CS5010
Artificial Intelligence Principles:
Lecture 6

Neural networks

Chapter 20, Section 5

Outline

◇ Brains

◇ Neural networks

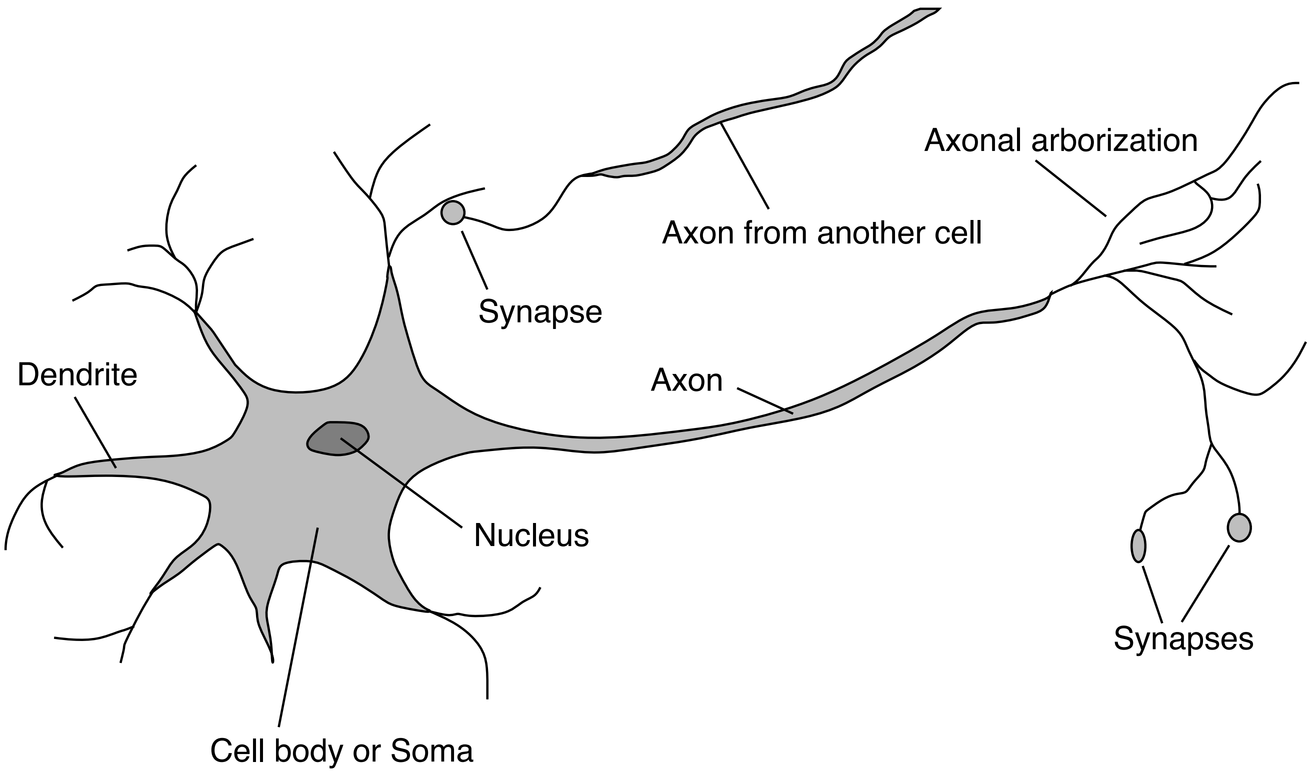
◇ Perceptrons

◇ Multilayer perceptrons

◇ Applications of neural networks

Brains

10^{11} neurons of > 20 types, 10^{14} synapses, 1ms–10ms cycle time
Signals are noisy "spike trains" of electrical potential

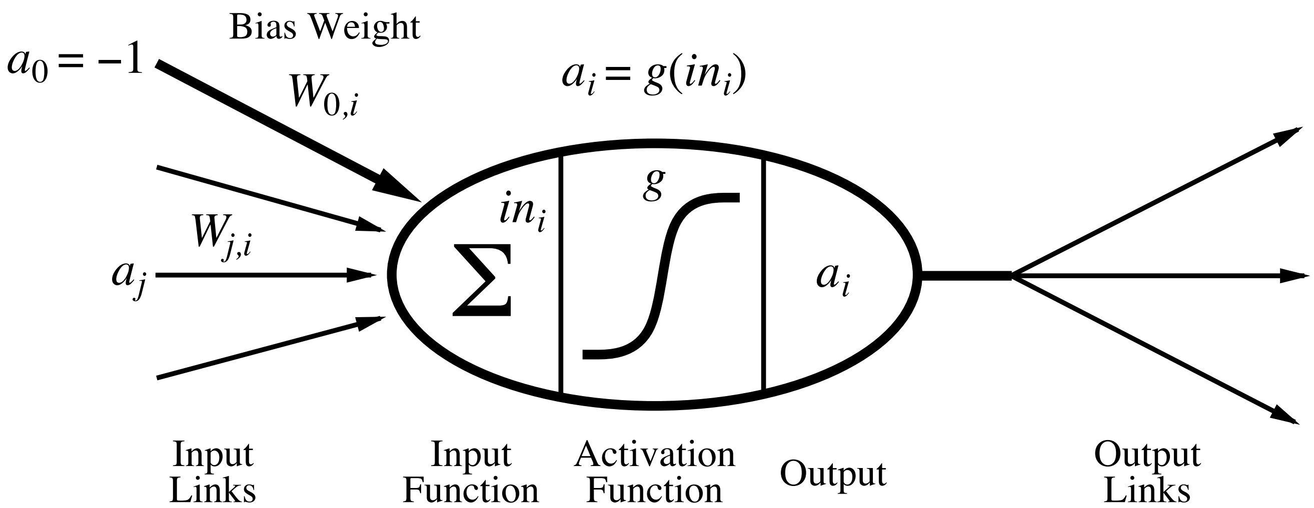


McCulloch–Pitts "unit"

