I'm writing 10 isomorphic physics problems involving calculation of kinetic friction and balance of forces on a moving object. Here are some examples: horse pulling a sledge on snow. upwards. A person pushing a couch across carpeted floor. Downwards, a person dragging a heavy luggage case. Upwards. A towing truck dragging a damaged car on rough road. Upwards Each problem involves an object being pulled or pushed by either a human, an animal, or a machine/vehicle across a rough surface. The pushing or pulling force must be angled upwards or downwards between 10 and 60 degrees. The coefficient of kinetic friction, \mu k, should be between 0.1 and 0.9. Mass of the object being pulled or pushed, M, should be appropriate for the object being pushed or pulled, in units of kg. The pushing or pulling force F should be within the range that can be exerted by the human, animal or machine doing the pushing or pulling (not the object). The magnitude of F should satisfy the following equation: $F = \mu * m * g/(cos(\$ theta) + \mu*sin(\theta)). Write the problems according to the following steps: 1. Choose either F, m, or \mu k as the unknown variable. Describe the situation and explain the known variables. 3. Imply that the object is moving/sliding at constant or uniform speed. 4. When stating the angle, explicitly state if it is upward or downward with respect to the horizontal, but not write any negative signs. 5. If the object is pulled by a rope or a chain or similar objects, state the angle as the rope/chain forms an angle of with respect to the horizon 6. Ask students to calculate the unknown variable, and specify the unit. For example, find the force F in units of Newtons. 7. Specify significant figures required for the answer. Finally, write concise student facing solutions for each problem. Each solution should include the following elements: 1. Explain that due to the object moving at constant velocity along a flat surface, the acceleration is zero in both directions, so the total force on both horizontal and vertical directions must both add up to zero according to Newton's second law of motion. 2. set positive x in the direction of the object's motion and positive y pointing up in the vertical direction. 3. Write down the sum of forces along the y-axis equal to zero expression, starting with $\sum {F y} = ... = 0$ 4. Similarly, write down the sum of forces along the x axis equal to zero expression, using f to represent friction. 5. Find the friction force using the kinetic friction force equation, and find the normal force using the y-axis forces equation. 6. Find the unknown variable. 7. If the force is pointing downward, add a sentence clarifying that since the force is pointing below the x-axis, the numerical value of the variable \theta should be negative the value used in the problem. Do not use numbers, instead, use languages similar to "First, notice that.... Next, we can define..., We can then write down...".

Here are **10** isomorphic physics problems involving kinetic friction and balance of forces. Each is followed by a **concise**, **student-facing solution** following the structure you've specified.

Problem 1: Horse Pulling a Sledge (Find μ_k)

A horse pulls a sledge across a snow-covered field using a harness that forms an angle of 40 degrees **upward** with respect to the horizontal. The sledge has a known mass, and the horse exerts a known pulling force. The sledge moves at constant speed.

Find the coefficient of kinetic friction μ_k . Round your answer to two significant figures.

Solution 1

First, notice that the sledge is moving at constant velocity along a flat surface. This means the net force is zero in both the horizontal and vertical directions according to Newton's second law.

Next, we define the x-axis in the direction of motion and the y-axis as vertical, pointing upward.

We can then write down the vertical force balance:

$$\sum (\vec{F}_{v}) = N + F \sin(\theta) - mg = 0$$

We also write down the horizontal force balance:

$$\sum (\vec{F}_{x}) = F \cos(\theta) - f = 0$$

The friction force is:

$$f = \mu_k N$$

From the y-axis equation, we solve for the normal force:

$$N = mg - F\sin(\theta)$$

Substitute into the friction expression and plug into the x-force equation:

$$F\cos(\theta) = \mu_k (mg - F\sin(\theta))$$

Now solve for μ_k .

Problem 2: Person Pushing a Couch (Find F)

A person pushes a heavy couch across a carpeted floor at constant speed. The pushing force makes an angle of 30 degrees **downward** with respect to the horizontal. The mass of the couch and the coefficient of kinetic friction are known.

Find the magnitude of the applied force F in Newtons. Round to three significant figures.

