Lecture 20, Oct. 27

20.1 Definition. $L = \lim_{x\to\infty} f(x)$ if for every $\epsilon > 0$, there exists $M > 0$ such that $x \geq M$, then

$$|f(x) - L| < \epsilon.$$

20.2 Example.

1. If $p > 0$, we have $\lim_{x\to\infty} \frac{1}{x^p} = 0$
2. $\lim_{x\to\infty} \frac{lnx}{x} = 0$

Variants

1. If $p > 0$, we have $\lim_{x\to 0} \frac{lnx}{x^p} = 0$
2. For all p, $\lim_{x\to 0} \frac{(lnx)^p}{x} = 0$
3. $\lim_{x\to\infty} \frac{x}{e^x} = 0$

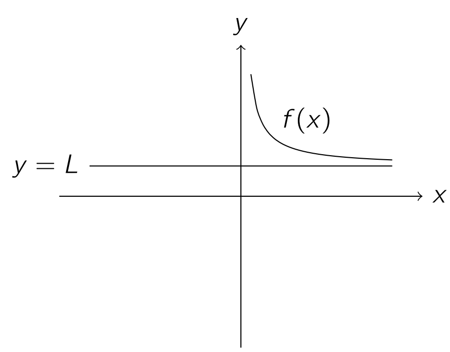
20.3 Definition. We say that L is the limit of $f(x)$ as x approaches $-\infty$ if for every $\epsilon > 0$ there exists $M > 0$ such that if $x < -M$, then $|f(x) - L| < \epsilon$. We write

$$\lim_{x\to-\infty} f(x) = L.$$

20.4 Example. By Squeeze Theorem, we have

$$\lim_{x\to\infty} \frac{\sin x}{x} = 0$$

20.5 Definition (Asymptote)**.** Assume $\lim_{x\to\pm\infty} f(x) = L$, then the line $y = L$ is called a horizontal asymptote of f(x).



Infinite Limits

20.6 Definition. We say that $f(x)$ approaches ∞ at $x = a$ if for every $M > 0$ there exists $\delta > 0$ such that if $|x - a| < \delta$, then $f(x) > M$. We write

$$\lim_{x\to a} f(x) = \infty$$

20.7 Definition (Vertical Asymptote)**.** If $\lim_{x\to a^+} f(x) = \pm\infty$ or $\lim_{x\to a^-} f(x) = \pm\infty$, then $x = a$ is called a vertical asymptote for $f(x)$