Output is a "squashed" linear function of the inputs:

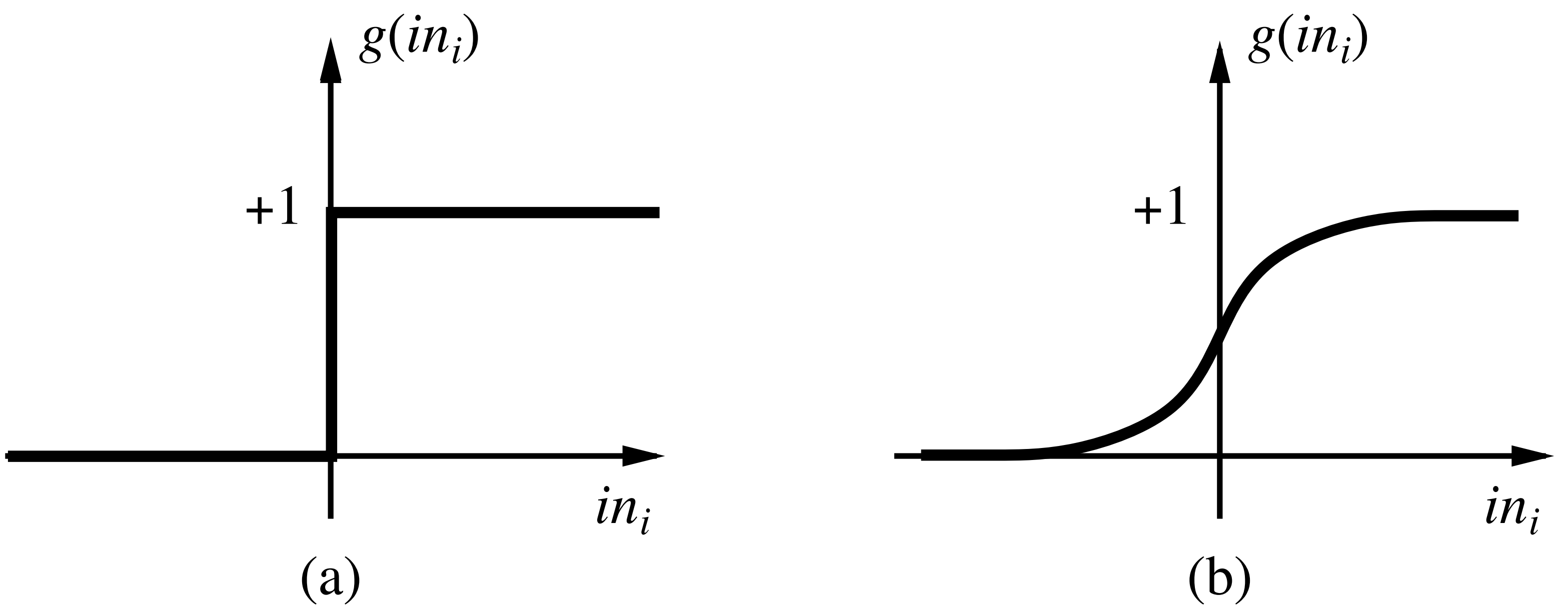
$$a_i \leftarrow g(in_i) = g\left(\Sigma_j W_{j,i} a_j\right)$$

i = 1:N is the number of units in the net
j = 1:M is the number of input sources



A gross oversimplification of real neurons, but its purpose is to develop understanding of what networks of simple units can do

Activation functions



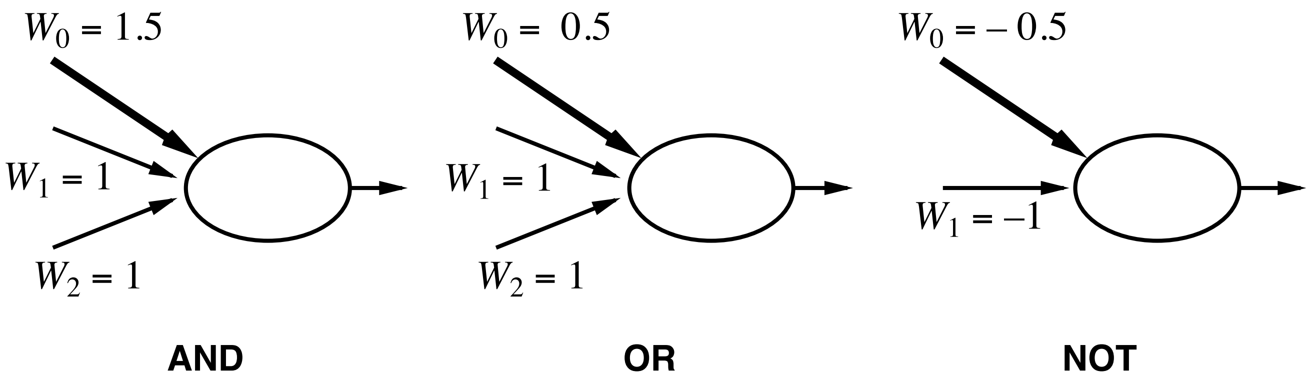
(a) is a step function or threshold function

