Tutorial 1, Sept. 15

Office Hours M 9:30-10:45
W 1:45-2:30
Th 2:30-4:00
F 9:30-10:45

Functions

1.1 Definition. A function is a rule that assigns to each element x in a set X a single value y in a set Y.

Notation.

$$f\colon X \to Y$$

where X is the domain of f and Y -is the codomain of f.

1.2 Example. $X = \mathbb{R}$ and $Y = \mathbb{R}$,

$$y = f(x) = x^2$$

1.3 Definition. Given $f\colon X \to Y$, the range of f is

$$ran(f) = \{y \in Y \mid y = f(x) \text{ for some } x \in X\}$$

We say that $f\colon X \to Y$ is onto if $ran(f) = codomain(f)$.

1.4 Example. $f(x) = x^3$, $f\colon \mathbb{R} \to \mathbb{R}$. f is onto.

$f\colon X \to Y$ is 1-1 if whenever $x_1, x_2 \in X$ with $x_1 \neq x_2$, then $f(x_1) \neq f(x_2)$

If f is 1-1 and onto, for each $y \in Y$ we define

$$g(y) = x \iff y = f(x)$$

This gives us a function $g\colon Y \to X$ which is called the inverse of f and denoted by f^{-1}

Properties Suppose $f\colon X \to Y$, $g\colon Y \to Z$, we get

$$g \circ f = g(f(x))$$

and

$$g \circ f\colon X \to Z$$

Suppose that $f\colon X \to Y$ is 1-1 and onto with inverse g,

$$g \circ f(x) = x$$