人工智能

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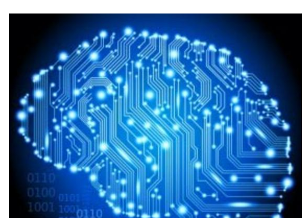
Knowledge 4



First-Order Logic: Inference

A brief history of reasoning

450B.C.	Stoics	propositional logic, inference (maybe)
322B.C.	Aristotle	"syllogisms" (inference rules), quantifiers
1565	Cardano	probability theory (propositional logic + uncertainty)
1847	Boole	propositional logic (again)
1879	Frege	first-order logic
1922	Wittgenstein	proof by truth tables
1930	Gödel	$\exists$ complete algorithm for FOL
1930	Herbrand	complete algorithm for FOL (reduce to propositional)
1931	Gödel	$\neg\exists$ complete algorithm for arithmetic
1960	Davis/Putnam	"practical" algorithm for propositional logic
1965	Robinson	"practical" algorithm for FOL—resolution

First-Order Logic: Inference

Propositionalization

Universal Instantiation (UI)

- Every instantiation of a universally quantified sentence is entailed by it:

$$\frac{\forall v \;\; \alpha}{SUBST(\{{}^{v}/_{g}\}, \alpha)}$$

- For any variable v and ground term g

- E.g., $\forall x \; King(x) \wedge Greedy(x) \Rightarrow Evil(x)$ yields
$King(John) \wedge Greedy(John) \Rightarrow Evil(John)$
$King(Richard) \wedge Greedy(Richard) \Rightarrow Evil(Richard)$
$King(Father(John)) \wedge Greedy(Father(John)) \Rightarrow Evil(Father(John))$

Existential Instantiation (EI)

- For any sentence α,variable v, and constant symbol k
  That does not appear elsewhere in the knowledge base:

$$\frac{\exists v \quad \alpha}{SUBST(\{v/k\}, \alpha)}$$

- E. g. , $\exists x \; Crown(x) \wedge OnHead(x, John)$ yields
  $Crown(C_1) \wedge OnHead(C_1, John)$
- Provided C_1 is a new constant symbol, called a **Skolem constant**
- Another example: from $\exists x \; d(x^y)/dy = x^y$ we obtain $d(e^y)\,/dy = e^y$
- Provided e is a new constant symbol

Reduction to propositional inference

- Suppose the KB contains just the following:

$\forall x\ King(x) \wedge Greedy(x) \Rightarrow Evil(x)$

$King(John)$

$Greedy(John)$

$Brother(Richard, John)$

- Instantiating the universal sentence in all possible ways, we have

$King(John) \wedge Greedy(John) \Rightarrow Evil(John)$

$King(Richard) \wedge Greedy(Richard) \Rightarrow Evil(Richard)$

$King(John)$

$Greedy(John)$

$Brother(Richard, John)$

- The new KB is propositionalized: proposition symbols are

$King(John), Greedy(John), Evil(John), King(Richard)$etc.

Reduction to propositional inference

- Claim: a ground sentence is entailed by new KB iff entailed by original KB
- Claim: every FOL KB can be propositionalized so as to preserve entailment
- Idea: propositionalize KB and query, apply resolution, return result
- Problem: with function symbols, there are infinitely many ground terms,

  e.g., $Father(Father(Father(John)))$
- Theorem: Herbrand (1930). If a sentence α is entailed by an FOL KB, it is entailed by a finite subset of the propositional KB
- Idea: For $n = 0$ to ∞ do

  create a propositional KB by instantiating with depth-n terms see if α

  is entailed by this KB
- Problem: works if α is entailed, loops if α is not entailed
- Theorem: Turing(1936), Church(1936), entailment in FOL is semi-decidable

Problems with Propositionalization

- Propositionalization seems to generate lots of irrelevant sentences.
- E.g., from

  $\forall x\ King(x) \land Greedy(x) \Rightarrow Evil(x)$

  $King(John)$

  $\forall y\ Greedy(y)$

  $Brother(Richard, John)$
- It seems obvious that $Evil(John)$, but propositionalization produces lots of facts such as $Greedy(Richard)$ that are irrelevant
- With p k-ary predicates and n constants, there are $p \cdot n^k$ instantiations
- With function symbols, it gets much much worse!

First-Order Logic: Inference

Resolution in FOL

Unification

- We can get the inference immediately if we can find a substitution θ such that $King(x)$ and $Greedy(x)$ match $King(John)$ and $Greedy(y)$
- $\theta = \{x/John, y/John\}$ works
- $UNIFY(\alpha, \beta) = \theta$ if $\alpha\theta \; equals \; to \; \beta\theta$

p	q	θ
$Knows(John, x)$	$Knows(John, Jane)$	$\{x/Jane\}$
$Knows(John, x)$	$Knows(y, OJ)$	$\{x/OJ, y/John\}$
$Knows(John, x)$	$Knows(y, Mother(y))$	$\{y/John, x/Mother(John)\}$
$Knows(John, x)$	$Knows(x, OJ)$	$fail$

Standardizing apart eliminates overlap of variables, e.g., $Knows(z_{17}, OJ)$

Resolution: brief summary

- Full first-order version

$$\frac{l_1 \vee \dots \vee l_k, \qquad m_1 \vee \dots \vee m_n}{(l_1 \vee \dots \vee l_{i-1} \vee l_{i+1} \vee \dots \vee l_k \vee m_1 \vee \dots \vee m_{j-1} \vee m_{j+1} \vee \dots \vee m_n)\theta}$$

