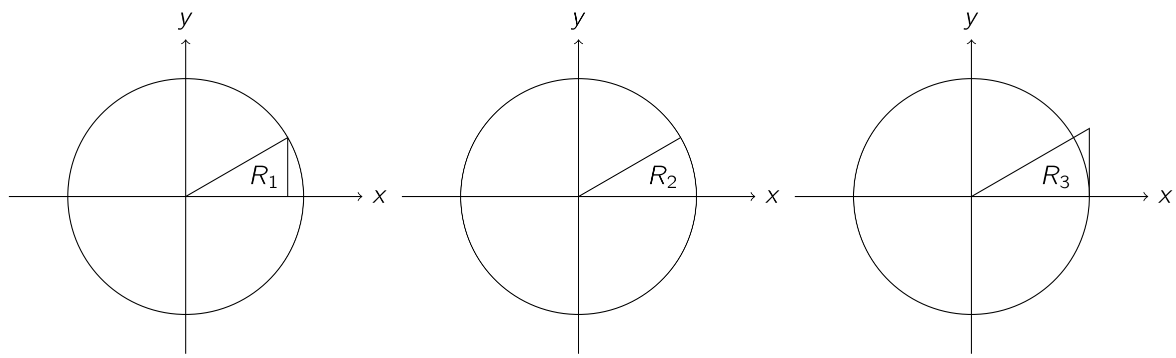
Lecture 19, Oct. 26

Written Assignment 3 Due Wed, Nov. 9

19.1 Theorem (Fundamental Trig Limit)**.**

$$\lim_{x\to 0}\frac{\sin x}{x}=1$$

Proof. Note that $f(x)$ is even. Hence we need only $\lim_{x\to 0^+}\frac{\sin x}{x}=1$



We have $R_1=\sin x\cos x/2$, $R_2=x/2$ and $R_3=\sin x/(2\cos x)$.

Since $R_1\le R_2\le R_3$, we get

$$\cos x\le\frac{x}{\sin x}\le\frac{1}{\cos x}.$$

Hence

$$\cos x\le\frac{\sin x}{x}\le\frac{1}{\cos x}.$$

By Squeeze Theorem, $\lim_{x\to 0^+}\frac{\sin x}{x}=1$.

□

19.2 Example. Find

$$\lim_{x\to 0}\frac{\sin 3x}{\sin 4x}$$

Solution.

$$\begin{aligned}\lim_{x\to 0}\frac{\sin 3x}{\sin 4x}&=\lim_{x\to 0}\frac{\sin 3x}{3}\cdot\lim_{x\to 0}\frac{4}{\sin 4x}\cdot\frac{3}{4}\\&=1\cdot 1\cdot\frac{3}{4}\\&=\frac{3}{4}\end{aligned}$$

19.3 Example. Find

$$\lim_{x\to 0}\frac{\tan x}{x}.$$

19.4 Example. Find

$$\lim_{x\to 0} \frac{\tan \pi x}{\sin 2x}.$$

Asymptotes and Limits at ∞

19.5 Definition. We say that L is the limit as x approaches infinity of $f(x)$ if for every $\epsilon > 0$, there exists $M > 0$ such that if $x \geq M$, then $|f(x) - L| < \epsilon$. We write

$$\lim_{x\to\infty} f(x) = L.$$

19.6 Example. If $f(x) = 1/x$, then $\lim_{x\to\infty} f(x) = 0$.

Note. Arithmetic Rules, Sequential Characterization and Squeeze Theorem carry through.

19.7 Theorem (Fundamental Log Limit)**.**

$$\lim_{x\to\infty} \frac{ln(x)}{x} = 0$$

Proof.

$$\frac{ln(x)}{x} = \frac{2ln(x^{1/2})}{x^{1/2}\cdot x^{1/2}} = \frac{2ln(x^{1/2})}{x^{1/2}} \cdot \frac{1}{x^{1/2}} < \frac{2}{x^{1/2}}$$

By squeeze theorem, $\lim_{x\to\infty} \frac{ln(x)}{x} = 0$ ∎

19.8 Example. Find

$$\lim_{x\to\infty} \frac{ln(x)}{x^{1/100}}$$