



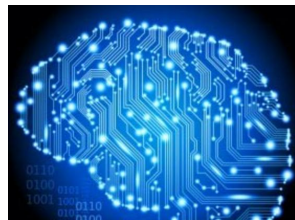
人工智能



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Knowledge 3



First-order Logic: syntax and semantics

First-order logic

- Whereas propositional logic assumes world contains **facts**, first-order logic (like natural language) assumes the world contains
- **Objects:** people, house, numbers, theories, Ronald McDonald, colors, baseball games, wars, centuries.....
- **Relations:** red, round, bogus, prime, multistoried..., brother of, bigger than, inside, part of, has color, occurred after, owns, comes between,...
- **Functions:** father of, best friend, third inning of, one more than, end of...

Syntax of FOL: Basic elements

Constants	<i>KingJohn, 2, UCB,...</i>
Predicates	<i>Brother, >,...</i>
Functions	<i>Sqrt, LeftLegOf,...</i>
Variables	<i>x, y, a, b,...</i>
Connectives	$\wedge \quad \vee \quad \neg \quad \Rightarrow \quad \Leftrightarrow$
Equality	$=$
Quantifiers	$\forall \quad \exists$

Atomic sentences

Atomic sentence = *predicate*(*term*₁, ..., *term*_{*n*})
or *term*₁ = *term*₂

Term = *function*(*term*₁, ..., *term*_{*n*})
or *constant* or *variable*

E.g., *Brother*(*KingJohn*, *RichardTheLionheart*)
 > (*Length*(*LeftLegOf*(*Richard*)), *Length*(*LeftLegOf*(*KingJohn*)))

Complex sentences

Complex sentences are made from atomic sentences using connectives

$$\neg S, \quad S_1 \wedge S_2, \quad S_1 \vee S_2, \quad S_1 \Rightarrow S_2, \quad S_1 \Leftrightarrow S_2$$

E.g., $Slibing(KingJohn, Richard) \Rightarrow Slibing(Richard, KingJohn)$

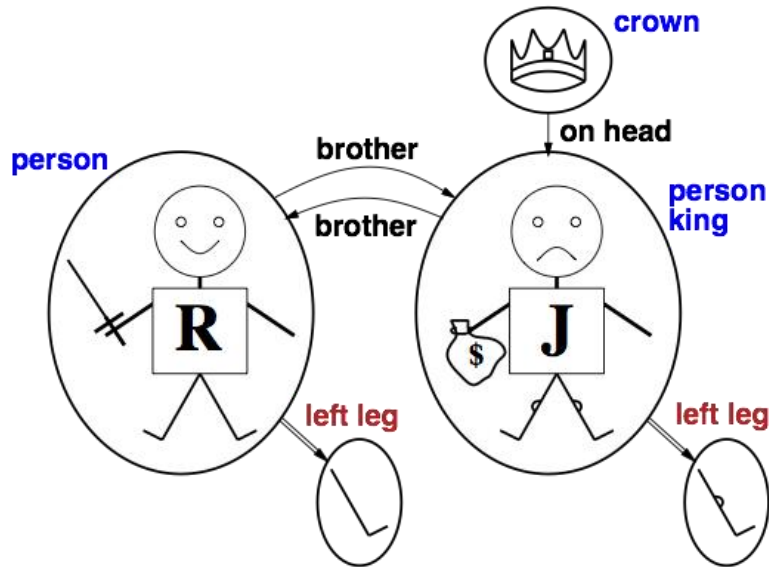
$$> (1,2) \vee \leq (1,2)$$

$$> (1,2) \wedge \neg > (1,2)$$

Truth in first-order logic

- Sentences are true with respect to a **model** and an **interpretation**
- Model contains ≥ 1 objects(**domain elements**) and relations among them
- Interpretation specifies referents for
 - constant symbols** \rightarrow **objects**
 - predicate symbols** \rightarrow **relations**
 - function symbols** \rightarrow **function**
- An atomic sentence *predicate*(*term*₁, ..., *term*_{*n*}) is true iff the **objects** referred to by *term*₁, ..., *term*_{*n*} are in the **relation** referred to by *predicate*

Models for FOL: Example



- Consider the interpretation in which
 - *Richard* → Richard the Lionheart
 - *John* → the evil King John
 - *Brother* → the brotherhood relation
-
- Under this interpretation, *Brother*(*Richard*, *John*) is true just in case Richard the Lionheart and the evil King John are in the brotherhood relation in the model

Models for FOL: Lots!

