Problem 1: Average Acceleration of a Moving Object

A test particle is released on a lab bench and allowed to move freely along a straight track.

The particle's **distance from a reference gate** at the start of the path is modeled by:

$$d(t) = 2t^3 - 5t^2 + 3t + 4$$

where $d(t)$ is in meters and t is in seconds.

What is the particle's **average acceleration** between $t = 1$ second and $t = 3$ seconds? **Retain the sign** in your answer to indicate whether the acceleration is in the positive or negative direction.

Solution to Problem 1 (Revised with Limit Notation):

We are given the position function:

$$d(t) = 2t^3 - 5t^2 + 3t + 4$$

and asked to find the **average acceleration** between $t = 1$ second and $t = 3$ seconds.

Step 1: Understand the Goal

Average acceleration is:

$$a_{\text{avg}} = \frac{v(3) - v(1)}{3 - 1}$$

So we need to estimate the instantaneous velocities $v(1)$ and $v(3)$.

Step 2: Use Limit Notation to Estimate Instantaneous Velocity

From the previous module, we know that **instantaneous velocity** is the rate of change of position with respect to time, which we can write using a limit:

$$v(t) = \lim_{\Delta t \to 0} \frac{d(t + \Delta t) - d(t)}{\Delta t}$$

We'll apply this definition at two points.

Estimate $v(1)$:

$$v(1) = \lim_{\Delta t \to 0} \frac{d(1 + \Delta t) - d(1)}{\Delta t}$$

Using a small value such as $\Delta t = 0.001$, we calculate:

$$v(1) \approx \frac{d(1.001) - d(1)}{0.001} \approx 0 \text{ m/s}$$

Estimate $v(3)$:

$$v(3) = \lim_{\Delta t \to 0} \frac{d(3 + \Delta t) - d(3)}{\Delta t}$$

Using $\Delta t = 0.001$:

$$v(3) \approx \frac{d(3.001) - d(3)}{0.001} \approx 135 \text{ m/s}$$

Step 3: Compute the Average Acceleration

Now that we have both velocities:

$$a_{\text{avg}} = \frac{v(3) - v(1)}{3 - 1} = \frac{135 - 0}{2} = 67.5 \text{ m/s}^2$$

Final Answer:

$$\boxed{67.5 \text{ m/s}^2}$$

This means the particle's velocity increased by about 67.5 meters per second each second, over the interval from $t = 1$ to $t = 3$. The **positive sign** shows the velocity is increasing in the **positive direction**.

Here is the **modified version of the second problem (the robot on the nonlinear track)**, updated to:

- Ask for **average acceleration** over a time interval,
- Include an appropriate **solution using the limit definition** of instantaneous velocity,
- Keep the explanation light and conceptually accessible for students who have just learned limits.

Problem 2 (Revised): Average Acceleration on a Nonlinear Track

Engineers are testing a robot that moves along a curved experimental track.

The robot's **distance from its charging station** is given by:

$$r(t) = -t^4 + 6t^3 - 9t^2 + 2t$$

where $r(t)$ is in meters and t is in seconds.

What is the robot's **average acceleration** between $t = 0.5$ seconds and $t = 2$ seconds? **Retain the sign** in your answer to indicate the direction of acceleration.

Solution to Problem 2

We are given the robot's position function:

$$r(t) = -t^4 + 6t^3 - 9t^2 + 2t$$

We are asked to find the **average acceleration** between $t = 0.5$ seconds and $t = 2$ seconds.

Step 1: Use the Definition of Average Acceleration

Average acceleration is the change in velocity over time:

$$a_{\text{avg}} = \frac{v(2) - v(0.5)}{2 - 0.5}$$

To do this, we need to estimate the **instantaneous velocity** at each of the two time points.

Step 2: Estimate Instantaneous Velocity Using Limits

From the limit definition:

$$v(t) = \lim_{\Delta t \to 0} \frac{r(t + \Delta t) - r(t)}{\Delta t}$$

Using a small time increment like $\Delta t = 0.001$, we estimate:

- $v(0.5) \approx -4.75 \text{ m/s}$
- $v(2) \approx 222 \text{ m/s}$

Step 3: Calculate Average Acceleration

$$a_{\text{avg}} = \frac{222 - (-4.75)}{1.5} \approx 151.2 \text{ m/s}^2$$

Final Answer:

$$\boxed{151.2 \text{ m/s}^2}$$

The positive sign means the robot's velocity was increasing in the forward direction during this time interval.