- Where $UNIFY\left(l_i, \neg m_j\right) = \theta$
- For example,

$$\frac{\neg Rich(x) \vee Unhappy(x)\ , \qquad Rich(Ken)}{Unhappy(Ken)}$$

- With $\theta = \{\,x/Ken\,\}$
- Apply resolution steps to CNF(KB$\wedge \neg\alpha$); complete for FOL

Conversion to CNF

- Everyone who loves animals is loved by someone:
  $\forall x \ [\forall y \, Animal(y) \Rightarrow Loves(x,y)] \Rightarrow [\exists y \ Loves(y,x)]$
- Eliminate biconditionals and implications $\forall x \neg [\forall y \neg Animal(y) \vee Loves(x,y)] \vee [\exists y \ Loves(y,x)]$
- Move $\neg$ inwards: $\neg\forall \, x, \, p \ \equiv \exists x \neg p, \ \ \neg\exists x, p \ \equiv \forall x \neg p$:
  $\forall x \ [\exists y \neg(\neg Animal(y) \vee Loves(x,y))] \vee [\exists y \ Loves(y,x)]$
  $\forall x \ [\exists y \neg\neg Animal(y) \wedge \neg Loves(x,y))] \vee [\exists y \ Loves(y,x)]$
  $\forall x \ [\exists y \, Animal(y) \wedge \neg Loves(x,y))] \vee [\exists y \ Loves(y,x)]$
- Standardize variables: each quantifier should use a different one
  $\forall x \ [\exists y \, Animal(y) \wedge \neg Loves(x,y))] \vee [\exists z \ Loves(z,x)]$
- Skolemize: a more general form of existential instantiation.
  Each existential variable is replaced by a Skolem function of the enclosing universally quantified variables:
  $\forall x \ [\, Animal(F(x)) \wedge \neg Loves(x, F(x)))] \vee [\, Loves(G(x), x)]$

Conversion to CNF

- Drop universal quantifiers:

$$[\,Animal(F(x)) \land \neg Loves(x, F(x))] \lor [\,Loves(G(x), x)]$$

- Distribute ∧ over ∨ :

$$[\,Animal(F(x)) \lor \neg Loves(G(x), x))] \land [\neg Loves(x, F(x)) \lor Loves(G(x), x)]$$

Example knowledge base

The law says that it is a crime for an American to sell weapons to hostile nations. The country Nono, an enemy of America, has some missiles, and all of its missiles were sold to it by Colonel West, who is American.

Prove that Col. West is a criminal

… it is a crime for an American to sell weapons to hostile nations:

$American(x) \land Weapon(y) \land Sells(x, y, z) \land Hostile(z) \Rightarrow Criminal(x)$

Nono … has some missiles, i.e., $\exists x\ Owns(Nono, x) \land Missile(x)$:

$Owns(Nono, M_1)$ and $Missile(M_1)$

… all of its missiles were sold to it by Colonel West

$Missile(x) \land Owns(Nono, x) \Rightarrow Sells(West, x, Nono)$

Missiles are weapons:

$Missile(x) \Rightarrow Weapon(x)$

An enemy of America counts as "hostile":