- Entailment in propositional logic can be computed by enumerating models

- We **can** enumerate the FOL models for a given KB vocabulary:

- For each number of domain elements n from 1 to ∞

For each k -ary predicate P_k in the vocabulary

For each possible k -ary relation on n objects

For each constant symbol C in the vocabulary

For each choice of referent for C from n objects ...

- Computing entailment by enumerating FOL models is not easy!

Universal quantification

- $\forall < \text{variables} > < \text{sentence} >$
- Everyone at Berkeley is smart:
 $\forall x \text{ At}(x, \text{Berkeley}) \Rightarrow \text{Smart}(x)$
- $\forall x P$ is true in a model m iff P is true with x being **each** possible object in the model
- **Roughly** speaking, equivalent to the **conjunction** of **instantiations** of P

$$\begin{aligned} & (\text{At}(\text{KingJohn}, \text{Berkeley}) \Rightarrow \text{Smart}(\text{KingJohn})) \\ & \wedge (\text{At}(\text{Richard}, \text{Berkeley}) \Rightarrow \text{Smart}(\text{Richard})) \\ & \wedge (\text{At}(\text{Berkeley}, \text{Berkeley}) \Rightarrow \text{Smart}(\text{Berkeley})) \\ & \wedge \dots \end{aligned}$$

A common mistake to avoid

- Typically, \Rightarrow is the main connective with \forall
- Common mistake: using \wedge as the main connective with \forall

$$\forall x \text{ } \textit{At}(x, \textit{Berkeley}) \wedge \textit{Smart}(x)$$

- Means “Everyone is at Berkeley and everyone is smart”

Existential quantification

- $\exists < \text{variables} > < \text{sentence} >$
- Someone at Stanford is smart:
 $\exists x \text{ At}(x, \text{Stanford}) \wedge \text{Smart}(x)$
- $\exists x P$ is true in a model m iff P is true with x being **some** possible object in the model
- **Roughly** speaking, equivalent to the **disjunction** of **instantiations** of P

$$\begin{aligned} & (\text{At}(\text{KingJohn}, \text{Stanford}) \wedge \text{Smart}(\text{KingJohn})) \\ \vee & (\text{At}(\text{Richard}, \text{Stanford}) \wedge \text{Smart}(\text{Richard})) \\ \vee & (\text{At}(\text{Berkeley}, \text{Stanford}) \wedge \text{Smart}(\text{Berkeley})) \\ \vee & \dots \end{aligned}$$

Another common mistake to avoid

- Typically, \wedge is the main connective with \exists
- Common mistake: using \Rightarrow as the main connective with \exists :
$$\exists x \text{ } At(x, Stanford) \Rightarrow Smart(x)$$
- Is true if there is anyone who is not at Stanford!

Properties of quantifiers

- $\forall x \forall y$ is the same as $\forall y \forall x$ (why??)
- $\exists x \exists y$ is the same as $\exists y \exists x$ (why??)
- $\exists x \forall y$ is not the same as $\forall y \exists x$
- $\exists x \forall y \text{ Loves}(x, y)$
“There is a person who loves everyone in the world”
- $\forall y \exists x \text{ Loves}(x, y)$
“Everyone in the world is loved by at least one person”
- Quantifier duality: each can be expressed using the other

$$\begin{array}{ll} \forall x \text{ Likes}(x, \text{IceCream}) & \neg \exists x \neg \text{Likes}(x, \text{IceCream}) \\ \exists x \text{ Likes}(x, \text{Broccoli}) & \neg \forall x \neg \text{Likes}(x, \text{Broccoli}) \end{array}$$