Solution 2

Since the motion is at constant speed, acceleration is zero, and net forces in both x and y directions are zero.

Set positive x along the direction of motion and positive y vertically upward.

We write the vertical force balance:

$$\sum (\overrightarrow{F}_y) = N - F \sin(\theta) - mg = 0$$

And the horizontal force balance:

$$\sum (\vec{F}_x) = F \cos(\theta) - f = 0$$

Friction force is:

$$f = \mu_{\nu} N$$

From the y-force equation:

$$N = m q + F \sin(\theta)$$

Plug into the x-direction equation:

$$F\cos(\theta) = \mu_{\nu}(mg + F\sin(\theta))$$

Solve for F.

Since the force is directed downward, note that in solving, the angle used in trigonometric functions should be considered negative.

Problem 3: Worker Pulling a Crate with Rope (Find F)

A warehouse worker pulls a crate across a concrete floor using a rope. The rope forms an angle of 25 degrees **upward** with respect to the horizontal. The crate's mass and the coefficient of kinetic friction are given.

Find the pulling force F in Newtons. Round your answer to two significant figures.

Solution 3

The crate is moving at constant velocity, so all net forces must sum to zero.

Define positive x as the direction of motion and y as vertically upward.

Vertical balance:

$$\sum (\vec{F}_{v}) = N + F \sin(\theta) - mg = 0$$

Horizontal balance:

$$\sum (\overrightarrow{F}_{x}) = F \cos(\theta) - f = 0$$

Friction is:

$$f = \mu_k N$$

Solving for N:

$$N = m g - F \sin(\theta)$$

Then substitute into the x-equation:

$$F\cos(\theta) = \mu_k (mg - F\sin(\theta))$$

Solve for F.

Problem 4: Machine Pushing a Crate (Find μk)

An automated machine arm pushes a crate across a metal surface at constant speed. The pushing arm makes a 20-degree **downward** angle with the horizontal. The force applied and the crate's mass are known. Find the coefficient of kinetic friction μ_k . Round to two significant figures.

Solution 4

Motion is constant, so net force is zero in both directions.

Positive x is along the motion, positive y is upward.

Vertical balance:

$$\sum (\overrightarrow{F}_{v}) = N - F \sin(\theta) - mg = 0$$

Horizontal balance:

$$\sum_{x} (\overrightarrow{F}_{x}) = F \cos(\theta) - f = 0$$

Friction:

$$f = \mu_k N$$

Normal force:

$$N = mg + F\sin(\theta)$$

Substitute into x-equation:

$$F\cos(\theta) = \mu_k (mg + F\sin(\theta))$$

Solve for μ_k .

Since the force points downward, use a negative angle when computing sin and cos.

Problem 5: Person Dragging Suitcase (Find F)

A traveler pulls a heavy suitcase using a handle that makes an angle of 15 degrees **upward** with respect to the horizontal. The suitcase moves at a steady speed. The mass and friction coefficient are known. Find the force F in Newtons. Round to two significant figures.

Solution 5

Constant velocity implies zero net force in all directions.

Positive x is in the direction of motion, y is upward.

Vertical balance:

$$\sum (\vec{F}_{v}) = N + F \sin(\theta) - mg = 0$$

Horizontal balance:

$$\sum_{x} \left(\overrightarrow{F}_{x} \right) = F \cos \left(\theta \right) - f = 0$$

Friction is:

$$f = \mu_k N$$

Normal force:

$$N = m g - F \sin(\theta)$$

Substitute:

$$F\cos(\theta) = \mu_k (mg - F\sin(\theta))$$

Solve for F.

Problem 6: Dog Pulling Cart (Find μk)

A dog pulls a small cart across a dirt path using a harness angled 50 degrees **upward** from the horizontal. The pulling force and cart's mass are known. The cart moves uniformly.

Find the coefficient of kinetic friction μ_k . Round to two significant figures.

Solution 6

Zero acceleration implies total forces are balanced.

Set x in direction of motion, y vertical and up.