$Enemy(x, America) \Rightarrow Hostile(x)$

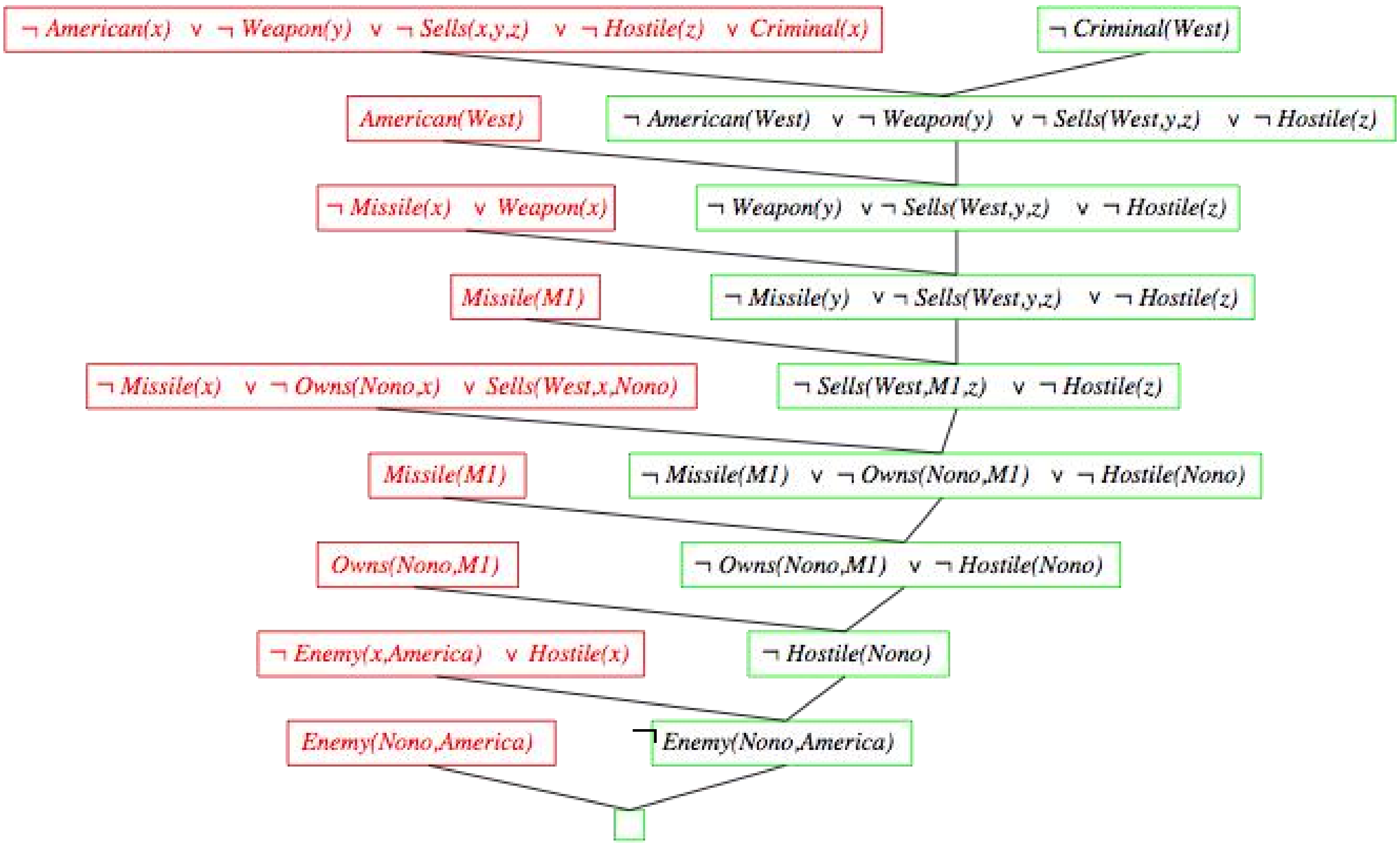
West, who is American …

$American(West)$

The country Nono, an enemy of America …

$Enemy(Nono, America)$

Resolution proof



Completeness of FOL resolution

- Resolution is refutation-complete. If a set of sentences is unsatisfiable, resolution always derives a contradiction.
- It can find all answers of a given question, $Q(x)$, by proving that $KB \wedge \neg Q(x)$ is unsatisfiable
- Check out AIMA for the (brief) proof:

> If *S* is an unsatisfiable set of clauses, then the application of a finite number of resolution steps to *S* will yield a contradiction

First-Order Logic: Inference

Generalized Modus Ponens

Generalized Modus Ponens (GMP) 前见推理

$$\frac{p_1', p_2', \ldots, p_n', \ (p_1 \wedge p_2 \wedge \ldots \wedge p_n \Rightarrow q)}{q\theta}$$

where $p_i'\theta$ $equals\ to\ p_i\theta$ for all i

p_1' is $King(John)$

p_2' is $\mathrm{Greedy}(y)$

Θ is $\{x/John, y/John\}$

$q\theta$ is $Evil(John)$

p_1 is $King(x)$

p_2 is Greedy(x)

q is $Evil(x)$

- GMP used with KB of definite clauses (exactly one positive literal)
  All variables assumed universally quantified

Soundness of GMP

- Need to show that

$$p'_1, \ldots, p'_n, \ (p_1 \land \ldots \land p_n \Rightarrow q) \vDash q\theta$$

- Provided that $p'_i\theta = p_i\theta$ for all i
- Lemma: For any definite clause p, we have $p \vDash p\theta$ by UI
- 1. $(p_1 \land \ldots \land p_n \Rightarrow q) \vDash (p_1 \land \ldots \land p_n \Rightarrow q)\theta = (p_1\theta \land \ldots \land p_n\theta \Rightarrow q\theta)$
- 2. $p'_1, \ldots, p'_n \vDash p'_1 \land \ldots \land p'_n \vDash p'_1\theta \land \ldots \land p'_n\theta$
- 3. From 1 and 2, $q\theta$ follows by ordinary Modus Ponens

the soundness of ordinary Modus Ponens

Forward chaining algorithm

```
function FOL-FC-ASK(KB, α) returns a substitution or false
  repeat until new is empty
      new ← { }
      for each sentence r in KB do
          (p1 ∧ ... ∧ pn ⇒ q) ← STANDARDIZE-APART(r)
          for each θ such that (p1 ∧ ... ∧ pn)θ = (p'1 ∧ ... ∧ p'n)θ
                          for some p'1, ..., p'n in KB
              q' ← SUBST(θ, q)
              if q' is not a renaming of a sentence already in KB or new then do
                  add q' to new
                  φ ← UNIFY(q', α)
                  if φ is not fail then return φ
      add new to KB
  return false
```

Forward chaining proof

Facts

R_1 $Owns(Nono, M_1)$

R_2 $Missile\,(M_1)$

R_3 $American(West)$

R_4 $Enemy(Nono, America)$

Implications

R_5 $American(x) \wedge Weapon(y) \wedge Sells(x, y, z) \wedge Hostile(z) \Rightarrow Criminal(x)$

