0.1 Matrix storage

Let A be a square matrix of order n, with nnz non-zero coefficients.

1. For each of the following three matrices (of respective dimensions 5×5, 4×4, and 5×4) for which the non-zero coefficients are represented by crosses, choose with a short justification which is the most appropriate storage format between DIA, COO, and CSR.

$$A_1 = \begin{bmatrix} \times & \times & & & \\ \times & \times & \times & & \\ & \times & \times & \times & \\ & & \times & \times & \times \\ & & & \times & \times \end{bmatrix} \qquad A_2 = \begin{bmatrix} \times & \times & \times & \times \\ & & & \\ & & & \\ \times & & \times & \times \end{bmatrix} \qquad A_3 = \begin{bmatrix} \times & & & \\ & & & \times \\ & & \times & \\ & \times & & \times \\ \times & & & \end{bmatrix}$$

2. Write in pseudo-code the algorithm calculating the transposed product $y = A^T x$, where A is stored in the CSR format, x and y are two vectors of size n.
3. Choose and justify in a few sentences an appropriate data structure to calculate a matrix-vector product $y = Ax$ on a distributed memory architecture.

0.2 Domain decomposition method and preconditioning

Let $\Omega = [0;1]^2$. We are interested in preconditioning a linear system with a Schwarz method. Ω is being decomposed in two subdomains $\{\Omega_i\}_{i \in \{1,2\}}$. Let us assume that $\Omega_1 = [0;1] \times [0;2h]$ and $\Omega_2 = [0;1] \times [h;1]$, with $h = \frac{1}{3}$.

1. What is the action of $\{R_i\}_{i \in \{1,2\}}$ (respectively $\{R_i^T\}_{i \in \{1,2\}}$) on a vector u (respectively $\{u_i\}_{i \in \{1,2\}}$) ?
2. Write down the restriction matrix R_1 (there are more than a single choice).
3. What parameter(s) influence the convergence of an overlapping Schwarz method ?
4. Briefly justify which type of parallelism (distributed memory or shared memory) is most appropriate for the block Jacobi and block Gauss–Seidel methods.