人工智能

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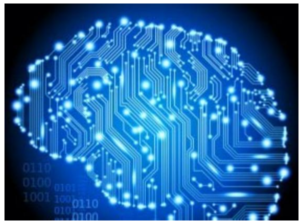
Knowledge 2



RESOLUTION 归结原理

Resolution （消解、归结）

- **Conjunctive Normal Form** (CNF—universal)

  conjunction of **disjunctions of literals** (clauses)

  E.g., $(A \vee \neg B) \wedge (B \vee \neg C \vee \neg D)$
- **Resolution** inference rule (for CNF): complete for propositional logic

$$\frac{\ell_1 \vee \ldots \vee \ell_k, \qquad m_1 \vee \ldots \vee m_n}{\ell_1 \vee \ldots \vee \ell_{i-1} \vee \ell_{i+1} \vee \ldots \vee \ell_k \vee m_1 \vee \ldots \vee m_{j-1} \vee m_{j+1} \vee \ldots \vee m_n}$$

where ℓ_i and m_j are complementary literals. E.g.,

$$\frac{P_{1,3} \vee P_{2,2}, \qquad \neg P_{2,2}}{P_{1,3}}$$

- Resolution is sound and complete for propositional logic

J. A. Robinson. A machine-oriented logic based on the resolution principle. **Journal of the ACM**, 1965, 12(1):23-41

Conversion to CNF

- $B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$
- 1. Elimate$\Leftrightarrow$,replacing $\alpha \Leftrightarrow \beta$ with$(\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)$

  $$(B_{1,1} \Rightarrow (P_{1,2} \vee P_{2,1})) \wedge ((P_{1,2} \vee P_{2,1}) \Rightarrow B_{1,1})$$
- 2.Elimate $\Rightarrow$,replacing $\alpha \Rightarrow \beta$ with $\neg\alpha \vee \beta$.

  $$(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge (\neg(P_{1,2} \vee P_{2,1}) \vee B_{1,1})$$
- 3.Move $\neg$ inwards using de Morgan's rules and double-negation

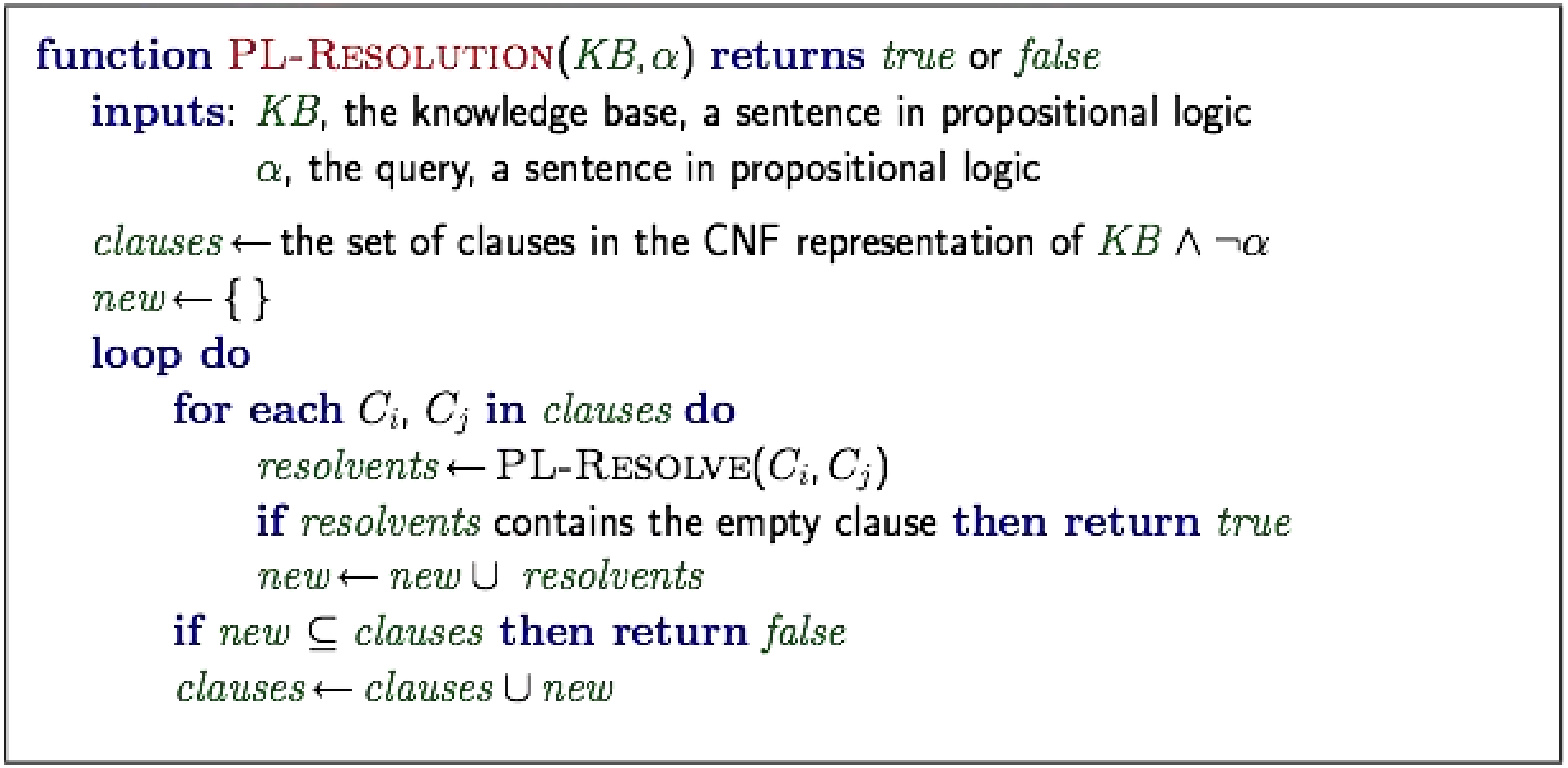
  $$(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge ((\neg P_{1,2} \wedge \neg P_{2,1}) \vee B_{1,1})$$
- 4.Apply distributivity law ($\vee$ over $\wedge$) and flatten:

  $$(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge (\neg P_{1,2} \vee B_{1,1}) \wedge (\neg P_{2,1} \vee B_{1,1})$$

多项式时间复杂度

Resolution algorithm

- Proof by contradiction, i.e., show $KB \wedge \neg\alpha$ unsatisfiable



```
function PL-RESOLUTION($KB, \alpha$) returns true or false
  inputs: $KB$, the knowledge base, a sentence in propositional logic
          $\alpha$, the query, a sentence in propositional logic

  clauses ← the set of clauses in the CNF representation of $KB \wedge \neg\alpha$
  new ← { }
  loop do
     for each $C_i$, $C_j$ in clauses do
        resolvents ← PL-RESOLVE($C_i, C_j$)
        if resolvents contains the empty clause then return true
        new ← new ∪ resolvents
     if new ⊆ clauses then return false
     clauses ← clauses ∪ new
```

Resolution

- 证明：若KB $\nvdash \emptyset$

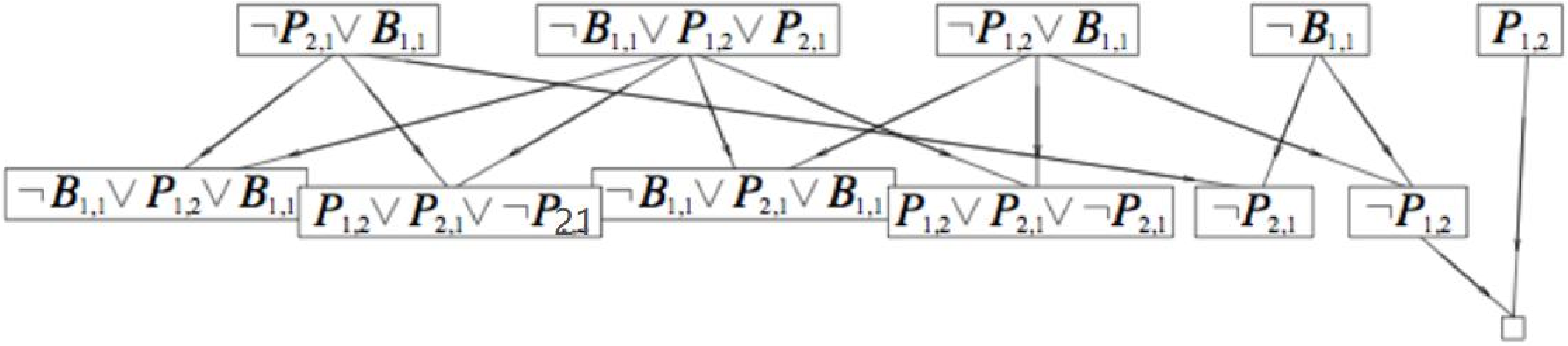
$$\text{KB} \vdash \alpha \text{ 当且仅当 } \{\text{KB}, \neg\alpha\} \vdash \emptyset$$

其中$\vdash$仅使用归结法则获得新子句

Resolution example

- 证明：KB ⊢ α

$$KB = \left(B_{1,1} \Leftrightarrow \left(P_{1,2} \vee P_{2,1}\right)\right) \wedge \neg B_{1,1} \qquad \alpha = \neg P_{1,2}$$



$\neg P_{2,1} \vee B_{1,1}$
$\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}$
$\neg P_{1,2} \vee B_{1,1}$
$\neg B_{1,1}$
$P_{1,2}$
$\neg B_{1,1} \vee P_{1,2} \vee B_{1,1}$
$P_{1,2} \vee P_{2,1} \vee \neg P_{2,1}$
$\neg B_{1,1} \vee P_{2,1} \vee B_{1,1}$
$P_{1,2} \vee P_{2,1} \vee \neg P_{2,1}$
$\neg P_{2,1}$
$\neg P_{1,2}$
$\square$

Resolution is sound

- 证明：**Resolution规则是可靠的。即证明：**

- $(l_1 \vee \cdots \vee l_k) \wedge (m_1 \vee \cdots \vee m_n)$
  $\vDash (l_1 \vee \cdots \vee l_{i-1} \vee l_{i+1} \vee \cdots \vee l_k \vee m_1 \vee \cdots \vee m_{j-1} \vee m_{j+1} \vee \cdots \vee m_n)$

Resolution规则：

$$\frac{\ell_1 \vee \ldots \vee \ell_k, \qquad m_1 \vee \ldots \vee m_n}{\ell_1 \vee \ldots \vee \ell_{i-1} \vee \ell_{i+1} \vee \ldots \vee \ell_k \vee m_1 \vee \ldots \vee m_{j-1} \vee m_{j+1} \vee \ldots \vee m_n}$$

where ℓ_i and m_j are complementary literals.

