Multi-stage Robust Reactive Power Optimization in ADNs with Renewables Considering Discrete Intertemporal Constraints

ONLINE APPENDIX

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This is the online appendix of our submitted manuscript entitled Multi-stage Robust Reactive Power Optimization in ADNs with Renewables Considering Discrete Intertemporal Constraints. In this appendix, the proof of **Claim 1** is given in detail. It should be noted that the notations and symbols in this appendix follow the main manuscript.

1 Preliminaries

1.1 The P3

$$\max_{\xi_1 \in \Xi_1} \min_{x_1 \in \mathcal{F}_1(\xi_1) \bigcap \mathcal{F}_1^\star(\alpha_1)} c_1^\top x_1 + \max_{\xi_2 \in \Xi_2} \min_{x_2 \in \mathcal{F}_2(\xi_2) \bigcap \mathcal{F}_2^\star(\alpha_2)} c_2^\top x_2 \\ + \cdots + \max_{\xi_T \in \Xi_T \bigcap \Xi_T'(\xi_{[T-1]})} \min_{x_T \in \mathcal{F}_T(\xi_T) \bigcap \mathcal{F}_T^\star(\alpha_T)} c_T^\top x_T \tag{\textbf{P3}}$$

1.2 The Claim 1

Claim 1: P3 is equivalent to

$$\max_{\forall \xi \in \Xi \bigcap \Xi'} \min_{x_t \in \mathcal{F}_t(\xi_t) \bigcap \mathcal{F}_t^\star(\alpha_t), \forall t \in \mathbb{T}} \sum_{t=1}^{T} c_t^\top x_t \tag{1}$$

2 Proof of Claim 1

Proof: Consider periods $T-1$ and T in **P3**:

$$\cdots \max_{\xi_{T-1} \in \Xi_{T-1}} \min_{x_{T-1} \in \mathcal{F}_{T-1}(\xi_{T-1}) \bigcap \mathcal{F}_{T-1}^\star(\alpha_{T-1})} c_{T-1}^\top x_{T-1} \\ + \max_{\xi_T \in \Xi_T \bigcap \Xi_T'(\xi_{[T-1]})} \min_{x_T \in \mathcal{F}_T(\xi_T) \bigcap \mathcal{F}_T^\star(\alpha_T)} c_T^\top x_T \tag{2}$$

Since the minimization over x_{T-1} is independent from the following two stages over Ξ_T and x_T, we can change the position of inner minimization and maximization operators; as a result, problem (2) is equivalent to

$$\cdots \max_{\xi_{T-1} \in \Xi_{T-1}} \max_{\xi_T \in \Xi_T \bigcap \Xi_T'(\xi_{[T-1]})} \left\{ \min_{x_{T-1} \in \mathcal{F}_{T-1}(\xi_{T-1}) \bigcap \mathcal{F}_{T-1}^\star(\alpha_{T-1})} c_{T-1}^\top x_{T-1} \right. \\ \left. \min_{x_T \in \mathcal{F}_T(\xi_T) \bigcap \mathcal{F}_T^\star(\alpha_T)} c_T^\top x_T \right\} \tag{3}$$

Taking one more step by combining adjacent two maximization/minization operators, problem (3) is equivalent to

$$\cdots \max_{\substack{\xi_{T-1} \in \Xi_{T-1} \\ \xi_T \in \Xi_T \\ \xi_{(T-1)} \in \Xi_T'(\xi_{[T-2]})}} \left\{ \min_{\substack{x_{T-1} \in \mathcal{F}_{T-1}(\xi_{T-1}) \bigcap \mathcal{F}_{T-1}^\star(\alpha_{T-1}) \\ x_T \in \mathcal{F}_T(\xi_T) \bigcap \mathcal{F}_T^\star(\alpha_T)}} \sum_{t=T-1}^{T} c_t^\top x_t \right\} \tag{4}$$

It is not hard to find that the max-min problem in period $T-2$ and problem (4) constitute a max-min-max-min sequence that is similar with (3), which thus can be reformulated into a max-min problem like what we have done above. By a backward induction towards period 1, we can obtain

$$\max_{\substack{\xi_1 \in \Xi_1 \\ \xi_2 \in \Xi_2 \\ \vdots \\ \xi_T \in \Xi_T \\ \xi \in \Xi'}} \left\{ \min_{\substack{x_1 \in \mathcal{F}_1(\xi_1) \bigcap \mathcal{F}_1^\star(\alpha_1) \\ x_2 \in \mathcal{F}_2(\xi_2) \bigcap \mathcal{F}_2^\star(\alpha_2) \\ \vdots \\ x_T \in \mathcal{F}_T(\xi_T) \bigcap \mathcal{F}_T^\star(\alpha_T)}} \sum_{t=1}^{T} c_t^\top x_t \right\} \tag{5}$$

It is obvious that problem (5) is equivalent to problem (1), which completes the proof. ■