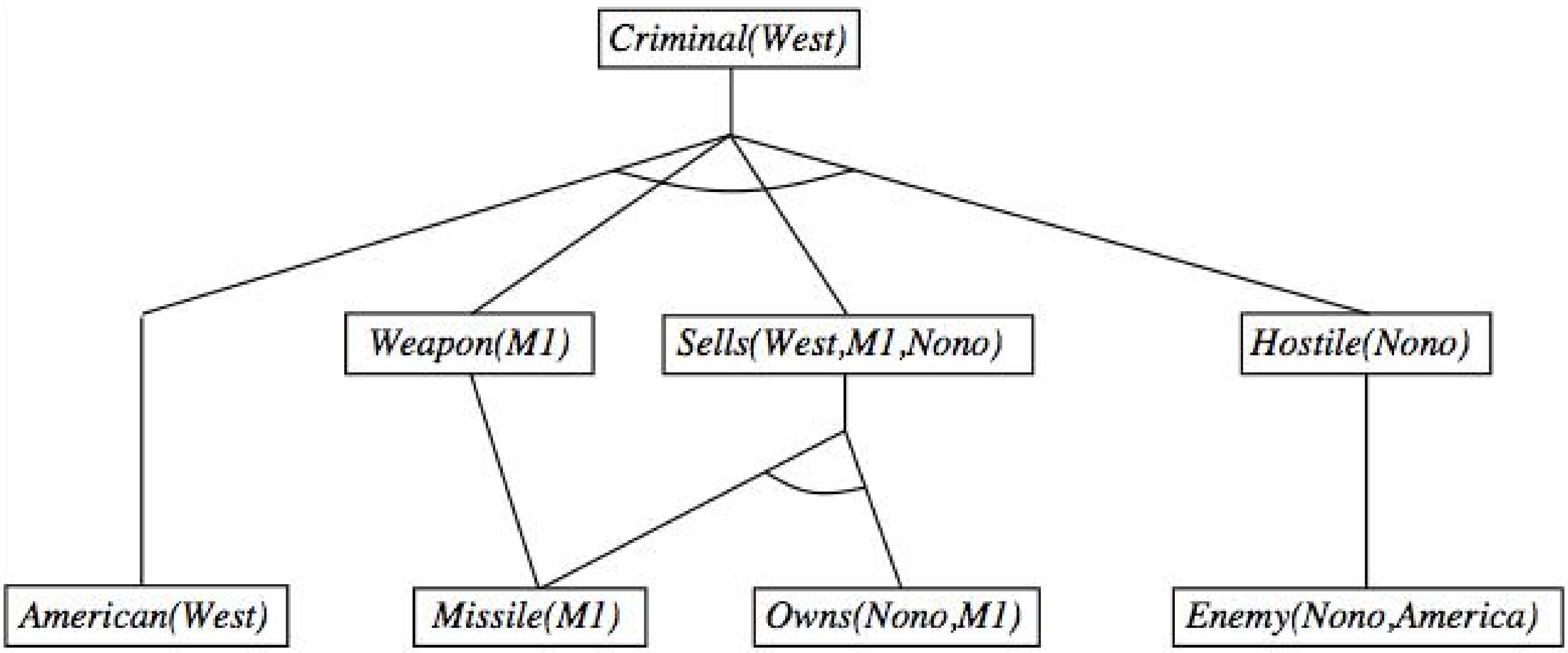
R_6 $Missile(x) \wedge Owns(Nono, x) \Rightarrow Sells(West, x, Nono)$

R_7 $Missile(x) \Rightarrow Weapon(x)$

R_8 $Enemy(x, America) \Rightarrow Hostile(x)$

- 1^{st} iteration, R_5 has unsatisfied premises
  R_6 is satisfied with $\{x/M_1\}$, $Sells\ (West,\ M_1, Nono)$ is added.
  R_7 is satisfied with $\{x/M_1\}$, $Weapon(M_1)$ is added.
  R_8 is satisfied with $\{x/Nono\}$, $Hostile(Nono)$ is added.
- 2^{nd} iteration, R_5 is satisfied with $\{x/West, y/M_1, z/Nono\}$, $Criminal(West)$ is added

Forward chaining proof



Properties of forward chaining

- Sound and complete for first-order definite clauses
  (proof similar to propositional proof)
- May not terminate in general if α is not entailed
- This is unavoidable: entailment with definite clauses is semi-decidable

- Datalog = first-order definite clauses + no functions(e.g., crime KB)
- FC terminates for Datalog in poly iterations

Efficiency of forward chaining

- Simple observation: no need to match a rule on iteration k if a premise wasn't added on iteration $k-1$

  ⇒ match each rule whose premise contains a newly added literal
- Matching itself can be expensive
- Database indexing allows $O(1)$ retrieval of known facts

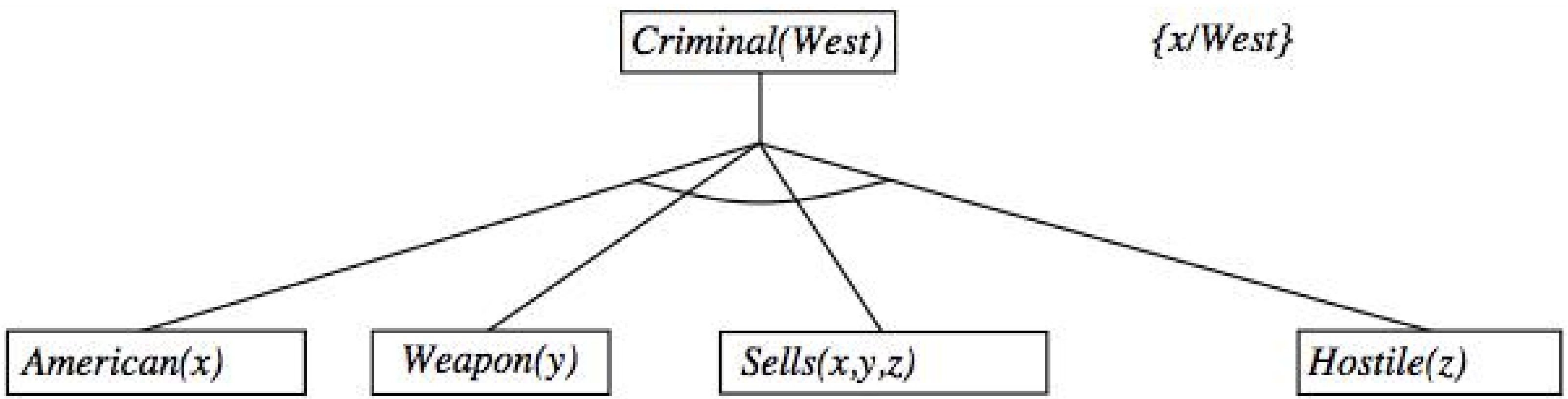
  e.g., query $Missile(x)$ retrives $Missile(M_1)$
- Matching conjunctive premise against known facts is NP-hard
- Forward chaining is widely used in deductive databases

