

## Logical Agents – Lecture 7 (Oggie)

Please read this along with the lecture slides (he actually follows the slides here)

**Deductive reasoning** – use the example of Wumpus World to illustrate some points

### Knowledge Base

- Set of sentences in a formal language – domain specific content

### Inference Engine

- Domain-independent
- How to combine different sentences to arrive at new sentences

Thus, gather information → Update the representation of the world → Deduce a new conclusion

### Modern example of the Wumpus World PEAS description

- 1972 (Oggie was being sarcastic...)
- There is a grid of cells and you can move from one cell to another
- (Read slide 5 for the rules of the Wumpus game, which is rather simple)
- The goal is to pick up as much gold

So how do you reason in this kind of a maze?

Oggie: Is this intelligence? No the Wumpus is just following the rules dictated by codes

The purpose of the Wumpus example is to make you think about the design (of what Oggie?)

- Observable? No, only local perception
- Deterministic? Yes, outcomes are exactly specified
- Episodic? No, sequential at the level of actions
- Static? Yes, Wumpus and Pits do not move
- Discrete? Yes, you have only set number of squares
- Single-agent? Yes

### Exploring a Wumpus world

- From the current state of knowledge, you deduce that the two movements are okay (Please refer to page 14 of the slide)

### Slides 14-16

- How you can deduce where Wumpus is (Perhaps no need to read this, it's rather simple)
- You can also deduce where there is a pit

- Chances are uniformly distributed
  - o i.e. equal chance in each square

## Logic in General

- See page 22 for the definition

### Logic in general

**Logics** are formal languages for representing information such that conclusions can be drawn

**Syntax** defines the sentences in the language

**Semantics** define the “meaning” of sentences; i.e., define **truth** of a sentence in a world

E.g., the language of arithmetic

$x + 2 \geq y$  is a sentence;  $x^2 + y >$  is not a sentence

$x + 2 \geq y$  is true iff the number  $x + 2$  is no less than the number  $y$

$x + 2 \geq y$  is true in a world where  $x = 7, y = 1$

$x + 2 \geq y$  is false in a world where  $x = 0, y = 6$

### Entailment

**Entailment** means that one thing **follows from** another:

$KB \models \alpha$                       Sequence of percepts entailed Wumpus in [1,3]

Knowledge base  $KB$  entails sentence  $\alpha$   
if and only if

$\alpha$  is true in all worlds where  $KB$  is true

E.g., the KB containing “the Giants won” and “the Reds won” entails “Either the Giants won or the Reds won”

E.g.,  $x + y = 4$  entails  $4 = x + y$

Entailment is a relationship between sentences (i.e., **syntax**) that is based on **semantics**

Note: brains process **syntax** (of some sort)

= is syntax. The meaning “equals” is semantics. Could mean string concatenation in javascript, etc.

Oggie was reluctant to explain this slide so most likely not important but here we go:

## Models

Logicians typically think in terms of **models**, which are formally structured worlds with respect to which truth can be evaluated

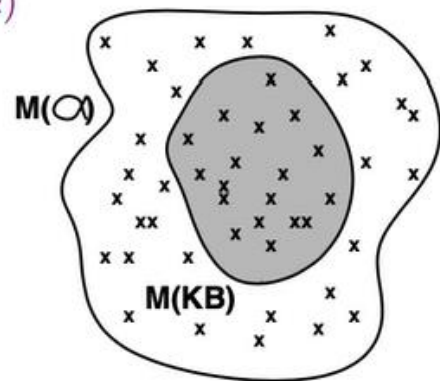
We say  $m$  is a **model** of a sentence  $\alpha$  if  $\alpha$  is true in  $m$

$M(\alpha)$  is the set of all models of  $\alpha$

Then  $KB \models \alpha$  if and only if  $M(KB) \subseteq M(\alpha)$

E.g.  $KB = \text{Giants won and Reds won}$   
 $\alpha = \text{Giants won}$

I



### Entailment in the Wumpus world

Page 27

KB = rule + observations

- Then you can expand this knowledge base with some inference rule or with new perceptions

So what is inference?

Soundness is important

- When we use inference to deduce from the knowledge base
  - o It is true that KB entails A

Completeness

- Every time that something is entailed by the knowledge base,
  - o It is true that A can be inferred from KB
  - o So if the rules are entirely sound, but they can't derive everything, it's incomplete

### Inference

$KB \vdash_i \alpha$  = sentence  $\alpha$  can be derived from  $KB$  by procedure  $i$

Consequences of  $KB$  are a haystack;  $\alpha$  is a needle.

Entailment = needle in haystack; inference = finding it

**Soundness:**  $i$  is sound if

whenever  $KB \vdash_i \alpha$ , it is also true that  $KB \models \alpha$

**Completeness:**  $i$  is complete if

whenever  $KB \models \alpha$ , it is also true that  $KB \vdash_i \alpha$

Preview: we will define a logic (first-order logic) which is expressive enough to say almost anything of interest, and for which there exists a sound and complete inference procedure.

That is, the procedure will answer any question whose answer follows from what is known by the  $KB$ .

