Lecture 3, Sept. 16

Well Ordering Property

3.1 Theorem. *If $S \in \mathbb{N}$ and $S \neq \emptyset$, then S contains a least element.*

The following are equivalent

1. Principle of Mathematical Induction
2. Strong Induction
3. Well Ordering Principle

Note. A function f such that $f : \mathbb{N} \times \mathbb{N} \to \mathbb{N}$ can be defined by $f((m,n)) = 7^n 13^m$

Properties of $\mathbb{R}$

Interval

3.2 Theorem. *A set $I \in \mathbb{R}$ is an interval if for each $x, y \in I$ with $x \leq y$ and $z \in I$ with $x \leq y \leq z$, we have $z \in I$*

3.3 Question. 1. Is $\emptyset$ an interval? Yes

2. Is $\{3\}$ an interval? Yes

Other Intervals

1. $[a, b] = x \in \mathbb{R} \mid a \leq x \leq b \to$ Closed Interval
2. $(a, b) = x \in \mathbb{R} \mid a < x < b \to$ Open Interval
3. $[a, b) = x \in \mathbb{R} \mid a \leq x < b \to$ Half Open Half Closed Interval
4. $[a, \infty) = x \in \mathbb{R} \mid a \leq x \to$ Closed Ray
5. $(\infty, b] = x \in \mathbb{R} \mid x \leq b \to$ Closed Ray
6. $(0, \infty)$
7. $(-\infty, b)$
8. $(-\infty, \infty) = \mathbb{R}$