(b) is a sigmoid function $1/(1+e^{-x})$ tanh is also often used

Changing the bias weight $W_{0,i}$ moves the threshold location

Recall y = mx + c as a simple regression model
m is the slope or rate of change of the prediction model
c moves the threshold location - it is a bias term

Implementing logical functions



McCulloch and Pitts: every Boolean function can be implemented

Both arguments and results are from a two-valued set, normally denoted {0,1} or {T,F}

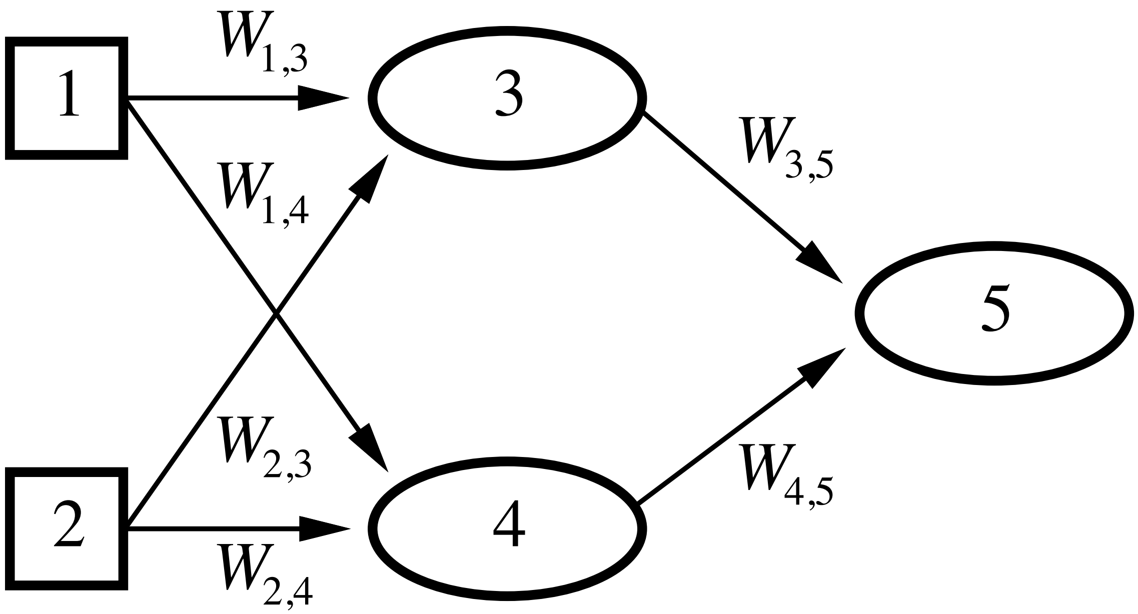
Network structures

Feed-forward networks:
- single-layer perceptrons
- multi-layer perceptrons

Feed-forward networks implement functions, have no internal state

Recurrent networks:
- Hopfield networks have symmetric weights ($W_{i,j} = W_{j,i}$)
  $g(x) = \mathsf{sign}(x)$, $a_i = \pm 1$; **holographic associative memory**
- Boltzmann machines use stochastic activation functions,
  $\approx$ MCMC in Bayes nets
- recurrent neural nets have directed cycles with delays
  $\Rightarrow$ have internal state (like flip-flops), can oscillate etc.

Feed-forward example



Feed-forward network = a parameterized family of nonlinear functions:

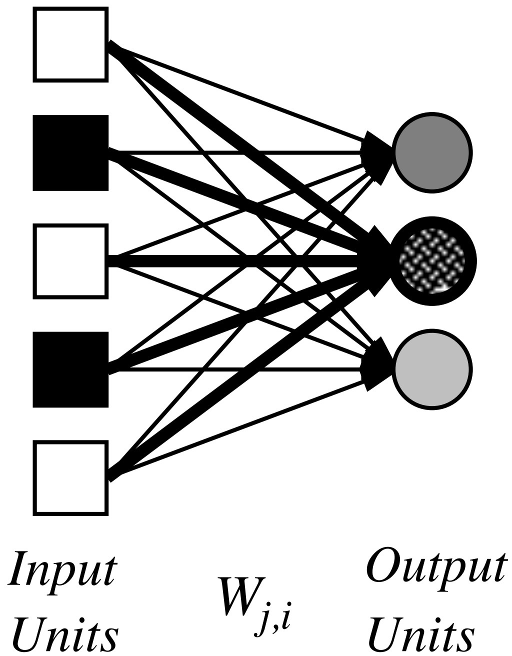
$$\begin{aligned} a_5 &= g(W_{3,5} \cdot a_3 + W_{4,5} \cdot a_4) \\ &= g(W_{3,5} \cdot g(W_{1,3} \cdot a_1 + W_{2,3} \cdot a_2) + W_{4,5} \cdot g(W_{1,4} \cdot a_1 + W_{2,4} \cdot a_2)) \end{aligned}$$

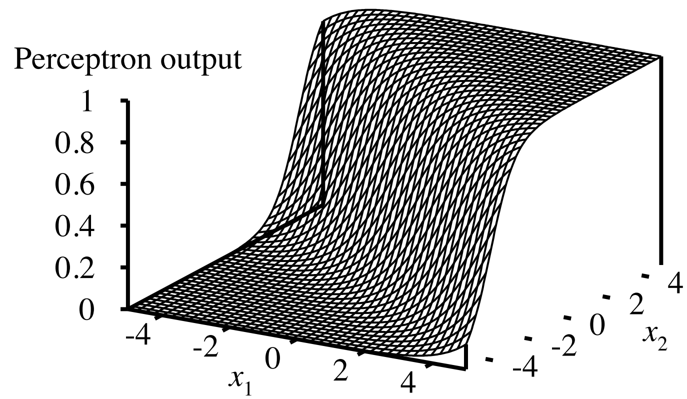
Adjusting weights changes the function: do learning this way!