Conjunction (and), Disjunction (or), Biconditional (implies in both directions)

### Propositional logic: Syntax

Propositional logic is the simplest logic—illustrates basic ideas

The proposition symbols  $P_1, P_2$  etc are sentences

If  $S$  is a sentence,  $\neg S$  is a sentence (**negation**)

If  $S_1$  and  $S_2$  are sentences,  $S_1 \wedge S_2$  is a sentence (**conjunction**)

If  $S_1$  and  $S_2$  are sentences,  $S_1 \vee S_2$  is a sentence (**disjunction**)

If  $S_1$  and  $S_2$  are sentences,  $S_1 \Rightarrow S_2$  is a sentence (**implication**)

If  $S_1$  and  $S_2$  are sentences,  $S_1 \Leftrightarrow S_2$  is a sentence (**biconditional**)

Page 33 and 34 are just an elaboration of the previous slide on the definitions.

It's not complicated.

Page 36 is an elaboration of these rules in the Wumpus world sentences. It's quite straightforward and Oggie didn't elaborate anything in addition to the contents of the slide.

Page 37 – Truths tables for inference based on the previous slides

Oggie makes a good point that the real world is extremely complex

- Very seldom can you represent it in truth tables

### Truth tables for inference

$B_{1,1}$	$B_{2,1}$	$P_{1,1}$	$P_{1,2}$	$P_{2,1}$	$P_{2,2}$	$P_{3,1}$	$R_1$	$R_2$	$R_3$	$R_4$	$R_5$	$KB$
false	false	false	false	false	false	false	true	true	true	true	false	false
false	false	false	false	false	false	true	true	true	false	true	false	false
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
false	true	false	false	false	false	false	true	true	false	true	true	false
false	true	false	false	false	false	true	true	true	true	true	true	true
false	true	false	false	false	true	false	true	true	true	true	true	true
false	true	false	false	false	true	true	true	true	true	true	true	true
false	true	false	false	true	false	false	true	false	false	true	true	false
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
true	true	true	true	true	true	true	false	true	true	false	true	false

Enumerate rows (different assignments to symbols),  
if  $KB$  is true in row, check that  $\alpha$  is too

7 Boolean variables so  $2^7$  rows in the truth table  
In three of these the  $KB$  is true  
In those three, there is no pit in [1,2]  
So we have derived no pit in [1,2] from the  $KB$  by  
exhaustive case analysis

Chapter 7 37

Oggie skipped page 38 entirely

Page 39

- According to Oggie, the only non-intuitive ones are the De Morgan\*
- Which I disagree, I think they are quite intuitive

Page 40

- Oggie basically just read through these
- They are not unimportant though

Reductio ad Absurdum

- Proof by contradiction

Mathematicians don't like proof by contradiction because it doesn't really give you much insight as to why something is true.

Page 41 – Proof Methods (Oggie sort of skipped this slide, so likely not important)

Modus Ponens is important

- If B implies Q
- If B is true, then we can imply Q
- Sounds very trivial but important

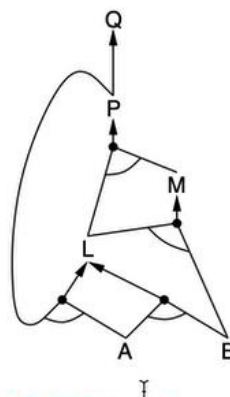
Forward Chaining

- Start with something you know and go forward to derive new things

### Forward chaining

Idea: fire any rule whose premises are satisfied in the *KB*,  
add its conclusion to the *KB*, until query is found

$P \Rightarrow Q$   
 $L \wedge M \Rightarrow P$   
 $B \wedge L \Rightarrow M$   
 $A \wedge P \Rightarrow L$   
 $A \wedge B \Rightarrow L$   
 $A$   
 $B$



A and B are facts

AND-OR graph  
arcs denote a conjunction  
no arc denotes disjunction

Forward chaining logic explained from slides p 43 onwards

Need graphical explanation – Oggie didn't really add anything, the slides are enough

### Proof of completeness

FC derives every atomic sentence that is entailed by  $KB$  atomic means one variable or its negation

1. FC reaches a **fixed point** where no new atomic sentences are derived
2. Consider the final state as a model  $m$ , assigning true/false to symbols
3. Every clause in the original  $KB$  is true in  $m$   
**Proof:** Suppose a clause  $a_1 \wedge \dots \wedge a_k \Rightarrow b$  is false in  $m$   
Then  $a_1 \wedge \dots \wedge a_k$  is true in  $m$  and  $b$  is false in  $m$   
Therefore the algorithm has not reached a fixed point!
4. Hence  $m$  is a model of  $KB$   $\quad \text{I}$
5. If  $KB \models q$ ,  $q$  is true in **every** model of  $KB$ , including  $m$

**General idea:** construct any model of  $KB$  by sound inference, check  $\alpha$

**FC is data-driven reasoning: start with what we know and build forwards**

**Exercises 7.4, 7.5, 7.6**

Intelligent modelling involves thinking about a theorem first.

And work backwards perhaps...

Rather than starting with axioms

Oggie says