Formula for logical agents part

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1 Entailment

Entailment is a relationship between sentences.
Knowledge base KB entails sentence α
iff(if and only if)
α is true in all worlds where KB is true
$KB \models \alpha$

i.e., when m is a model of a sentence α if α is true in m,
$M(\alpha)$ is the set of all models of α
Then $M(KB) \subseteq M(\alpha)$

2 Inference

$KB \vdash_i \alpha$ means α can be derived from KB by procedure i

i is sound if
whenever $KB \vdash_i \alpha$, it is also true that $KB \models \alpha$

i is complete if
whenever $KB \models \alpha$, it is also true that $KB \vdash_i \alpha$

3 Semantics

$\neg S$ is true **iff** S is false
$S_1 \wedge S_2$ is true **iff** S_1 is true **and** S_2 is true
$S_1 \vee S_2$ is true **iff** S_1 is true **or** S_2 is true
$S_1 \Rightarrow S_2$ is true **iff** S_1 is false **or** S_2 is true (S_1 belongs to S_2 based on false belongs to true)
$S_1 \Leftrightarrow S_2$ is true **iff** $S_1 \Rightarrow S_2$ is true **and** $S_2 \Rightarrow S_1$ is true

4 Logical equivalence

$\alpha \equiv \beta$ means α logical equals β.
$\alpha \equiv \beta$ **iff** $\alpha \models \beta$ and $\beta \models \alpha$

rules:

- commutativity:

$$(\alpha \wedge \beta) \equiv (\beta \wedge \alpha)$$
$$(\alpha \vee \beta) \equiv (\beta \vee \alpha)$$

- associativity:

$$((\alpha \land \beta) \land \gamma) \equiv (\alpha \land (\beta \land \gamma))$$
$$((\alpha \lor \beta) \lor \gamma) \equiv (\alpha \lor (\beta \lor \gamma))$$

- double-negation elimination:

$$\neg(\neg\alpha) \equiv \alpha$$

- contraposition:

$$(\alpha \Rightarrow \beta) \equiv (\neg\beta \Rightarrow \neg\alpha)$$

- implication elimination:

$$(\alpha \Rightarrow \beta) \equiv (\neg\alpha \lor \beta)$$

- biconditional elimination:

$$(\alpha \Leftrightarrow \beta) \equiv ((\alpha \Rightarrow \beta) \land (\beta \Rightarrow \alpha))$$

- De Morgan:

$$\neg(\alpha \land \beta) \equiv (\neg\alpha \lor \neg\beta)$$
$$\neg(\alpha \lor \beta) \equiv (\neg\alpha \land \neg\beta)$$

- distributivity:

$$(\alpha \land (\beta \lor \gamma)) \equiv ((\alpha \land \beta) \lor (\alpha \land \gamma))$$
$$(\alpha \lor (\beta \land \gamma)) \equiv ((\alpha \lor \beta) \land (\alpha \lor \gamma))$$

5 First-order logic

5.1 Basic elements

- Constants: e.g., KingJohn, 2, UCB, ...
- Predicates: e.g., Brother, $>$, ...
  A predicate can be regarded as a function which only returns true or false. Hence a relationship was defined.
- Functions: e.g., Sqrt, LeftLegOf, ...
- Variables: e.g., x, y, a, b ...
- Connectives: $\land$, $\lor$, $\neg$, $\Rightarrow$, $\Leftrightarrow$, ...
- Equality: $=$
- Quantifiers: $\forall$, $\exists$, ...

5.2 Atomic sentences and terms

e.g.s for atomic sentences,
$predicate(term_1, ..., term_n)$
$term_1 = term_2$

e.g.s for term,
$function(term_1, ..., term_n)$

5.3 Quantifiers

5.3.1 Universal quantification

$\forall < variables > \quad < sentence >$
Typically, $\Rightarrow$ is the main connective with $\forall$

Wrong example: $\forall x \quad At(x, Berkeley) \wedge Smart(x)$
It should be : $\forall x \quad At(x, Berkeley) \Rightarrow Smart(x)$

5.3.2 Existential quantification

$\exists < variables > \quad < sentence >$
Typically, $\wedge$ is the main connective with $\exists$

Wrong example: $\exists x \quad Crown(x) \Rightarrow OnHead(x, John)$
It should be : $\exists x \quad Crown(x) \wedge OnHead(x, John)$

5.3.3 Properties of quantifiers

$\forall x \forall y \equiv \forall y \forall x$
$\exists x \exists y \equiv \exists y \exists x$
$\exists x \forall y \not\equiv \forall y \exists x$

$\forall < variables > \quad < sentence > \equiv \neg \exists < variables > \quad \neg < sentence >$
$\exists < variables > \quad < sentence > \equiv \neg \forall < variables > \quad \neg < sentence >$