Resolution is complete

- Soundness is not surprising since inference rules are sound(check the truth table)
- Resolution is also complete.
  - Resolution closure *RC(S)* of a set of clauses *S*: the set of all clauses derivable by resolution and all the sentences in *S*
  - Final value of *clauses* in PL_RESOLUTION is *RC(S)*
  - *RC(S)* is finite, and hence PL_RESOLUTION always terminates.

Ground Resolution Theorem

S is unsatisfiable $\Rightarrow RC(S)$ contains the empty clause.

$$S = \{KB, \neg\alpha\}$$

$$KB \vDash \alpha$$

Ground resolution theorem

$RC(S)$ does not contain the empty set $\Rightarrow$ S is satisfiable.

- 证明：针对**S**中的原子命题$R_1, R_2, \cdots, R_l$，我们构造如下的model：
- 首先，因为***RC(S)***中不包含空集，即***RC(S)***中不包含永假的子句。

从$\boldsymbol{i} = \mathbf{1}$ 到 $\boldsymbol{l}$，顺序的指派$R_1, R_2, \cdots, R_l$的真值：

如果***RC(S)***中包含一个子句，此子句包含$\neg R_i$，且此子句的其它文字都已经被指派为**False**（在之前的步骤中进行的）或不包含其它文字，则把$\boldsymbol{R_i}$指派为**False**；

否则，把R_i指派为**True**

- 我们用反证法证明：这个真值指派使得**RC(S)**中的子句都为真。假设，在此过程的第$\boldsymbol{i}$步，我们这样来指派$\boldsymbol{R_i}$使得某个子句**C**为**False**，且假设这是首次出现**False**的子句；此时，子句**C**只能是如下两种形式之一：

�����∨�����⋯∨�����∨ R_i 或者 �����∨�
����⋯∨�����∨ $\neg R_i$

- 显然，如果**RC(S)**中只包含以上两个子句之一，子句**C**是不会在此真值指派中为**False**的。因此， **RC(S)**此时应该同时包含了以上两个子句。
- 以上两个子句显然是满足归结条件的，也就是说，它归结后的子句也应该在**RC(S)**中；同时，该子句已经被指派为**False**了；这与我们之前的假设矛盾。

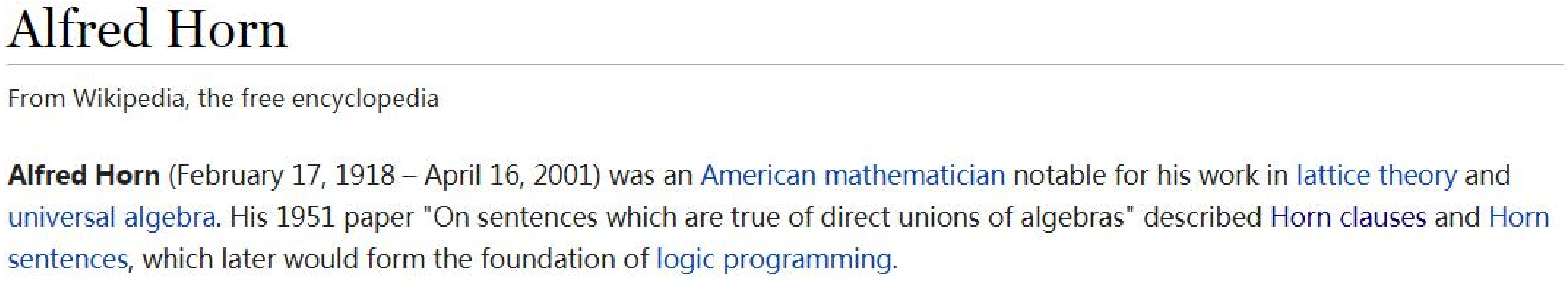
Process of Resolution: Search

Path-based Search: goal, actions
Requirement: optimal solution in terms of the number of resolution steps

Homework: design a heuristic for A* search

Requirements: formally define what the states are (single clause or a set of clauses (preferred))

Inference over
Horn and Definite Clauses


Alfred Horn

From Wikipedia, the free encyclopedia

Alfred Horn (February 17, 1918 – April 16, 2001) was an American mathematician notable for his work in lattice theory and universal algebra. His 1951 paper "On sentences which are true of direct unions of algebras" described Horn clauses and Horn sentences, which later would form the foundation of logic programming.

https://en.wikipedia.org/wiki/Alfred_Horn

Horn and Definite Clauses

什么是正文字和负文字？

- The completeness of resolution is good.
- For many real-world applications, if we add some restrictions, more efficient inference can be achieved.
- Definite clause: a disjunction of literals where exactly one is positive
- Horn clause: a disjunction of literals where at most one is positive
- Horn clauses are closed under resolution:

  Resolving two Horn clauses yields a Horn clause.
- Another way to view Horn clauses:

  TRUE $\Rightarrow$ symbol

  (Conjunction of symbols) $\Rightarrow$ symbol
- Deciding entailment with Definite clauses can be done in linear time!

  Forward and backward chaining

缩小propositional logic的表达范围，以换取更好的inference的时间效率

Forward and backward chaining

- Horn Form (restricted)

  KB = conjunction of definite clauses

  definite clause =

  - Proposition symbol; or
  - (conjunction of symbols) $\Longrightarrow$ symbol

  E.g., $C \wedge (B \Rightarrow A) \wedge (C \wedge D \Rightarrow B)$

- Modus Ponens (for Horn Form): complete for Horn KBs

（肯定式推理）

$$\frac{\alpha_1, \ldots, \alpha_n, \qquad \alpha_1 \wedge \ldots \wedge \alpha_n \Rightarrow \beta}{\beta}$$

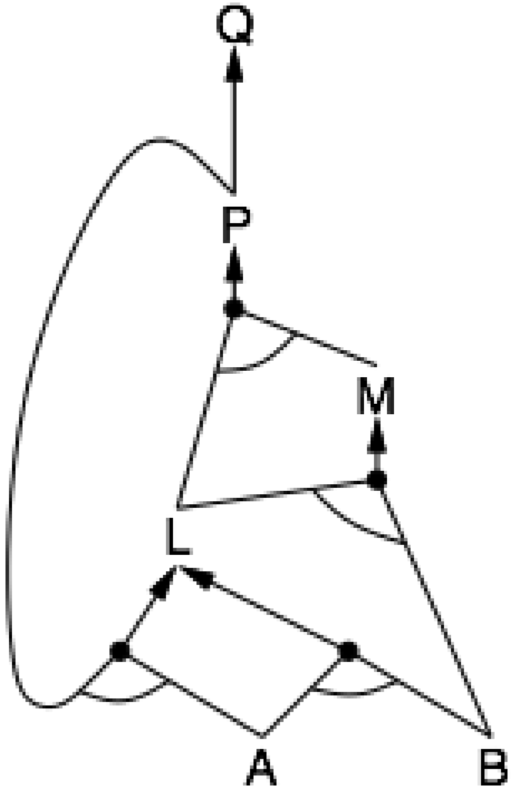
归结的一种形式

- Can be used with forward chaining or backward chaining. These algorithms are very natural and run in linear time