As before, g is the activation function

This example has no bias term

Single-layer perceptrons





Output units all operate separately—no shared weights

Adjusting weights moves the location, orientation, and steepness of cliff

Expressiveness of perceptrons

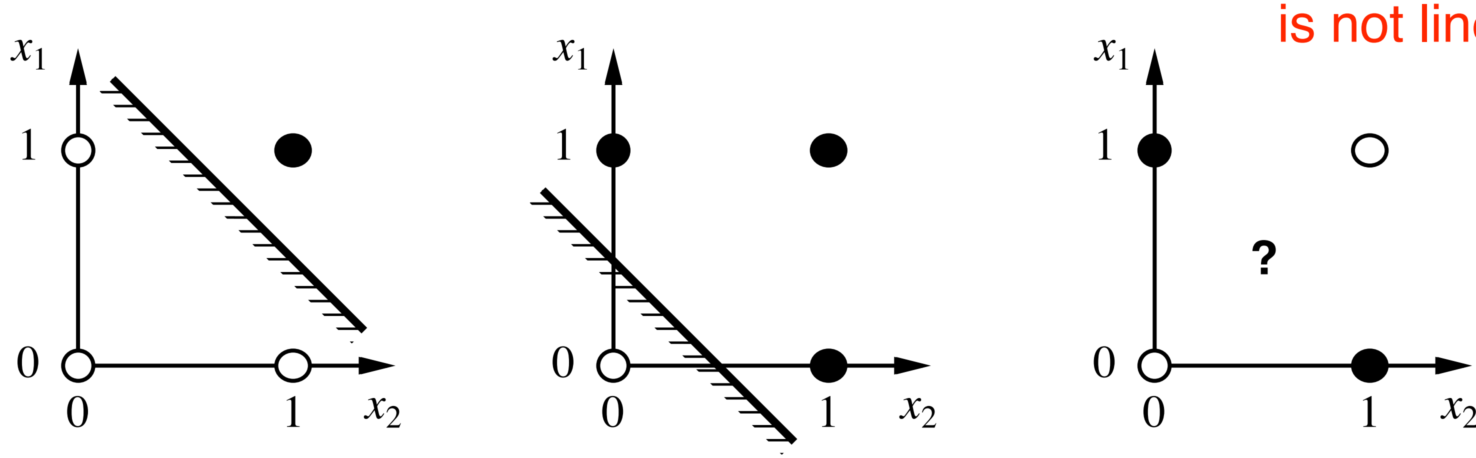
Consider a perceptron with g = step function (Rosenblatt, 1957, 1960)

Can represent AND, OR, NOT, majority, etc., but not XOR

Represents a linear separator in input space:

$\sum_j W_j x_j > 0$ or $\mathbf{W} \cdot \mathbf{x} > 0$

We say that the XOR space is not linearly separable



(a) x_1 **and** x_2 (b) x_1 **or** x_2 (c) x_1 **xor** x_2

Minsky & Papert (1969) pricked the neural network balloon

Most Boolean functions are not linearly separable, so these networks have very limited applicability

Perceptron learning

Learn by adjusting weights to reduce error on training set

The squared error for an example with input $\mathbf{x}$ and true output y is

$$E = \frac{1}{2}Err^2 \equiv \frac{1}{2}(y - h_{\mathbf{W}}(\mathbf{x}))^2 ,$$

h is a function that combines weights and activation to produce outputs from inputs x

Perform optimization search by gradient descent:

$$\frac{\partial E}{\partial W_j} = Err \times \frac{\partial Err}{\partial W_j} = Err \times \frac{\partial}{\partial W_j}\left(y - g(\Sigma_{j=0}^{n} W_j x_j)\right)$$
$$= -Err \times g'(in) \times x_j$$

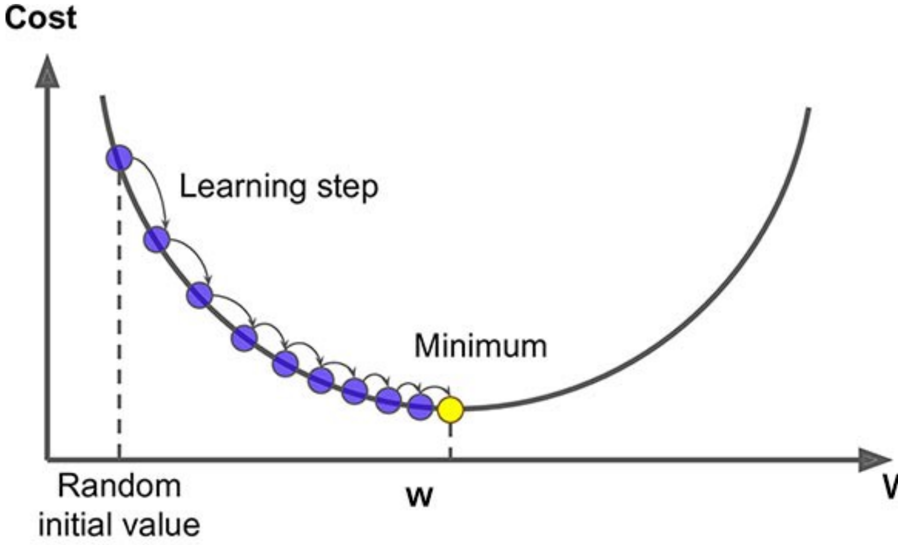
We square the error so that the error function is a quadratic curve for which the minimum will be where the derivative is zero

