Lecture 2, Sept. 14

New Section 12:30-1:20 CPH 3604
Tutorial Moved to DC 1350 Th 4:30-5:20

Greek Letters

- α - alpha
- β - beta
- δ - delta
- ϵ - epsilon
- γ - gamma

Properties of $\mathbb{N}$

$$\mathbb{N} = \{1, 2, 3, 4, \dots\}$$

Mathematical Induction

2.1 Axiom. Assume $S \in \mathbb{N}$ such that

1. $1 \in S$
2. If $k \in S$, then $k + 1 \in S$

Then $S = \mathbb{N}$

Proof by Induction

1. Establish for each $n \in \mathbb{N}$ a statement $P(n)$ to be proved.
   Example. Let $P(n)$ be the statement that $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$, show this is true for all $n \in \mathbb{N}$.
   Let $S = \{n \in \mathbb{N} \mid P(n) \text{ is true}\}$, show $S = \mathbb{N}$
2. Base Case: show that $P(1)$ is true. ie): $1 \in S$
3. Inductive Step: Assume that $P(k)$ is true for some k (Inductive Hypothesis). Use this to show that $P(k+1)$ is also true. ie): $k \in S \Rightarrow k + 1 \in S$

By the Principle of Mathematical Induction, $S = \mathbb{N}$

2.2 Example. Prove that $\displaystyle\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

Proof. Step.1 Let $P(n)$ be the statement that $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

Step.2 Let $n = 1$ then $P(1) = 1 = \frac{1(1+1}{2}$. Hence $P(1)$ is true.

Step.3 Assume that $P(k)$ is rue for some k

$$P(k)\frac{k(k+1}{2}$$

Step.4

$$\begin{aligned}\sum_{i=1}^{k+1} i &= \sum_{i=1}^{k} i + (k+1) \\ &= \frac{k(k+1)}{2} + (k+1) \\ &= \frac{(k+1)(k+2)}{2}\end{aligned}$$

Hence $P(k+1)$ is true

Step.5 By Principle of Mathematical Induction, $P(n)$ is true for all $n \in \mathbb{N}$ □

2.3 Example. Prove that $3^n + 4^n$ is divisible by 7 for every odd n

Proof. Let $P(k)$ be the statement that $3^{2k-1} + 4^{2k-1}$ is divisible by 7.

Base case: $k = 1$, $P(1)$ is true.

Inductive Step: Assume $P(j)$ is true.

$$\begin{aligned}&3^{2(j+1)-1} + 4^{2(j+1)-1} \\ =&9(3^{2j-1}) + 16(4^{2j-1}) \\ =&9(3^{2j-1} + 4^{2j-1}) + 7(4^{2j-1})\end{aligned}$$

Hence $P(j+1)$ is true.

By Principle of Mathematical Induction, $P(k)$ is true for all n

□