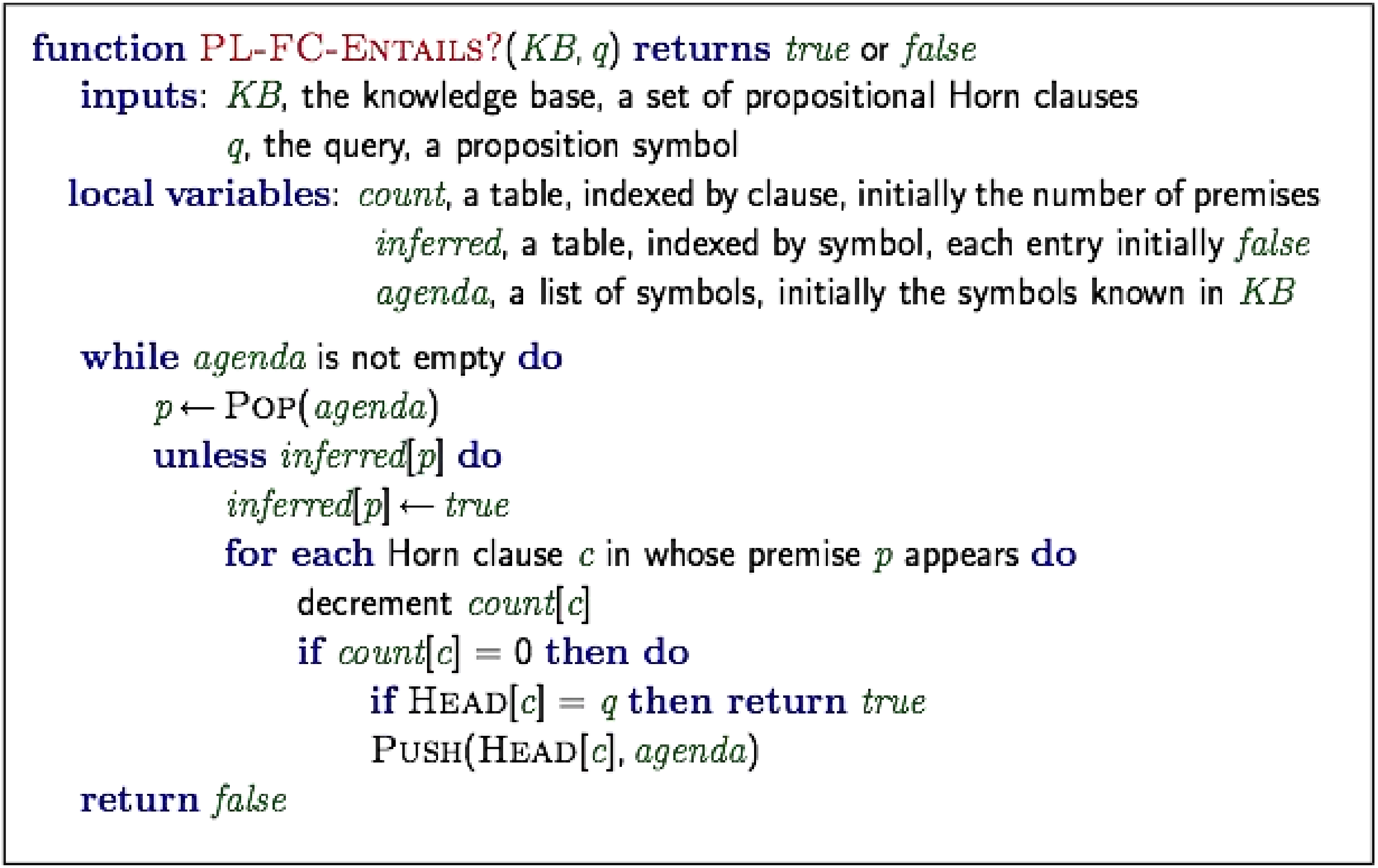
Forward chaining （前向推理）

- Idea : fire any rule whose premises are satisfied in the *KB*, add its conclusion to the *KB*, until query is found

$$P \Rightarrow Q$$
$$L \wedge M \Rightarrow P$$
$$B \wedge L \Rightarrow M$$
$$A \wedge P \Rightarrow L$$
$$A \wedge B \Rightarrow L$$
$$A$$
$$B$$



Forward chaining algorithm

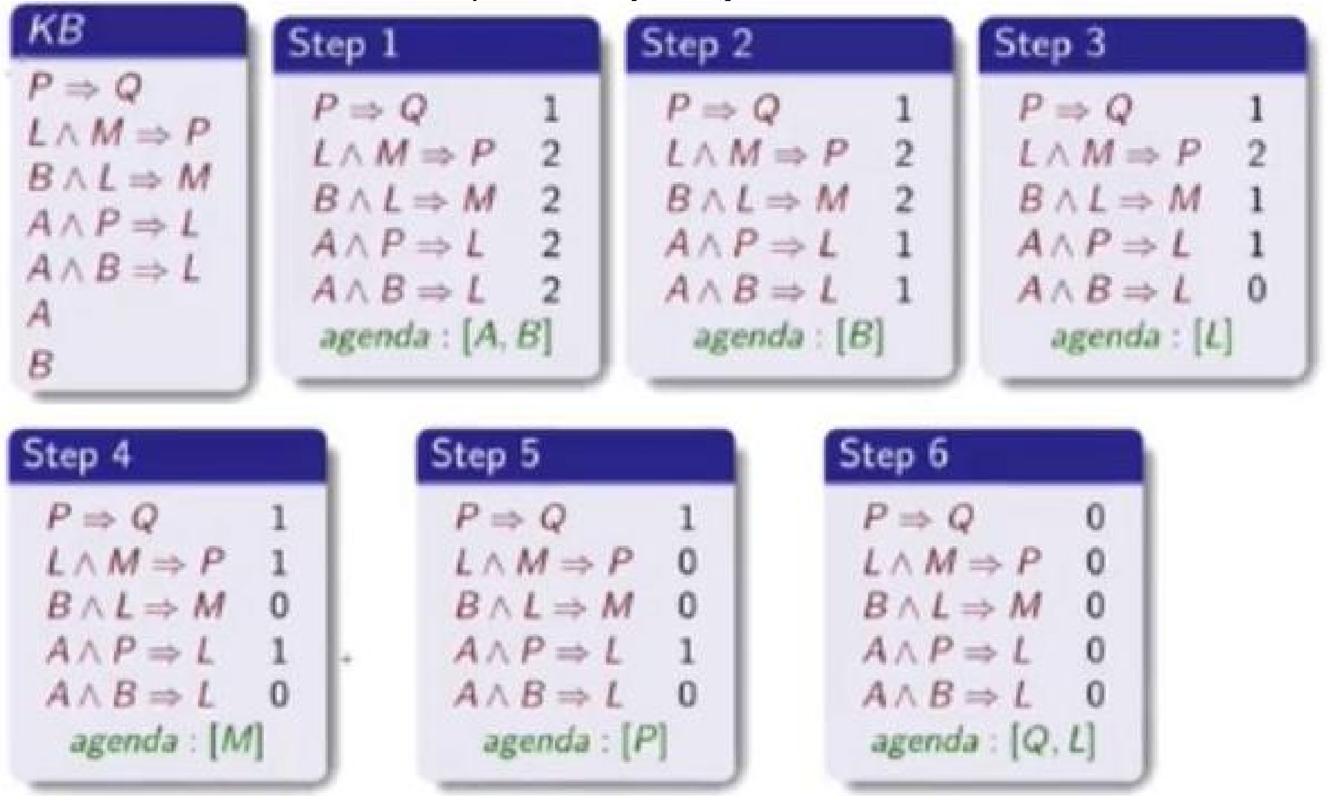


```
function PL-FC-ENTAILS?($KB$, $q$) returns true or false
  inputs: $KB$, the knowledge base, a set of propositional Horn clauses
          $q$, the query, a proposition symbol
  local variables: count, a table, indexed by clause, initially the number of premises
                   inferred, a table, indexed by symbol, each entry initially false
                   agenda, a list of symbols, initially the symbols known in $KB$

  while agenda is not empty do
      $p \leftarrow$ POP(agenda)
      unless inferred[$p$] do
          inferred[$p$] $\leftarrow$ true
          for each Horn clause $c$ in whose premise $p$ appears do
              decrement count[$c$]
              if count[$c$] $= 0$ then do
                  if HEAD[$c$] $= q$ then return true
                  PUSH(HEAD[$c$], agenda)
  return false
```

Forward chaining example

- Idea : fire any rule whose premises are satisfied in the KB, add its conclusion to the KB, until query is found



KB

$P \Rightarrow Q$
$L \wedge M \Rightarrow P$
$B \wedge L \Rightarrow M$
$A \wedge P \Rightarrow L$
$A \wedge B \Rightarrow L$
A
B

Step 1

$P \Rightarrow Q$	1
$L \wedge M \Rightarrow P$	2
$B \wedge L \Rightarrow M$	2
$A \wedge P \Rightarrow L$	2
$A \wedge B \Rightarrow L$	2

agenda : [A, B]

Step 2

$P \Rightarrow Q$	1
$L \wedge M \Rightarrow P$	2
$B \wedge L \Rightarrow M$	2
$A \wedge P \Rightarrow L$	1
$A \wedge B \Rightarrow L$	1

agenda : [B]

Step 3

$P \Rightarrow Q$	1
$L \wedge M \Rightarrow P$	2
$B \wedge L \Rightarrow M$	1
$A \wedge P \Rightarrow L$	1
$A \wedge B \Rightarrow L$	0

agenda : [L]

Step 4

$P \Rightarrow Q$	1
$L \wedge M \Rightarrow P$	1
$B \wedge L \Rightarrow M$	0
$A \wedge P \Rightarrow L$	1
$A \wedge B \Rightarrow L$	0

agenda : [M]

Step 5

$P \Rightarrow Q$	1
$L \wedge M \Rightarrow P$	0
$B \wedge L \Rightarrow M$	0
$A \wedge P \Rightarrow L$	1
$A \wedge B \Rightarrow L$	0

agenda : [P]

