CS 332: Theory of Computation, Fall 2024

Homework 5

Due: 11:00PM, Tuesday, November 26, 2024 on Gradescope

COLLABORATION POLICY REMINDER: You may **verbally** collaborate on the required problems (e.g. discuss ideas and approach), however, **you must write your solutions independently**. If you choose to collaborate, you are allowed to discuss it with a small group of your classmates (say, no more than 3), and you must acknowledge all of your collaborators. Before collaborating on a problem, it is highly encouraged that you should think about it yourself for a some time or better yet, try solving it. **Any violation of this policy will be dealt with according to University regulations.**

READING ASSIGNMENT: Section 5.3 of our textbook, about mapping reducibility, pages 234-238. And Section 7.4, pages 299-304 and 312-314.

PROBLEMS: Do each of the 5 problems below, each counts 8 points. Try to keep your answers to the problems short and to the point. **You need to give a formal proof only for problems 3, 4, and 5.** The list of problems is below.

Problem 1. (Decidability and Polynomial Time Computation) Give an example for each of (i) and (ii) below. **No explanation needed**

(i) A subset of the 5-Clique problem which is infinite and computable in polynomial time.

(ii) A subset of the Halting Problem which is infinite and decidable.

Problem 2. (Big-O Notation) Determine whether each of (a), (b), (c), and (d) below is **True** or **False**. **No explanation needed**

(a) $2n^2 = O(n^3)$

(b) $4n + 7 = O(n)$

(c) $5(3^n) = 2^{O(n)}$

(d) $(\log n)^2 = O(n \log n)$

Problem 3. (P vs NP) Show that if $\mathbf{P} = \mathbf{NP}$, then every language $A \in \mathbf{P}$, except $A = \emptyset$ and $A = \Sigma^*$, is $\mathbf{NP}$-complete.

Problem 4. (P vs NP) **PATH** is defined to be the set of $\langle G, s, t\rangle$ such that G is a directed graph, s and t are nodes in G, and that G has a directed path from s to t.

Explain the reason why many people believe **PATH** is not **NP**-complete. Show that proving **PATH** is not **NP**-complete would imply $\mathbf{P} \neq \mathbf{NP}$.

Problem 5. (NP-completeness) Let **d-SAT** be the set of all Boolean formulas with at least 2 different satisfying assignments. Formally,

$$\textbf{d-SAT} = \{\langle\varphi\rangle \mid \varphi \text{ is a Boolean formula with at least 2 different satisfying assignments}\}$$

Show that **d-SAT** is **NP**-complete.