Lecture 30, Nov. 16

Maxima, Minima and Critical Points

30.1 Definition (Global Maximum and Minimum)**.** Let f be defined on an interval I. We say that, $d \in I$ is a global maximum for f on I if

$$f(x) \leq f(d) \text{ for all } x \in I$$

and $f(d)$ is the global maximum value.

Similarly we define the global minimum and global minimum value.

30.2 Example. $f(x) = x$ on $(0, 1)$ has no global maximum or minimum on $(0, 1)$.

30.3 Definition (Local Maximum and Minimum)**.** We say that c ks a local maximum for $f(x)$ if there exists an open interval (a, b) containing c with

$$f(x) \leq f(c) \text{ for all } x \in (a, b)$$

Similarly we define the local minimum.

30.4 Theorem (The Might-be-on-the-exam Theorem)**.**

1. *Assume that $f(x)$ has a local maximum at $x = c$. If $f(x)$ is differentiable at $x = c$ then $f'(c) = 0$*
2. *Assume that $f(x)$ has a local minimum at $x = c$. If $f(x)$ is differentiable at $x = c$ then $f'(c) = 0$. (Might be on the exam)*

Proof.

1. Since $x = c$ is a local maximum for $f(x)$ there exists $\delta > 0$ such that if $c - \delta < x < c + \delta$, then $f(x) \leq f(c)$. Then if $c - \delta < x < c$,
$$\frac{f(x) - f(c)}{x - c} \geq 0$$
and if $c < x < c + \delta$,
$$\frac{f(x) - f(c)}{x - c} \leq 0$$
Thus
$$\frac{f(x) - f(c)}{x - c} = 0$$
Hence $f'(c) = 0$. □

30.5 Definition (Critical Point)**.** Assume that f is defined on an open interval I. We call $c \in I$ a critical point for f if either

1. $f'(c) = 0$
2. f is not differentiable at $x = c$.

Note. Given f continuous on $[a.b]$, then the global max (min) will be at

1. either $x = a$ or $x = b$ or
2. a critical point in (a, b).