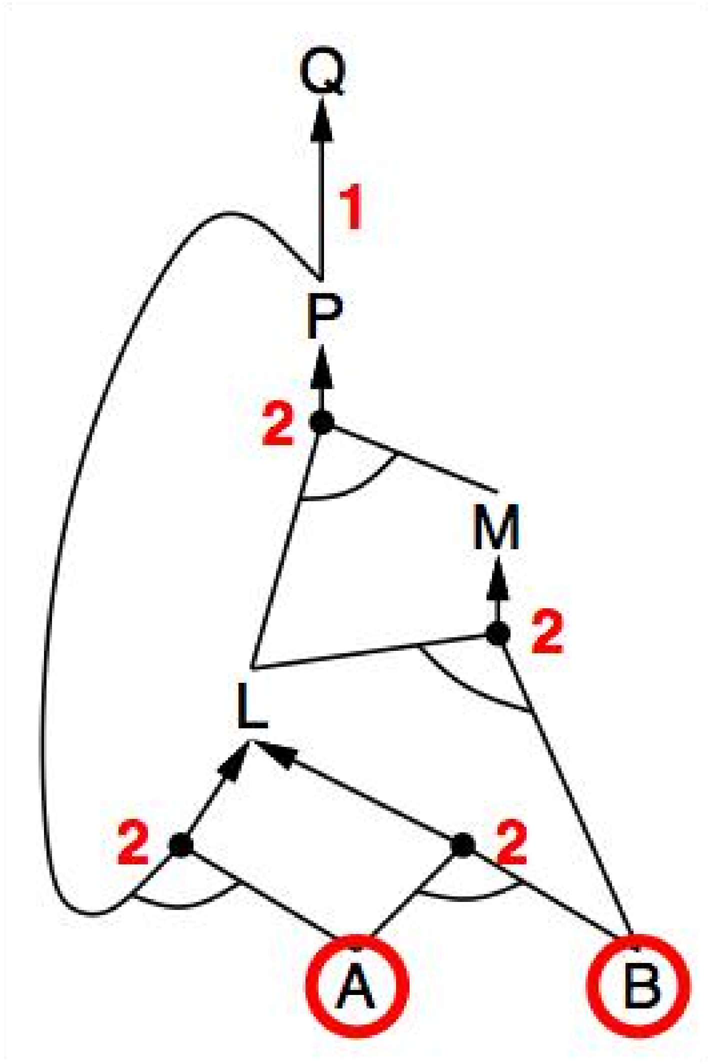
Step 6

$P \Rightarrow Q$	0
$L \wedge M \Rightarrow P$	0
$B \wedge L \Rightarrow M$	0
$A \wedge P \Rightarrow L$	0
$A \wedge B \Rightarrow L$	0

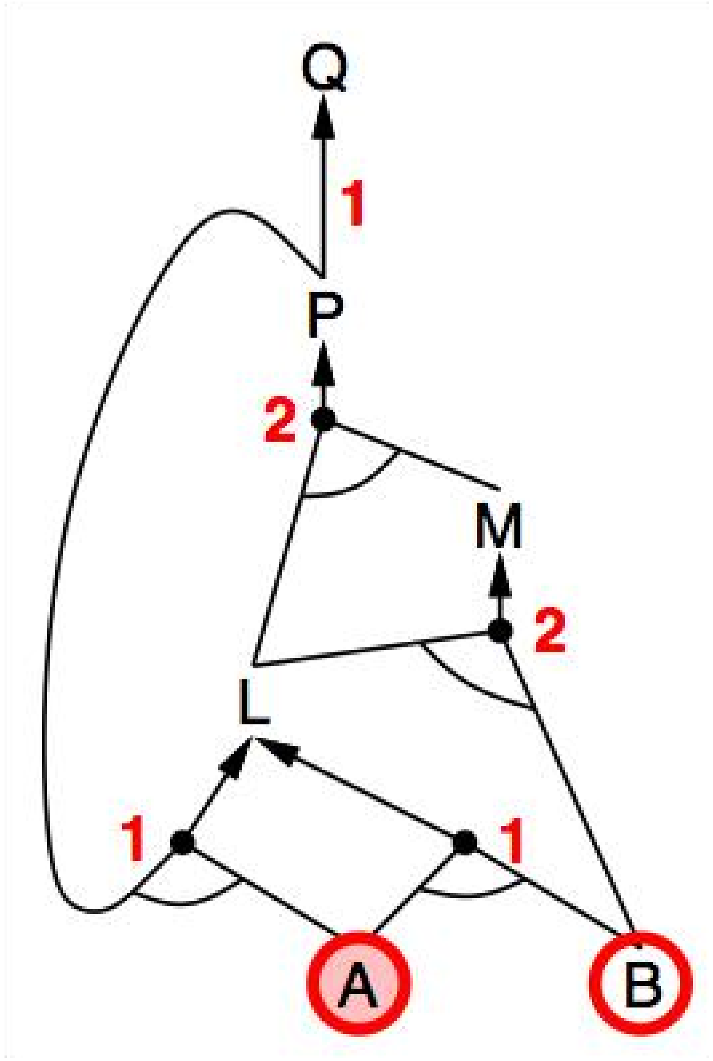
agenda : [Q, L]

Linear to the number of what?