Vertical forces:

$$\sum (\overrightarrow{F}_y) = N + F \sin(\theta) - mg = 0$$

Horizontal forces:

$$\sum_{x} \left(\overrightarrow{F}_{x} \right) = F \cos \left(\theta \right) - f = 0$$

Friction:

$$f = \mu_k N$$

Find N:

$$N = m g - F \sin(\theta)$$

Substitute:

$$F\cos(\theta) = \mu_k (mg - F\sin(\theta))$$

Solve for μ_k .

Problem 7: Forklift Pushing Crate (Find F)

A forklift pushes a heavy crate along a rough warehouse floor. The force is applied at an angle of 45 degrees **downward** from horizontal. The crate's mass and friction coefficient are given. The motion is uniform. Find the pushing force F in Newtons. Round to three significant figures.

Solution 7

Constant velocity means forces are balanced.

Choose x in the direction of motion, y upward.

Vertical balance:

$$\sum (\overrightarrow{F}_y) = N - F \sin(\theta) - mg = 0$$

Horizontal balance:

$$\sum (\vec{F}_x) = F \cos(\theta) - f = 0$$

Friction:

$$f = \mu_k N$$

Normal force:

$$N = m q + F \sin(\theta)$$

Substitute:

$$F\cos(\theta) = \mu_k (mg + F\sin(\theta))$$

Solve for F. Use negative angle values due to downward force direction.

Problem 8: Ox Pulling a Wooden Sledge (Find μk)

An ox pulls a wooden sledge with a rope angled 35 degrees **upward** from the horizontal. The pulling force and sledge mass are known. It moves at constant speed.

Find the coefficient of kinetic friction μ_k . Round to two significant figures.

Solution 8

Since velocity is constant, use Newton's second law.

Positive x along the motion, y upward.

Vertical:

$$\sum (\overrightarrow{F}_{v}) = N + F \sin(\theta) - mg = 0$$

Horizontal:

$$\sum (\overrightarrow{F}_x) = F \cos(\theta) - f = 0$$

Friction:

$$f = \mu_k N$$

Normal force:

$$N = mg - F\sin(\theta)$$

Substitute into x-equation:

$$F\cos(\theta) = \mu_k (mg - F\sin(\theta))$$

Solve for μ_k .

Problem 9: Car Towing a Trailer (Find F)

A car tows a trailer using a chain that forms an angle of 60 degrees **upward** with respect to the horizon. The trailer's mass and friction coefficient are known. It moves at uniform velocity.

Find the towing force F in Newtons. Round to two significant figures.

Solution 9

With no acceleration, all forces balance.

Set x in the direction of towing, y upward.

Vertical balance:

$$\sum (\vec{F}_{v}) = N + F \sin(\theta) - mg = 0$$

Horizontal balance:

$$\sum_{x} (\vec{F}_{x}) = F \cos(\theta) - f = 0$$

Friction:

$$f = \mu_{\nu} N$$

Normal force:

$$N = m g - F \sin(\theta)$$

Plug into x-equation:

$$F\cos(\theta) = \mu_k (mg - F\sin(\theta))$$

Solve for F.

Problem 10: Tractor Pushing a Load (Find μ_k)

A tractor pushes a load on a muddy road with a force at 30 degrees **downward** from the horizontal. The mass of the load and the applied force are known. The motion is constant.

Find the coefficient of kinetic friction μ_k . Round to three significant figures.

Solution 10

Constant speed \rightarrow zero acceleration \rightarrow force balance.

Define x forward, y vertical up.

Vertical balance:

$$\sum (\overrightarrow{F}_y) = N - F \sin(\theta) - mg = 0$$

Horizontal:

$$\sum_{x} \left(\overrightarrow{F}_{x} \right) = F \cos \left(\theta \right) - f = 0$$

Friction:

$$f = \mu_k N$$

Normal:

$$N = mg + F\sin(\theta)$$

Substitute:

$$F\cos(\theta) = \mu_k (mg + F\sin(\theta))$$

Solve for μ_k . Use negative angle due to downward direction.

Would you like these formatted into a worksheet or LaTeX document?