Lecture 4, Sept. 14

Class

4.1 Definition. A class is a collection of sets of the form

$$\{x \mid F(x) \text{ is true }\}$$

Where $F(x)$ is a mathematical statement about an unknown set x.

4.2 Example. The collection of all sets is the class $\{x \mid x = x\}$

4.3 Example. If A is a set then $A = \{x \mid x \in A\}$ which is also a class

Mathematical Statement

4.4 Definition. In the languages of Propositional logic we use symbols from the symbol set

$$\{\neg, \wedge, \vee, \rightarrow, \leftrightarrow, (,)\}$$

together with propositional variable symbols such as $P, Q, R, \ldots$

The variable symbols are intended to represent mathematical statements which are either true or false.

In propositional logic, a formula is a non-empty, finite string of symbols (from the above set of symbols) which can be obtained by applying the following rules.

1. Every propositional variable is a formula.
2. If F is a formula, then so is the string $\neg F$.
3. If F and G are formulas then so is each of the following strings
   - $(F \vee G)$
   - $(F \wedge G)$
   - $(F \rightarrow G)$
   - $(F \leftrightarrow G)$

A derivation for a formula F is a list of formulas

$$F_1, F_2, F_3, \ldots$$

with $F = F_k$ for some index k and for each index l, either F_l is a propositional variable, or F_l is equal to $F_l = \neg F_i$ for some $i < l$, or $F_l = (F_i * F_j)$ for some $i, j < l$ and for some symbol $* \in \{\wedge, \vee, \rightarrow, \leftrightarrow\}$

4.5 Example.

$$(\neg(\neg P \rightarrow Q) \leftrightarrow (R \vee \neg S))$$

is a formula and one possible derivation, with justification on each line, is as follows

1. P

2. Q
3. R
4. S
5. $\neg P$
6. $(\neg P \to Q)$
7. $\neg S$
8. $R \vee \neg S$
9. $\neg(\neg P \to Q)$
10. $(\neg(\neg P \to Q) \leftrightarrow (R \vee \neg S))$

4.6 Definition. An **assignment** of truth-values to the propositional variables is a function $\alpha\colon \{P, Q, R \dots\} \to \{0, 1\}$

For a propositional variable X when $\alpha(X) = 1$ we say $X$is true under α and when $\alpha(X) = 0$ we say X is false under α

Given an assignment $\alpha\colon \{\text{propositional variables}\} \to 0, 1$ we extend α to a function $\alpha\colon \{\text{formulas}\} \to 0, 1$ by defining $\alpha(F)$ for all formulas F recursively as follows:

When $F = X$ where X is a propositional variable symbol, the value of $\alpha(X)$is already known

When $F = \neg G$ where G is a formula, define $\alpha(F)$ according to the following table

G	$\neg G$
1	0
0	1

When $F = (G * H)$ where G and H are formulas and where $* \in \{\wedge, \vee, \to, \leftrightarrow\}$

we define $\alpha(F)$ according to the following table

G	H	$G \wedge H$	$G \vee H$	$G \to H$	$G \leftrightarrow H$
1	1	1	1	1	1
1	0	0	1	0	0
0	1	0	1	1	0
0	0	0	0	1	1