

2D Range Tree

A JavaScript Implementation & Visualization

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Range Tree Recap

Range Tree Complexity Overview

- Construction: $O(n \log^{d-1} n)$
- Space: $O(n \log^{d-1} n)$
- Search Time Complexity: $O(\log^d n + k)$
- Adding fractional cascading
 - Search time Complexity: $O(\log^{d-1} n + k)$
- KD Tree Size: $O(n)$, Time Complexity: $O(n^{1-1/d} + k)$

Range Tree Construction - Time Complexity

- Construct a OneD Range tree, and construct another OneD Range tree for each of the node. Do this recursively for k-dimensions.

- Uses the median value of that dimension as the node -> A balanced tree

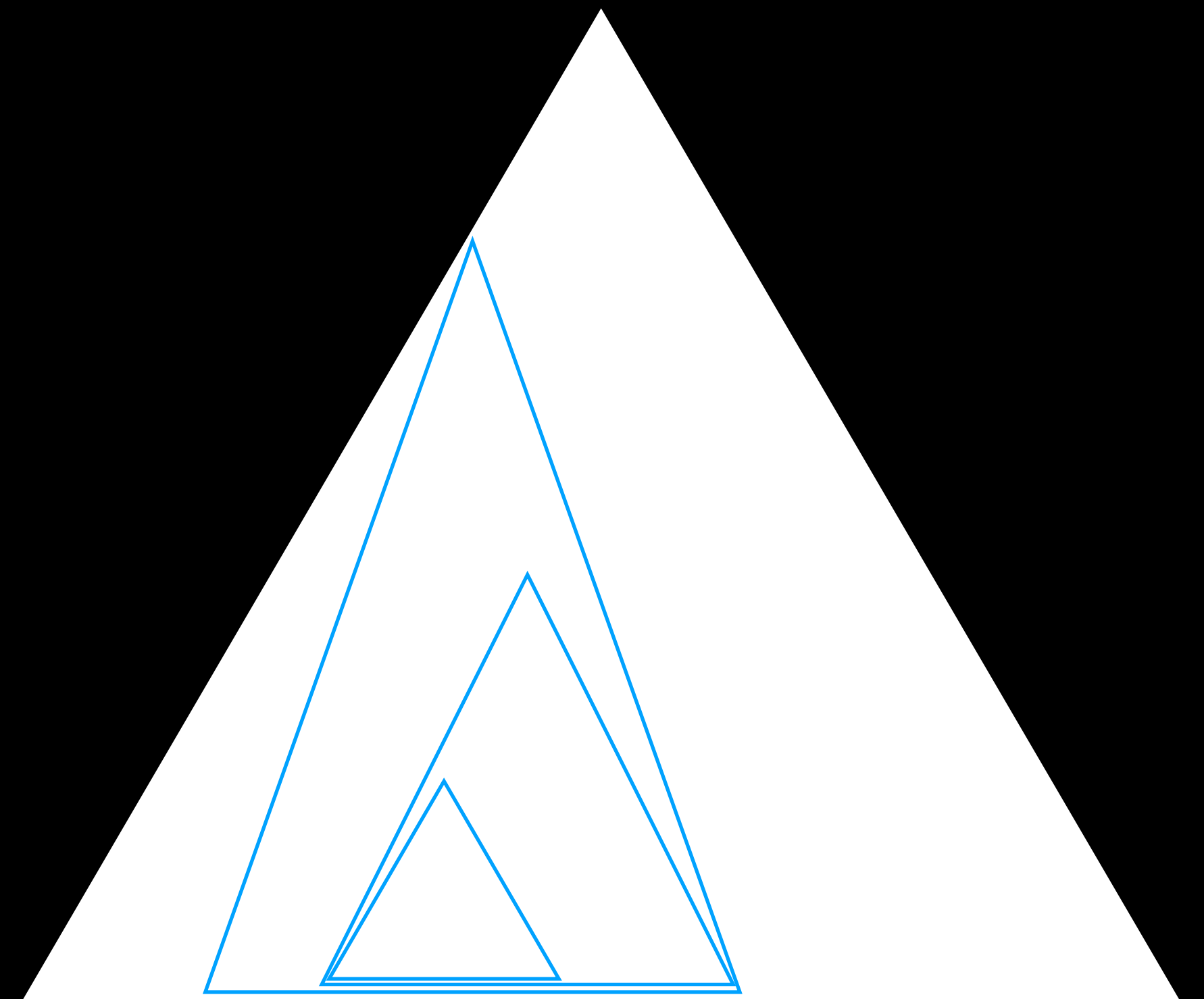
- $$T_d(n) = O(n \log n) + O(\log n) \cdot T_{d-1}(n).$$

- With Optimization:

$$T_2(n) = O(n \log n), \text{ this recurrence solves to } O(n \log^{d-1} n)$$

Range Tree Storage Complexity

- Each point would appear in $\lceil \log^{d-1} n \rceil$ trees
- The number of nodes in the tree is linear in respects to the number of leafs
- $O(n \log^{d-1} n)$



Range Tree Search Review

- Searching in a first-level tree, takes $O(\log n)$, and query $\log n$ number of $(d - 1)$ -dimensional range trees.
- $$Q_d(n) = O(\log n) + O(\log n) \cdot Q_{d-1}(n)$$

where $Q_2(n) = O(\log^2 n)$
- $$Q_d(n) = O(\log^d n)$$
- Report points, +k

My Project Demo

- Takes in points & query from user
- Draw the 2D range tree
- Visualize the search