Backward chaining algorithm

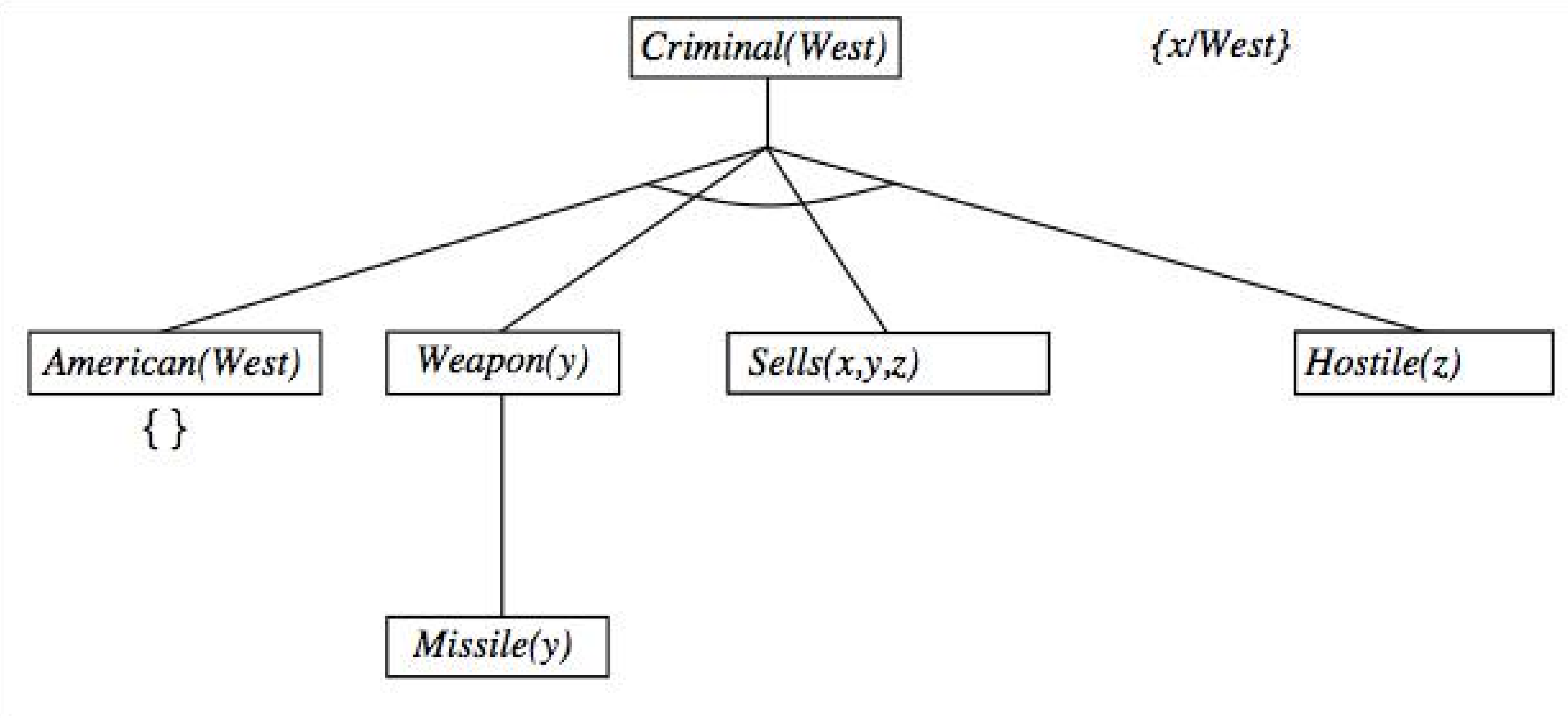
function FOL-BC-ASK(KB, $goals$, θ) **returns** a set of substitutions
  inputs: KB, a knowledge base
    $goals$, a list of conjuncts forming a query (θ already applied)
    θ, the current substitution, initially the empty substitution $\{\ \}$
  local variables: $answers$, a set of substitutions, initially empty

  if $goals$ is empty **then return** $\{\theta\}$
  $q' \leftarrow$ SUBST(θ, FIRST($goals$))
  for each sentence r **in** KB
    where STANDARDIZE-APART(r) $= (p_1 \wedge \ldots \wedge p_n \Rightarrow q)$
    and $\theta' \leftarrow$ UNIFY(q, q') succeeds
   $new\_goals \leftarrow [p_1, \ldots, p_n | \text{REST}(goals)]$
   $answers \leftarrow$ FOL-BC-ASK(KB, new_goals, COMPOSE(θ', θ)) $\cup$ $answers$
  return $answers$

