasible sets of ξ and λ are separable, problem (5) can be rearranged as

$$\begin{aligned}\max_{\lambda}\ & \lambda^\top\left(l+J\widehat{x}\right)+\left\{\begin{aligned}&\max_{\xi}\ \lambda^\top\left(L\xi+U\widehat{x}V\xi\right)\\ &\text{ s.t. } G\xi\le g:\gamma\end{aligned}\right\} \\ \text{s.t. } & N^\top\lambda=0,-1\le\lambda\le 0\end{aligned} \tag{6}$$

Viewing λ as a parameter, the inner problem becomes a linear program over ξ, whose optimality conditions are

$$\begin{aligned}& G^\top\gamma-\left(L+U\widehat{x}V\right)^\top\lambda=0 \\ & 0\le\gamma\perp\left(g-G\xi\right)\ge 0\end{aligned}$$

By strong duality, the objective values of the pair of primal and dual problems are equal, so the inner problem can be replaced by the objective function with its optimality conditions. The complementarity and slackness condition can be linearized by

$$0\le\gamma\le M(1-\theta),0\le g-G\xi\le M\theta,\theta \text{ is binary}$$

where M is a large enough constant and θ has the same dimension as vector g.

Finally, problem (6) is equivalent to the following MILP:

$$\begin{aligned}\max_{\lambda,\gamma,\xi,\theta}\ & \lambda^\top\left(l+J\widehat{x}\right)+g^\top\gamma \\ \text{s.t. } & N^\top\lambda=0,-1\le\lambda\le 0,\theta\in\{0,1\}^{|g|} \\ & G^\top\gamma-\left(L+U\widehat{x}V\right)^\top\lambda=0 \\ & 0\le\gamma\le M(1-\theta) \\ & 0\le g-G\xi\le M\theta\end{aligned} \tag{7}$$

In the same way, the optimality subproblem can be reformulated as an MILP.