1 Definitions

Node proves the statement "distance is within a threshold" (less or equal), for node coordinates (x_n, y_n, z_n), given location (x_l, y_l, z_l), and some threshold d (all integers):

$$d^2 - ((x_n - x_l)^2 + (y_n - y_l)^2 + (z_n - z_l)^2) = a_1^2 + a_2^2 + a_3^2 + a_4^2 \tag{1}$$

We rely on 4-squares Lagrange theorem to prove equality statement (Lipmaa). Proofs for integer relations are possible in hidden group order setup (Camenisch-Stadler).

2 Proof setup

Let g be a generator of a proper group of a hidden order, and h be a group element. We use multiplicative group of invertible residue classes modulo a composite n such that $n = pq$, $p = 2p' + 1$, $q = 2q' + 1$ and p, q, p', q' primes.

3 Signals harvesting

Node creates commitment (s_x, s_y, s_z) to coordinates:

$$s_x = g^{x_n} h^{r_x},\ s_y = g^{y_n} h^{r_y},\ s_z = g^{z_n} h^{r_z} \tag{2}$$

and keeps coordinates-randoms pairs (x_n, y_n, z_n), (r_x, r_y, r_z) private.

4 Proof

Sigma-protocol with 3 messages. Public information is node location commitment, given location, threshold, proof parameters. Private information is node location and randomness to commitment.

1. Prover (node) picks random $\alpha_j, \beta_x, \beta_y, \beta_z, \rho_0, \rho_1, \eta_x, \eta_y, \eta_z$ and computes initial commitments b_0, b_1, t_x, t_y, t_z
   (f_0 and f_1 explained at Background section)

$$b_0 = g^{f_0} h^{\rho_0},\ b_1 = g^{f_1} h^{\rho_1} \tag{3}$$

$$t_x = g^{\beta_x} h^{\eta_x},\ t_y = g^{\beta_y} h^{\eta_y},\ t_z = g^{\beta_z} h^{\eta_z} \tag{4}$$

2. Challenge c

3. Prover computes responses

$$X_n = cx_n + \beta_x,\ Y_n = cy_n + \beta_y,\ Z_n = cz_n + \beta_z \tag{5}$$

$$R_x = cr_x + \eta_x,\ R_y = cr_y + \eta_y,\ R_z = cr_z + \eta_z \tag{6}$$

$$A_j = ca_j + \alpha_j,\ j = 1..4 \tag{7}$$

$$R_a = c\rho_1 + \rho_0 \tag{8}$$

4. proof verification

$$g^{c^2 d - ((X_n - cx_l)^2 + (Y_n - cy_l)^2 + (Z_n - cz_l)^2) - (A_1^2 + A_2^2 + A_3^2 + A_4^2)} h^{R_a} b_1^{-c} b_0^{-1} = 1 \tag{9}$$

$$g^{X_n} h^{R_x} s_x^{-c} t_x^{-1} = 1,\ g^{Y_n} h^{R_y} s_y^{-c} t_y^{-1} = 1,\ g^{Z_n} h^{R_z} s_z^{-c} t_z^{-1} = 1 \tag{10}$$

5 Background

Consider quadratic (degree 2 in v) polynomial

$$\begin{aligned} f_V(v) = f_2 v^2 + f_1 v + f_0 = \\ v^2 d - (((vx_n + \beta_x) - vx_l)^2 + ((vy_n + \beta_y) - cy_l)^2 + ((vz_n + \beta_z) - cz_l)^2) \\ - ((va_1 + \alpha_1)^2 + (va_2 + \alpha_2)^2 + (va_3 + \alpha_3)^2 + (va_4 + \alpha_4)^2) \end{aligned} \tag{11}$$

Major point is, this polynomial is actually linear ($f_2 = 0$, degree-one in v) if, and only if statement about distance (1) holds for hidden node coordinates. We evaluate this polynomial at a random point chosen as challenge of verifier. It follows, distance verification equation (9) only needs constant b_0 and degree-one b_1^c components.

$$f_0 = -\beta_x^2 - \beta_y^2 - \beta_z^2 - \alpha_1^2 - \alpha_2^2 - \alpha_3^2 - \alpha_4^2 \tag{12}$$

$$f_1 = -2x_n\beta_x - 2y_n\beta_y - 2z_n\beta_z - 2a_1\alpha_1 - 2a_2\alpha_2 - 2a_3\alpha_3 - 2a_4\alpha_4 \tag{13}$$