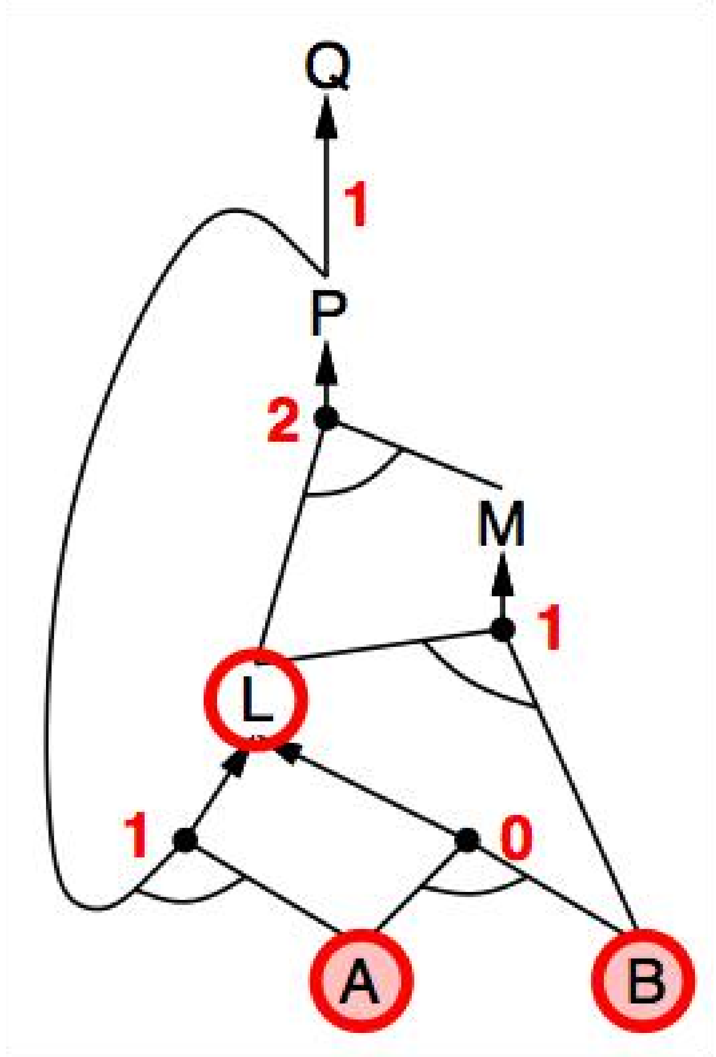
Forward chaining example



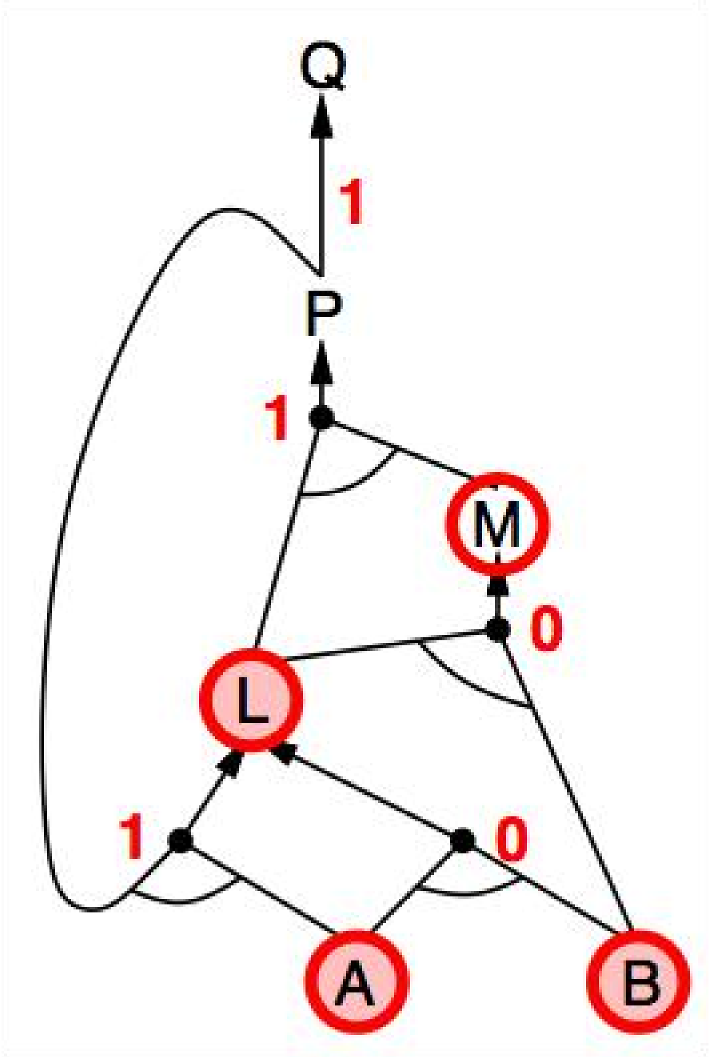
Forward chaining example



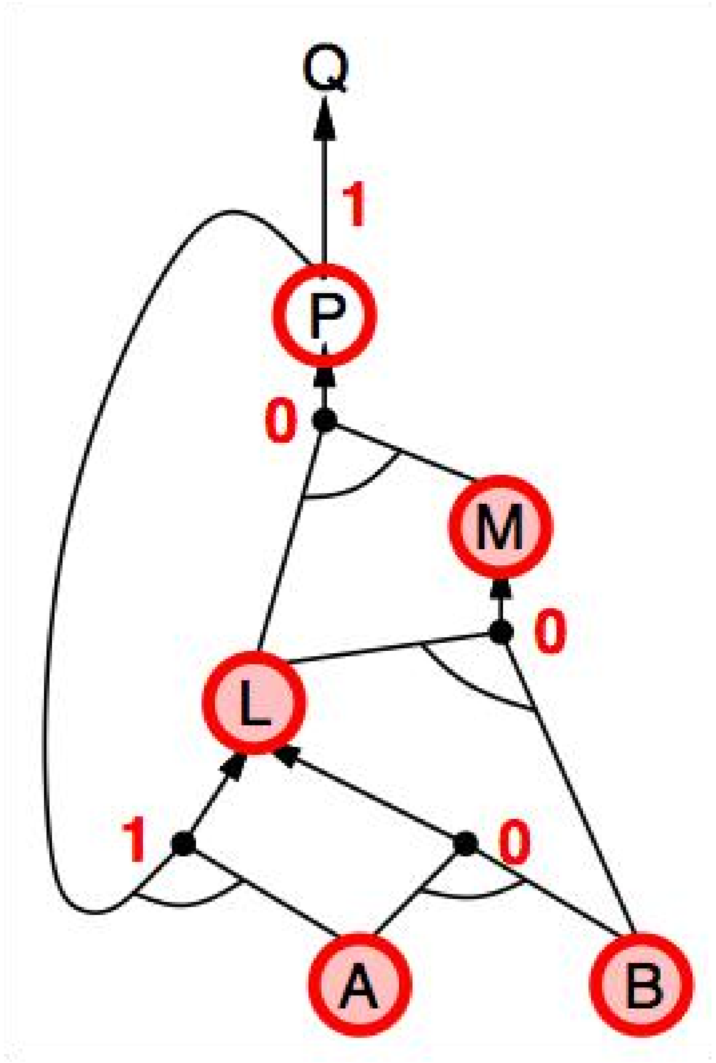
Forward chaining example



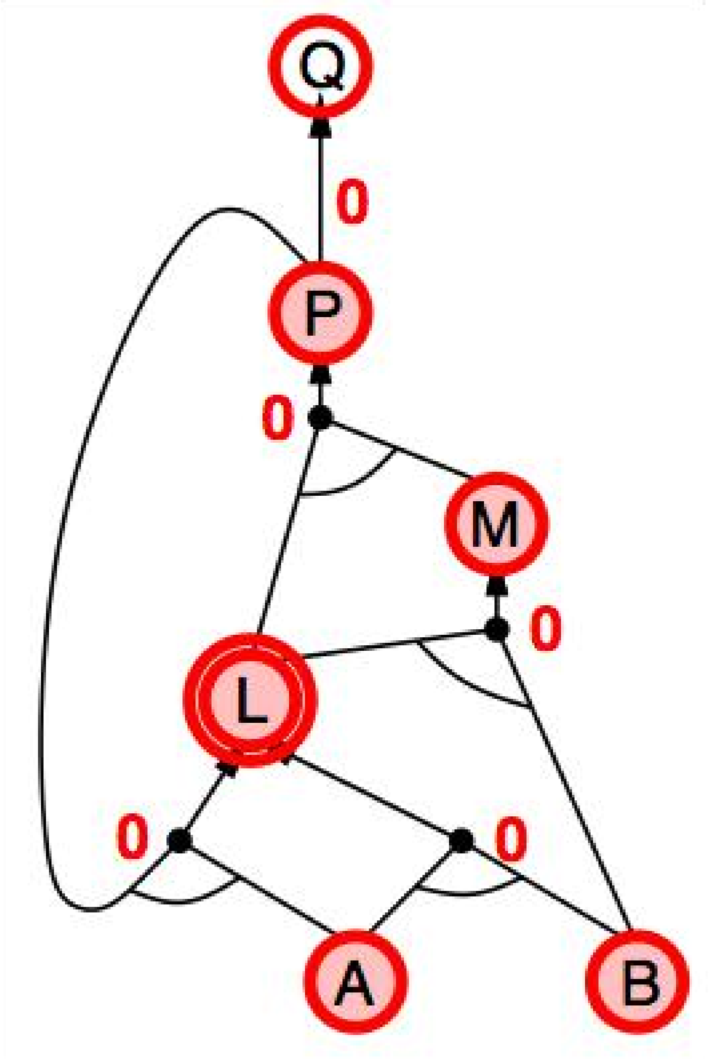
Forward chaining example



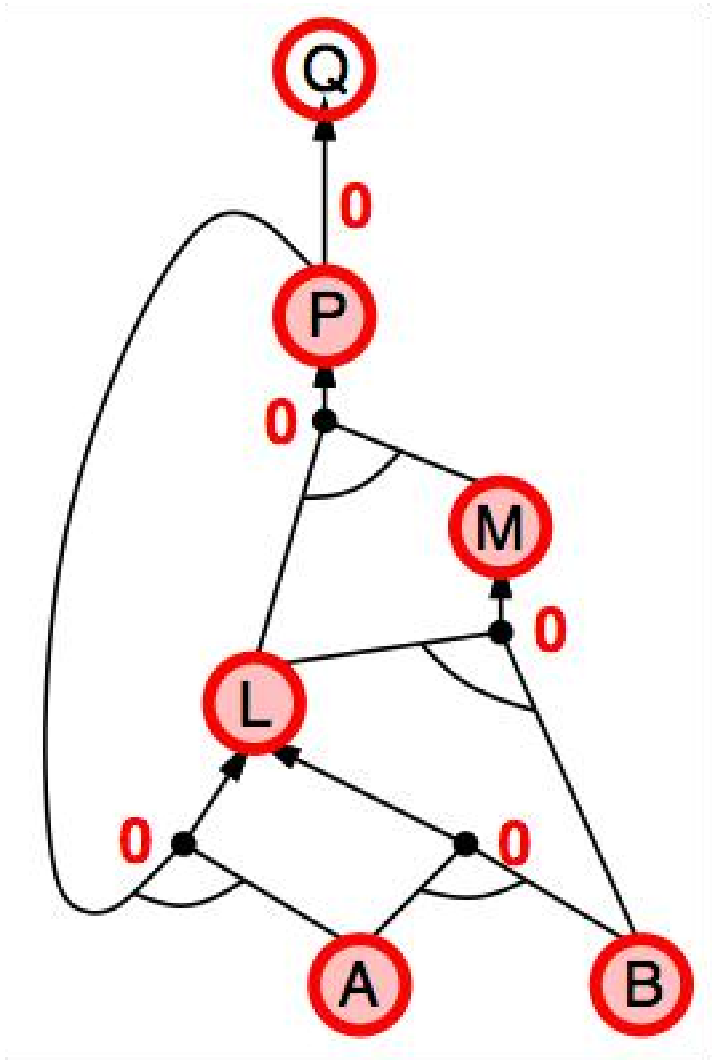
Forward chaining example



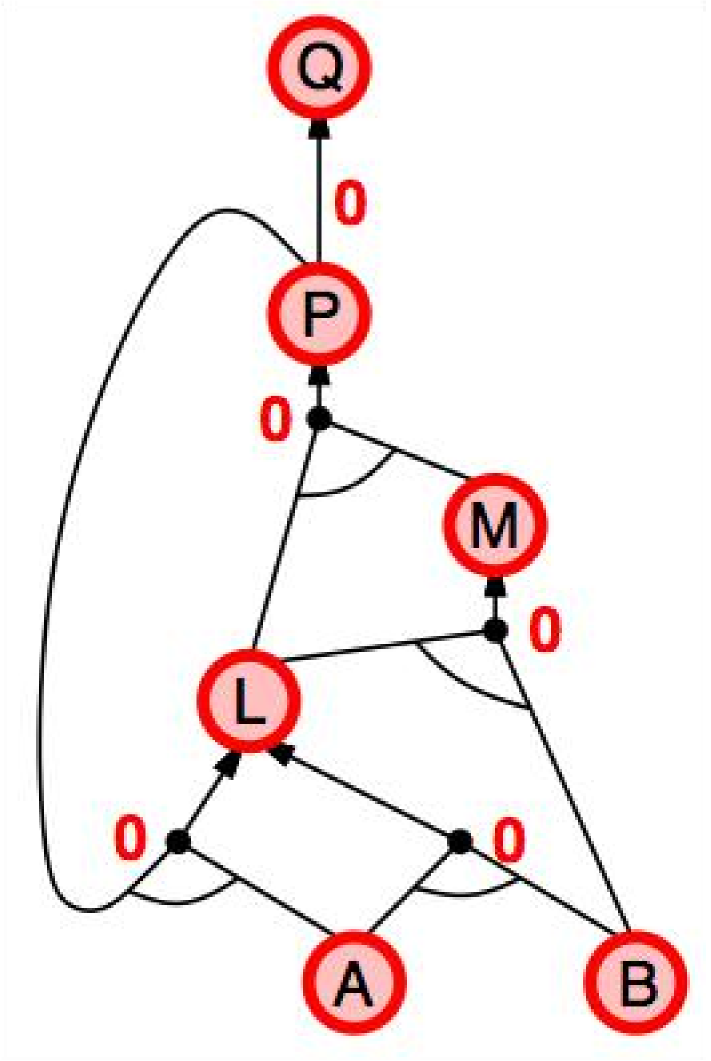
Forward chaining example



Forward chaining example



Forward chaining example



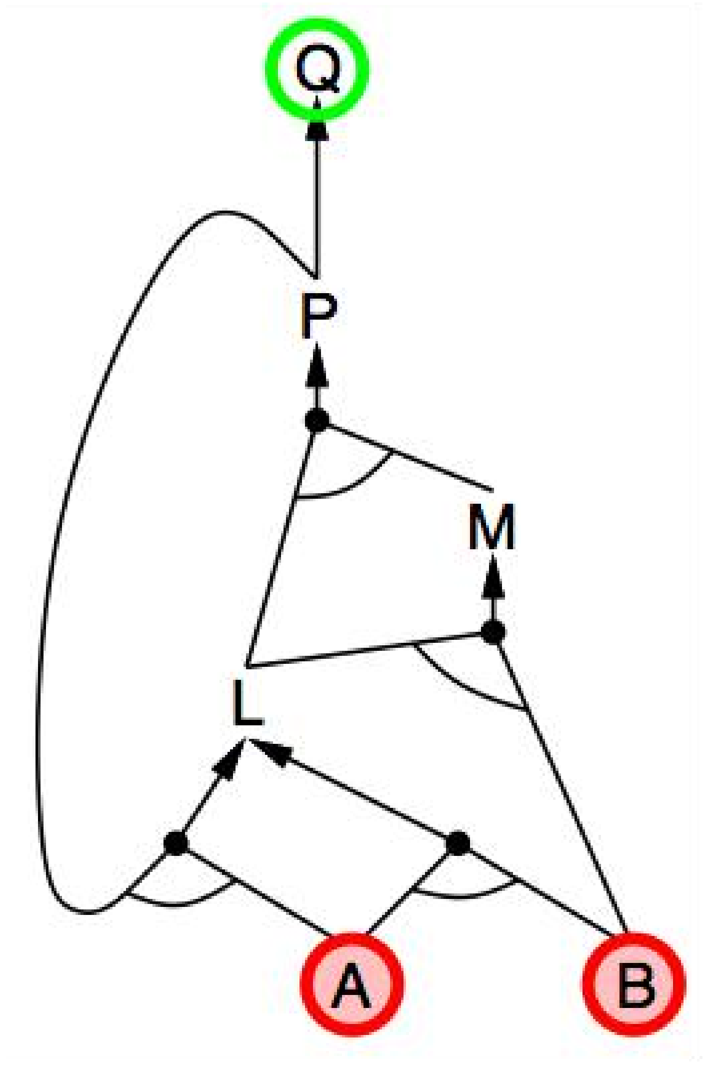
Backward chaining（后向推理）

- Idea: work backwards from the query *q*:

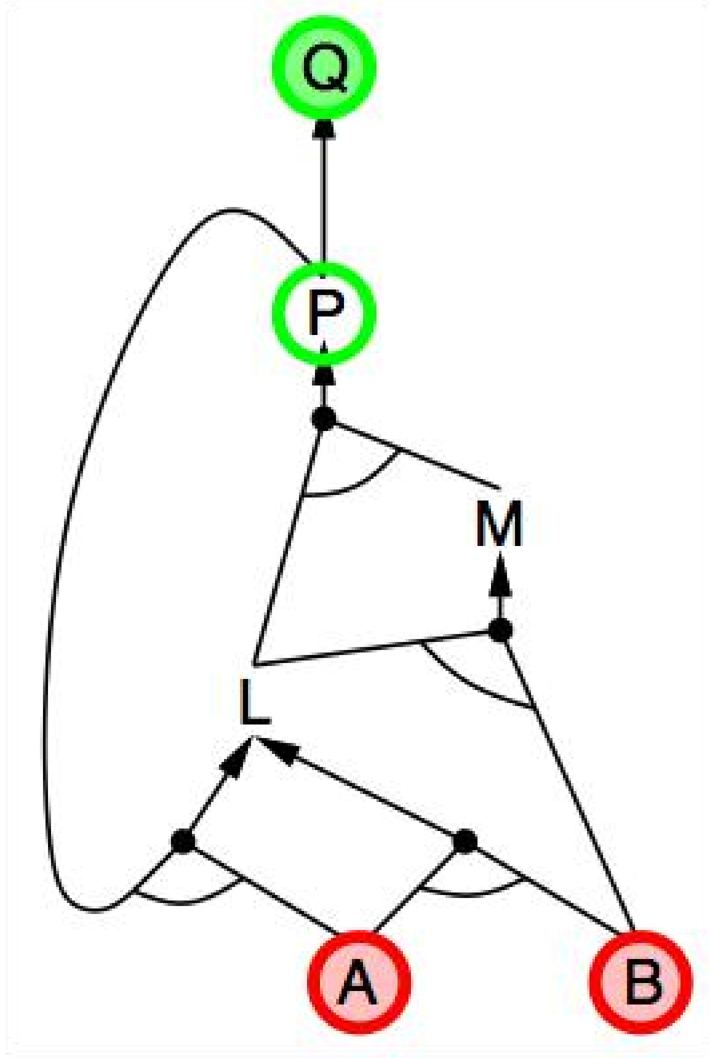
  to prove *q* by BC,

  - Check if q is known already, or
  - prove by BC all premises of some rule concluding *q*
- Avoid loops: check if new subgoal is already on the goal stack
- Avoid repeated work: check if new subgoal
  - 1) has already been proved true, or
  - 2) has already failed

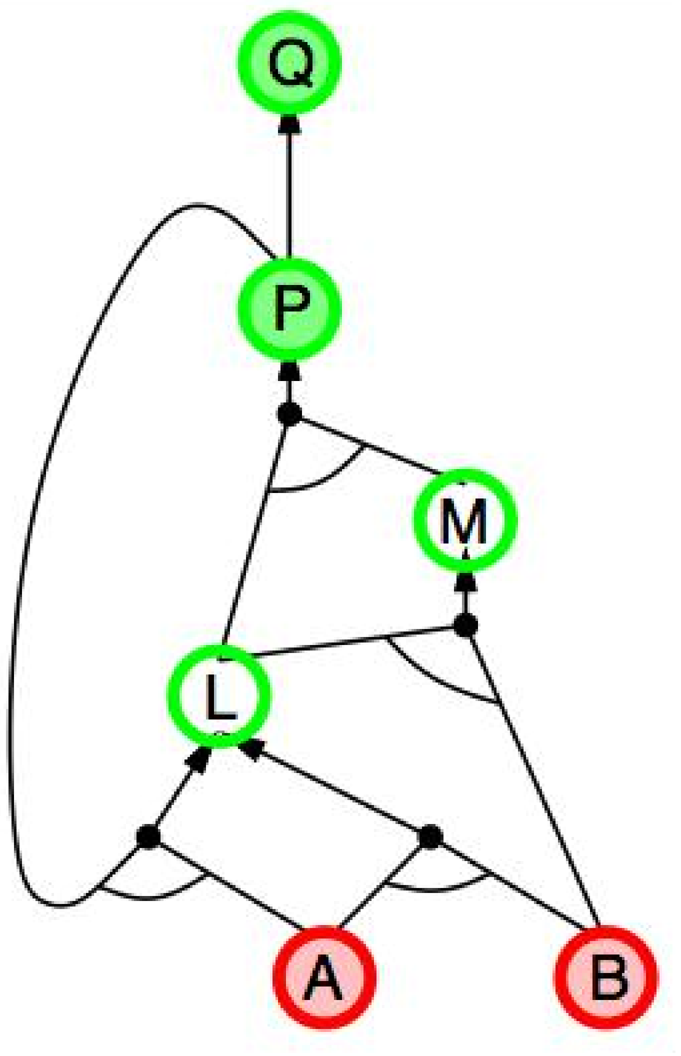
Backward chaining example



Backward chaining example



Backward chaining example



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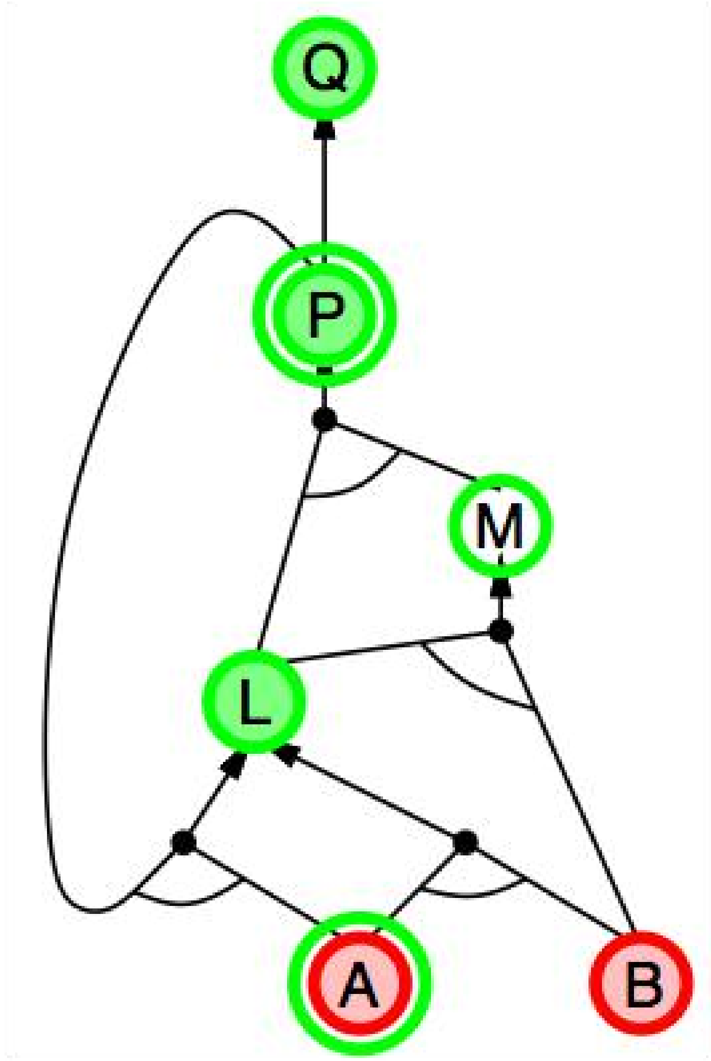