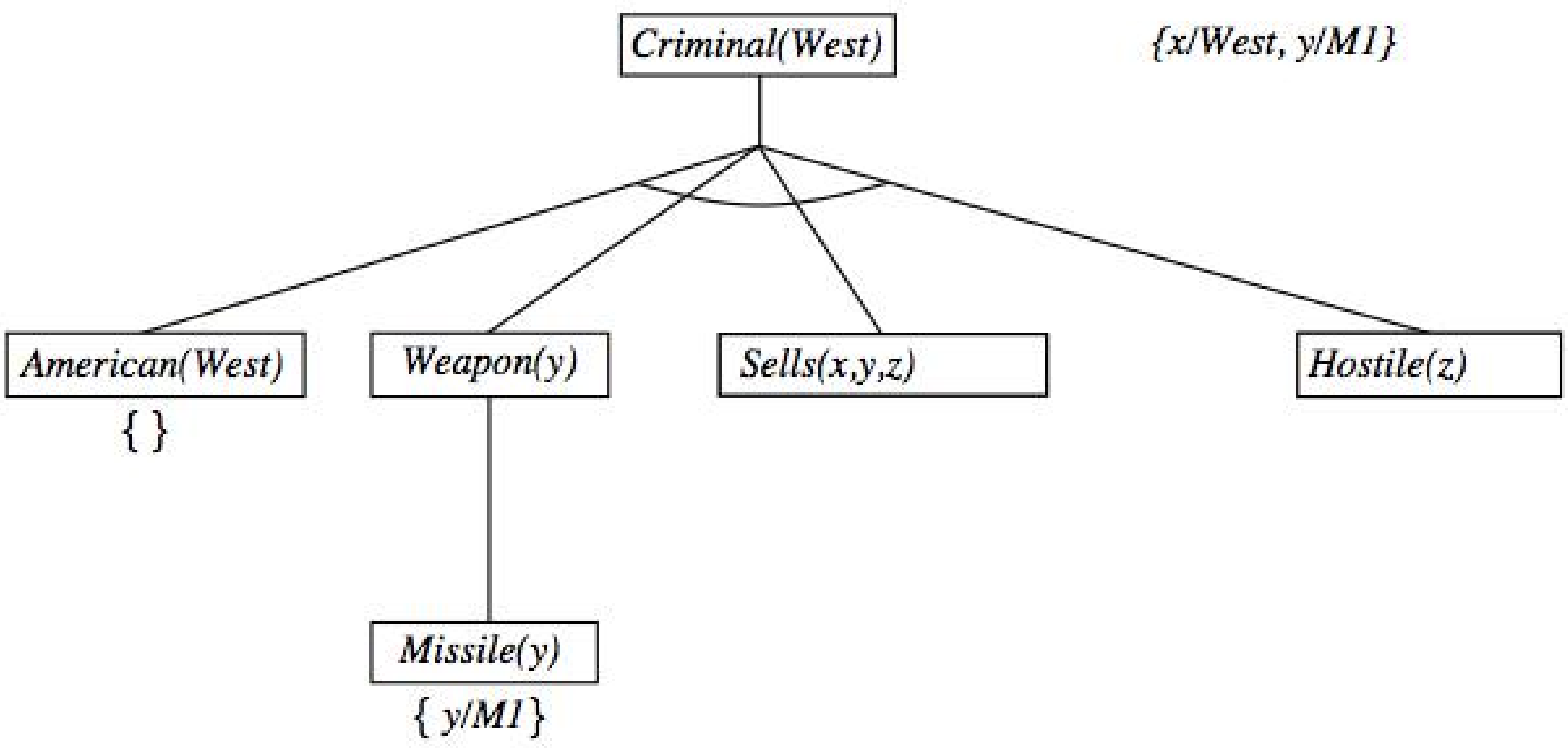
Backward chaining example



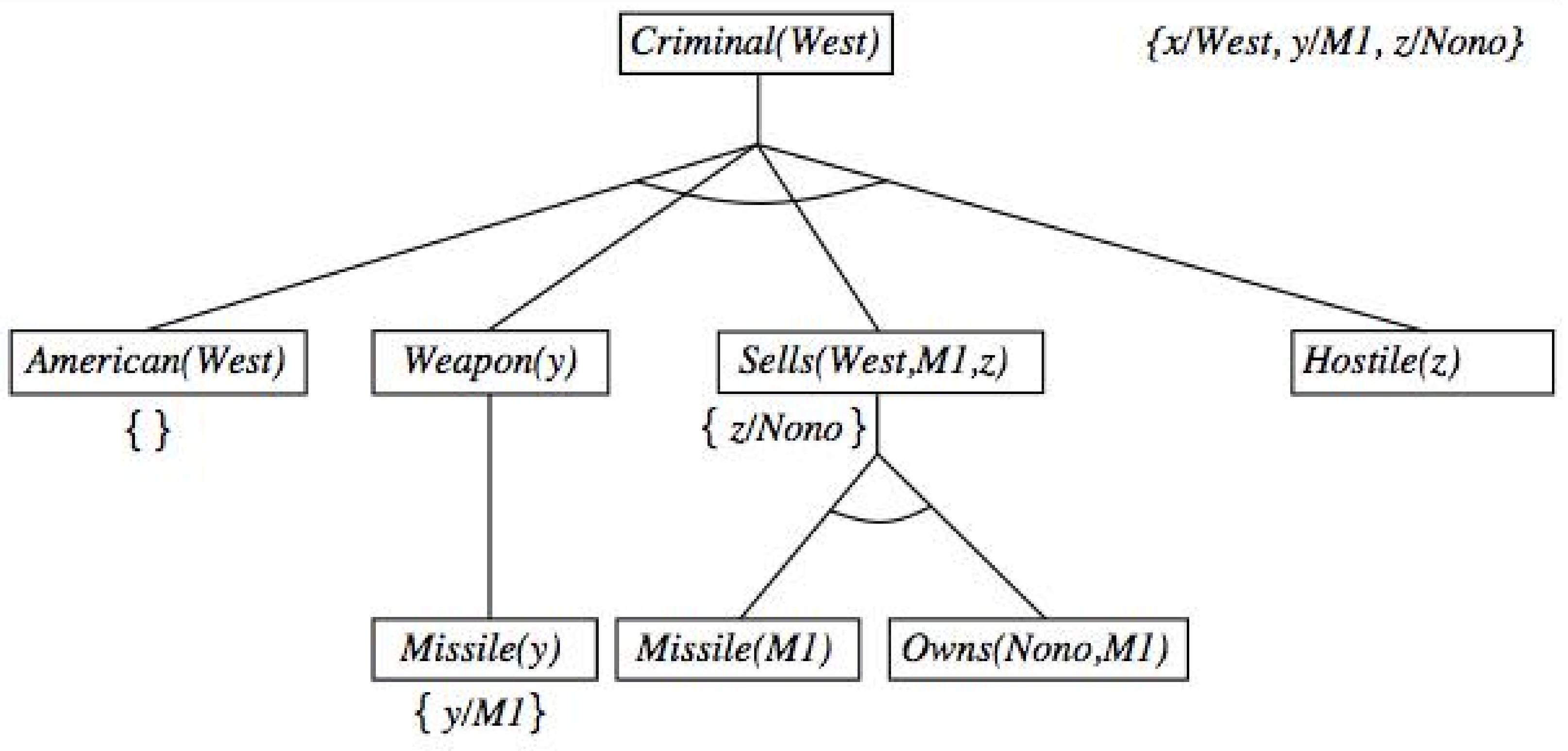
Backward chaining example



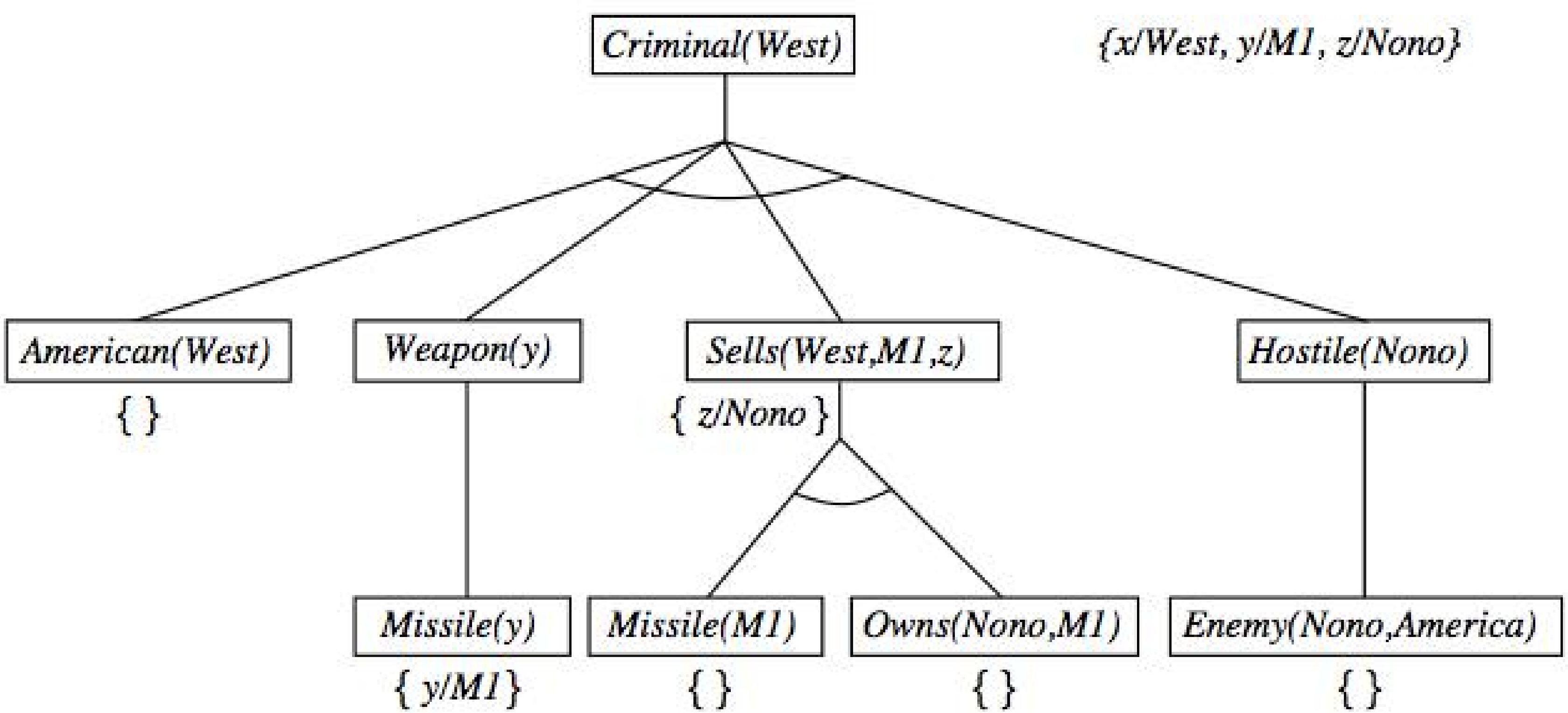
Backward chaining example



Backward chaining example



Backward chaining example



Properties of backward chaining

AND-OR search: AND for all premises; OR since the goal query can be proved by any rules

