Lecture 2, Sept. 12

Mathematics Contest

Big Contests

- Small C
- Big E/Special K
- Putnam
- Bernoulli Trials

Students Run

- Integration Bee
- over 6000

Others

- Recreational Problem Sessions

ZFC Axioms

- Empty Set: there exist a set, denoted by $\emptyset$, with no elements.
- Equality: two sets are equal when they have the same elements. $A = B$ when for every set x, $x \in A \iff x \in B$
- Pair Axiom: if A and B are sets then so is $\{A, B\}$. In particular, taking $A = B$ shows that $\{A\}$ is a set.
- Union Axiom: if S is a set of sets then $\cup_S = \bigcup_{A \in S} A = \{x \mid x \in A \text{ for some } A \in S\}$. If A and B are sets, then so is $\{A, B\}$ hence so is $A \cup B = \cup_{\{A,B\}}$
- Power Set Axiom: if A is a set, then so is its Power Set $P(A)$. $P(A) = \{X \mid X \subseteq A\}$. In particular, $\emptyset \subseteq X$, $X \subseteq X$
- Axiom of Infinity: if we define

$$\begin{aligned} 0 &= \emptyset \\ 1 &= \{0\} = \{\emptyset\} \\ 2 &= \{0, 1\} = \{\emptyset, \{\emptyset\}\} \\ 3 &= \{0, 1, 2\} = \{\emptyset, \{\emptyset, \{\emptyset\}\}\} \\ &\vdots \\ n + 1 &= n \cup \{n\} \end{aligned}$$

  Then $\mathbb{N} = \{0, 1, 2, 3, \dots\}$ is a set (called the set of natural numbers)