Simple weight update rule:

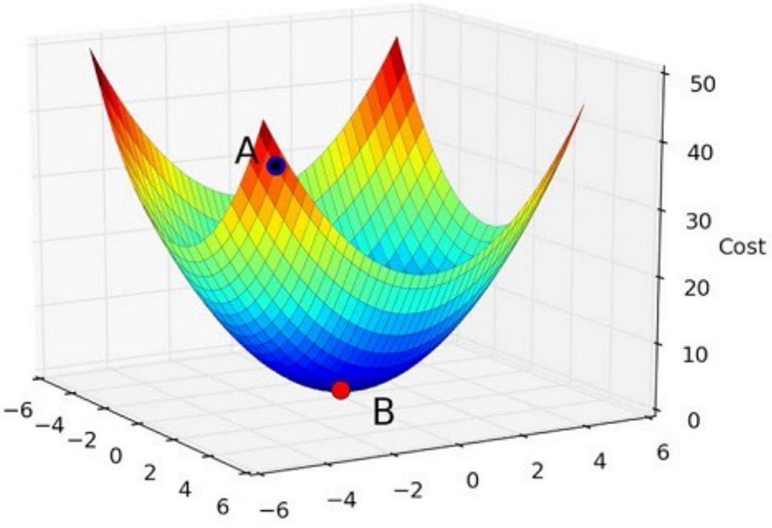
$$W_j \leftarrow W_j + \alpha \times Err \times g'(in) \times x_j$$

E.g., +ve error $\Rightarrow$ increase network output
$\Rightarrow$ increase weights on +ve inputs, decrease on -ve inputs

Gradient Descent 1

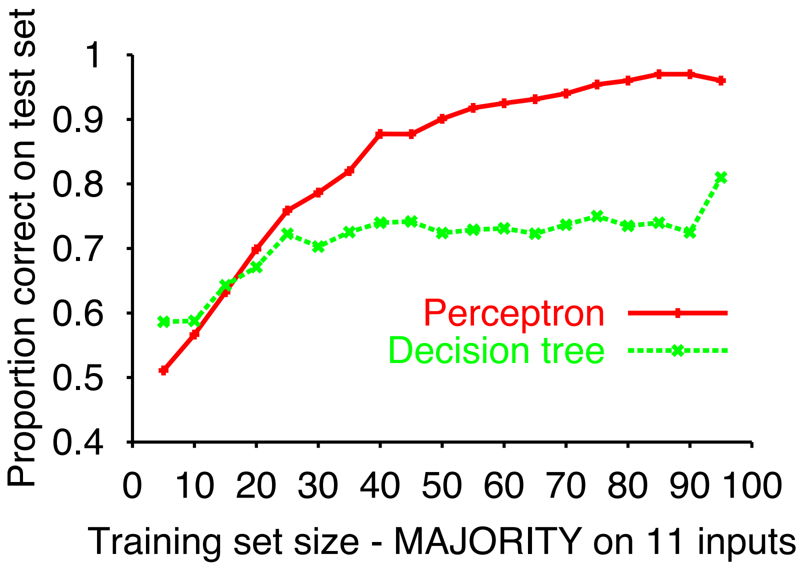


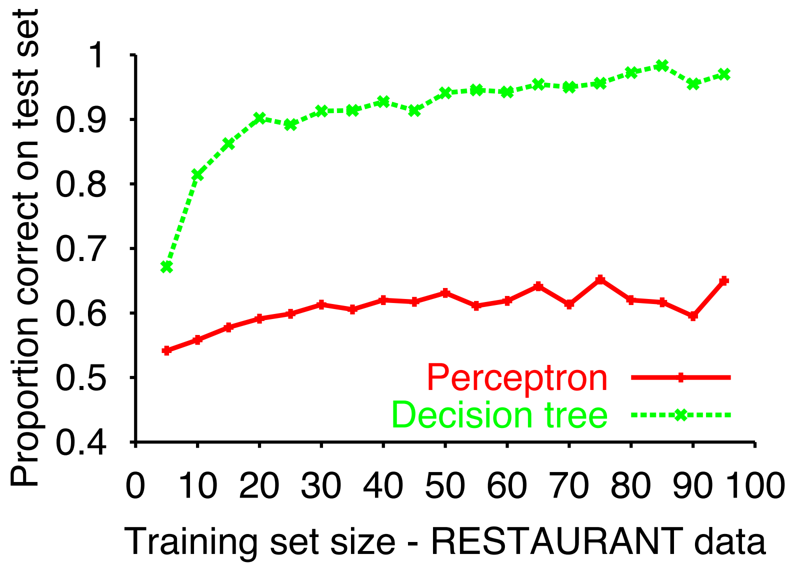
Gradient Descent 2



Perceptron learning contd.