Fun with sentences

- Brothers are siblings
- $\forall x, y \text{ Brother}(x, y) \Rightarrow \text{Sibling}(x, y).$
- “Sibling” is symmetric
- $\forall x, y \text{ Sibling}(x, y) \Leftrightarrow \text{Sibling}(y, x).$
- One’s mother is one’s female parent
- $\forall x, y \text{ Mother}(x, y) \Leftrightarrow (\text{Female}(x) \wedge \text{Parent}(x, y)).$
- A first cousin is a child of a parent’s sibling
- $\forall x, y \text{ Firstcousin}(x, y) \Leftrightarrow \exists p, ps \text{ parent}(p, x) \wedge \text{Sibling}(ps, p) \wedge \text{Parent}(ps, y).$

Fun with sentences

- 不到长城非好汉。
- 到了长城就是好汉。
- 理发师只给那些不给自己理发的人理发
 - 理发师悖论

Equality

- $term_1 = term_2$ is true under a given interpretation if and only if $term_1$ and $term_2$ refer to the same object
- E.g., $1 = 2$ and $\forall x \times (Sqrt(x), Sqrt(x)) = x$ are satisfiable

$2 = 2$ is valid

- E.g., definition of (full) *Sibling* in terms of *Parent*:

$$\forall x, y \text{ Sibling}(x, y) \Leftrightarrow [\neg(x = y) \wedge \exists m, f \neg(m = f) \wedge \\ \text{Parent}(m, x) \wedge \text{Parent}(f, x) \wedge \text{Parent}(m, y) \wedge \text{Parent}(f, y)]$$

Back to the wumpus world again

- Define adjacency:

$$\forall x, y, a, b \text{ Adjacent}([x, y], [a, b]) \Leftrightarrow$$

$$(x = a \wedge (y = b - 1 \vee y = b + 1)) \vee (y = b \wedge (x = a - 1 \vee x = a + 1)).$$

- Location predictor, x is at square s at time t:

$$\forall t \text{ At}(WUMPUS, [2,2], t).$$

$$\forall x, s_1, s_2, t \text{ At}(x, s_1, t) \wedge \text{At}(x, s_2, t) \Rightarrow s_1 = s_2$$

- Define property for squares:

$$\forall s, t \text{ At}(AGENT, s, t) \wedge \text{Breeze}(t) \Rightarrow \text{Breezy}(s).$$

$$\forall s, t \text{ At}(PIT, s, t) \Rightarrow \text{Pit}(s).$$

$$\forall s, t \text{ At}(WUMPUS, s, t) \Rightarrow \text{Wumpus}(s).$$

- Rules of the Wumpus world can be defined.

$$\forall s \text{ Breezy}(s) \Leftrightarrow \exists r \text{ Adjacent}(r, s) \wedge \text{Pit}(r)$$

$$\forall t \text{ HaveArrow}(t + 1) \Leftrightarrow (\text{HaveArrow}(t) \wedge \neg \text{Action}(\text{shoot}, t)).$$

Short Summary

- First-order logic:
 - Objects and relations are semantic primitives
 - Syntax: constants, functions, predicates, equality, quantifiers
- Increased expressive power: sufficient to define Wumpus world

研究形式逻辑的目的是什么？

- 知识表示

- 将一组知识形式化为符号

- 知识推理

- 通过形式推演，自动推出结论
 - 可靠
 - 完备

Homework

8.15 Explain what is wrong with the following proposed definition of the set membership predicate \in :

$$\begin{aligned} &\forall x, s \quad x \in \{x|s\} \\ &\forall x, s \quad x \in s \Rightarrow \forall y \quad x \in \{y|s\} . \end{aligned}$$

8.20 Arithmetic assertions can be written in first-order logic with the predicate symbol $<$, the function symbols $+$ and \times , and the constant symbols 0 and 1 . Additional predicates can also be defined with biconditionals.

- a. Represent the property “ x is an even number.”
- b. Represent the property “ x is prime.”
- c. Goldbach’s conjecture is the conjecture (unproven as yet) that every even number is equal to the sum of two primes. Represent this conjecture as a logical sentence.