Lecture 1, Sept. 9

Course Orientation and Organization

About the Professor

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Recommended Textbook

- An Introduction to Mathematical Thinking by Will J. Gilbert, Scott A. Vanstone
- Lecture Notes: Integers, Polynomials and Finite Fields by K. Davidson

Some Paradoxes

There are lots of paradoxes in English, such as "This statement is false".

There are also some paradoxes in Mathematical world.

Russell's Paradox

Let X be the set of all sets. Let $S = \{A \in X | A \notin A\}$. Is $S \in S$?

Some Question

To avoid such paradoxes, some question was raised.

1. What is an allowable mathematical object?
2. What is an allowable mathematical statement?
3. What is an allowable mathematical proof?

Mathematical Object

Essentially all mathematical objects are (mathematical) sets. In math, a set is a certain specific kind of collection whose elements are sets. Not all collection of sets are called sets. For a collection to be a set, it must be constructable using specific rules. These rules are called the ZFC axioms of set theory (or the Zermelo–Fraenkel axioms along with the Axiom of Choice)

These axioms include (imply) the following:

- Empty Set: there exist a set, denoted by $\emptyset$, with no elements.

- Equality: two sets are equal when they have the same elements. $A = B$ when for every set x, $x \in A \iff x \in B$
- Pair Axiom: if A and B are sets then so is $\{A, B\}$
- Union Axiom: if S is a set of sets then $\cup_S = \{x | x \in A \text{ for some } A \in S\}$. If A and B are sets, then so is $\{A, B\}$ hence so is $A \cup B = \cup_{\{A,B\}}$