Perceptron learning rule converges to a consistent function
for any linearly separable data set



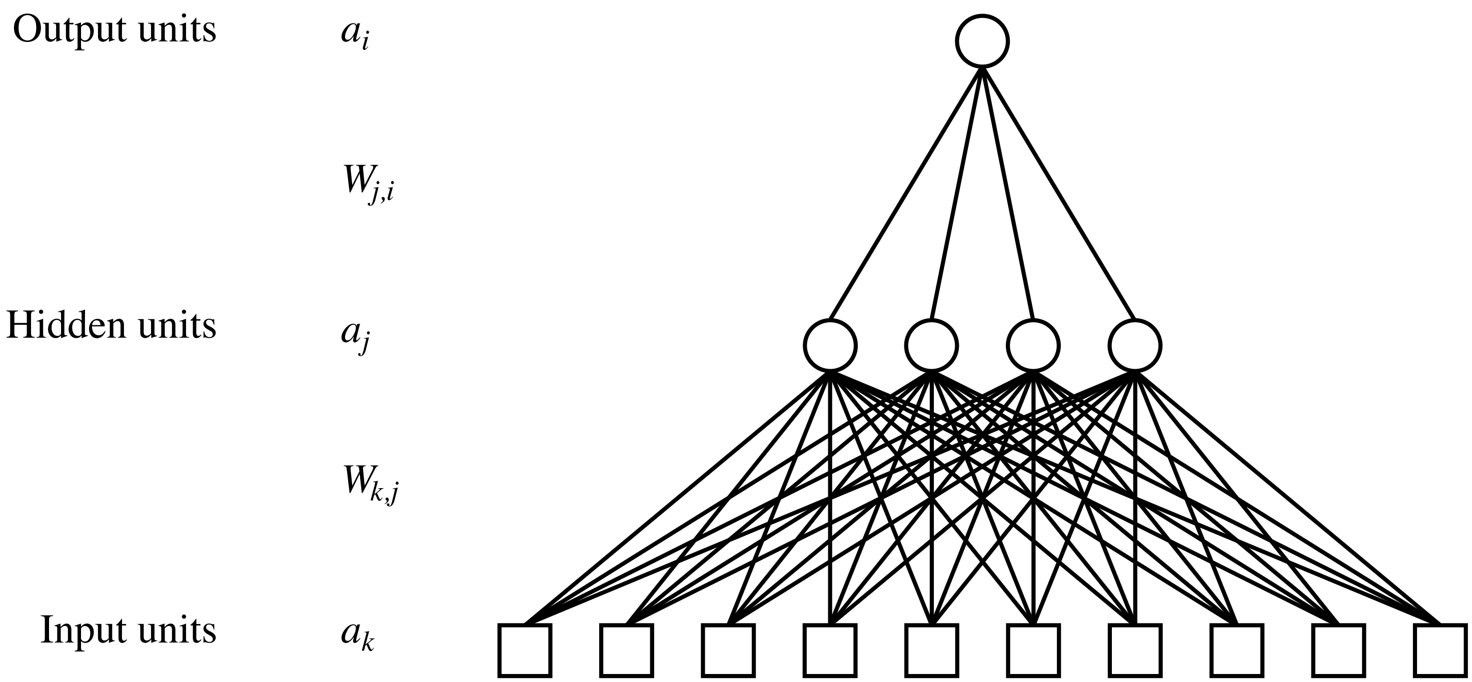


Perceptron learns majority function easily, DTL is hopeless

DTL learns restaurant function easily, perceptron cannot represent it

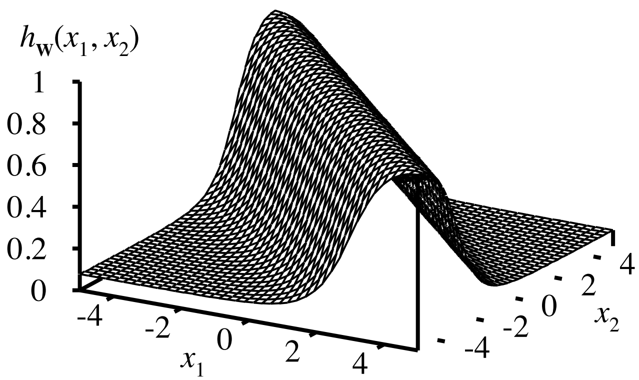
Multilayer perceptrons

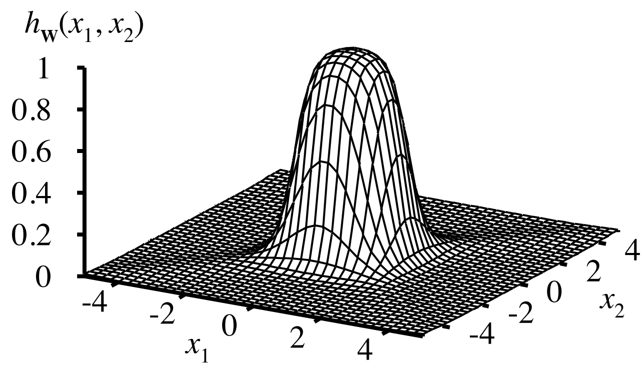
Layers are usually fully connected;
numbers of hidden units typically chosen by hand



Expressiveness of MLPs

All continuous functions w/ 2 layers, all functions w/ 3 layers





Combine two opposite-facing threshold functions to make a ridge

Combine two perpendicular ridges to make a bump

Add bumps of various sizes and locations to fit any surface

Proof requires exponentially many hidden units (cf DTL proof)

Back-propagation learning

Output layer: same as for single-layer perceptron,

$$W_{j,i} \leftarrow W_{j,i} + \alpha \times a_j \times \Delta_i$$

where $\Delta_i = Err_i \times g'(in_i)$

Hidden layer: **back-propagate** the error from the output layer:

$$\Delta_j = g'(in_j) \sum_i W_{j,i} \Delta_i \ .$$

Update rule for weights in hidden layer:

$$W_{k,j} \leftarrow W_{k,j} + \alpha \times a_k \times \Delta_j \ .$$

(Most neuroscientists deny that back-propagation occurs in the brain)

