Lecture 1, Sept. 12

Mathematical tools $\LaTeX$

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Basics on Sets and Functions

1.1 Definition. Basic Sets

- $\mathbb{N}$ = Natural numbers = $\{1, 2, 3, \ldots\}$
- $\mathbb{Z}$ = Integers = $\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$
- $\mathbb{Q} = \{\frac{m}{n} \mid n \in \mathbb{N}, m \in \mathbb{Z}, gcd(n, |m|) = 1\}$
- $\mathbb{R}$ = Real Numbers
- $\mathbb{R} \backslash \mathbb{Q} = \{x \in \mathbb{R} \mid x \text{ is not in } \mathbb{Q}\}$

Notation.

$S \subset X \rightarrow S$ is a subset of X

If $S, T \subset X$ then $S \cup T = \{x \in X \mid x \in S \text{ or } x \in T\}$

If $S, T \subset X$ then $S \cap T = \{x \in X \mid x \in S \text{ and } x \in T\}$

Given a collection $\{A_\alpha\}_{\alpha \in I}$ of subsets of X

$$\bigcup_{\alpha \in I} A_\alpha = \{x \in X \mid x \in A_\alpha \text{ for some } \alpha \in I\}$$
$$\bigcap_{\alpha \in I} A_\alpha = \{x \in X \mid x \in A_\alpha \text{ for all } \alpha \in I\}$$

$\emptyset$ = empty set, $\emptyset \subset X$

What if $I = \emptyset$, what is $\bigcup\limits_{\alpha \in \emptyset} A_\alpha$

Define

$$\bigcup_{\alpha \in \emptyset} A_\alpha = \emptyset$$

Then

$$\bigcap_{\alpha \in \emptyset} A_\alpha = ??$$

Given $S, T \subset X$ we define

$$S \backslash T = \{x \in X \mid x \in S, x \text{ does not belong to } T\}$$

We denote $X \backslash T$ by T^c = compliment of T in $X = \{x \in X \mid x \text{ does not belong to } T\}$

Note.
$$(S \cup T)^c = S^c \cap T^c$$

De Morgans Law

1.2 Theorem.
$$(\bigcup_{\alpha \in I} A_\alpha)^c = \bigcap_{\alpha \in I} A_\alpha^c$$

Proof.
$$\begin{aligned} x \in (\bigcup_{\alpha \in I} A_\alpha)^c &\iff x \text{ is not a member of } \bigcup_{\alpha \in I} A_\alpha \\ &\iff x \text{ is not in } A_\alpha \quad \forall \alpha \in I \\ &\iff x \in A_\alpha^c \quad \forall \alpha \in I \\ &\iff x \in \bigcap_{\alpha \in I} A_\alpha^c \end{aligned}$$

□

Note. From this we really should have

$$\begin{aligned} \bigcap_{\alpha \in \emptyset} A_\alpha &= (\bigcup_{\alpha \in \emptyset} A_\alpha^c)^c \\ &= \emptyset^c \\ &= X \end{aligned}$$

Power Set

1.3 Definition. Given X, the Power Set of X is the set of all subset of X

Notation.

$$\begin{aligned} P(X) &= \text{ power set of } X \\ &= \{S \mid S \subset X\} \end{aligned}$$

Note. We can observe that
$$\emptyset, X \in P(X)$$