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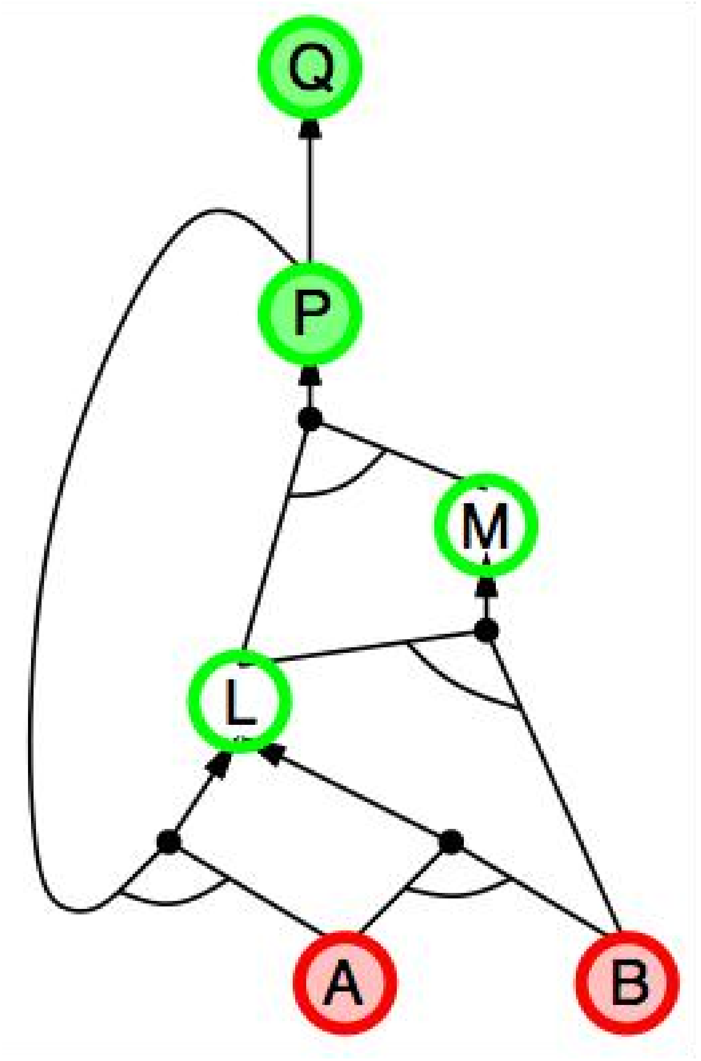

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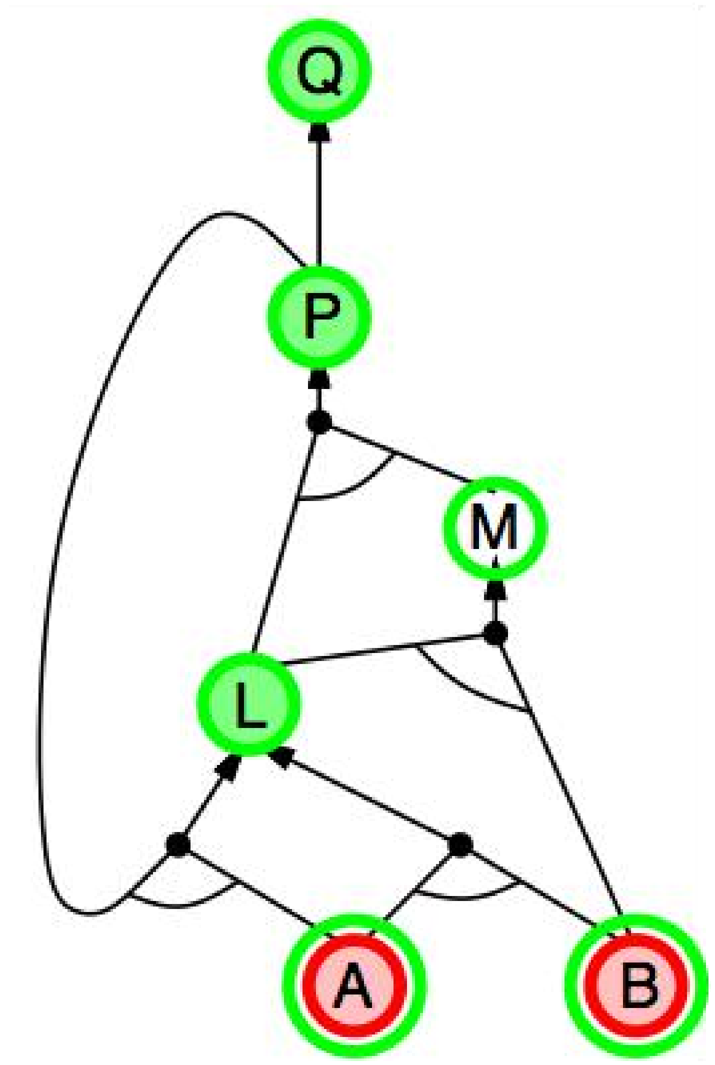

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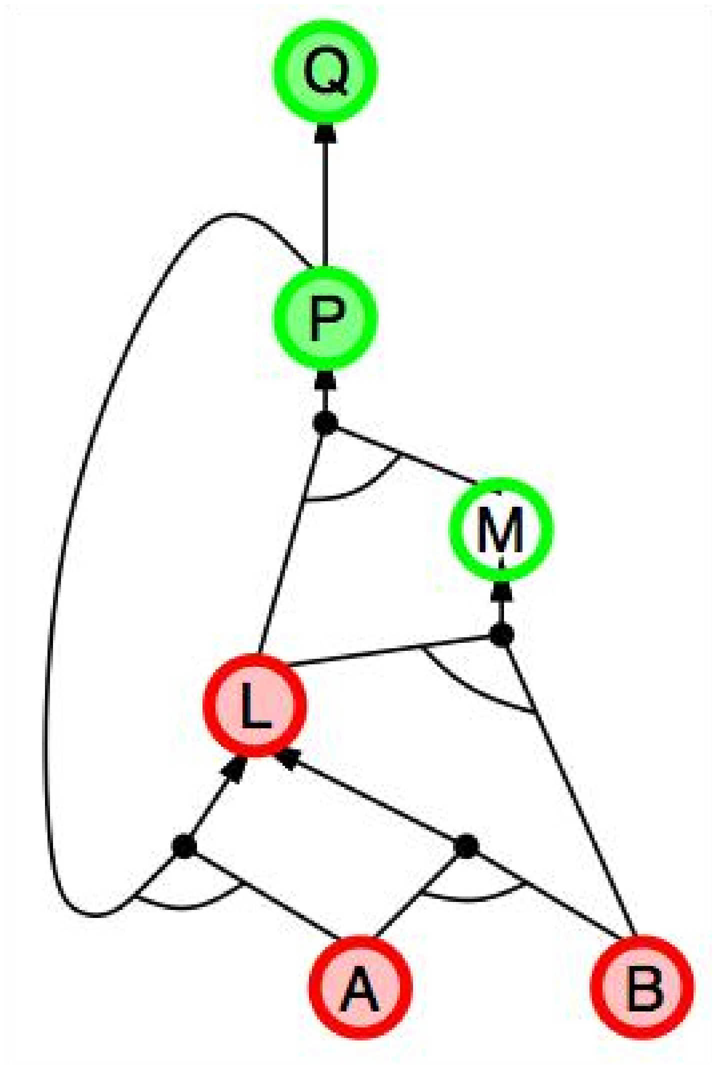
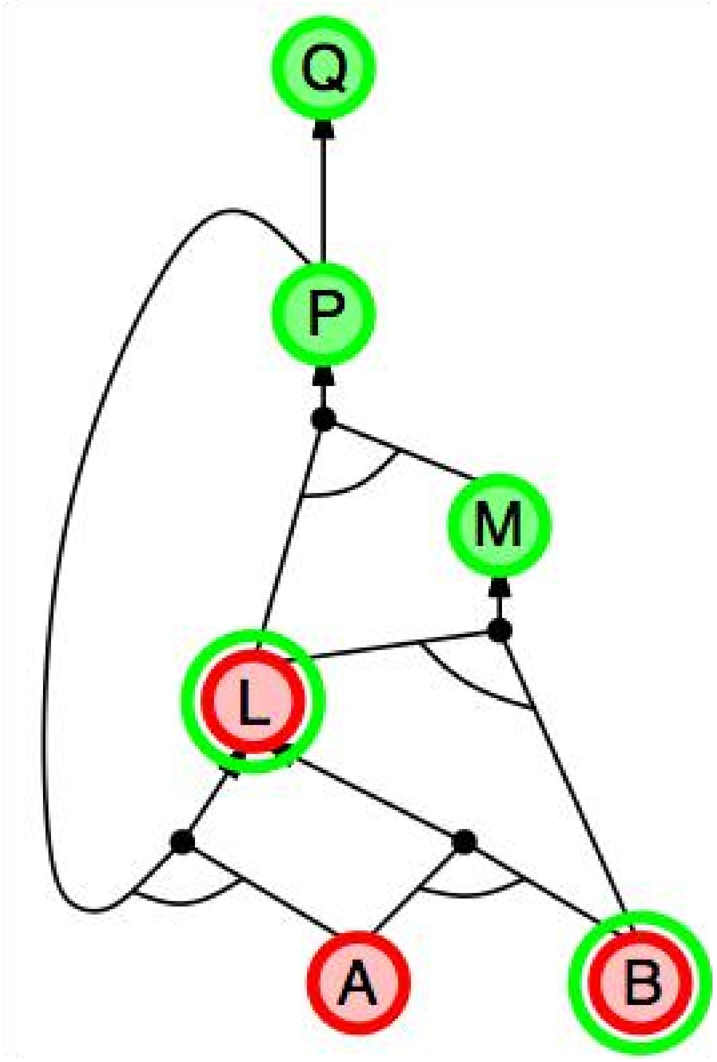