Back-propagation derivation

The squared error on a single example is defined as

$$E = \frac{1}{2}\sum_i (y_i - a_i)^2 \ ,$$

where the sum is over the nodes in the output layer.

$$\begin{aligned}
\frac{\partial E}{\partial W_{j,i}} &= -(y_i - a_i)\frac{\partial a_i}{\partial W_{j,i}} = -(y_i - a_i)\frac{\partial g(in_i)}{\partial W_{j,i}} \\
&= -(y_i - a_i)g'(in_i)\frac{\partial in_i}{\partial W_{j,i}} = -(y_i - a_i)g'(in_i)\frac{\partial}{\partial W_{j,i}}\left(\sum_j W_{j,i}a_j\right) \\
&= -(y_i - a_i)g'(in_i)a_j = -a_j\Delta_i
\end{aligned}$$

Same idea: quadratic error surface and find weights where error is (close to) zero

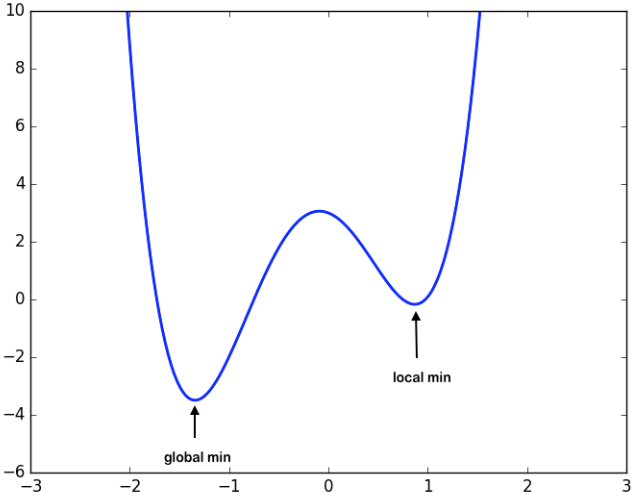
The hidden layers make this difficult: our input data doesn't tell us what the values at internal nodes should be, only the values at the output layer

Back-propagation derivation contd.

$$
\begin{aligned}
\frac{\partial E}{\partial W_{k,j}} &= -\sum_i (y_i - a_i)\frac{\partial a_i}{\partial W_{k,j}} = -\sum_i (y_i - a_i)\frac{\partial g(in_i)}{\partial W_{k,j}} \\
&= -\sum_i (y_i - a_i) g'(in_i)\frac{\partial in_i}{\partial W_{k,j}} = -\sum_i \Delta_i \frac{\partial}{\partial W_{k,j}}\left(\sum_j W_{j,i} a_j\right) \\
&= -\sum_i \Delta_i W_{j,i}\frac{\partial a_j}{\partial W_{k,j}} = -\sum_i \Delta_i W_{j,i}\frac{\partial g(in_j)}{\partial W_{k,j}} \\
&= -\sum_i \Delta_i W_{j,i} g'(in_j)\frac{\partial in_j}{\partial W_{k,j}} \\
&= -\sum_i \Delta_i W_{j,i} g'(in_j)\frac{\partial}{\partial W_{k,j}}\left(\sum_k W_{k,j} a_k\right) \\
&= -\sum_i \Delta_i W_{j,i} g'(in_j) a_k = -a_k \Delta_j
\end{aligned}
$$

In English, hidden node j is responsible for some fraction of the error at each output node it is connected to. So we divide total error by the strengths of connections and propagate our search for zero derivatives back through the layers

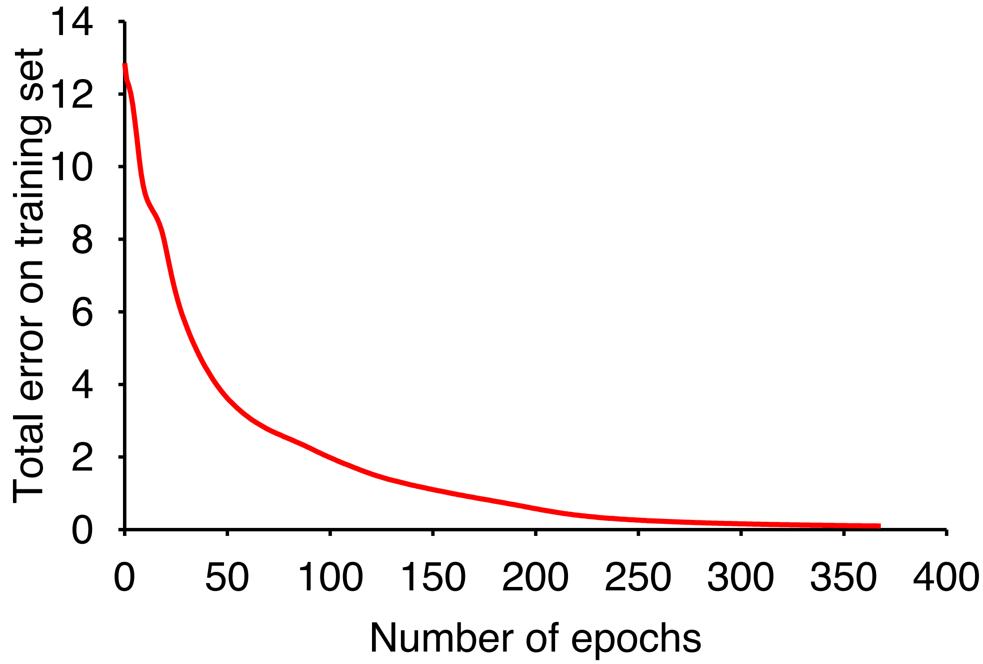
Local minimum



Back-propagation learning contd.

