Uncertainty

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1 Two rules in probability theory

1.1 summation rule or marginalisation

$$P(X) = \sum_y P(X, Y = y)$$

$$P(Y) = \sum_x P(X = x, Y)$$

1.2 product rule

$$P(X, Y) = P(X)P(Y|X) = P(Y)P(X|Y)$$

$$P(X, Y, Z) = P(X)P(Y|X)P(Z|X, Y)$$

2 Conditional Probability

$$P(X|Y) = \frac{P(X, Y)}{P(Y)} = \frac{P(X, Y)}{\sum_x P(X = x, Y)}$$

3 Independence

if $P(X)$ and $P(Y)$ are independent,

$$P(X, Y) = P(X)P(Y) \Longleftrightarrow P(X|Y) = P(X)$$

4 Baye's rule

$$P(Y|X) = \frac{P(X, Y)}{P(X)} = \frac{P(X|Y)P(Y)}{P(X)} = \frac{P(X|Y)P(Y)}{\sum_y P(X, Y = y)} = \frac{P(X|Y)P(Y)}{\sum_y P(Y = y)P(X|Y = y)}$$

i.e.,

$$P(Y|X) = \alpha P(Y)P(X|Y)$$