Backward chaining example



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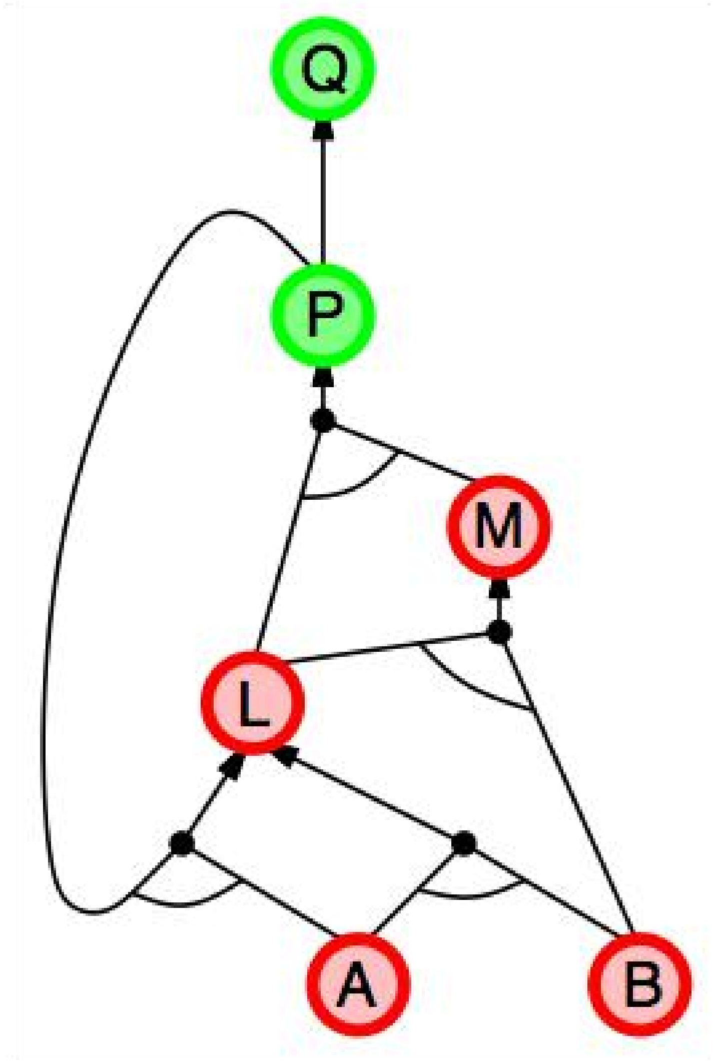
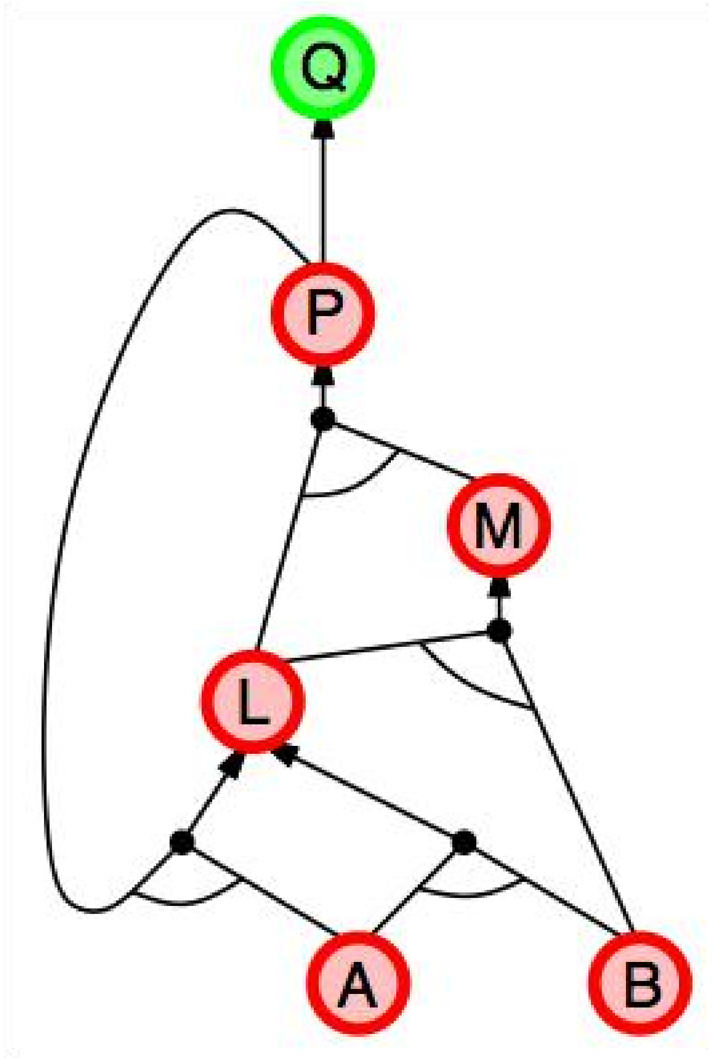


Backward chaining example



Comparison

- FC is data-driven, cf. automatic unconscious processing,

  e.g., object recognition, routine decisions
- May do lots of work that is irrelevant to the goal
- BC is goal-driven, appropriate for problem-solving

  e.g., Where are my keys? How do I get into a PhD program?
- Complexity of BC can be much less than linear in size of KB

Proof of soundness

- 证明：**Modus Ponens**规则是可靠的。即证明：

$$\alpha_1 \wedge \cdots \wedge \alpha_n \wedge (\alpha_1 \wedge \cdots \wedge \alpha_n \Rightarrow \beta) \vDash \beta$$

Modus Ponens规则：

$$\frac{\alpha_1, \ldots, \alpha_n, \qquad\qquad \alpha_1 \wedge \ldots \wedge \alpha_n \Rightarrow \beta}{\beta}$$

Proof of completeness

- 若$KB \vDash \alpha$，$KB \vdash \alpha$。此时，KB中仅包含**definite**子句，$\vdash$仅使用**Modus Ponens**规则，且α是一个正文字
- 证明：***RC(KB)*** **is the set of all clauses derived by Modus Ponens and all the original clauses inside *KB***
- 1) 构造如下的真值指派 m：对于任意的symbol a，

  a指派为True 当且仅当 � ∈� � (� �)
- 2) 接下来，证明：在m下，KB为真。

  反证：若此时KB为False，那么：

  必存在一个definite子句，在m下为False。

  i) 若该子句为 $a_1 \wedge \cdots \wedge a_k \Rightarrow b$

  也就是说，在m中，$a_1,\cdots\ a_k$ 均为True，且� 为False。

  根据1)中的定义，a_i ∈� � (� �) (� =1,⋯ �)

  又根据Modus Ponens规则，� ∈� � (� �)

  根据1)中的定义，在m中，� 为True。推出矛盾。

  ii) 若该子句为 � ，在m下为b为False，则� ∉ � � (� �)，矛盾
- 3)若� � ⊨ � ，根据蕴含的定义：在m中，� 为真；

  则根据1)中的定义，� ∈� � (� �)

  也就是说：� � ⊢ �

Summary on Propositional Logic

Logical agents apply inference to a knowledge base to derive new information and make decisions

Basic concepts of logic:

- syntax: formal structure of sentences
- semantics: truth of sentences wrt models
- entailment: necessary truth of one sentence given another
- inference: deriving sentences from other sentences
- soundess: derivations produce only entailed sentences
- completeness: derivations can produce all entailed sentences

Wumpus world requires the ability to represent partial and negated information, reason by cases, etc

Forward, backward chaining are linear—time, complete for definite clauses

Resolution is complete for propositional logic.

Propositional logic lacks expressive power

Pros and Cons of Propositional Logic

- Pro
  - Propositional logic is declarative: pieces of syntax correspond to facts.
  - Propositional logic allows partial/disjunctive/negated information (unlike most data structures and databases)
  - Propositional logic is compositional:

    Meaning of $B_{1,1} \wedge P_{1,2}$ is derived from meaning of $B_{1,1}$ and of $P_{1,2}$
  - Meaning in propositional logic is context-independent (unlike natural language, where meaning depends on context)
- Con
  - Propositional has very limited expressive power
    - Cannot say “pits cause breezes in adjacent squares” except by writing one sentence for each square

Homework

7.17 A propositional *2-CNF* expression is a conjunction of clauses, each containing *exactly* 2 literals, e.g.,

$$(A \vee B) \wedge (\neg A \vee C) \wedge (\neg B \vee D) \wedge (\neg C \vee G) \wedge (\neg D \vee G) .$$

a. Prove using resolution that the above sentence entails G.

b. Two clauses are *semantically distinct* if they are not logically equivalent. How many semantically distinct 2-CNF clauses can be constructed from n proposition symbols?

c. Using your answer to (b), prove that propositional resolution always terminates in time polynomial in n given a 2-CNF sentence containing no more than n distinct symbols.

d. Explain why your argument in (c) does not apply to 3-CNF.