CS 332: Theory of Computation, Fall 2024

SAMPLE EXAM 1

EXAM 1 LOGISTICS REMINDER: The exam will take place on **Thursday, October 24th** during regular lecture time **2PM-3:15PM**. There are two different locations. The first location is in the regular lecture hall, the second one is in CGS 421. If you had been assigned to the second location, then you would have received an email from the TA telling you so on Monday, 22th. If you received no such email, then it means you will take the exam at the first location. The exam is closed-book, closed-notes, and electronic devices are strictly prohibited. You are allowed to use one cheatsheet in standard notebook size, and you can write absolutely anything on both sides of the sheet.

PROBLEMS: Try to keep your answers to the 3 problems short and to the point. Follow the specific instructions in each problem carefully. The list of problems is below, **they are not meant to hint out anything that will appear in the actual exam, but rather serve as some extra practice**.

Problem 1. Consider the following 7-part program of the following Turing Machine M:

1. Input alphabet: $\Sigma = \{1, 2\}$
2. Tape alphabet: $\Gamma = \{1, 2, B\}$
3. Set of state $Q = \{q_0, q_1, q_2, q_4, q_5, q_a, q_r\}$
4. q_0 is the starting state
5. q_a is the accepting state
6. q_r is the rejecting state
7. The transition function: $\delta : Q \times \Gamma \to Q \times \Gamma \times \{L, R\}$ which is defined as follows
   - for state q_0: $\delta(q_0, 1) = (q_1, B, R)$, $\delta(q_0, 2) = (q_2, B, R)$, $\delta(q_0, B) = (q_r, B, R)$
   - for state q_1: $\delta(q_1, 1) = (q_1, 1, R)$, $\delta(q_1, 2) = (q_1, 2, R)$, $\delta(q_1, B) = (q_4, B, L)$
   - for state q_2: $\delta(q_2, 1) = (q_2, 1, R)$, $\delta(q_2, 2) = (q_2, 2, R)$, $\delta(q_2, B) = (q_5, B, L)$
   - for state q_4: $\delta(q_4, 1) = (q_a, B, R)$, $\delta(q_4, 2) = (q_r, B, R)$, $\delta(q_4, B) = (q_a, B, R)$
   - for state q_5: $\delta(q_5, 1) = (q_r, B, R)$, $\delta(q_5, 2) = (q_5, 2, R)$, $\delta(q_5, B) = (q_5, B, R)$

(The problem continues on the next page)

Answer both parts (i) and (ii) below

(i) What does M do on input 212? And what does M do on input 1211? You can draw a picture to trace the computation.

(ii) Define $H(M)$ to be the set of input which M **halts**. What is $H(M)$ for the specification of M above? Just write down the description for $H(M)$, **no justification needed**.

Problem 2. Answer both parts (i) and (ii) below

(i) Briefly explain what it means to say that the accepting problem is recognizable, but undecidable. **Note:** you need not prove anything here, just state clearly what recognizable and what undecidable mean in this case

(ii) Give an example of a specific language D which is recognizable and whose complement is also recognizable. **Note:** you need not prove anything here, but be precise in defining the set of strings D. Use set theory notations is fine.

Problem 3. In a homework problem, we proved that if a language V is enumerable in increasing order, then it is decidable. In this problem, we connect enumerability with recognizability. **Prove:** if a language L is enumerable, then it is recognizable.

Note: in this case, there is no increasing order. So, enumerable here just means there is a TM that enumerates it, and this enumeration may output elements of L in any order, not necessarily increasing.