- Depth-first recursive proof search: space is linear in size of proof
- Incomplete due to infinite loops
  - ⇒ fix by checking current goal against every goal on stack
- Inefficient due to repeated sub-goals (both success and failure)
  - ⇒ fix using caching of previous results (extra space!)
- Widely used (without improvements !) for logic programming

Homework

- Reading Chapter 8.4

9.9 Suppose you are given the following axioms:

1. $0 \leq 3$.
2. $7 \leq 9$.
3. $\forall x \quad x \leq x$.
4. $\forall x \quad x \leq x + 0$.
5. $\forall x \quad x + 0 \leq x$.
6. $\forall x, y \quad x + y \leq y + x$.
7. $\forall w, x, y, z \quad w \leq y \land x \leq z \;\Rightarrow\; w + x \leq y + z$.
8. $\forall x, y, z \quad x \leq y \land y \leq z \;\Rightarrow\; x \leq z$

a. Give a backward-chaining proof of the sentence $7 \leq 3 + 9$. (Be sure, of course, to use only the axioms given here, not anything else you may know about arithmetic.) Show only the steps that leads to success, not the irrelevant steps.

b. Give a forward-chaining proof of the sentence $7 \leq 3 + 9$. Again, show only the steps that lead to success.

Logic Programming: Prolog

Logic programming

- Sound bite: computation as inference on logical KBs

	Logic programming	Ordinary programming
1.	Identify problem	Identify problem
2.	Assemble information	Assemble information
3.	Tea break	Figure out solution
4.	Encode information in KB	Program solution
5.	Encode problem instance as facts	Encode problem instance as data
6.	Ask queries	Apply program to data
7.	Find false facts	Debug procedural errors

- Should be easier to debug $Capital(NewYork, US)$ than $x := x + 2$

Prolog systems

Basis: backward chaining with Horn clauses + bells & whistles
Widely used in Europe, Japan (basis of 5th Generation project)
Compilation techniques ⇒ approaching a billion LIPS

Program = set of clauses = `head :- literal`$_1$`, ... literal`$_n$`.`

```
criminal(X) :- american(X), weapon(Y), sells(X,Y,Z), hostile(Z).
```

Efficient unification by open coding
Efficient retrieval of matching clauses by direct linking
Depth-first, left-to-right backward chaining
Built-in predicates for arithmetic etc., e.g., `X is Y*Z+3`
Closed-world assumption ("negation as failure")
e.g., given `alive(X) :- not dead(X).`
`alive(joe)` succeeds if `dead(joe)` fails

Prolog examples

- Depth-first search from a start state X:

```
dfs(X)  :— goal(x).
dfs(X)  :— successor(X,S),dfs(S).
```

- No need to loop over S: successor succeeds for each

- Appending two lists to produce a third:

```
append({}, Y, Y).
append( [X|L], Y, [X|Z])   :—  append(L, Y, Z).
```

第二个input放到第一个的右边

- Query: append(A, B, [1:2]) ?
- Answers: A=[]   B=[1, 2]
  A=[1]   B=[2]
  A=[1, 2] B=[]

Prolog systems

- Unification without the occur check, may results in unsound inferences. But almost never a problem in practice.
- Depth-first, left-to-right backward chaining search with no checks for infinite recursion.
- Built-in predicates for arithmetic etc., e.g., $X\ is\ Y * Z + 3$; no arbitrary equation solving. e.g., $5\ is\ X + Y$ fails
- Database semantics instead of first-order semantics
  - Closed-world assumption — anything not known to be true is false.
  - Unique-names assumption— different names refer to distinct objects.
  - Domain closure— only those mentioned exist in the domain.

Summary

- For small domains, we can use UI and EI to propositionalize the problem
- Unification identifies proper substitutions, more efficient than instantiation.
- Forward and backward chaining uses the generalized Modus Ponens on a sets of definite clauses.
- GMP is complete for definite clauses, where the entailment is semi-decidable; for Datalog KB (function-less definite clauses), entailment can be decided in $\mathcal{P}$-time (with forward-chaining)
- Backward chaining is used in logic programming systems; inferences are fast but may be unsound or incomplete.
- Resolution is sound and (refutation-) complete for FOL, using CNF KB

Homework

member(1,[1,2,3,4,5])
member(3,[1,2,3,4,5])　　要求：给出一个集合，列出其所有元素

subset([2,4],[1,2,3,4,5])　　要求：给出一个集合，列出其所有子集

Homework

事实的集合：有三种事实

is_relation (fact_ID, company_name, time, index-name, value)

supplier (fact_ID, time, company_A, company_B, k, value)

client (fact_ID, time, company_A, company_B, k, value)

检测冲突的事实（facts）

给定一个事实的集合，找出所有“冲突”的fact ID对

思考：什么情形“发生冲突”

fact2.pl