At each epoch, sum gradient updates for all examples and apply

Training curve for 100 restaurant examples: finds exact fit

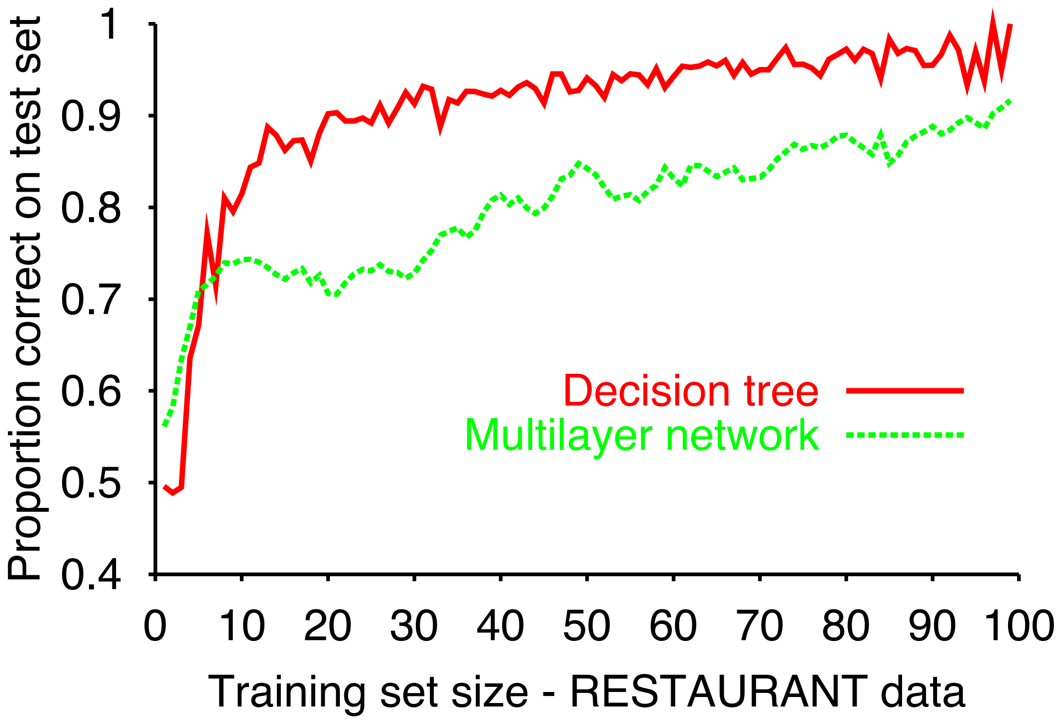


Typical problems: slow convergence, local minima

The error surface is quadratic near a minimum,
but there can be many minima

Back-propagation learning contd.

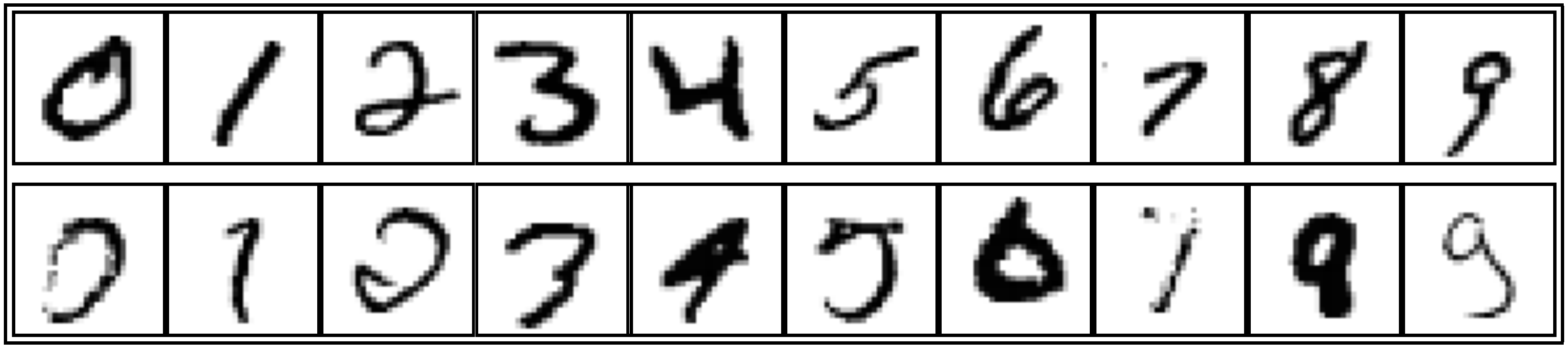
Learning curve for MLP with 4 hidden units:



MLPs are quite good for complex pattern recognition tasks,
but resulting hypotheses cannot be understood easily

White Box AI and Black Box AI:
I can explain DT predictions to a 10 year old child -
I typically can't understand the MLP predictions myself

Handwritten digit recognition



3-nearest-neighbor = 2.4% error
400–300–10 unit MLP = 1.6% error
LeNet: 768–192–30–10 unit MLP = 0.9% error

Current best (kernel machines, vision algorithms) ≈ 0.6% error

Current best changes all the time. The MLP, SVM, RF, etc. communities are in competition

Summary

Most brains have lots of neurons; each neuron $\approx$ linear–threshold unit (?)

Perceptrons (one-layer networks) insufficiently expressive

Multi-layer networks are sufficiently expressive; can be trained by gradient descent, i.e., error back-propagation

Many applications: speech, driving, handwriting, fraud detection, etc.

Engineering, cognitive modelling, and neural system modelling
subfields have largely diverged